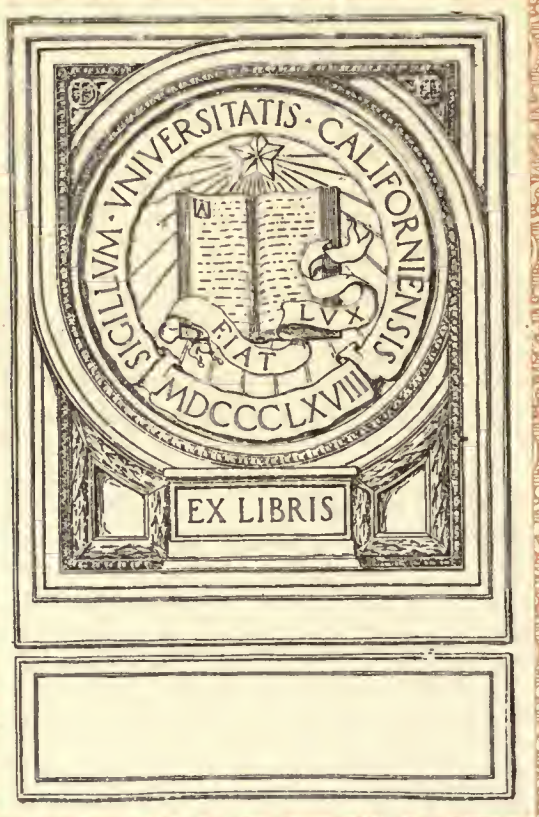
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THE CONSTRUCTOR

A HAND-BOOK OF MACHINE DESIGN

BY

F. REULEAUX

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and of the American Society of Mechanical Engineers

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AUTHORIZED TRANSLATION

COMPLETE AND UNABRIDGED

FROM THE FOURTH ENLARGED GERMAN EDITION

BY

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Translator's Preface.

In presenting to the engineering profession of England and America this translation of Reuleaux's Constructor, a few prefatory remarks may be permitted. Although the first edition of the German work appeared as long ago as 1861, and translations have been made into French, Swedish and Russian, no English translation has hitherto been made, notwithstanding the fact that repeated editions and enlargements of the original German work have appeared.

The translation here given, therefore, is the first presentation to English speaking engineers of a work which during the past thirty years has acquired the highest reputation over all Europe, and is so well known to German reading engineers and students in this country that no excuse is needed for its present appearance.

The freedom with which the author has drawn from English and American sources as well as from Continental practice gives the work a value not found in other treatises upon machine design, while the vast improvement which has been made by the introduction of the kinematic analysis and the resulting classification of the details of the subject, cannot fail to appeal to the instructor as well as to the practising engineer.

The translation has been made from the Fourth Enlarged German Edition of 1889, the last which has appeared in the original, and is complete and unabridged in every respect. The introduction to this edition is especially worthy of note, as it contains the author's summary of the principles set forth in his larger work on Theoretical Kinematics,* and the more so as it includes a brief glance at the still wider subject included in his work on Applied Kinematics, as yet unpublished in Germany, and embodying a mass of manuscript which it is trusted will at no distant day be given to the public.

The work of translation has been done with the especial sanction and exclusive authorization of Prof. Reuleaux, by whom also the portrait and special introduction to the American edition have been furnished.

The transformation of the notation of the work from the metric system to the English values has involved much labor and while it is too much to expect entire freedom from errors, notwithstanding the care which has been given to this portion of the work, it is trusted that but few errors will be found. It is especially requested that any corrections which may be found necessary will kindly be sent to the translator for future use.

HENRY HARRISON SUPLEE.

Philadelphia, September, 1893.

* It is to be regretted that Prof. Kennedy's translation of this valuable work is now out of print, and it is hoped that a new edition may be issued.

82997

Author's Introduction to the American Edition.

The present translation of the Constructor places my book before a large circle of readers who have been practically active and energetic in the development of machine design, for no one of the technical professions has been followed by the English-speaking race with more activity and success than that of the construction of machinery. I therefore take pleasure in prefacing this book with a few words of special introduction.

During the series of years in which my Constructor has grown from a small beginning to a large volume, the practice of machine construction has also been continuously developing, so that in every new edition changes and additions have been necessary. Much new matter has been added in this edition to the theoretical portion; first, in the section on Graphical Statics, enabling many numerical calculations to be dispensed with, using in their places graphical methods; second, by the introduction of the methods of Kinematics, or the science of controlled movements, a science which reduces the apparently inexhaustible complexity of machine forms to a few simple and fundamental principles, the command of which may be of extraordinary value to the engineer. I am still constantly engaged with the subject of Kinematics, especially with its practical applications, but on account of the pressure of other occupations I have not as yet been able to carry out my intention of treating this portion of the subject in a separate work, corresponding to my work on Theoretical Kinematics. The work already published on this subject I have therefore characterized as an "outline" of a theory of machines." *

The simplification of the conceptions concerning machines to which these kinematical studies led me, was of such importance that I have introduced the kinematical treatment into the Constructor in various places, especially in the latter portion of the book. Even where no special reference has been made to it, the theory has been followed, although the proof has been omitted in order to avoid burdening the non-theoretical reader with details not absolutely necessary for the practical application. It is in this manner that kinematical axioms have been introduced into Chapter XVIII., where the subject of ratchets is treated. These were formerly considered as devices of only minor importance, but the application of kinematical investigation reveals the fact that they are of the very greatest importance, occupying a position in machine construction superior to that of any other element or combination, and this notwithstanding the apparent simplicity and almost insignificant appearance of the original contrivance. A similar treatment has been given to the subject of Pressure organs, Chapter XXIII. Hitherto fluids, such as water, steam, gas, etc., have been considered as something apart from the machine, not belonging to it, but rather introduced from the outside. The idea that fluids, broadly considered, are but the exact opposites of tension organs, such as ropes, chains, belts, wire cables, etc., is wholly contrary to earlier conceptions, and yet it is just this introduction of the kinematic method which has led to an unexpected insight and very great simplification. An illustration of this is seen in the manner in which valves for pressure organs are treated as ratchets. In Chapter XVIII. ratchets formed of rigid elements only are considered, but the principles there deduced are applied in Chapters XXIII. to XXVI. to fluid elements with most satisfactory results. Since

* F. Reuleaux, The Kinematics of Machinery. Outlines of a Theory of Machines. Translated and edited by Alex. B. W. Kennedy, C. E., London. Macmillan & Co., 1876.

To Henry Harrison Suplee, Esqre.
the excellent interpreter of
the authors modest efforts
to serve Science and Practice

F. Reuleaux.

Berlin 14/2 93.

the kinematic analysis has shown that such devices as pneumatic tubes, canal locks, and the like, both ancient and modern, belong to precisely the same class of constrained combinations as steam engines and water wheels, the whole subject has been condensed and simplified in a manner not possible under the earlier conceptions. The value of the kinematic method is evident in in Section 333, where fifty different combinations of pressure organs are gathered together under a few and simple fundamental principles. Another instance is shown near the end of the book in the discussion of what I have called "Fluid valves." From the time of Hero of Alexandria down to the present day, these fluid valves have been used in what is now seen to be a continuous series of applications of a simple kinematical principle. These important simplifications will both excuse and justify the wide departure from previous conceptions which characterizes the latter part of the volume.

In regard to the other and principal object of the work, namely, the treatment of the practical construction of machine details, this has not been as consistently and fully revised as I had intended and desired; chiefly owing to the long delay in the completion of the last edition. In my lectures I have been able to follow the the technical advances which have been made in the detailed construction of bearings, levers, cranks, connecting rods, etc., and discuss them accordingly, but in the book itself many of these subjects still appear in the older dress. For these imperfections the kind indulgence of the reader is requested, and in the next edition an earnest endeavor will be made to bring these subjects up to date.

To Mr. HENRY HARRISON SUPLEE, to whom I have given the exclusive right of translation, I take this opportunity to express my particular appreciation of the great care and extraordinary accuracy which he has displayed in the production of this English version, and also my gratification at the care which has been given to the printing and the reproduction of the illustrations. Mr. Suplee has recalculated and transformed all the formulæ and numerous tables into the English system of measurements, and also reworked all the examples, and has shown in this portion of the work a patience that deserves especial recognition. It is a matter of regret that the time has not yet arrived for the general acceptance of the metric system in England and America, and until such time comes tedious transformations of this sort will often be necessary and will merit our gratitude.

I can only add that it is my earnest desire that the friendly acceptance of my book by English speaking engineers may correspond to the magnitude of the labor which has been expended in the preparation of this translation.

F. REULEAUX,

Honorary Member, American Society of Mechanical Engineers.

BERLIN, February, 1893.

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Introduction to the Fourth German Edition.

The fourth enlarged edition of the Constructor is presented to its readers much later than I had hoped. As some excuse for the delay I plead the great labor involved in the re-arrangement of more than half the book. As already explained, it has been my intention to re-arrange the matter upon a kinematical basis. It was not, however, entirely due to this re-arrangement that the work was delayed, but also to the fact that nearly one-half of the work had to be re-written. In many places I found almost everything lacking to make what I had previously determined upon, namely, a complete and consistent whole, and much more was needed than I had imagined. In addition to these shortcomings the spirit of invention has been more active than ever during the past few years and advanced at such a rapid rate that I could by no means overtake it. It is hoped that these conditions may be accepted as at least a partial excuse for the delay and for the shortcomings of the work.

The first point to which I desire to call attention in the new matter is the subject of Ratchets, which upon closer examination will be found to be the most important of all forms of driving mechanism. This subject has not until now been treated as an element of construction, it having been apparently overlooked that those forms of driving mechanism in which pawls and ratchet wheels form a part, are in reality a most important and prolific class. Special forms have indeed been treated mainly as checking devices but without any attempt to indicate the general principles, or wide extent of the construction. Locks, in spite of their universal use and of the high order of inventive talent devoted to them, have had no analytical treatment, but have been relegated to the domain of technology rather by accident than otherwise, and from Prechtl to Karmarsch and his followers, have been given an intelligent but by no means fundamental treatment. Gun locks, although having a similar name to door locks, have a very different construction, but have found no resting place in technical literature. It has often been observed that while we place in the hands of our soldiers the modern rifles and cannon, there is no place in the head for them, either in machine shop training, in machine design, in applied mechanics or in technology, or indeed anywhere. In §252 I have placed them in that class which I have termed Locking Ratchets where they fall into their proper place as members of the great division of ratchet gearing. The safety devices for elevators and hoisting machines, —Checking Ratchets, I have termed them—have been entirely overlooked; books have been written about them, catalogues and price lists issued, but the fundamental principle of their construction quite overlooked. As for escapements of clocks and watches, these have been sent hither and thither, now in mechanical text books, now in kinematics, now in applied mechanics, again in encyclopædias, where their fundamental principle has been entirely lost, their intimate relation to ratchet mechanism being hardly noticed. They will here be found classified in their proper place in §258.

Many of the readers of previous editions may shake their heads at this statement, but an examination of the fourth edition will show how the action of the piston engine is similar in principle to a watch escapement, the action of the slide valve being practically identical with the anchor of the escapement, see §§324, 325, pp. 228–232. It has only been by more recent investigations that I have become convinced of the relations of these various forms of escapements. The correctness of this position will be confirmed by comparing the the pneumatic postal tubes, canal locks, sluices, hydraulic cranes and numerous other hydraulic devices, hydraulic riveting machines, and all the many kinds of direct-acting steam pumps; these and many others, when considered from the present point of view, arrange themselves in a complete and orderly manner as true escapements. The similarity is especially well marked in the case of a deep mine pump, of which the successive puffs of the exhaust are not infrequently used by neighboring dwellers to indicate intervals of time; the steam end practically as well as theoretically becoming a time-piece. Nay, more: I am convinced that it is not a pure accident that throughout the centuries in which the delicate clock escapement has been known, the steam engine has so slowly developed; for although both the clock and the engine are in principle escapements, yet in the clock there is an escapement of precision, and in the steam engine an escapement of force,* but both devices are theoretically a solution of the same problem. Closely allied to the steam engine are the various water pressure engines, and water pumps, which as I have shown in § 319, are truly continuous ratchet trains. From the ratchet to the escapement, however, what a long, long gap! The water pump and hydraulic pressure engine differ from each other only in the different motion and action of the valves—and yet the inventive genius of

*In my Theoretical Kinematics, I have considered the steam engine as a reciprocating running ratchet train, but I have since perceived this classification to be incorrect, and therefore desire to emphasize its proper classification here.

mankind required over two thousand years to make that little step, (see § 325). How important, then, to make this fundamental connection clear!

Another important, and hitherto neglected subject, is that of the more recent steering devices, which move in either direction, or remain at rest, as required. This principle has found many applications in power steering gear for vessels, and has even made possible the solution of the difficult problem of guiding the automatic fish-torpedo at a determinate depth. It is not surprising that uncertainty should exist as to the theoretical classification of these devices. I have, however, shown that they are properly considered as escapements, and, in fact, as escapements of a special kind, which I have termed "adjustable" escapements. Such adjustable escapements of rigid construction are shown in § 259, and those constructed with pistons and fluids, in § 329.

The chapter upon Ratchet Gearing is not only entirely new, but it has also involved a new and more elaborate treatment of many subjects discussed in earlier chapters of the book. These I here only name: Screw thread systems in Chapter IV.; Thrust-bearings for screw propeller shafts; Columns; Long distance shafting transmission, etc., in § 351; Couplings, Friction gearing; Transmission of motion by toothed gearing (p. 128); Spiral gears (p. 141); Globoid gearing (p. 142), Proportions of gearing, (§ 226–§ 228). Ratchet wheels are treated in a similar manner to spur gear wheels, to which they bear a close relation, (§ 246).

From this point the book takes a fresh start, with the discussion of another species of machine elements, namely, Tension organs, as I have termed them, (Chapters XIX. to XXII). While the elements previously considered approximate so closely to rigidity that they may properly be termed rigid elements, those which follow possess the peculiarity that they are only adapted to resist tension; these elements include cords, ropes, wire, bands, belts, chains, etc. In § 262 it is shown how these are used in connection with other elements in three distinct ways, as for "guiding," for "winding," and for "driving." An examination of pages 182 to 176 will make the importance of this subject evident, and shows its scope to be far greater than might at first have been expected. The important distinction between the functions of "driving" and "guiding" is shown in the discussion of the differential tackle and the ordinary system in connection with Fig 813, (p. 176).

In discussing Cord Friction (§ 264) I have attempted to show by a graphical representation relations not otherwise easy to make clear. In § 268 I have called attention to some points which should be considered in connection with stiffness of ropes. The subject of pocketed sheaves has been treated in connection with chains, and also the chain system of boat propulsion.

In the chapter upon Belt Transmission, is introduced a new subject and one which appears to me of great importance, and which I have called "Specific Capacity." By its use it is possible to facilitate very greatly the calculations of Belting, Rope Transmission, Water Transmission and even Shafting, and bring them to a comparable basis, (see § 349 and § 351).

The discussions of Hemp and Cotton Rope transmission are both new, and that of Wire Rope greatly enlarged over previous editions. By the introduction of the subject of the "mean deflection" (p. 198) and the diagram (Fig. 884) the question of the deflection is greatly simplified, and a graphical solution is also given. Transmission with inclined cables, which in previous editions was only given an approximate solution, unsuited for long spans, is here accurately discussed (assuming the catenary as a parabola) and extended to long stretches of cable. This has been done in view of the use of rope transmissions and telegraph cables over valleys, etc.

Next follows my system of "Ring Transmission" by wire rope. This offers great advantages over the previous system of line transmission, and has met with much success in Germany, Austria, and Switzerland, as well as in America; and further discussion of it will be given hereafter. The use of chain transmission in mines, both in Germany and elsewhere, is discussed. The subject of brakes brings the book to another point where a fresh start must be made.

The third group of machine elements includes those called "Pressure Organs," and those are treated in Chapters XXIII. to XXVI. These are directly opposed to tension organs, since they are only capable of resisting compression, and include not only fluids, both liquid and gaseous, but also granular materials, etc. (§ 308).

Although these elements have been primarily arranged in a manner adapted for a practical hand book, I believe that my theoretical treatment of the subject will also find acceptance, and hence have here included the essentials of the theory also (see § 319). Pressure organs are serviceable not only in machines, but also for the transmission of force and motion; by them we can control the motion of a force in a determinate path and with a determinate velocity quite as well as with rigid elements, and indeed upon closer inspection we perceive that pressure organs are used in nearly all the most important prime movers, (steam engines and hydraulic motors), and hence they are surely entitled to be classed among machine elements. The extent to which this conception facilitates the subject of machine construction will be seen by an examination of the latter part of this volume.

I have thought it advisable to give also at this time a general review of the result of my labors in the field of Kinematics. These have been fully and thoroughly given in my lectures for the past twenty-five years, and are therefore not new to my immediate

pupils, while the publication of my Theoretical Kinematics has placed the the theory before a larger circle of scientific readers. I cannot assume, however, that the readers of this practical hand book are all familiar with the above mentioned work, and I therefore give the following abstract, covering the most important portions of my treatment of the subject.

* * * * * *

Motion and the effects which are dependent upon motion form the subject of the study of Scientific Mechanics; and hence to it belongs properly the problems of motion in machinery. The motions in a machine, however, may be distinguished from others in that they can be treated independently of the material parts of the machine, and of the forces acting upon them. The important bearing which this separation gives to the subject of machine construction was perceived about one hundred years ago, but has made small progress during the century and has only recently been taken up [10–23].*

I took up this subject in 1862, laying down the principles in my lectures; in 1864 first propounded them publicly before the convention of the Swiss "*Naturforscher*" and their German guests; first published them serially in the *Berliner Verhandlungen* in 1865, and finally in 1872–75 published my book entitled "*Theoretische Kinematik.*"

The modern discussion of these principles begins with the publication, by the celebrated physicist Ampère, in 1830, of his *Essai sur la Philosophie des Sciences*, in which he gave the subject the distinctive name Kinematics *(Cinématicque)*, which name is well derived from the Greek *kinéo*, to drive, to constrain, since it treats of constrained or controlled motions.

I have defined the term *Kinematics* [40] as "the study of those arrangements of the machine by which the mutual motions of its parts. considered as changes of position are determined." This I have divided into to parts: "Theoretical" and "Applied" Kinematics, the former treating of the general and fundamental principles, and the latter of their practical applications.

a. Theoretical Kinematics.

It is this branch of the subject which is treated in my well-known book "The Kinematics of Machinery." The following is a condensed analysis of the treatment there expanded at greater length:

1. A material system having motion within itself, I call a machinal system, as may be determined according as the motion is constrained or not [32].

2. Motions can only be constrained by forces. These forces differ in the two systems, since in the pure machinal system sensible and latent forces enter into equilibrium with each other, while in the pure cosmical system sensible forces enter into equilibrium with sensible forces, [33]. It therefore follows that the two systems can not always be accurately determined [34].* The terms "latent" and "sensible" are here used in a similar sense as in thermal physics. Latent forces are those which exert internal resistance to deformation of a body under the action of external forces; sensible forces are those which act upon the body from without [33].

3. The motions of the machine can be logically controlled according to a predetermined conception, since the action of all external forces which do not tend to produce the desired end can be opposed and neutralized by latent forces [35].

4. From the preceding follows the definition of a machine:—

A machine is a combination of resistent bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions [35, 50, 203].

5. If we consider the machine to be made of rigid materials and neglect its mass, we need only take into account geometrical considerations [42]. If a body A, by means of latent forces, is to be prevented from being put in motion by any external forces (case 3), it must be held in a stationary position by at least one body B. The body B, then acts as the envelope of A, and conversely A is the envelope of B, the relation being a reciprocal one. There are also reciprocal envelope forms possible between the bodies A and B for a relative motion, which shall exclude all other relative motions [43]. Such a pair of bodies, I have called a kinematic pair of elements and a machine consists solely of bodies which thus correspond, pairwise, reciprocally [43].

6. In order to obtain a determinate motion in a given space by means of a kinematic pair of elements, one of the elements of the pair must be held at rest with regard to the given portion of space under consideration. The relative motion of the moving element to the fixed one will then be that of absolute motion, so far as the given portion of space is concerned [43].

7. The choice between the two elements as to which shall be stationary and which movable is not limited; the substitution of the fixed for the moving element I have called the *inversion* of the pair [93].

8. The control which can be exercised over a determinate motion in this manner is not mathematically exact but only approximate (case 5) because the latent forces of bodies can only be brought into action by their deformation. If, however, the elements are made of materials which possess a high degree of resistance and are given proper dimensions (machine construction) the deformation can be kept within such small limits as to be practically insignificant, and the result considered as determinate [33]. (Compare cases 46 to 49, below).

* The numbers in brackets refer to the pages of Reuleaux's "Kinematics of Machinery. Translated by Prof. A. B. W. Kennedy. London, Macmillan & Co., 1876.

* The internal forces of a moving system form the subject of d' Alembert's principle.

9. Each element of a kinematic pair may be rigidly combined with an element of another similar pair without interfering with the relative motion of the separate pairs. In this manner a large number of pairs of elements may be arranged in a series, so that each element of a pair is firmly connected with an element of another pair. Such a series of pairs of elements returns upon itself, resembling a chain [46], consisting of links connected together. I have called such a series a kinematic chain, and the body which is formed by the junction of the elements of two different pairs is a link of the kinematic chain [46]. There are therefore as many links as there are pairs of kinematic elements.

10. A kinematic chain may close or return upon itself in various ways; among these is one in which every alteration in the position of a link relatively to the one next to it is accompanied by an alteration in the position of every other link relatively to the first [46]. In such a chain each link has only a single relative motion with regard to every other link. Such a kinematic chain I call a constrained closed—or simply a closed chain [46].

11. A constrained closed kinematic chain compels a definite determinate motion in a given portion of space when one link of the chain is fixed, with regard to this given portion of space. A closed kinematic chain of which one link is thus made stationary, is called a mechanism [47].

12. A constrained closed kinematic chain, therefore, can be formed into a mechanism in as many ways as it has links [47]. The substitution of the stationary link of a kinematic chain for another link I have called its inversion.

13. A kinematic chain may have so few members and be closed in such a manner that the links can have no motion relative to each other, and that the pairs themselves do not have their own motion. This I have termed fixed closure [485].

14. The manner of closure of the chain can be chosen so that adjoining links can have more than one relative motion. This I have called unconstrained closure [485].

15. A kinematic chain in which a series of pairs of elements are arranged in the stated manner, but of which the first and last elements are not connected, I have called an open chain.

16. Kinematic chains of the kinds above mentioned can be combined with each other, forming constructions which may be called compound chains. These may have constrained, unconstrained, or fixed closure or may be open chains. The same conditions exist for these as for the previously described chains, which may for sake of distinction, be called simple chains.

17. From the preceding we may give the following general definition of a mechanism, as follows:

A mechanism is a closed kinematic chain of which one link is fixed: this chain is compound or simple and consists of kinematic pairs of elements; these carry the envelopes required for the motion which the bodies in contact must have, and by these all motions other than those desired in the mechanism are prevented [50].

18. From all that has preceded, it is apparent that the investigation of the motions in machinery is a subject which is based in great part upon geometry. This has been treated as a separate subject of Phoronomy, or the study of geometrical motion. The most important principles of this subject I have treated in Chapter II. of my "Theoretical Kinematics," with applications to constrained as well as cosmical motions [56 to 85]. It is there shown that all relative motion can be considered as that of a pair of ruled surfaces, so that the motion is reduced to a rolling of the two ruled surfaces upon each other, and under certain circumstances with a simultaneous endlong sliding upon each other of the generators which are in contact. These rolling surfaces, for which previously no special name had been used, I have called *axoids*, the combined sliding and rolling motion being termed *twisting*. When rolling motion is absent only sliding remains, when on the contrary, the sliding is omitted only the motion of rolling remains. In the latter case certain sections through the axoids give curves which twist upon each other, or roll with a cross sliding action. The combined points of these curves form centres of rotation or poles about which, as instantaneous centres, both bodies turn. These centres or poles travel in the paths of the aforesaid curves whence the latter may be called *pole-paths* (Polbahnen) or centroids.* The study of axoids and centroids will greatly extend the range of phoronomic researches.

19. In order to pass from the general principles to the special applications of kinematics, further consideration must be given to the elementary pairs. The simplest form must necessarily be that in which the corresponding envelopes actually surround, one the other, and such I have called a closed pair. Of this there are but three forms: 1, the twisting pair (screw and nut); 2, the turning pair (pin and collar); 3, the sliding pair (full and open prism, or prism pair [91]. The two latter may be considered as particular cases of the first. In all three no change in the character of the motion is caused by inversion (case 7).

20. In a pair of elements it is not always necessary to use all of both envelope forms. The question of the minimum number of points necessary to insure resistence to disturning forces, I have discussed in § 17 of my Kinematics, under the title: "The Necessary and Sufficient Restraint of Elements."

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* The term "centroid," due to Prof. Clifford, and used by Prof. Kennedy in his translation of the "Kinematics," will be hereafter used as the translation of Polbahn.—TRANS.

21. We have thus far omitted from consideration such elementary pairs as are not closed. These possess the general property of giving a change in the character of the motion when they are inverted. I have called them "higher" pairs of elements [115], and conversely the closed pairs may be termed "lower" pairs. It is only in special cases that no change occurs in the character of the motion by inversion of higher pairs. A series of higher pairs, for the most part entirely new, has been discussed in § 21 of my Kinematics.

22. I have given (§ 30 to § 39) seven geometric methods of determining the restraining bodies for higher pairs, many of which were already known, but which were then for the first time grouped into one general system.

23. Incomplete pairs [169] are those which are not entirely closed by the latent forces, but are partly closed in some other manner. Examples of these are half-journal bearings, in which the weight of the parts is used to keep the journal down in its bearing; knife-bearings for scale beams, the V bearings for the beds of planing machines, etc. Pairs may also be closed by the action of springs or other external forces. The closure of a pair of elements in this manner I have termed "force closure." This form of closure can only be used when the disturbing forces are not sufficiently great to overcome the closing force.

24. Force closure also finds application in higher pairs of elements. An important example is found in the driving wheels of locomotive engines, and another still more important, in the axoid rolling action of friction wheels. (See Chapter XVI. of this volume.)

25. The application of force closure can be carried still further. By its application we are enabled to utilize two classes of elements which are only capable of opposing resistance in one direction (case 8). These are what I have called "tension organs" and "pressure organs," (see § 261 and § 308 of this volume). These I have grouped together as "flectional" kinematic elements [173]. They include a long series of most useful machines, such as belt and rope transmission systems, pumps, water-wheels, etc., all involving the principles of force closure.

26. Force closure may be used in a dynamical as well as in a statical manner, as in the case of an engine crank which is carried over the dead centre by the action of the fly wheel [186].

27. In such cases the closure may also be effected by means of another kinematic chain used in combination with the first [178]. This I have called chain closure. An example is found in a double engine with cranks at right angles.

28. The preferable form of chain-closure is that in which similar elements are employed. This occurs (case 25) when one force-closing chain is used in connection with another of the same kind, the two being so combined that each supplies the necessary closing force for the other; whence it follows that the sensible and latent forces in the two chains counteract each other in the same manner as if they were composed of rigid elements. [§ 44, "Complete Kinematic Closure of the Flectional Elements."] Examples of this are found in the ordinary belt transmission, and in the so-called "water rod." By means of this method of closure, which is destined to be much more widely used than heretofore, the applications of flectional elements have been greatly extended for purposes to which rigid elements are not adapted, such as the transmission of force in a path of constantly changing direction, as in the use of high pressure water transmission systems through pipe conductors.

29. Finally a kinematic chain may be closed by the application of springs [176]. These may be so constructed as to oppose resistance in a number of chosen directions, but not in all directions; e. g., both tension and compression, also bending in one plane, but not in a second plane at right angles to the first. This latter condition is seen in the case of flat or plate springs, also in the plate link shown in Fig. 507 of this book, where the spring acts as a substitute for a pin connection. In the plate link the force closure and complete kinematic closure are replaced by chain closure. Another example is found in the Emery Scale, Fig. 789c.

30. The pairing of flectional with rigid elements may be assumed, *a priori*, to be practicable in the same manner as that of rigid elements [544].

31. If the principles of investigation, however they may be set forth, are correctly based, they should when applied to the historical development of machines, shed a light upon the whole subject from the rude attempt at invention to the highest attainments of mechanical ingenuity. This subject I have discussed as a "Sketch of the History of Machine Development, [201 to 246], in which the substitution of pair closure for force closure is made most apparent.

32. In order to facilitate the elucidation of the action of machinery, and to abridge the labor of the application of the preceding methods, it became necessary to devise a system of kinematic notation. This is given in Chapter VII., pages 247–273, of my Kinematics of Machinery. The elements are designated by capital letters, of which twelve are required, and the relations of these are indicated by auxiliary symbols derived for the most part from those already used in mathematical notation. For the symbolical representation of the kinematic chain I have also introduced the conception of an order in which each pair in the chain is numbered from 1 upwards, and the links represented by the small letters from *a* onwards [270–273]. The pair and the link at which the numbering and lettering is to begin may be agreed upon previously, as well as the direction in which they are to proceed. The link between the pairs 1 and 2 will then be indicated by *a*; that between

2 and 3, by *b*, etc. For instance, the connecting rod and crank device, shown in Fig. 1022 of this book, is indicated by the formula $(C_3'' P\perp) \frac{a}{c}$. Translated, this means that the kinematic chain of the mechanism consists of three parallel, closed cylinder pairs, and one closed prism pair at right angles to them; that it contains four links, which I have called the crank, the coupler, the slide and the link, and designated by a, b, c, and d; that this chain is converted into a mechanism by the link d being held fast; that the right line from the centre of the bearing 2 to the end of the coupler b (the connecting rod) moves around the axis 1 of the crank shaft, and that the crank a is driven, by means of the coupler b, by the slide (cross-head) c. This is certainly expressing very much by means of very few symbols, dispensing with long and comprehensive definition. According to case 12, this chain can be converted into three other kinds of machines, symbolically indicated by: $(C_3'' P\perp)^a$, $(C_3'' P\perp)^b$, etc. These symbols have as yet been used but little by practical designers, but those who have made use of them have found them brief and accurate both for writing and for description otherwise requiring much longer explanation.

33. The application of the system of symbols leads to what I have termed "Kinematic Analysis," [Chapter VIII.] The application of this analysis to the so-called "mechanical powers," [275–283] leads to interesting conclusions, this is also the case with the cylindric crank chain [283–341], which taken in connection with Chapter V., yields a wealth of valuable results.

34. This is followed by an analysis of "chamber-crank" trains, Chapter IX. In this, it is shown that upwards of a hundred pressure organ machines, hitherto considered as separate inventions, have a systematic relationship dependent largely upon kinematic inversion; and a number of difficulties are cleared up.

35. In Chapter X. the subject of the so-called "chamber-wheel" trains is analyzed; the principles of which I had previously investigated in 1868.

36. Finally, in Chapter XI., is given the Analysis of the Constructive Elements of Machinery, including a brief investigation of ratchet mechanisms. At the time this portion was written my investigation of that subject, however, had not been carried to any great extent, and in the present volume for the first time have I set forth the extraordinary and varied importance of ratchet mechanism.

37. To this subject is added an analysis of the complete machine [486–526], in which the strict limits of theoretical kinematics are frequently overstepped and encroachments made upon the domain of applied kinematics. The older ideas of the "receptor" the "communicator," and the "tool" are examined and rejected and machines classified as "place-changing" and "form-changing" machines. This classification will be found to possess a decided value and will be referred to again. (Cases 42 to 49.)

38. Kinematic Analysis has as a necessary counterpart Kinematic Synthesis. This has been already seen (cases 19, 21, 30) in the application of pairs, chains and mechanisms to given machinal purposes. Kinematic synthesis may also be called a theory of the invention of mechanisms. This it can only be, however, in a limited sense. It can in no case enable the genius of the inventor to be dispensed with, but by the aid of this theory his scope can be greatly extended. The application of synthesis to problems which have already been solved may also point the way to the solution of others as yet undetermined.

In discussing this synthesis, I have grouped the pairs of kinematic elements into 21 orders [538–544] by means of which the determination of the greater number of kinematic chains and dependent mechanisms may be made; also eight classes of simple chains. The application of synthesis may be made in two forms, the direct and the indirect, and these again into general and special synthesis. Of these the indirect synthesis is the most useful [529]. It is my expectation that this theoretical exposition of the subject, which I cannot expect to extend further, but by means of which I have been able to devise a number of new mechanisms, may find many successful applications by others.

b. Applied Kinematics.

39. Applied Kinematics is not so much to be considered as standing in opposition to theoretical kinematics as it is included in it. In fact, applied kinematics has existed as a study for a long time, as in the treatise of Monge, without the existence of any theoretic foundation. That such a treatment of kinematics may be very useful for a time is readily admissible, but an *ex post facto* theoretical discussion may seem of little value to the practical man. Indeed my highly esteemed former preceptor, Redtenbacher, considered an actual theoretical treatment of the movements of machinery to be an impossibility.

Under these circumstances I did not feel inclined to follow the "Theoretical Kinematics" hastily with a treatise on the applied science. For this purpose it was not possible to arrange all the various forms of machines under the new classification hurriedly and properly in permanent form. Notwithstanding the simplicity of the preceding system, its application developed many difficulties and required a succession of researches with which even my immediate pupils are not fully acquainted. A not inexcusable impatience on their part has led me to have my investigations in applied kinematics multiplied for a limited circulation although the matter was incomplete. I gave this permission reluctantly and with the condition that only a limited number of impressions, to be considered as "manuscript," should be circulated. In this manner

four parts of the work have appeared, the last consisting entirely of the application of the symbols to lecture room models. The result of such premature publication cannot always be foreseen by those who have urged it, but for the misunderstandings which have arisen from this source I can only express my regret.

In the meantime I have since 1882 been engaged in the partial application of the principles of kinematics to this book in such a manner as to avoid burdening the reader with theoretical matter, which would be contrary to the purpose of the work. The most important subjects to which the kinematic method has been applied are here briefly noted.

40. With the great extension of modern mechanical engineering we find that the various mechanisms, (the number of which as we have seen is not great), are given a great variety of applications. It is the object of applied kinematics to furnish a clear distinction between the various methods of practical application. It is apparent that the preceding analysis does not extend to this point, since it does not include the subject of the method of constraint, but only treats of the combination of the elementary parts which are involved. We may therefore properly term it the Elementary Kinematic Analysis. As a counterpart for this in applied kinematics we may place the subject of another analysis which relates to the conditions of motion in a given train, and which may be called Train Analysis, or the Analysis of Trains. This analysis is not intended to solve anew the construction of the various trains, but rather to elucidate clearly their method of action; a train consisting of a closed group of elements and bearing the same relation to a machine as an atom does to a molecule.

41. Train analysis does not admit of an arrangement logically similar to the elementary analysis, but possesses a new and different order. This is due to the fact that the elements of which trains are composed occur only in pairs, while the trains of which machines are composed are considered singly. In Vose's pump, for example, Fig. 979[a], there are two ratchet trains combined in one machine, while in Downton's pump, Fig. 979[c], there are three trains.

42. The various methods of tain action may be divided into four principal kinematic divisions, viz.: Guiding, Storing, Driving, and Forming, §333.* The first three divisions are "Place-changing" and the last is "Form-changing."

43. Various forms of guiding devices may be mentioned; linkages by means of which curved paths are obtained, parallel and straight line motions, also "position motions," as I have termed those by means of which a system of points may be transferred to another position parallel to the first. Guiding devices can be constructed from kinematic chains of every kind. It was by means of examples with chains for this purpose that the general conditions of motion in theoretical kinematics were illustrated, and the same conditions belong also to applied kinematics.

34. Storing includes those especial machine organs by means of which work can be accumulated and the supply drawn upon for later use. This, until now has not been considered as a special mechanical conception, although it has had numerous applications. Storage of power may be accomplished in three quite different ways.

a. By means of rigid elements, this being statical or dynamical. Examples of statical storage are found in elevated weights, compressed springs, etc., and of dynamical storage in fly wheels, or pendulums. One of the oldest forms of dynamical storage is the old-fashioned spindle [216].

b. By means of tension organs, acting by winding the tension organ upon a drum or pulley. Examples are seen in tower clocks, etc.

c. By means of pressure organs. These are the most frequently used, and examples include tanks for water, oil, gas, air, steam, also hydraulic accumulators.

45. Driving. In this term I include the transmission of motion within a single train and also from one system to another. As "guiding" includes the control of the path of a point, "driving" considers the control of the velocities of various points in their paths. Examples in this branch of applied kinematics are those which take into consideration the velocity of the various parts of a mechanism. (See the close of Case 38).

46. Forming, includes the working of materials by means of machine tools. This fourth division is the richest of all, and offers the widest range to the genius of invention. This operation takes place by the action of the tool upon the material, or as I have called it, the "work piece" [495]. In form changing machines, the work-piece is a part or the whole of a kinematic link, and is paired or chained with the tool by so arranging the latter that it itself changes the original form of the work-piece into that of the envelope corresponding to the motion in the pair or linkage employed [495]. We can distinguish between three forms in which this action can occur.

a. The tool is hard and operates by cutting the material from the work-piece which lies without the envelope of the desired form. Examples are found in lathes, planers, grinding machines, etc.

b. The tool is of high resistance so as to be able to maintain its form, but does not act by cutting, but by pressure upon the yielding work-piece. It follows that the material which lies outside of the desired form is forced into another part of the work-piece without being removed from it. Examples are found in coining presses, rolling mills, wire drawing benches, etc.

c. The tool and the work-piece are both alike yielding, and act each upon the other, each being the

* In § 333, the second of these has been translated "Supporting," and the English language lacks a suitable equivalent for "*Haltung*," but in a correspondence with the author, the above has been adopted.—TRANS.

tool for the other piece. Examples are the various kinds of spinning, weaving, and other textile machinery. All three forms are described in this volume, many examples being given among the pressure organs.

47. It may appear from the preceding as if the theory of the action of the tool breaks through the logical arrangement given in the theoretical kinematics, since in Case *a*, one of the elements, the tool, cuts away and destroys its partner because it is enough harder to cut it. We must here distinguish between yielding and unyielding elements. This looks like a return to empiricism. The defect in the logic, how- is only apparent. All elementary pairs without exception involve the idea that both of the partners evoke the latent forces by the action of deformation; and at the same time the friction between the moving parts induces wear. Applied mechanics takes friction into account in considering elementary pairs and investigates and provides for the consequent wear. The machine constructor endeavors by all means within his power to reduce the alteration of form at points where it is not desired, but where it is the end to be accomplished he takes every opportunity to increase it. The form-changing action which occurs between the tool and the work-piece differs in degree only and not in kind, from the action taking place between the elements of every other pair in the machine [503].

48. A similar idea may arise in connection with the method of form-changing given above under *(b)*, in which an alteration of form takes place without an actual removal of any of the particles. In this case the the correspondence of the kinematic to the mechanical action is evident. In case 8, as already noted, the deformation which takes place in non-rigid bodies makes it only practicable to obtain approximate solutions. This only involves a quantitative, and not a qualitative distinction [502].

Examples of this occur in the construction of instruments of precision. It is not possible to construct even a simple cylindrical pair (case 19) such as a centre for a theodolite, or for an astronomical telescope, entirely free from error. By the use of a variety of methods the errors are kept as small as possible, and then by other methods, nearly always kinematic, the residual errors are determined and the proper corrections made.

49. In other instances the designer may utilize the elastic yielding of the members of a kinematic chain, as for instance in the method of Adolph Hirn, by which the springing of the beam of a steam engine is used to produce the indicator diagram of the steam pressure; or the torsional deflection of a large shaft to measure the power transmitted.*

This method is also found in Gidding's device for measuring valve friction (p. 285), and also in the Emery scale, in which a very small deflection of a diaphragm measures accurately weights of many tons.

Although in many instances the deformations of material may be neglected, yet we should never permit ourselves to forget that they have been neglected. Otherwise important errors may creep into theoretical deductions, as well as in practical construction. This subject of the yielding of materials is receiving more attention at present than formerly.

50. The "order" of a system of transmission is a subject of importance since there are several methods by which the various parts may be kinematically arranged. I have applied the term "order" to the method of arrangement, and distinguish between three different methods.

a. "Series Order." This "order" exists when a number of transmissions are arranged in series, so that each acts upon the following one. If in a single machine, two, three, four or "n" transmissions are thus arranged in series, I call the whole a system of the second, third, fourth or n^{th} order. Examples are found in Figs. 766, 767.

A transmission can return upon itself. This I have called a "ring" system of transmission. (See p. 208). This return to the original must always occur in the kinematic chain of any mechanism since the elements exist only in the relation of pairing (Case 5). In the system under consideration (Case 41), the groups of elements follow each other in a series, or line as it may be termed, whence I have termed such a series a "line" transmission (p. 257). Ring transmission may also be combined with line transmission, the line being divided into two or more parts. An example of the first kind is seen on page 229, in which the pump mechanism is combined with the steam mechanism, as a line with a ring system. An intermediate form between ring and line transmission is referred to on page 208.

b. Combined Order. By this title is meant a combination of transmissions in which each transmission is connected to the next, but in which any one can be stopped without stopping the others. An example of this is shown in the ring transmission in Fig. 917.

Under certain circumstances a number of the driven pulleys T_1, T_2, T_3 -- T_n, may be allowed to run empty, in which case they become merely supporting sheaves (Case 43); as soon, however, as any load is thrown on any of them, the entire system is influenced by the increased stress upon the rope.

Another example of "compound" order, is the multiple expansion steam engine. Here each engine of the compound, triple, or multiple expansion engine may be considered singly as a separate chain, and the entire machine as a series of transmissions. Each engine, T_1, T_2, T_3, etc., exerts an influence upon the action of the others, but is not indispensable to their action, as would be the case if arranged in "series" order. Compound, Triple, Quadruple expansion en-

* See *Berliner Verhandlungen.*

gines are therefore, respectively of the second, third, and fourth order, but should also be considered to belong to the class of "Compound order."

c. "Parallel Order." This arrangement is the oldest and the one which occurs most frequently. It occurs when a number of different machines are all driven from one and the same transmission, this being the usual arrangement in manufacturing establishments. Any of the machines can be stopped or started independently of the others without affecting the motion, a suitable regulator being assumed. This principle may also be applied to the motors by which the transmission is driven, automatic couplings, such as shown on page 101, being used. A "parallel" order occurs in rope transmission when a number of ropes are used on the same pulley; another instance is that of a train which is pulled by two locomotive engines.

The three different "orders" are not always sharply defined, but the distinction will be found of material assistance in the study of transmissions. An example in which all three "orders" are used is found in the engine shown in diagram in Fig. 1023. Here the cylinder, piston, valve and steam form an escapement: the connection c l r being driven, and in turn operating a second r_1 l_1 b, and thence the valve. These three transmissions therefore form a "series" order, this also returning to itself and being thus a ring system, and of the third order. The fly-wheel and its bearings form a dynamical power storage system, absorbing and giving out power in response to the irregularities of the action of the piston, this being of the "compound" order. Frequently such an engine is made with an additional cut-off valve gear, with governor, also of "compound" order, also possibly a feed pump, ("parallel" order) and the engine usually drives an extensive transmission system by which a number of machines are operated ("parallel" order).

In § 260 is shown the manner in which physical and chemical trains are arranged in series, the action of heat, of gases and electricity being considered; the steam engine being the most notable example.

51. The magnitude of the exponent of the order of any train has an important influence upon the hurtful resistance of a machine, especially in a series order of a high degree. In such cases the injurious resistance increases at least directly as the exponent, and frequently more rapidly. It is therefore important in machine design to keep the degree of the order as low as practicable. In the system of pneumatic clocks of Mayrhofer (p. 171) the mechanism for several years was as high as the 17th order, but the degree subsequently reduced to the 8th order. It may safely be affirmed that the simplicity of a machine may be measured by the closeness which the exponent of its order approaches unity. Examples are found in the Giffard injector, in which the guiding and driving mechanisms are united in one, and exponent becomes =1; the same is true of Siemens Geyser pump, Fig. 971a. The apparatus of Morrison & Ingram, Fig. 1181, is a device of the 2nd order, which acts by a combination of guiding and driving.

52. The preceding pages have shown that applied kinematics, by means of the separation of the controlled motion into the forms of Guiding, Storing, Driving and Forming, and by means of the division of the various "orders," has enabled the machine problem to be solved as a whole. Theoretical kinematics has assisted in this solution by enabling the various problems to be investigated in a purely scientific manner. Without such a theoretical investigation, a system of applied kinematics would be an impossibility. At the same time practical instruction must be given by actual daily work as well. A clear understanding of the principles of the applied science cannot but be useful to the practical man, and as I believe, welcome also.

The fundamental principles of machine construction as I have sought to lay them down in the preceding pages, coincide in many points with the practical methods already in use. The practical mechanic is well acquainted with crank trains, gear trains, and the like, or if he is not familiar with them he is readily taught, but in combining these and arranging them so as to act upon each other the theory comes into play and shows clearly the best arrangement for the end in view. This is well shown in the case of the various valve gears, which have been in fact developed independently, instead of being the result of a theoretical analysis of various combinations of kinematic chains. The application of the kinematic analysis will facilitate work of this sort, making it clearer and simpler the more fully the fundamental principles are understood. For this reason I have introduced the kinematic principles into this work, not to reduce invention to an art to be taught, but rather to bring the principles of science to its assistance.

I am ready to admit that the general view of theoretical kinematics which I have placed before the practical man, may not be accepted without further proof being demanded. It may be considered only as an ingenious form of theorizing, of but little practical value. For the present I must ask my readers to prove by the test of practical application how far the principles of kinematics may be made of genuine practical value.

The principles included in cases 40 to 51 are practically applied in the latter half of this volume. The application of the analysis to the subject of ratchet gearing has produced an extensive series of results. Storage is clearly shown to be a form of ratchet gear; the discussion of the degree of "order" of ratchet trains will also, I believe, be found very useful. In the discussion of pressure organs (Chapter XXIII. and following) the subject of storage is highly developed. The notion of the two divisions of guiding, and driv-

ing will also be found most useful. In like manner the methods of analysis as applied to ratchet trains, are found capable of equally prolific results when applied to pressure organ trains, not, to my knowledge, otherwise attainable. The great number of applications in this direction will be seen in § 333, these being the result of the application of the theory sketched under Case 46, above.

Since the subject of friction was considered in connection with rigid elements, it was also necessary to to take into account this resistance to the motion of fluids (§ 340), as also the loss of heat in steam pipes (briefly discussed in § 338). In § 362 the very important subject of boiler design is only generally considered.

The closing chapter relates to valves. These are treated as ratchets, not only from the theoretical standpoint, but also practically, and much more fully than in previous editions. The section on "fluid valves" will, I trust, be found of use to the practical man, as a subject worthy of further investigation.

In closing, I may refer to the increasing size of this volume. In spite of my earnest efforts, it has not been possible to reduce its bulk. In many places evidence will be found of attempts at condensation, but nevertheless the work can hardly be called properly a "hand book" any longer. When discussing purely technical matters I can be brief, but in a practical work, it is above all things necessary to be clear and intelligible. In this I have endeavored to be guided by the dictum of Boileau: "*Un ouvrage ne doit point paraître trop travaillé, mais il ne saurait être trop travaillé.*"

FUNCHAL, *February, 1889.*

F. REULEAUX.

TABLE OF CONTENTS.

SECTION I.

STRENGTH OF MATERIALS.

Introductory 1
Co-efficients of Resistance 1
Resistance to Tension and Compression 2
Bodies of Uniform Strength 2
Resistance to Shearing 2
Resistance to Bending 2
Table of Sections 5
Value of the Quantity S 8
Sections of Uniform Resistance 8
Bodies of Uniform Resistance to Bending 8
Resistance to Shearing in the Neutral Plane 10
Beams with a Common Load 11
Resistance to Torsion 11
Polar Moment of Inertia and Section Modulus 11
Bodies of Uniform Resistance to Torsion 13
Resistance to Buckling 13
Columns of Uniform Resistance 13
Compound Stresses 13
Resistance of Walls of Vessels 15
Calculation of Springs 18

SECTION II.

THE ELEMENTS OF GRAPHOSTATICS.

Introductory 22
Multiplication by Lines 22
Division by Lines 23
Multiplication and Division Combined 23
Area of Triangles 23
Area of Quadrilateral Figures 23
Area of Polygons 24
Graphical Calculation of Powers 24
Powers of Trigonometrical Functions 25
Extraction of Roots 26
Addition and Subtraction of Forces. 26
Isolated forces in One Plane—Cord Polygon 26
Equilibrium of External Forces of Cord Polygon 27
Equilibrium of Internal Forces of Cord Polygon 28
Resultant of Isolated Forces in One Plane 29
Conditions of Equilibrium of Isolated Forces 29
Force Couples 29
Equilibrium between Three Parallel Forces 30
Resultant of Several Parallel Forces 31
Decomposition of Forces 31
Uniformly Distributed Parallel Forces 32
Twisting and Bending Movements .. 33
Determination of Centre of Gravity 33
Resultant of Load on Water Wheel. 34
Force Plans for Framed Structures. 35
Force Plans for Roof Trusses 36
Graphical Determination of Wind Stresses 37
Force Plans for Framed Beams 38
Remarks 38

SECTION III.

THE CONSTRUCTION OF MACHINE ELEMENTS.

Introductory 39

CHAPTER I.

RIVETING.

Rivets 39
Strength of Riveted Joints 40
Table and Proportional Scale 40
Riveting disposed in Groups 40
Steam Boiler Riveting 42
Table for Boiler Riveting 42
Table of Weights of Sheet Metal ... 43
Especial Forms of Riveted Joints ... 43

CHAPTER II.

HOOPING.

Hooping by Shrinkage 45
Cold Hooping 45
Examples of Forced Connections ... 46
Dimensions of Rings for Cold Forcing 47

CHAPTER III.

KEYING.

Keyed Connections 47
Cross Keyed Connections 48
Longitudinal Keys 48
Edge Keys 49
Methods of Keying Screw Propellers 49
Unloaded Keys 49
Methods of Securing Keys 50

CHAPTER IV.

BOLTS AND SCREWS.

Geometrical Construction of Screw Thread 50
Whitworth Screw System 51
Sellers' Screw Thread System 52
Metrical Screw Systems 52
New Systems 53
Nuts, Washers and Bolt Heads 54
Table for Metrical Bolts and Nuts .. 55
Weight of Round Iron 55
Special Forms of Bolts 55
Wrenches 56
Nut Locks 56
Special Forms of Screw Threads ... 58
Screw Connections, Flange Joints .. 59
Unloaded Bolt Connections 60

CHAPTER V.

JOURNALS.

Various Kinds of Journals 60

A.—LATERAL JOURNALS.

Overhung Journals 61
Example of Table of Journals 62
Neck Journals 62
Fork Journals 63
Multiple Journals 63
Half Journal 64
Friction of Journals 64

B.—THRUST BEARINGS.

Proportions of Pivots 65
Friction of Flat Pivot Bearings 66
Collar Thread Bearings 66
Multiple Collar Thread Bearings ... 66
Compound Link as Thrust Bearing. 67
Attachment of Journals 67

CHAPTER VI.

BEARINGS.

Design and Proportion 68

A.—LATERAL BEARINGS.

Pillow Blocks 68
Proportional Scale for Pillow Blocks 68
Various Forms of Journal Boxes ... 69
Narrow Bases—Large Pillow Blocks 69
Adjustable Pillow Blocks 70
Bearings with Three Part Boxes 70
Wall Bearings 71
Yoke Bearings 72
Wall Brackets 72
Hangers 73
Adjustable Hangers 74
Special Forms of Bearings 74

B.—THRUST BEARINGS.

Step Bearings 75
Wall Step Bearings 75
Independent Step Bearings 76
Thrust Bearings with Wooden Surfaces 76
Multiple Collar Bearings 77
Examples of Thrust Bearings 78

CHAPTER VII.

SUPPORTS FOR BEARINGS.

General Considerations 79
Simple Supports 79
Multiple Supports for Bearings 80
Calculation for Iron Columns 82
Forms for Iron Columns 84

CHAPTER VIII.

AXLES.

Various Kinds of Axles 85

A.—AXLES WITH CIRCULAR SECTION.

Simple Symetrical Axles 85
Non-Symmetrical Simple Axles 86
Graphical Calculation of Simple Loaded Axles 86
Proof Diagrams 87
Axles Loaded at Two Points 78
Railway Axles, Crane Pillars 88
Axles with Three or More Bearings. 89
Axles with Inclined Loads 90

B.—AXLES WITH COMBINED SECTION.

Annular Section 90
Axles with Cruciform Section 90
Modified Ribbed Axle 91
Compound Axles for Water Wheels. 91
Construction of Rib Profiles 91
Wooden Axles 92

CHAPTER IX.

SHAFTING.

Calculations for Cylindrical Shafting 92
Wrought Iron Shafting 93
Line Shafting 93
Determination of the Angle of Torsion 93
Journals for Shafting—Round Rolled Shafting 94
Combined Sections—Wooden Shafting 94
Shafting Subjected to Deflection ... 94

CHAPTER X.

COUPLINGS.

Various Kinds of Couplings 95
I. Rigid Couplings 95
II. Flexible Couplings 96
Various Kinds of Flexible Couplings 96
Couplings for Lengthwise and Parallel Motion 96
Jointed Couplings 97
III. Clutch Couplings 98
Toothed Clutch Couplings 98

Friction Clutches 99
Automatic Couplings 101

CHAPTER XI.

SIMPLE LEVERS.

Journals for Levers 101
Cast Iron Rock Arms 102
Rock Arm Shafts 102
Lever Arms for Rectangular Section 103
Lever Arms for Combined Section .. 103

CHAPTER XII.

CRANKS.

Various Kinds of Cranks 104
Single Wrought Iron Cranks 104
Graphostatic Calculation of Single Crank 104
Cast Iron Cranks 105
Return Cranks 105
Graphostatic Calculation of Return Crank 105
Simple Crank Axle 106
Multiple Crank Shafts 107
Locomotive Axles 107
Hand Cranks 109

CHAPTER XIII.

COMBINED LEVERS.

Various Kinds of Combined Levers. 110
Walking Beams 110
Scale Beams 111

CHAPTER XIV.

CONNECTING RODS.

Various Parts of Connecting Rods. 112
Connections for Overhung Crank Pins 112
Stub Ends for Fork Journals 114
Connections for Neck Journals 114
Round Connecting Rods 116
Rods for Rectangular Section 116
Channeled and Ribbed Connecting Rods 117
Cast and Wrought Iron Rods 118

CHAPTER XV.

CROSS HEADS.

Various Kinds of Cross Heads 118
Free Cross Heads 119
Cross Heads for Link Connections .. 119
Cross Heads for Guides 119
Guides and Guide Bars 121

CHAPTER XVI.

FRICTION WHEELS.

Classification of Wheels 122
The Two Applications of Friction Wheels 123
Friction Wheels for Parallel Axes .. 123
Friction Wheels for Inclined Axes .. 124
Wedge Friction Wheels 125
Special Applications of Friction Wheels 126
Roller Bearings 126

CHAPTER XVII.

TOOTHED GEARING.

Classification of Gear Wheels 127
A. The Construction of Spur Teeth 128
General Considerations 128
Pitch Radius, Circumferential Division 128
Table of Radii of Pitch Circles 128
General Solution of Tooth Outlines 129
The Action of Gear Teeth 129
The Cycloidal Curves 130
The Generation of Cycloidal Curves 130
Tooth Outlines of Circular Arcs 131
Evolute Teeth for Interchangeable Gears 131
Pin Teeth 132
Disc Wheels with Pin Teeth 133
Mixed Tooth Outlines, Thumb Teeth 133
Tooth Friction in Spur Gearing 134
General Remarks 135
B. Conical Gear Wheels 135
General Considerations 135
Construction Circles for Bevel Gears 135
The Plane Gear Wheel 136
C. Hyperboloidal Gear Wheels
Base Figures for Hyperboloidal Teeth for Hyperboloidal Gears 138
D. Spiral Gears
Cylindrical Spiral Gears 138
Approximately Cylindrical Spiral Gears 139
Spiral Gear Teeth and their Friction 140
Spiral Bevel Gears 141
Globoid Spiral Gears 142
E. Calculation of Pitch and Force of Gearing
Pitch of Gear Wheels, Tooth Section 144
Pitch and Face of Hoistiug Gears .. 144
Table of Cast Iron Hoisting Gears. 145
Pitch and Face of Gearing for Transmission 145
Examples and Comments 147
F. Dimensions of Gear Wheels
The Rim 147
The Arms of Gear Wheels 149
Table of Gear Wheel Arms 149
Gear Wheel Hubs 150
Weight of Gear Wheels 150

CHAPTER XVIII.

RATCHET GEARING.

Classification of Ratchet Gearing ... 150
Toothed Running Ratchet Gears ... 150
The Thrust upon the Pawl 152
The Sliding Flanks 153
Spring Ratchets, Quadrants 153
Methods of Securing Pawls, Silent Ratchets 153
Special Forms of Ratchet Wheels .. 154
Multiple Ratchets 154
Step Ratchets 155
Stationary Ratchets 156
Ratchets of Precision 157
General Form of Toothed Ratchets. 158
Dimensions of Parts of Ratchet Gearing 158
Running Friction Ratchets 158
Release of Friction Pawls 161
Stationary Friction Ratchets 161
Releasing Ratchets 162
Checking Ratchets 163
Continuous Running Ratchets 164
Continuous Ratchets with Locking Teeth 165
Locking Ratchets 166
Escapements, Their Varieties 167
Uniform Escapements 167
Periodical Fscapements 169
Adjustable Escapements 170
General Remarks upon Ratchet Mechanism 171

CHAPTER XIX.

TENSION ORGANS CONSIDERED AS MACHINE ELEMENTS.

Various Kinds of Tension Organs .. 172
Methods of Application 172
Technological Applications of Tension Organs 177
Cord Friction 177
Ropes of Organic Fibres 178
Wire Rope 179
Weight of Wire Rope and its Influence 180
Stiffness of Ropes 181
Rope Connections and Buffers 181
Stationary Chains 182
Running Chains 182
Calculations for Chains 183
Weight of Chain 183
Chain Couplings 184
Chain Drums and Sheaves 185
Ratchet Tension Organs 185

CHAPTER XX.

BELTING.

Self-Guiding Belting 186
Guide Pulleys for Belting 186
Fast and Loose Pulleys 188
Cone Pulleys 189
Cross Section and Capacity of Belts. 190
Examples of Belt Transmission 191
Belt Connections 191
Proportions of Pulleys 193
Efficiency of Belting 194

CHAPTER XXI.

ROPE TRANSMISSION.

Various Kinds of Rope Transmission 194
A. Hemp Rope Transmission. 194
Specific Capacity, Cross Section of Rope 195
Sources of Loss in Hemp Rope Transmission 195
Pressure and Wear on Hemp Rope. 196
B. Cotton Rope Transmission. 196
C. Wire Rope Transmission 196
Specific Capacity, Cross Section of Rope 196
Influence of Pulley Diameter 197
Deflection of Wire Ropes 198
Tightening Driving Ropes 200
Short Span Cable Transmission 200
Transmission with Inclined Cable .. 200
Construction of the Rope Curve 202
Arrangement of Pulleys 202
Construction of Rope Pulleys 202
Construction of Pulley Stations 204
Efficiency of Rope Transmission ... 205
Reuleaux's System of Rope Transmission 206

CHAPTER XXII.

CHAIN TRANSMISSION. STRAP BRAKES.

Specific Capacity of Driving Chains 211
Efficiency of Chain Transmission ... 213
Intermediate Stations for Transmission 213
Strap Brakes 214
Internal Strap Brakes 316

CHAPTER XXIII.

PRESSURE ORGANS CONSIDERED AS MACHINE ELEMENTS.

Various Kinds of Pressure Organs. 216
Methods of Using Pressure Orgaus. 216
Guiding by Pressure Organs 216
Guide Mechanism for Pressure Organs 217
Reservoirs for Pressure Organs 218
Motors for Pressure Organs 219
A. Running Mechanism for Pressure Organs 219
Running Mechanism operated by Weight 219
Running Mechanism Operated by Impact 219
Running Mechanism Operated against Gravity 221
Running Mechanism in which the Motor is Propelled 222
B. Ratchet Mechanism for Pressure Organs 223
Fluid Running Ratchet Trains 223
Fluid Trains with Stationary Ratchets 225
Escapements for Pressure Organs .. 226
A. Unperiodic Escapements for Pressure Organs 227
Fluid Escapements for Transportation 227
Hydraulic Tools 228
Pressure Escapements for Moving Liquids 228
B. Periodical Pressure Escapements 229
Pumping Machinery 229
Fluid Transmission at Long Distance 233
Rotative Pressure Engines 233
Valve Gears for Rotative Engines .. 234
C. Adjustable Power Escapements 236
Adjustable Pump Gears 236

Adjustable Gears for Rotative Motors..... 237
D. Escapements for Measurement of Volume.....
Running Measuring Devices..... 239
Escapements for Measurement of Fluids..... 239
Technological Applications of Pressure Organs..... 240

CHAPTER XXIV.

CONDUCTORS FOR PRESSURE ORGANS.

Formulæ for Cast Iron Pipes..... 242
Weights of Cast Iron Pipes..... 242
Pipes for High Pressures..... 242
Wrought Iron and Steel Pipes..... 243
Steam Pipes..... 245
Pipes of Copper and other Metals.. 246
Resistance to Flow in Pipes..... 246
Methods of Connecting Cast Iron Pipes..... 248
Connections for Wrought Iron and Steel Pipes..... 249
Connections for Pipes of Lead and other Metals..... 251
Flexible Pipes..... 252
Pistons..... 252
Plungers and Stuffing Boxes..... 253
Pistons with Valves..... 255
Piston Rods..... 255
Specific Capacity of Pressure Transmission systems..... 255
Ring System of Distribution, with Pipe Conductors..... 256
Specific Capacity of Shafting..... 257
Specific Value of Long Distance Transmissions..... 259

CHAPTER XXV.

RESERVOIRS FOR PRESSURE ORGANS.

Various Kinds of Reservoirs..... 260
Cast Iron Tanks..... 260
Riveted Tanks..... 260
Tanks with Concave Bottoms..... 262
Combination Forms for Tanks..... 264
High Pressure Reservoirs, or Accumulators..... 264
Steam Boilers..... 265
Boiler Details subjected to Internal Pressure..... 266
Boiler Flues subjected to External Pressure..... 269
Future Possibilities in Steam Boiler Construction..... 270
Reservoirs for Air and Gas..... 272
Other forms of Storage Reservoirs. 273

CHAPTER XXVI.

RATCHETS FOR PRESSURE ORGANS, OR VALVES.

The two Divisions of Valves..... 273
A. Lift Valves..... 274
Hinged or Flap Valves..... 274
Round Self-Acting Valves..... 275
Unbalanced Pressure on Lift Valves 277
Closing Pressure of Self-Acting Valves..... 278
Mechanically Actuated Pump Valves 278
Valves with Spiral Movement..... 279
Balanced Valves..... 279
B. Sliding Valves.....
Rotary Valves and Cocks..... 281
Gate Valves for Open and Closed Conductors..... 282
Slide Valves..... 282
Balanced Slide Valves..... 285
Fluid Valves..... 287
Stationary Valves..... 289
Stationary Machine Elements in General..... 289

SECTION IV.

Mathematical Tables..... 291–301

ERRATA.

Page 14, Case IV., first panel of table should read $P=4\,\pi^2\,\frac{JE}{l^2}$

Page 15, line 13 from bottom, second column, omit words "⅜" thick."

Page 53, line 31 from bottom, second column, read "interpolated diameter," instead of "interpolated meter."

Page 61, formula (89) substitute P, instead of p.

Page 61, line 11 from top, second column, after "Proportions of Journals," insert the formula number (93).

Page 63, line 39 from top, first column, after "Formula for Fork Journals" insert the formula number (98).

Page 64, the formulæ on lines 12 and 14, of § 96, should be numbered respectively (99) and (100).

Page 64, line 33 from top, second column, for "Prowny" read "Prony."

Page 89, line 17 from bottom, first column, for 85 mm.=8⅜" read 85 mm.=3⅜".

Page 89, illustration at the bottom of second column, the diagram to the left should be Fig. 409, and that to the right, Fig. 410.

Page 97, line 16 from bottom, second column, for "drawn" read "driven."

Page 103, the last formula on first column should be numbered (154) instead of (155).

Page 144, formula at bottom of first column, the cube root sign applies to the whole of the second member and not to the numerator only, as printed.

Page 175, line 17 from bottom. second column, for Harturcn, read Hartwich.

Page 195, line 29 from top, second column, for "can only be given by indeterminate results," read "can only give approximate results."

Page 206, title of § 301 read Reuleaux's instead of Reuleux's.

Page 255, example in second column, for 4 in. stroke, read 40 in. stroke.

Page 263. The following revisions of formulæ (385) and (386) have been communicated by Prof. Reuleaux and should be inserted:

$$s_5=\gamma\frac{R}{2}\left\{\left[1-\left(\frac{D}{D_0}\right)^2\right]H-\frac{f}{3}\left[1-\frac{D}{D_0}-2\left(\frac{D}{D_0}\right)^2\right]\right\}\qquad(385)$$

$$s_6=\gamma\frac{R}{2}\left\{\left[\left(\frac{D}{D_0}\right)^2-1\right]H-\frac{f}{3}\left[1+\frac{D}{D_0}-2\left(\frac{D}{D_0}\right)^2\right]\right\}\qquad(386)$$

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THE CONSTRUCTOR:

A HAND-BOOK OF MACHINE DESIGN.

BY F. REULEAUX.

SECTION I.—STRENGTH OF MATERIALS.

§ 1.

INTRODUCTORY.

The study of the streugth of materials ultimately depends upon the question of the resistance which rigid bodies oppose to the operatiou of forces, and the following definitions are to be noted:

SUPERFICIAL PRESSURE is the pressure upon a unit of surface.

TENSILE STRENGTH is the resistance per unit of surface, which the molecular fibres oppose to separation.

MODULUS OF RESISTANCE is the strain which corresponds to the limit of elasticity, compression and extension, each having a corresponding modulus.

MODULUS OF RUPTURE is the strain at which the molecular fibres cease to hold together.

MODULUS OF ELASTICITY is the measure of the elastic extension of a material, and is the force by which a prismatic body would be extended to its own length, supposing such extension were possible.

THEORETICAL RESISTANCE is the force which, when applied to any body, either as tension, compression, torsion or flexure, will produce in those fibres which are strained to the greatest extent a tension equal to the modulus of resistauce; or, in other words, it is the load which strains a body to its limit of elasticity.

THE PRACTICAL RESISTANCE often improperly termed merely Resistauce, is a definite but arbitrary working strain to which a body may be subjected within the limits of elasticity.

THE COEFFICIENT OF SAFETY is the ratio between the theoretical resistance and the actual load, or, what amounts to the same thing, the ratio between the elastic limit and the actual tension of the fibres.

THE BREAKING LOAD is that load which causes in those fibres which are subjected to the greatest strain, a tension equal to the modulus of rupture; in every case this is equal to the force necessary to tear, crush, shear, twist, break, or otherwise deform a body.

THE FACTOR OF SAFETY is the ratio between the breaking load and the actual load.

As a general rule, for machine construction, the Coefficient of Safety may be taken as double that which is used for construction subjected to statical forces. Circumstances may also require it to be takeu as either greater or less than the customary value, sometimes even narrower than is permitted for statical forces. Care must be takeu never to permit a material to be straiued iu use to its theoretical resistance; although, indeed, there are some materials, such as wrought iron, which have been strained slightly beyond the limit of elasticity, without reducing the breaking load, or causing any apparent injury. (See § 2.)

The determiuation of the breaking load, and consequently the use of the modulus of rupture, is limited to those cases in which the actual breaking of the structure must be considered; but for the actual calculations of working machinery the modulus of resistance, or limit of elasticity is of primary importance.

§ 2.

COEFFICIENTS OF RESISTANCE.*

The coefficients given in the followiug table are selected as the mean of uiauy experiments upon the various materials named.

Under the title "Wood" is given an average value from experiments made with oak, beech, fir and ash. Those materials which show the greatest difference between the modulus of rupture and the limit of elasticity also possess in the highest degree the property of toughness.

Experiments upon wrought iron show that a strain beyond the limit of elasticity, if not carried too far, although it will cause a permanent deformation, will not lower the modulus of elasticity, but will raise the modulus of resistance.

For example, a rod of wrought iron, subjected to a tensile strain of 28,400 lbs. per square inch, was subsequently found to have its limit of elasticity raised from 21,300 lbs. to 28,400 lbs. (This property is utilized in drawing wire).

Tenacity is a particularly desirable property for a material of construction, and it may generally be approximately measured by the ratios $K : T$ and $K_1 : T_1$.

If the rod above referred to be subject to compression it will return to its former limit of elasticity.

TABLE OF COEFFICIENTS.*

MATERIAL.	Modulus of Elasticity. E.	Modulus of Resistance. Tension. T.	Modulus of Resistance. Compression. T_1.	Modulus of Rupture. Tension. K.	Modulus of Rupture. Compression. K_1.
Wrought Iron	20,000 28,400,000	15 21,300	15 21,300	40 56,800	22 31,240
Iron Wire	20,000 28,400,000	30 42,600	—	70 99,400	—
Sheet Iron	17,000 24,140,000	—	—	32 45,440	—
Cast Iron	10,000 14,200,000	7.5 10,650	15 21,300	11 15,620	63 88,460
Spring Steel (hardened)	20,000 28,400,000	50 to 70 70 to 90,000	—	80 113,600	—
Cast Steel (not hardened)	20,000 28,400,000	25 35,500	—	80 113,600	—
Cast Steel (spring temper)	30,000 42,600,000	65 to 150 90 to 200,000	—	100 142,000	—
Copper (hammered)	11,000 15,620,000	2.5 3,550	—	30 42,600	70 99,400
Copper Wire	13,000 18,460,000	12 17,040	—	40 46,800	—
Brass	6,500 9,230,000	4.8 6,816	—	12 17,040	110 156,200
Brass Wire	10,000 14,200,000	13 18,460	—	50 71,000	—
Bell Metal (bronze)	3,200 4,544,000	9 12,780	—	13 18,460	—
Phosphor Bronze	—	15 21,300	—	36 51 120	—
Aich Metal	—	15 21,300	—	75 106,500	—
Lead	500 710,000	1 1,420	—	1.3 1,846	5 7,100
Wood	11,000 1,562,000	2 2,840	1.8 2,556	9 12,780	5 7,100
Hemp Rope (new)	250 (?) 355,000	5 (?) 7,100	—	12 17,040	—
Hemp Rope (old)	50 (?) 71,000	1 (?) 1,420	—	5 7,100	—
Belting	15 to 20 20 to 30,000	1.6 2,272	—	2.9 4,118	—
Granite	—	—	—	—	8 11,360
Limestone	—	—	—	—	5 7,100
Quartz	—	—	—	—	12 17,040
Sandstone	—	—	—	—	7 9,940
Brick	—	—	—	—	0.6 852
Limestone Masonry	—	—	—	—	5 7,100
Sandstone Masonry	—	—	—	—	1.5 2,130
Brickwork	—	—	—	—	0.4 568

* The upper figures are kilogrammes per square millimetre, and the lower figures are pounds per square inch.

* Throughout the original work all dimensions and quantities are given in the metric system, but these have been transformed into English units for English readers, except in the following table, where both are given. —*Trans.*

§ 3.

Resistance to Tension and Compression.

A body is said to be under tension when the action of a force P, tends to extend it in the direction of its length. When the force acts in the opposite direction the body is said to be under compression; but when the length is great in proportion to the cross section, a combined action occurs. (See § 16.)

Let q be the cross section of the member: S, the strain due to the action of the force P; then if we neglect the weight of the material we have:

$$P = S\,q \qquad (1)$$

*Example. A rafter exerts a horizontal thrust of 22,000 pounds, which must be borne by a rod of circular cross section. If we make $S = 7100$ pounds we have for the diameter of the rod d,

$$Sq = 7100\frac{\pi}{4}d^2 = 22{,}000$$

from which $d = 1.98''$ say $2''$.

The principal action which the application of a force to a member produces is the consequent elongation or compression. A prismatical body subjected to the action of a force P, will have its original length l increased by in amount λ, determined by the formula

$$\frac{\lambda}{l} = \frac{S}{E} \qquad (2)$$

and this holds good as long as S is not greater than the modulus of Resistance for tension T. This relation is also true for compression, in which case the limit depends upon the modulus of resistance T_1 for compression.

Example. Suppose the rod, whose diameter was determined in the preceding example, to have a length of 114 ft. 10 in. or 1378 inches, its elongation under those conditions would be

$$\lambda = \frac{1378 \times 7100}{28{,}400{,}000} = 0.3445 \text{ in. say } \frac{11}{32} \text{ in.}$$

The preceding formula (2) is a fundamental one, and upon it is constructed the whole systematical study of the strength of materials.

Formula (1) is of use when a section is strained beyond the limit of elasticity, as by it we may determine the force required to rend or crush a material, using the proper Modulus of Rupture.

Example. The force necessary to pull the above given rod asunder is

$$P = K\,q$$

or

$$P = 56{,}800 \times (2)^2\,\frac{\pi}{4} = 178{,}442 \text{ lbs.}$$

§ 4.

Bodies of Uniform Strength.

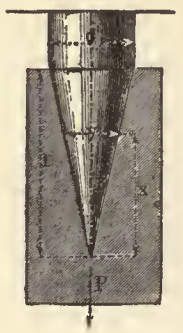
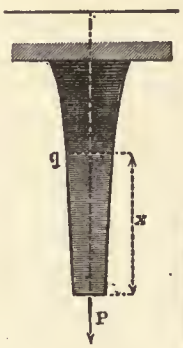
By bodies of uniform strength are meant those in which the shape is so made that the cross sections at various points are subjected to the same strain S, and consequently a proportionally economical distribution of material secured.

Such forms are not often employed in practice, although approximate shapes may often be adopted, but they serve in many cases to determine the general style of a structure, and give it the effect of proportioual strength without adhering too closely to the exact form. These forms will be found of value to the designer for both reasons: principally as a guide to the style of his work rather than for close determinations of economy.

If a designer has become thoroughly familiar with the resisting capacity of various shapes, and can keep them so clearly in his mind that he can perceive the general form of the proper curve to be used in any particular case, he will be able to produce, with an artistic freedom, designs which will approach the shapes indicated by mathematical analysis.

The following forms are alike suitable for tension and compression. As examples of their practical use, the first two are applicable to cast columns, and the third is suitable for chimneys of masonry as well as for high piers of bridges and viaducts.

*In all cases the quantities given in the original examples have been converted into their English equivalents, which will account for the unusual quantities chosen. (*Trans.*).

SHAPE.	EQUATION.	REMARKS
	$\frac{y}{d} = \sqrt{\frac{x}{l}}$ $d = \sqrt{\frac{4}{\pi}\cdot\frac{P}{S}}$	P, is distributed uniformly throughout the whole leugth of the figure. Cross section circular. Profile parabolic. Approximate form, a truncated cone with end diameter $= \frac{d}{2}$
	$\frac{y}{d} = \frac{x}{l}$ $d = \sqrt{\frac{4}{\pi}\cdot\frac{P}{S}}$	P, is uniformly decreased from above downward. Cross sectiou circular. Form conical.
	$q = \frac{P}{S}e^{\frac{\gamma}{S}x}$ $e = 2.718 =$ Base of natural logarithms. $\log q = \log \frac{P}{S} + 0.434\frac{\gamma}{S}x$	The body is strained by its own weight, γ being the weight of a unit of volume. The cross section increases with the increasing load in the logarithmic proportion given.

§ 5.

Resistance to Shearing.

A body is said to be subjected to a shearing strain in any cross section when the distorting force acts in the plane of that cross section.

Let q, be the sectional area, and S, the force acting upon it, so that we have as in the case of tensile and compressive strains

$$P = Sq \qquad (3)$$

The limit of elasticity will be reached when $S = \frac{1}{5}$ *of the lesser of the two Moduli of Resistance of the material,* in the case of wrought iron, where $T = T_1 = 21{,}300$, $S = 17{,}040$ lbs. while for cast iron $T < T_1$ and $= 10{,}650$ and $S = 8{,}520$. In this case the maximum strain is not in the plane of the cross section but at an angle of 45° with it, and is $\frac{5}{4}$ the value of S. The deformation which two surfaces suffer under a shearing strain is very slight within the limits of elasticity, but it is noticeable in the case of torsional distortion where numerous sections under shearing strain adjoin each other.

Equation (3) holds good for cases in which the division of the adjoining surfaces occurs, such as shearing, planing, boring, in fact, the work of the entire class of machines which act by the removal of a portion of the material upon which they are working.

The strain of separation S, in this case is somewhat less than the modulus of rupture for tension (K). This is due to the fact that in the case of shearing forces both K, and K_1 come into action. For the calculation of such machines a coefficient of rupture equal to 1.1 K may be taken.

§ 6.

Resistance to Bending.

Elasticity and Strength of Flexure.

A prismatic body upon which the external forces act in a direction at right angles to the axis of the prism, is said to be

subject to strains of flexure. As long as the elastic limit is not overstepped there will exist an equilibrium in each normal section of the prism between the moments of the external forces on the one hand and the moments of the opposing internal forces on the other hand, both acting about the neutral axis of the given section. This may be considered as a sort of equator of each section since it passes through its centre of gravity at right angles to the plane in which the bending takes place. It thus divides the section into two portions, in one of which all the fibres which are parallel to the axis of the prism are subjected to a tension proportional to their distance from the neutral axis (the tension side of the section), while in the other portion the corresponding fibres are subjected to compression in a like proportion (the compression side of the section). It follows that fibres which are at equal distances from the neutral axis will be deformed to the same extent. The resistance to bending is therefore a combination of the resistances to tension and to compression, both acting in a peculiar manner, that is, in rotation about an axis.

Let:

M=the statical moment of a cross section subjected to a bending force, taken with reference to the neutral axis of the section, that is, the so-called moment of force.

J=the moment of inertia of the section with regard to the neutral axis.

a=the distance from the neutral axis of the fibres which are subjected to greatest tension or compression; *i. e.*, those which are farthest from the neutral axis.

S=the corresponding strain in these fibres, then:

$$M = S\frac{J}{a} \qquad (4)$$

The product $S\frac{J}{a}$ is the statical moment of the entire collection of fibres of the cross section with reference to the neutral axis and is the strain moment of the section under consideration. For any prismatic shape subjected to a bending force P, whose lever arm is x, we may put $M=Px$ for each cross section, and for each section this will have a different value. The section at which the product Px has its greatest value is the section of danger, and the bending force P, which produces in it the strain S, is the resisting strength of the shape for the strain S, so that in this case we have

$$P = \frac{SJ}{x_m a} \qquad (5)$$

in which x_m, is that value of x, which makes Px a maximum.

The axis passing through the centres of gravity of successive cross sections of a figure is not subject to any change of length under flexure, but is only curved, and to find its radius of curvature under any load we have the formula

$$= \frac{JE}{M} \qquad (6)$$

This curve is called the elastic curve, and is determined by the general equation

$$\frac{d_2 y}{dx^2} = \frac{M}{JE} \qquad (7)$$

In the following tables are given the values of the quantities for calculations of flexure under the various conditions shown in the figures, being:

The Bending Moment M, for any point x,

The Bending Load P, according to formula (5),

The Co-ordinates x, y, for the elastic curve,

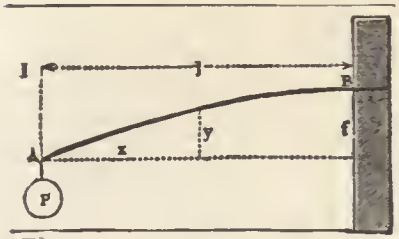
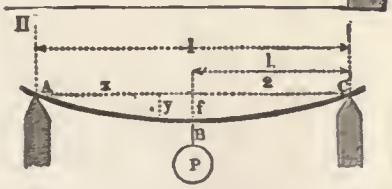
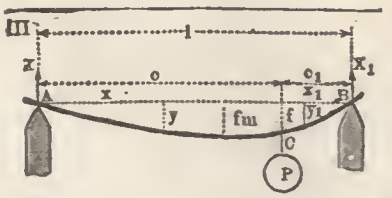
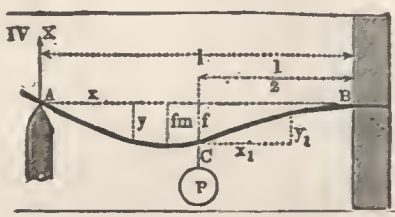
The value, f, of the abscissa y, at the point of application of the force as shown in Figures I to VI, and the value of the greatest deflection f, in the cases shown in Figures VII to XIII.

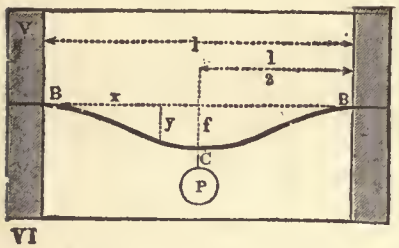
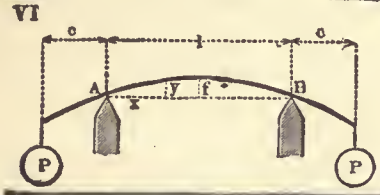
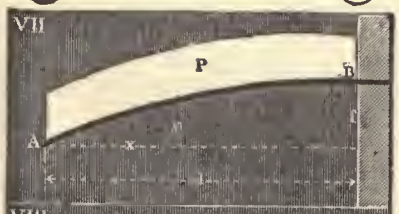
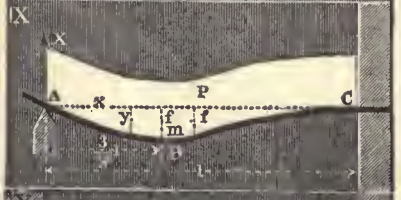
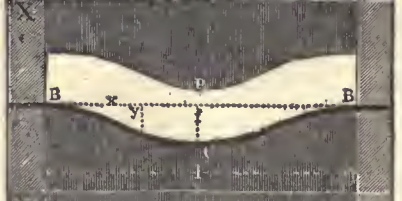
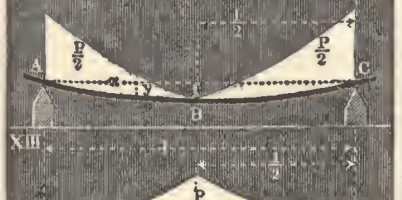
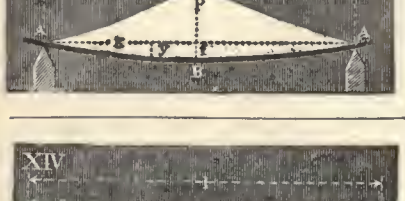
In all the examples the weight of the beam itself has been neglected, as this may usually be done in machine construction, although not in bridge work.

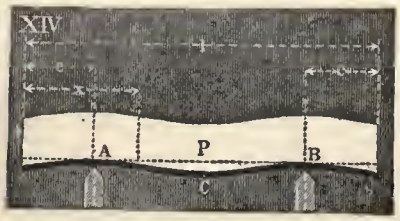
Figures VII to X are suitable for the latter purpose, as in them the weight of the beam may readily be taken into account.

In Figures XI and XII may be seen how unequal distribution of the load affects the sustaining power, as a beam loaded like Fig. XI or XII has 1½ times the sustaining power of one loaded like VII or VIII, or for the same load a correspondingly reduced deflection. These are important considerations in connection with the distribution of weights in buildings. The distribution in XIII is also unfavorable, as it has only ¾ the sustaining power of VIII, with a greater deflection.

It is to be noticed that the deflection f, increases as the third power of the length, and that it varies greatly under the various conditious given.

Example.	Bending Moment M.	Sustaining Power P.	Equation of the Elastic Curve.	Deflection f.	Remarks.
I 	$M = Px$	$P = \frac{SJ}{l\,a}$	$y = \frac{P}{JE}\frac{l^3}{2}\left[\frac{x}{l} - \frac{1}{3}\frac{x^3}{l^3}\right]$	$f = \frac{P}{JE}\frac{l^3}{3}$	Weakest Section at B.
II 	$M = \frac{Px}{2}$	$P = 4\frac{SJ}{a\,l}$	$y = \frac{P}{JE}\frac{l^3}{16}\left[\frac{x}{l} - \frac{4}{3}\frac{x^3}{l^3}\right]$	$f = \frac{P}{JE}\cdot\frac{l^3}{48}$	Weakest Section in the Middle.
III 	For $A\,C$ $M = \frac{Pc_1 x}{l}$ For $B\,C$ $M = \frac{Pc\,x_1}{l}$	$P = \frac{l}{c\,c_1}\frac{SJ}{a}$	$y = \frac{P}{JE}\frac{c^2 c_1^2}{6\,l}\left[2\frac{x}{c} + \frac{x}{c_1} - \frac{x^3}{c^2 c_1}\right]$ $y_1 = \frac{P}{JE}\frac{c^2 c_1^2}{6\,l}\left[2\frac{x_1}{c_1} + \frac{x_1}{c_1} - \frac{x_1^3}{c_1^2 c}\right]$	$f = \frac{P}{JE}\frac{l^3}{3}\frac{c^2}{l^2}\frac{c_1^2}{l^2}$ f= maximum when $x = c\sqrt{\frac{1}{3} + \frac{2}{3}\frac{c_1}{c}}$.	Weakest Section at C Reactions $X = P\frac{c_1}{l}$ $X_1 = P\frac{c}{l}$
IV 	For $A\,C$ $M = \frac{5}{16}Px$ For $B\,C$ $M = Pl\left(\frac{5}{32} - \frac{11}{16}\frac{x_1}{l}\right)$	$P = \frac{16}{3}\frac{SJ}{la}$	$y = \frac{P}{JE}\frac{l^3}{32}\left[\frac{x}{l} - \frac{5}{3}\frac{x^3}{l^3}\right]$ $y_1 = \frac{P}{JE}\frac{l^3}{32}\left[\frac{1}{4}\frac{x_1}{l} + \frac{5}{2}\frac{x_1^2}{l^2} - \frac{11}{3}\frac{x_1^3}{l^3}\right]$	$f = \frac{P}{JE}\frac{7\,l^3}{768}$ $f\max = \sqrt{\frac{1}{5}}\frac{P\,l^3}{48\,JE}$ when $x = l\sqrt{\frac{1}{5}}$	Weakest Section at B. Reaction at X $= \frac{5}{16}P$

Example.	Bending Moment M.	Sustaining Power P.	Equation of Elastic Curve.	Deflection f.	Remarks.
V 	$M = \frac{Pl}{2}\left(\frac{x}{l} - \frac{1}{4}\right)$	$P = 8\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{16}\left[\frac{x^2}{l^2} - \frac{4}{3}\,\frac{x^3}{l^3}\right]$	$f = \frac{P}{JE}\,\frac{l^3}{192}$	Weakest Sections at A and C.
VI 	For $A\,B$ $M = Pc$	$P = \frac{SJ}{ca}$	$y = f - \rho + \sqrt{\rho^2 - x^2 + l\left(x - \frac{l}{4}\right)}$ in which $\rho = \frac{JE}{Pc}$	$f = \frac{P}{JE}\,\frac{l^3}{8}\,\frac{c}{l}$	Weakest Section at an indeterminate point between A and B.
VII 	$M = \frac{Px}{2}\,\frac{x}{l}$	$P = 2\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{6}\left[\frac{x}{l} - \frac{1}{4}\,\frac{x^4}{l^4}\right]$	$f = \frac{P}{JE}\,\frac{l^3}{8}$	Weakest Section at B.
VIII	$M = \frac{Px}{2}\left(1 - \frac{x}{l}\right)$	$P = 8\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{24}\left[\frac{x}{l} - 2\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right]$	$f = \frac{P}{JE}\,\frac{5\,l^3}{384}$	Weakest Section in the Middle.
IX 	$M = \frac{Px}{2}\left(\frac{3}{4} - \frac{x}{l}\right)$	$P = 8\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{48}\left[\frac{x}{l} - 3\,\frac{x^3}{l^3} + 2\,\frac{x^4}{l^4}\right]$	$f = \frac{P}{JE}\,\frac{l^3}{192}$	Weakest Section at C. Greatest Deflection when $x = \frac{l}{16}\left(1 + \sqrt{33}\right)$ Reaction at $X = \frac{3}{8}\,P$ Turning point $x = \frac{3}{4}\,l$
X 	$M = \frac{Pl}{2}\left(\frac{1}{6} - \frac{x}{l} + \frac{x^2}{l^2}\right)$	$P = 12\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{24}\left[\frac{x^2}{l^2} - 2\,\frac{x^3}{l^3} + \frac{x^4}{l^4}\right]$	$f = \frac{P}{JE}\,\frac{l^3}{384}$	Weakest Section at B. Point of reversal at $x = \frac{l}{2}\left(1 - \sqrt{\frac{1}{3}}\right)$
XI	$M = \frac{P}{3}\,\frac{x}{l}\,\frac{x^2}{l^2}$	$P = 3\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{12}\left[\frac{x}{l} - \frac{1}{5}\,\frac{x^5}{l^5}\right]$	$f = \frac{P}{JE}\,\frac{l^3}{15}$	Weakest Section at B.
XII 	$M = Px\left(\frac{1}{2} - \frac{x}{l} + \frac{2}{3}\,\frac{x^2}{l^2}\right)$	$P = 12\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{12}\left[\frac{3}{8}\,\frac{x}{l} - \frac{x^3}{l^3} + \frac{x^4}{l^4} - \frac{2}{5}\,\frac{x^5}{l^5}\right]$	$f = \frac{P}{JE}\,\frac{3\,l^3}{320}$	Weakest Section in the Middle.
XIII 	$M = Px\left(\frac{1}{2} - \frac{2}{3}\,\frac{x^2}{l^2}\right)$	$P = 6\,\frac{SJ}{la}$	$y = \frac{P}{JE}\,\frac{l^3}{12}\left[\frac{5}{8}\,\frac{x}{l} - \frac{x^3}{l^3} + \frac{2}{5}\,\frac{x^5}{l^5}\right]$	$f = \frac{P}{JE}\,\frac{l^3}{60}$	Weakest Section in the Middle.

XIV 

XIV. In the case of a beam supported upon two symmetrically placed supports A and B, and carrying a uniformly distributed load P, we have for the bending moment $M = \frac{Px}{2}\left(\frac{x}{l} - 1 + \frac{c}{x}\right)$.

The supporting power varies according to the position of the supports, and also with the relation of c to l; it will become a maximum when $c = 0.207\,l$ [that is, $l\left(\sqrt{\frac{1}{2}} - \frac{1}{2}\right)$].

The supporting power will then approximate to

$$P = 47 \frac{S J}{l a},$$

or nearly six times as great as in case VIII, showing the advantage of this method of support. The weakest sections are at *A*, *B* and *C*.

§ 7.

Table of Sections.

The value of $\frac{J}{a}$ in equation (4) depends almost entirely upon the shape of the cross section of the beam, and this we shall hereafter call the Section Modulus.

The following table shows a large number of sections in use for various purposes, and gives the corresponding values of the following quantities:

The equatorial moment of inertia *J*, for the neutral axis, shown in the figures by the dotted line.

The greatest distance *a*, of the fibres under tension and compression, or their separate values *a'*, and *a''*, when the section is not symmetrical about both axes.

The equatorial section modulus $Z = \frac{J}{a}$, for which two values are given, when *a'*, and *a''*, are different; and

The sectioual area of the figure, which will be found of service in calculating weights.

To determine the value of *a*, experimentally or graphically, a model of the section may be cut ont of cardboard, and its centre of gravity found by balancing on knife edges, or else the graphostatic method given in § 46 may be employed.

The following example will serve to show the application of the table:

Example. Required the moment of inertia of a circular section 4 inches in diameter. According to No. XX in the table:

$$J = \frac{\pi}{64} d^4 = 0.0491\, d^4 = 0.0491 \times 256 = 12.5706$$

By making various combinations of the forms given in the tables other sections may be obtained to which the same formulæ will apply. As an example, the Section No. VIII may also be used for a rectangular tube, and No. XI for an *E* shaped section.

It is a matter of some importance for the designer to keep in mind some general conclusions, which may be drawn from the tables as to the influence of various shapes upon the strength. It will be plainly seen that the depth of a section is the dimension which has the greatest influence upon the strength, and also that those portions of the section which are furthest removed from the neutral axis are of the most service.

It is upon this point that the peculiar strengthening effect of ribs depends, and which makes their use so advantageous in cast iron constructions. These ribs do not act so much by the mere strength of their own cross section as by the fact that they strengthen those portions which are furthest from the neutral axis. This is a feature to be carefully watched, and its importance may be made clear by an example.

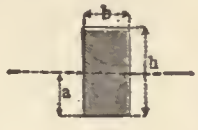
If we take a section of the form given in No. XV., and make its dimensions as follows: $b = 8\, b_1$, $h = 12\, b_1$, $h_1 = 11\, b_1$ (Fig. 1, § 9) and then divide it into two rectangular parts by a horizontal section, we have for the modulus of each section:

$$\frac{11^2 \times b_1^3}{6} = 20\tfrac{1}{6}\, b_1^3 \text{ and } \frac{8\, b_1^3}{6}$$

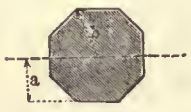
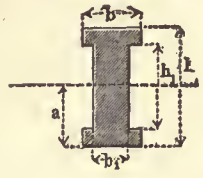
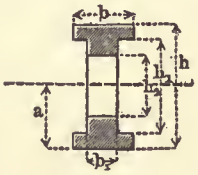
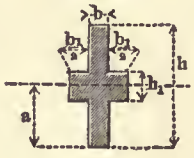
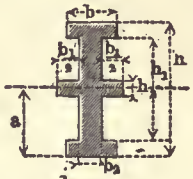
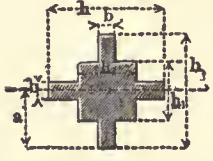
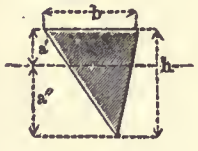
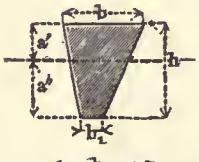
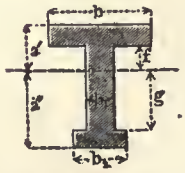
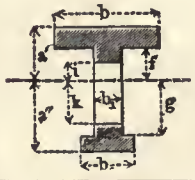
which, together, give $21.5\, b_1^3$.

The same material, when taken as a whole, in a single section (see § 9) would have a modulus $Z = 34.8\, b_1^3$, so that it has more than 1½ times the resistance of its separated portions, and as a matter of fact the right angle rib or *T* head is about ten times the value in that connection than if taken by itself. This is also found in a still higher degree in sections of other shapes.

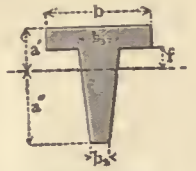
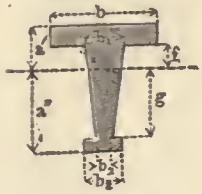
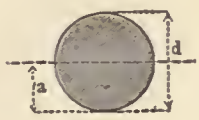
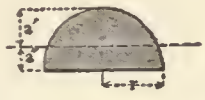
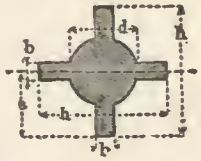
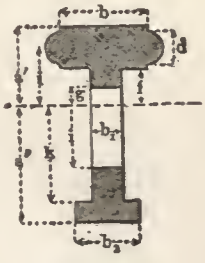
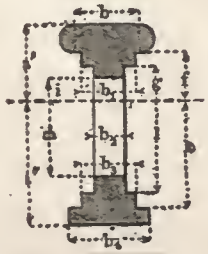
SECTION TABLE.

No.	Section.	Moment of Inertia *J*.	Distance *a*.	Section Modulus $Z = \frac{J}{a}$	Area, *F*.
I.		$\frac{b\,h^3}{12}$	$\frac{h}{2}$	$\frac{b\,h^2}{6}$	$b\,h$
II.		$\frac{b\,(h^3 - h_1^3)}{12}$	$\frac{h}{2}$	$\frac{b\,(h^3 - h_1^3)}{6\,h}$	$b\,(h - h_1)$
III.		$\frac{b^4}{12}$	$\frac{b}{2}$	$\frac{b^3}{6}$	b^2
IV.		$\frac{b^4}{12}$	$\frac{b}{\sqrt{2}}$	$\frac{\sqrt{2}}{12} b^3 = 0.118\, b^3$	b^2
V.		$\frac{5\sqrt{3}}{16} b^4 = 0.5413\, b^4$	$b\sqrt{\frac{3}{4}} = 0.866\, b$	$\frac{5}{8} b^3$	$\frac{3\sqrt{3}}{2} b^2 = 2.598\, b^2$
VI.		$\frac{5\sqrt{3}}{16} b^4$	b	$\frac{5\sqrt{3}}{16} b^3$	$\frac{3\sqrt{3}}{2} b^2$

SECTION TABLE—(*Continued*).

No.	Section.	Moment of Inertia J.	Distance a.	Section Modulus Z.	Area F.
VII.		$\frac{1+2\sqrt{2}}{6}b^4 = 0.638\,b^4$	$0.924\,b$	$0.677\,b^3$	$2.828\,b^2$
VIII.		$\frac{b\,h^3-(b-b_1)\,h_1^3}{12}$	$\frac{h}{2}$	$\frac{b\,h^3-(b-b_1)\,h_1^3}{6\,h}$	$b\,h-(b-b_1)\,h_1$
IX.		$\frac{b\,(h^3-h_1^3)+b_1\,(h_1^3-h_2^3)}{12}$	$\frac{h}{2}$	$\frac{b\,(h^3-h_1^3)+b_1\,(h_1^3-h_2^3)}{6\,h}$	$b\,(h-h_1)+b_1\,(h_1-h_2)$
X.		$\frac{b\,h^3+b_1\,h_1^3}{12}$	$\frac{h}{2}$	$\frac{b\,h^3+b_1\,h_1^3}{6\,h}$	$b\,h+b_1\,h_1$
XI.		$\frac{b\,h^3-(b-b_2)\,h_1^3+b_1\,h_2^3}{12}$	$\frac{h}{2}$	$\frac{b\,h^3-(b-b_2)\,h_1^3+b_1\,h_2^3}{6\,h}$	$b\,h-(b-b_2)\,h_1+b_1\,h_2$
XII.		$\frac{b\,h^3+(h_1-b)\,h_1^3+(h-h_1)}{12}$	$\frac{h}{2}$	$\frac{b\,h^3+(h_1-b)\,h_1^3+(h-h_1)\,b^3}{6\,h}$	$b\,h+(h_1-b)h_1+(h-h_1)b$
XIII.		$\frac{b\,h^3}{36}$	$a'=\frac{h}{3}$ $a''=\frac{2}{3}h$	$Z'=\frac{b\,h^2}{12}$ $Z''=\frac{b\,h^2}{24}$	$\frac{b\,h}{2}$
XIV.		$\frac{b^2+4\,b\,b_1+b_1^2}{36\,(b+b_1)}h^3$	$a'=\frac{b+2\,b_1}{b+b_1}\frac{h}{3}$ $a''=\frac{2\,b+b_1}{b+b_1}\frac{h}{3}$	$Z'=\frac{b^2+4\,b\,b_1+b_1^2}{12\,(b+2\,b_1)}h^2$ $Z''=\frac{b^2+4\,b\,b_1+b_1^2}{12\,(2\,b+b_1)}h^2$	$\frac{b+b_1}{2}h$
XV.		$\frac{1}{3}\left[b\,(a'^3-f^3)+b_1\,(f^3+a''^3)\right]$	$a'=\frac{b\,h_2^2+b_1\,h_1\,(h+h_2)}{2\,[b\,h-(b-b_1)\,h_1]}$ $a''=h-a'$	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b_1\,h_1+b\,h_2$
XVI.		$\frac{1}{3}\left[b\,(a'^3-f^3)+b_1\,(f^3+g^3)+b_2\,(a''^3-g^3)\right]$	Determined graphically or by experiment.	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b\,(a-f)+b_1\,(f+g)+b_2\,(a''-g)$
XVII.		$\frac{1}{3}\left[b\,(a'^3-f^3)+b_1\,(f^3+g^3-i^3-k^3)+b_2\,(a''^3-g^3)\right]$	Determined graphically or by experiment	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b\,(a'-f)+b_1(f+g-i-k)+b_2(a''-g)$

SECTION TABLE—(*Continued*).

No.	Section.	Moment of Inertia J.	Distance a.	Section Modulus Z.	Area F.
XVIII.		$\frac{1}{3}\left[\frac{b_1-b_2}{4(f+a'')}(a''^4-f^4)+b(a'^3-f^3)+b_2(f^3+a''^3)\right]$	Determined graphically or by experiment.	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b(a'-f)+\frac{b_1+b_2}{2}(f+a'')$
XIX.		$\frac{1}{3}\left[\frac{b_1-b_2}{4(f+g)}(g^4-f^4)+b(a'^3-f^3)+b_2(f^3+g^3)+b_3(a''^3-g^3)\right]$	Determined graphically or by experiment.	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b(a'-f)+\frac{b_1+b_2}{2}(f+g)+b_3(a''-g)$
XX.		$\frac{\pi}{64}d^4=0.0491\,d^4$	$\frac{d}{2}$	$\frac{\pi}{32}d^3$	$\frac{\pi}{4}d^2$
XXI.		$\frac{\pi}{64}(d^4-d_1^4)=0.0491(d^4-d_1^4)$	$\frac{d}{2}$	$\frac{\pi}{32}\frac{d^4-d_1^4}{d}$	$\frac{\pi}{4}(d^2-d_1^2)$
XXII.		$\frac{\pi}{64}bh^3$	$\frac{h}{2}$	$\frac{\pi}{32}bh^2$	$\frac{bh\pi}{4}$
XXIII.		$0.110\,r^4$	$a'=0.5755\,r$ $a''=0.4244\,r$	$Z'=0.19\,r^3$ $Z''=0.26\,r^3$	$\frac{r^2\pi}{2}$
XXIV.		(Parabolic section.) $\frac{8}{175}bh^3=0.0457\,bh^3$	$a'=\frac{3}{5}h$ $a''=\frac{2}{5}h$	$Z'=\frac{4}{35}bh^2=0.114\,bh^2$ $Z''=\frac{8}{105}bh^2=0.076\,bh^2$	$\frac{2}{3}bh$
XXV.		$\frac{1}{12}\left[\frac{3\pi}{16}d^4+b(h^3-d^3)+b^3(h-d)\right]$	$\frac{h}{2}$	$\frac{1}{6h}\left(0.589\,d^4+b(h^3-d^3)+b^3(h-d)\right)$	$\frac{\pi}{4}d^2+2b(h-d)$
XXVI.		$\frac{1}{3}\left[b(a'^3-f^3)+b_1(f^3-g^3+k^3-l^3)+b_2(a''^3-k^3)\right]+\frac{\pi}{64}(d^4+16\,d^2i^2)$	Determined graphically or by experiment.	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b(a'-f)+b_1(f-g+k-l)+b_2(a''-k)+\frac{\pi}{4}d^2$
XXVII.		$\frac{1}{3}\left[b(a'^3-f^3)+b_1(f^3-g^3)+b_2(g^3-i^3+l^3-m^3)+b_3(k^3-l^3)+b_4(a''^3-k^3)\right]+\frac{\pi}{64}\left[(a'-f)^4+8(a'+f)(a'-f)^2\right]$	Determined graphically or by experiment.	$Z'=\frac{J}{a'}$ $Z''=\frac{J}{a''}$	$b(a-f)+b_1(f-g)+b_2(g-i+l-m)+b_3(k-l)+b_4(a-k)+\frac{\pi}{4}(a'-f)^2$

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§ 8.

VALUE OF THE QUANTITY S.

The limit of elasticity in a deflected beam, both on the tension and compression sides, will be reached when their respective strains S become equal to the modulus of resistance. It is therefore of great importance to select such a value for S, that the modulus of resistauce may not be reached on either side. These conditions will be met for sections which are symmetrical about two axes, by taking the lesser of the two values of S, as in the case of cast iron, the modulus for tension should be used.

In those sections in which a', differs from a'', the first thing to be determined is the position of the tension and compression sides. Let

a = the greatest distance from the neutral axis on the tension side.

a_1 = the greatest distance on the compressiou side,

T = the modulus for tension.

T_1 = the modulus for compression,

M = the statical moment of the bending force,

m = the coefficient of safety, so that for double, triple, safety, etc., $m = 2$, or 3,

Then we may take:

$$\text{When } \frac{a}{a_1} > \frac{T}{T_1} \text{ then } M = \frac{T}{m}\frac{J}{a}$$

$$\text{When } \frac{a}{a_1} < \frac{T}{T_1} \text{ then } M = \frac{T_1}{m}\frac{J}{a_1}$$

$$\text{When } \frac{a}{a_1} = \frac{T}{T_1} \text{ then } M = \frac{TJ}{m\,a} \text{ or } \frac{T_1}{m}\frac{J}{a_1}$$

Example. For Cast Iron $\frac{T}{T_1} = \frac{1}{2}$.

Taking the parabolic section No. XXIV. $a = \frac{2}{3}h$, $a_1 \frac{1}{3}h$.

This gives $\frac{a}{a_1} = \frac{2}{3}$, so that $\frac{a}{a_1} > \frac{T}{T_1}$, and for S, we have $\frac{T}{m}$ or. $\frac{10{,}650}{m}$ and $M = \frac{10{,}650}{m}\ 0.114\ bh^2$.

With wrought iron, in which $T = T_1$ no investigation is necessary.

§ 9.

SECTIONS OF UNIFORM RESISTANCE.

In order to use the material to the greatest advantage to resist bending strains, it is necessary to pay especial attention to its distribution, particularly in those portions which are furthest from the neutral axis. The best economy is attained in this matter when the section is shaped so that the strains on both tension and compression sides shall reach the elastic limit simultaneously.

For this purpose it is necessary to make

$$\frac{a}{a_1} = \frac{T}{T_1}$$

Sections which are thus proportioned are known as Sections of Uniform Resistance. Wrought iron sections which are symmetrical about two axes fulfi' these conditions, since $T = T_1$. For cast iron, when the bending strain is exerted constantly in one direction, it is best to make $a_1 = 2\,a$, for $T_1 = 2\,T$.

Taking these conditions into consideration, the following sections (Figs. 1, 2, 3) have been drawn, in which b and b_1 may have any desired proportion to each other:

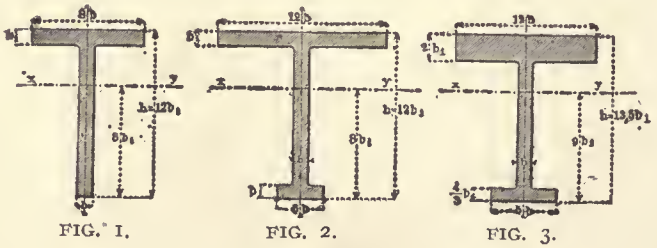


FIG. 1. FIG. 2. FIG. 3.

For these sections, when $b_1 = b$, we have:

J	$= 278b^4$	$440b^4$	$992b^4$
Z	$= 34.8b^3$	$55b^3$	$02.4b^3$
F	$= 19b^2$	$25b^2$	$40.8b^2$
ϕ	$= 1$	0.97	.04

The tension side is nearest to the neutral axis. Since the section modulus is determined from the value of $\frac{J}{a_1}$, S is always to be taken as $\frac{T_1}{m}$. F is the area of the section, and ϕ is the proportional economy of material, the cross section of Fig. 1 being taken as unity.

The value of ϕ may be determined thus:

$$\phi = \frac{\beta_1}{\beta}\left(\frac{a}{a_1}\right)^{\frac{2}{3}}\left(\frac{S}{S_1}\right)^{\frac{2}{3}} \qquad (8)$$

in which the sub-numbered letters belong to the required section and the un-numbered letters to the given section whose economy is to be taken as unity. In this equation $F = \beta b^2$, $Z = \alpha b^3$ and S is taken equal to S_1 except when the ratio of a to a_1 is not the same for both sections. It will be seen from an examinatiou of (8) that a slight variation from the exact proportions is not very material. When the bending force acts alternately in opposite directions, so that the strains are reversed, the sections which are symmetrical about two axes are the best for cast iron as well as for other materials, and the smaller value for S_1 should always be taken under such circumstances. If the force is constantly changiug its direction, so that the neutral axis passes through the centre of gravity, the most economical section is that of a circular ring, its resistance being greater than the cruciform or star-shaped sections, such as X., XII. and XXV., Table § 7, since there is in the former case a constant proportion of the sectiou and the greatest distance from the plane of the bending.

Example. A projecting beam of cast iron loaded as in No. I., § 6, carries a weight $P = 5{,}500$ pounds at its extremity, the length being 78.75 inches. Taking the cross section of the shape Fig. 2, we have by equation (4): $M = SZ$:

$$M = 5{,}500 \times 78.75, \qquad Z = 55b^3$$

To obtain double security we take $S = \frac{21{,}300}{2} = 10{,}650$. This gives:

$$5{,}500 \times 78.75 = 10{,}650 \times 55b^3$$

$$b = \sqrt[3]{\frac{5{,}500 \times 78.75}{10{,}650 \times 55}} = 0.9''$$

The sectional area will then be $25\ (0.9)^2 = 20.25$ sq. in., as determined by the constant given for the section Fig. 2. If the security be taken at $1\frac{1}{2}$,

$$S = \frac{21{,}300}{1.5} = 14{,}200.$$

This gives a lighter beam, and according to equation (8) its weight would be $\left(\frac{1.5}{2}\right)^{\frac{2}{3}} = 0.825$ of the preceding.

§ 10.

BODIES OF UNIFORM RESISTANCE TO BENDING.

A body is said to offer uuiform resistance to bending when its shape is so chosen that in all sections of its length the maximum strain, S, for tension or compression has the same value, and the general form of equation (4) for such bodies is

$$\frac{Ma}{J} = \text{Constant.} \qquad (9)$$

Bodies shaped so as to oppose a uniform resistance to bending are frequently used in machiue construction, approximations to the exact forms being often adopted, examples having already been shown in § 4. A variety of such shapes are given iu the following table.

The deflection in bodies of uniform resistance is of necessity greater than in prismatic bodies of the same strength. Iu many of the examples of the following table the deflection, f, is given, and in I. it is double, and in V. $1\frac{1}{2}$ times what it would be in prismatic bodies similarly loaded.

The elastic line for the following bodies, when exactly formed, is determined from the following equation:

$$\frac{d_2 y}{dx^2} = \frac{M_o}{EJ_o}\frac{a_o}{a_x} \qquad (10)$$

in which

M_o = the moment of the bending force for any given section,

J_o = its moment of inertia,

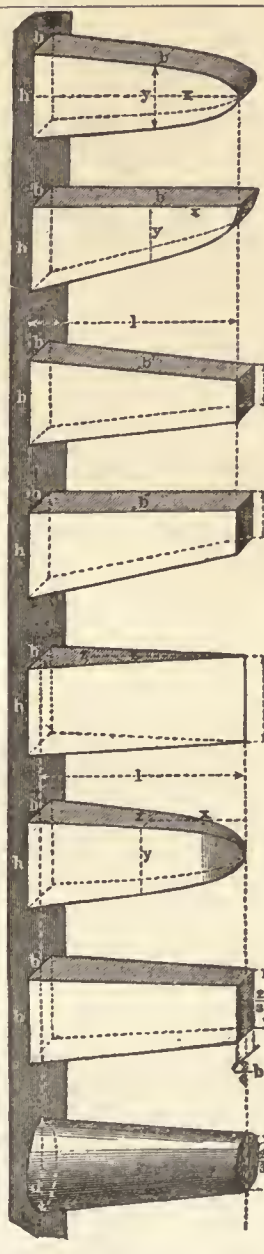
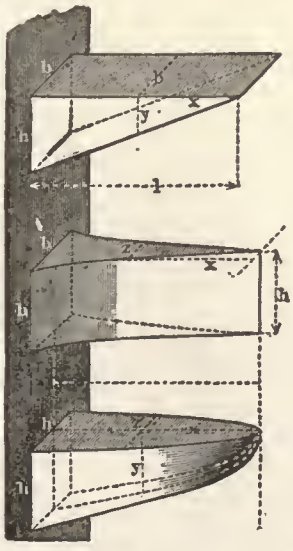
a_o = its greatest fibre distance,

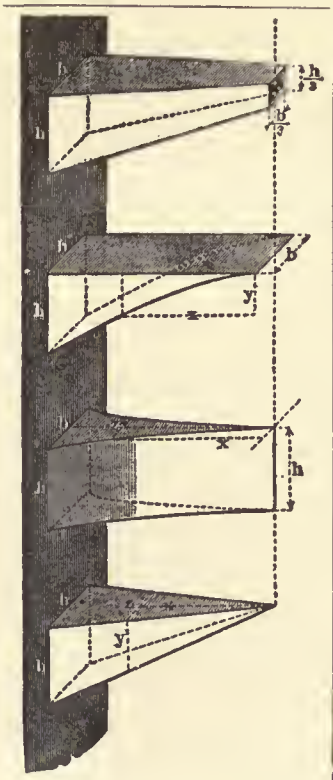
a_x = the greatest fibre distauce on the same side as a_o for any other section at a poiut x.

For the radius of curvature, ρ, of the elastic curve at a point whose co-ordinates are x, y, we have:

$$\rho = \frac{EJ_o}{M_o\,a_o}a_x \qquad (11)$$

which value is constant, and represents a circular arc when $a_x = a_o$; that is, when the section is of uniform height at all points, as in V., X., XIV.

No.	Form.	Application of Load.	Equation.	Sustaining Power.	Volume.	Remarks.
I.		The load is applied to the free end, as in Case I. of the table in § 6.	For rectangular section $\frac{z}{b}\frac{y^2}{h^2}=\frac{x}{l}$, In Cases I. and II., $z=b$, $\frac{y}{h}=\sqrt[3]{\frac{x}{l}}$ Parabolic truncated wedge.	$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{2}{3}\,bhl$	Deflection of the free end: $f=\frac{2}{3}\frac{P\,l^3}{J_0\,E}$; $J_0=\frac{b\,h^3}{12}$
II.				$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{2}{3}\,bhl$	The elastic curve is a parabola.
III.			Approximation to Form I., Truncated wedge.	$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{3}{4}\,bhl$	Weakest section at the base.
IV.			Approximation to Form II., Truncated wedge.	$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{3}{4}\,bhl$	In normal conditions the elastic curve bisects the angle of the wedge.
V.			$y=h$; $\frac{z}{b}=\frac{x}{l}$ Normal wedge.	$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{1}{2}\,bhl$	Elastic curve a circular arc. $f=\frac{1}{2}\frac{P\,l^3}{J_0\,E}$; $J_0=\frac{b\,h^3}{12}$
VI.			$\frac{z}{y}=\frac{b}{h}$; $\frac{y}{h}=\sqrt[3]{\frac{x}{l}}$ Cubic parabolic truncated pyramid.	$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{3}{5}\,bhl$	Equation $\frac{y}{h}=\sqrt[3]{\frac{x}{l}}$ is of greatest importance when all the sections are similar.
VII			Approximation to Form VI., Truncated Pyramid.	$P=\frac{S\,b\,h^2}{6\,l}$	$\frac{19}{27}\,bhl$	Weakest section at the base.
VIII.			Truncated cone; approximating to the form given by equation $\frac{y}{h}=\sqrt[3]{\frac{x}{l}}$	$P=\frac{S\,\pi}{32}\,\frac{d^3}{l}$	$\frac{19}{108}\,\pi\,l\,d^2$	To obtain the same strength as in Forms I. to VII., make $\frac{d}{h}=\sqrt[3]{\frac{16}{3\pi}\,\frac{b}{h}}$
IX.		Uniformly loaded, as in Case VII., in Table of § 6.	For rectangular section, $\frac{z}{b}\frac{y^2}{h^2}=\frac{x^2}{l^2}$ $z=b$; $\frac{y}{h}=\frac{x}{l}$ Wedge.	$P=\frac{S\,b\,h^2}{3\,l}$	$\frac{1}{2}\,bhl$	A very useful form when the sharp end is removed.
X.			$y=h$; $\frac{x}{l}=\sqrt[3]{\frac{z}{b}}$ Parabolic-sided wedge.	$P=\frac{S\,b\,h^2}{3\,l}$	$\frac{1}{3}\,bhl$	Elastic curve a circular arc. $f=\frac{1}{4}\frac{P\,l^3}{J_0\,E}$; $J_0=\frac{b\,h^3}{12}$
XI.			$\frac{z}{y}=\frac{b}{h}$ $\frac{y}{h}=\sqrt[3]{\frac{x^2}{l^2}}$ Truncated pyramid on semi-cubic parabola.	$P=\frac{S\,b\,h^2}{3\,l}$	$\frac{3}{7}\,bhl$	Applicable to stone brackets, etc., in architecture.

No.	Form.	Application of Load.	Equation.	Sustaining Power.	Volume.	Remarks.
XII.		Load uniformly diminishing, Case XI., Table § 6.	Approximation to Form XI.	$P=\frac{S\,b\,h^2}{3\,l}$	$\frac{13}{27}\,bhl$	Weakest section at the base.
XIII.			For rectangular section $\frac{z\,y^2}{b\,h^2}=\frac{x^3}{l^3}$; $z=b;\frac{x}{l}=\sqrt[3]{\frac{y^2}{h^2}}$ Wedge, on semi-cubic parabola.	$P=\frac{S\,b\,h^2}{2\,l}$	$\frac{2}{5}\,bhl$	Fundamental shape for architecture.
XIV.			$y=h$; $\frac{x}{l}=\sqrt[3]{\frac{z}{b}}$ Sides on cubic parabola.	$P=\frac{S\,b\,h^2}{2\,l}$	$\frac{1}{4}\,bhl$	Elastic curve a circular arc. $f=\frac{1}{6}\frac{P\,l^3}{J_0\,E}$; $J_0=\frac{b\,h^3}{12}$
XV.			$\frac{z}{y}=\frac{b}{h}$ $\frac{y}{h}=\frac{x}{l}$ Pyramid.	$P=\frac{S\,b\,h^2}{2\,l}$	$\frac{1}{3}\,bhl$	Value depends upon the simplicity of the form.

The preceding are only a few of the simpler forms which may be used, and it would be easy to multiply examples.

By altering the breadth, or height, the relations become more or less complicated, as the case may be.

For instance, in Case I., which is based on the parabola $\frac{z}{b}=\sqrt{\frac{x}{l}}$, it may be made the biquadratic parabola, $\frac{h}{y}=\sqrt[4]{\frac{x}{l}}$, etc. Combination sections give rise to new forms, and a great number of combinations may be made. Examples will be found in the chapter on axles and shafts.

The following discussion of springs will also give some instances of special forms, in which the neutral surface is irregular.

§ 11.

Resistance to Shearing in the Neutral Plane.

Since in a deflected beam there is on the tension side a continual tension, and on the compression side a continual compression of the respective fibres, it follows that the neutral plane is subjected to a shearing action, and this must not be neglected in determining the width of the beam.

The lower limit permissible is indeed a matter not likely to be reached, but at the same time it should be investigated.

Calling the least permissible width Z_o, and the mean force on either side of a given section R, then in order that the shearing strain at the neutral plane shall not exceed a value S_o, we must have :

$$z_o \geq \frac{R}{S_o}\,\frac{U}{2J} \qquad (14)$$

in which S_o should in no case exceed $\frac{4}{5}$ of the lesser modulus of resistance of the material under consideration (see § 5). J, as before, is the moment of inertia of the section, *i.e.*, the summation of the products of the elements of the section by the square of their distances from the neutral plane, while U is the statical moment of the section, *i.e.*, the summation of the products of the elements of the section by their distances from the neutral plane.

For the rectangular section No. I., Table (§ 7),

$$U=\frac{b\,h^2}{4},$$

and for the double T section, No. VIII.,

$$U=\frac{b\,h^2-(b-b_1)\,h_1^2}{4}$$

R is to be chosen according to the case under consideration, as, for example, in No. II. Table (§ 6) for all sections between B and C, it is equal to the reaction $\frac{P}{2}$, etc.

Equation (14) is not so much used to determine a value for z_o, as to find out in any case whether the breadth of the neutral plane has been taken too small. As a matter of fact, this is a question which very seldom arises in ordinary constructions, especially in machine construction.

If in (14) we give z_o any desired value, and make $S_o=\frac{4}{5}\,S$ we obtain

$$S=\frac{5}{4}\,\frac{R}{z_o}\,\frac{U}{2J}$$

and substituting this in equation (4) we get :

$$\frac{M}{R}=\frac{5}{8}\,\frac{U}{z_o\,a} \qquad (15)$$

$\frac{M}{R}$, is the lever arm of the force R; this we may call A. U: $z_o\,a$ contains one of the height dimensions of the section; hence equation (15) expresses a relation between two dimensions of the body under consideration. For a simple rectangular cross section, taking the value of U, given above, in which $z_o=b$, and $a=\frac{h}{2}$: $\frac{h}{A}=\frac{16}{5}$.

A greater value for h must not be taken if we do not wish the shearing strain to exceed the extension or compression in the tension and compression sides of the beam. These considerations are often of importance for the danger section, as, for example, in No. II., Table (§ 6) for the point B. In this case $A=\frac{l}{2}$ and we make $\frac{l}{h}\leq\frac{8}{5}$. This limit of height, however, is so great that it is very rarely reached in practice.

The most important application of this principle is found in the case of notched beams of wood, such as often occur in building construction. In such cases the resistance of the neutral plane is often very much reduced by the cutting of the notches, sometimes to one-half what it would be in the solid beam, and making a corresponding reduction in the value of $\frac{h}{l}$.

For the double T section we have :

$$\frac{h}{A}=\frac{16}{5\left[\frac{b}{b_1}-\left(\frac{b}{b_1}-1\right)\left(\frac{h_1}{h}\right)^2\right]}$$

If the brackets in the denominator contain an improper fraction the value of $\frac{h}{A}$ will approach the upper limit, but for

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all ordinary cases this value is very great. The nearest approach to this shearing action probably occurs in T beams where the flange joins the web, but examples are very rare.

§ 12.

BEAMS WITH A COMMON LOAD.

When two prismatic beams are united in the middle, and at that point subjected to a force P, the beams being supported at the ends, they will both be deflected, and the sum of their reactions P', and P'', enter into the support of P.

The double reactions are found from the formula in Table (§ 6), No. II., column 2, as follows:

$$\frac{P'}{P''} = \frac{J' E'}{J'' E''} \frac{l''^3}{l'^3},$$

and since

$$P' = 4\frac{S' J'}{a' l'} \text{ and } P'' = 4\frac{S'' J''}{a'' l''}$$

we get

$$\frac{S'}{S''} = \frac{E'}{E''} \frac{a'}{a''} \left(\frac{l''}{l'}\right)^2 \qquad (16)$$

If the two beams are of the same material ($E' = E''$), to obtain equal security, the product $\frac{a'}{a''}\left(\frac{l'}{l''}\right)^2 = 1$.

If the beams are not the same length then $a' = a''$, *i.e.*, the heights must be the same unless the breadths are equal to each other.

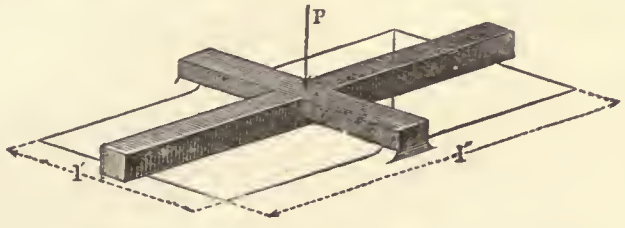


FIG 4.

Example. A cast iron support shaped like a cross. Fig. 4, must support a weight, P, at the intersection. The lengths of the arms are to each other as 3 : 2. In order to obtain equal security in the four arms, which are of prismatic shape. we have from (16)

$$\frac{a'}{a''} = \left(\frac{l''}{l'}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Hence the cross section of the short arms must be to that of the long arms as 4 : 9, and if the arms were of the same section the supporting power of the short arms would be to that of the long arms as 9 : 4.

It also follows from the preceding, that rectangular sheet metal plates carrying a uniformly distributed load are stronger parallel to their shorter axis than parallel to the longer axis.

For given loads and materials formula (16) may be used to govern the choice of dimensions and the relations of length to breadth.

For beams of cast or wrought iron resting upon each other, a suitable proportion may be secured by taking the sum of their several supporting powers as the supporting power of the combination. This is often a matter for consideration in strengthening existing structures.

§ 13.

RESISTANCE TO TORSION.

Resisting Power and Angle of Rotation.

A prismatic body which is subjected to the action of a force couple tending to rotate it about its geometric axis, opposes to such action its Resistance to Torsion. Under these conditions the elements in a normal section are subjected to a shearing strain, and until the elastic limit is reached there exists an equilibrium between the external rotating forces on the one hand and the strain moments of the various elements of the section on the other hand; both being taken with regard to the polar axis of the centre of gravity of the section, *i. e.*, the axis passing through the centre of gravity of the section and at right angles to it. Resistance to torsion may properly be considered a higher species of resistance to shearing, to which it bears the same relation that resistance to bending holds to tensile and compressive strength.

Let:

M = the statical moment for any giveu section of the rotating force,

J_p = the polar moment of inertia of the section, *i. e.*, its moment of inertia taken with regard to its polar axis (see § 14),

a = the distance of the farthest elements of the section from the centre of gravity,

S = the shearing strain in the elements at a distance a, then

$$M = \frac{S J_p}{a} \qquad (17)$$

If the body is of a uniform section, then $\frac{J_p}{a}$ is constant. Now if A be the lever arm of the rotating force P, for a moment M, the weakest or danger section will be that for which M is a maximum, and for it we have

$$P = \frac{S}{A_m} \frac{J_p}{a} \qquad (18)$$

in which A_m, is that value of A, which gives M, a maximum.

The limit of elasticity is reached, as in the case of shearing action, when $S = \frac{4}{5}$ of the lesser of the moduli of resistance for tension or compression (see § 5). This is plainly visible by a comparison between the action of bending and twisting.

The relative rotation which two sections of a prism at a given distance apart make with each other is called the angle of torsion. It is represented by the letter ϑ; and for two sections separated by a distance x, we have in general terms:

$$\frac{d\vartheta}{dx} = \frac{M}{J_p G} \qquad (19)$$

in which G is the modulus of torsion for the material used, and is equal to $\frac{2}{5}$ of the modulus of elasticity E.

In the following table will be found the values for:

The moment M, at a given point x, of the prism,
The force P, according to formula (18), and
The torsional deflectiou in terms of angular measure, or in other words, the angle of torsion ϑ.

These quantities are given for a variety of cases, as shown in the cuts, and from them total moment, PR, of the twisting force may be determined. In case IV., S is the point of application at which the collected forces, with a lever arm R, would act, if concentrated to produce an equivalent result to the sum of the separate efforts, l_o being the distance of the point S from the immovable end of the prism.

Questions relating to torsion are of varying importance in machine construction, and come especially into consideration in calculations relating to springs. Case IV. illustrates the conditions which occur in determination of mill shafting. Cases V. and VI. occur in machine framing.

§ 14.

POLAR MOMENT OF INERTIA AND SECTION MODULUS.

The polar moment of inertia, J_p, is easily determined, since we have

$$J_p = J_1 + J_2 \qquad (20)$$

in which J_1 and J_2 are the equatorial moments of inertia taken with regard to two axes at right angles to each other, and whose values are given for a variety of sections in the table of (§ 7). From this may be obtained the polar section modulus $\frac{J_p}{a} = Z_p$ for use in the preceding cases. An exception must be made for those sections in which we have not $J_1 = J_2$, as in cases III., VII., XII., XX., XXV., etc., § 7. For these it will be necessary to make a special correction in the values of J_p and $\frac{J_p}{a} = Z_p$, to provide for the warped surface which is assumed by the section under a heavy torsioual strain.

For a rectangle, which is a section of frequent occurrence in machine design, the corrected value of J_p and $Z_p = \frac{J_p}{a}$ is given in the following table, while for the circle and the square no correctious are necessary for the values obtained from equation 20.

Example. A cylindrical prism of wrought iron is subjected to a torsional strain applied as in case I. of the following table. The force $P = 1{,}000$ lbs., and the lever arm $R = 24''$; while the bar is $4''$ in diameter and $48''$ long.

These quantities give for S, the strain at the circumference

$$S = \frac{a}{J_p} PR = \frac{16}{\pi} \frac{PR}{d^3}$$

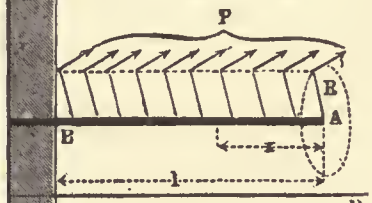
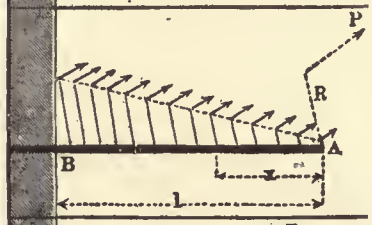
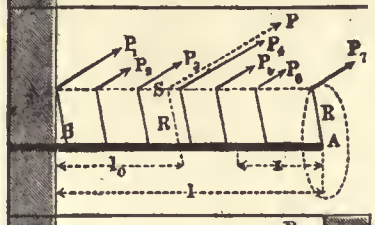
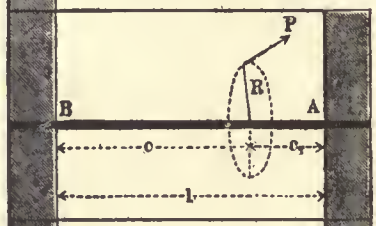
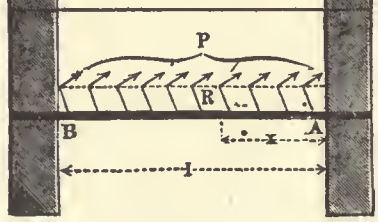
$$= \frac{16}{3.1416} \cdot \frac{1{,}000 \times 24}{64} = 1{,}909 \text{ lbs.}$$

and to get the angle of torsion we substitute this value in the formula:

$$\vartheta = \frac{S}{G} \cdot \frac{l}{a}$$

$$= \frac{1{,}909}{11{,}360{,}000} \cdot \frac{48}{2} = 0.004$$

which corresponds to an angle of about 0° 14′.

No.	Application.	Moment M.	Twisting Force P.	Angle of Torsion ϑ.	Remarks.
I.		$M = P\ R$ for all points between A, and B.	$P = \frac{S J_p}{a R}$	$\vartheta = \frac{P R l}{J_p G} = \frac{S}{G} \frac{l}{a}$	All sections between A, and B, are equally strong.
II.		$M = P R \frac{x}{l}$	$P = \frac{S J_p}{a R}$	$\vartheta = \frac{1}{2} \frac{P R l}{J_p G} = \frac{1}{2} \frac{S}{G} \frac{l}{a}$	Weakest section at B.
III.		$M = P R \frac{x^2}{l^2}$ $P R$ = the collected moment of all the twisting forces.	$P = \frac{S J_p}{a R}$	$\vartheta = \frac{1}{3} \frac{P R l}{J_p G} = \frac{1}{3} \frac{S}{G} \frac{l}{a}$	Twisting forces decrease uniformly from B, to A. Weakest section at B.
IV.		M = the sum of the moments within x.	$P = \frac{S J_p}{a R}$	$\vartheta = \frac{P R l_0}{J_p G} = \frac{S}{G} \frac{l_0}{a}$	General form of Cases I., II. and III. Weakest section at B. The value of ϑ in III. will be reached in IV., when $l_0 = \frac{l}{3}$.
V.		In the portion c: $M = P R \frac{c_1}{l}$ In the portion c_1: $M = P R \frac{c}{l}$	When $c_1 < c$ $P = \frac{S J_p}{a R} \frac{l}{c}$	$\vartheta = \frac{P R}{J_p G} \frac{c c_1}{l} = \frac{S}{G} \frac{c_1}{l}$	The shorter portion c_1 is the weaker.
VI.		$M = P R \left(\frac{1}{2} - \frac{x}{l}\right)$	$P = 2 \frac{S J_p}{a R}$	$\vartheta = \frac{1}{8} \frac{P R l}{J_p G} = \frac{1}{8} \frac{S}{G} \frac{l}{a}$	Weakest points at A_1 and B.

If we wish to reduce d, so that S shall be equal to one-half the modulus of resistance for torsion, *i. e.*, $= \frac{1}{2} \cdot \frac{4}{5} \cdot 21{,}300 = 8{,}520$ lbs., we make

$$d = \sqrt[3]{\frac{16}{\pi S} \cdot PR} = \sqrt[3]{\frac{16 \times 1{,}000 \times 24}{3.1416 \times 8{,}520}} = 2.42''$$

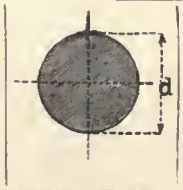
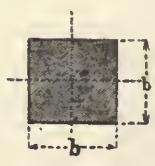
or about 2½ inches.

In this case the angle of torsion would be

$$\vartheta = \frac{8{,}520}{11{,}360{,}000} \cdot \frac{48}{1.25} = 0.0288''$$

which gives an angle of about 1° 39″.

SECTION TABLE.

No.	Section.	Polar Moment of Inertia J_p.	Polar Section Modulus, $Z_p = \frac{J_p}{a}$
I.		$\frac{\pi}{32} d^4$	$\frac{\pi}{16} a^3$
II.		$\frac{b^4}{6}$	$\frac{b^3}{3\sqrt{2}}$
III.		$\frac{1}{3} \frac{b^3 h^3}{b^2 + h^2}$	$\frac{b^2 h^2}{3\sqrt{b^2 + h^2}}$ Approximately $\frac{b^2 h^2}{3(0.4b + 0.96h)}$

§ 15.

BODIES OF UNIFORM RESISTANCE TO TORSION.

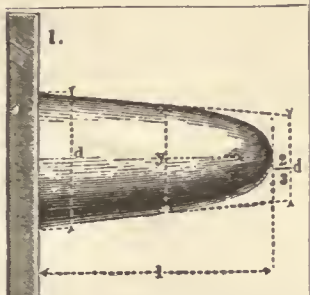
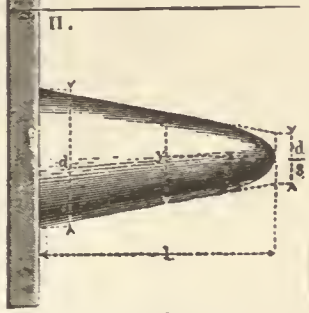
In order to make a body of uniform resistance to torsion it is necessary to take such sectional areas at various points as shall make in equation (17), S a constant, and also to take

$$\frac{M_a}{J_p} = \text{constant.} \qquad (21)$$

In case I. of the table in § 13, for all sections $M = PR$, and hence in this case the body should be prismatic in shape. For cases II. and III. the necessary formulæ are given in the following table. For such bodies the angle of torsion is greater than for those of prismatic shape. The angle for each is given in the table, and is derived from the following:

$$\frac{d\vartheta}{dx} = \frac{M}{J_x G} \qquad (22)$$

in which J_x is the polar moment of inertia for the section taken at the point x.

Form.	Application.	Equation and Angle of Torsion.
I. 	Case II., § 13.	Circular section $\frac{y}{d} = \sqrt[3]{\frac{x}{l}}$; $PR = S\frac{\pi}{16}d^3$; $\vartheta = 3\frac{S}{G}\cdot\frac{l}{a}$ Approximate form = a truncated cone, with extremity $= \frac{2}{3}d$.
II. 	Case III., § 13.	Circular section $\frac{y}{d} = \sqrt[3]{\frac{x^2}{l^2}}$; $PR = S\frac{\pi}{16}d^3$; $\vartheta = 6\frac{S}{G}\cdot\frac{l}{d}$ Approximate form = a truncated cone, with extremity $= \frac{d}{3}$

For other bodies of uniform resistance to torsion, see Torsion Springs (§ 20).

§ 16.

RESISTANCE TO BUCKLING.

Combined Bending and Compressive Strains.

A prismatic body is subjected to combined bending and compressive stresses, to which it yields by buckling, when its diameter is comparatively small in comparison with its length. Under these conditions a compression applied in the direction of the axis is opposed, both by the resistance of the body to compression and also to bending, with this difference, that in this case the lever arm of the bending force is not the abscissa, but the ordinate of the elastic curve. From this it follows that (neglecting some very small elements) any compressive force P, capable of producing a bending, would do so even up to the breaking point, provided that the laws of perfect elasticity held good until rupture occurred. This would only be true if the theoretical resistance and the breaking load were the same, and the elasticity of the prism held them in equilibrium until the final yielding of the point of application of the force P occurred.

In the following table (p. 14) the principal formulæ are given for a number of the most important applications of these buckling stresses. In the table

E = the modulus of elasticity of the material assumed to be of prismatic shape;

J = the least moment of inertia of its section taken with reference to a line of gravity, for example, in a rectangle of which the greater side is b and the lesser side h, according to § 7, $= \frac{h\,b^3}{12}$.

It may be remarked that the valuable experimental researches of Hodgkinson, as given in his rules, show a somewhat smaller breaking load than the formulæ in the table; this, however, does not detract from the value of the latter, since these are only strictly correct for perfectly elastic bodies, but at the same time they will be found practically reliable if the force P is not permitted to exceed a definite proportion of the breaking load. Different materials demand a different factor of safety. For cast iron, ¼ to ⅙ the breaking load, or less, and for wrought iron the same, and for wood ⅛ to $\frac{1}{10}$, or $\frac{1}{12}$, should be the limit. These inequalities often arise from the fact that it is not always easy to determine which of the applications of the table really meets the case in question. In order to determine the actual security from rupture, it is often necessary to make a comparison with other existing strains. From this standpoint the ratios of diameter to length in the following table have been determined in order that the resistance to compression and to buckling may be as nearly alike as possible.

In Hodgkinson's experiments it was shown that columns standing upon flat bases were nearly as strong as those which were firmly fixed at one end.

In the third section many applications of these formulæ will be given.

§ 17.

COLUMNS OF UNIFORM RESISTANCE.

Columns subjected to combined compressive and buckling stresses are said to be of uniform resistance when its various sections are so proportioned that a very small degree of buckling will produce the same strain in each section.

For case II. of the preceding table, when the section is circular, the following formulæ (by Redtenbacher) may be used:

$$\frac{x}{\left(\frac{l}{2}\right)} = \frac{2}{\pi}\left(\arcsin\frac{y}{h} - \frac{y}{h}\sqrt{1-\left(\frac{y}{h}\right)^2}\right)$$

This may be separated into a double equation by making:

$$\left.\begin{aligned} \frac{y}{h} &= \sin\phi \\ \text{which gives:}\quad \frac{x}{\left(\frac{l}{2}\right)} &= \frac{1}{\pi}(2\phi - \sin 2\phi) \end{aligned}\right\} \qquad (23)$$

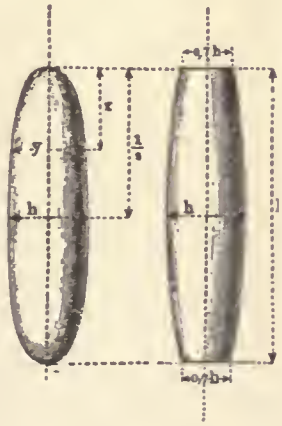


FIG. 5.

From these equations a limiting curve may readily be found, whose abscissas are those of a cycloid, and whose ordinates are those of a sinoide, and which may be called a cycloidal sinoide. A method of drawing this curve is given hereafter, in the discussion of connecting rods, and the approximate shape is also shown in the second form of Fig. 5, in which the outline is a circular curve, or at least a line of very slight curvature. The strength of these columns may be taken as ¾ that of a cylindrical column of a diameter h and length l.

§ 18.

COMPOUND STRESSES.

It very often occurs that a variety of forces act upon a body at the same time and in a variety of ways, so that, for instance, a section is subjected at the same time to tension and bending, or to torsion and bending, etc.

The resistance and the maximum strains are then to be determined in a different manner, according to circumstances.

In the following table (p. 15) are given the principal formulæ for some of the more commonly occurring cases.

Let:

S = the greatest strain at the weakest section;
Z = the section modulus at the weakest section, which latter is indicated at B in the figures;
F = the area of the section;
J = its moment of inertia (§ 7);
M_b = a beuding moment;
M_d = a twisting moment;
M_i = an ideal moment, so that
$(M_b)_i$ = an ideal bending moment, and
$(M_d)_i$ = an ideal twisting moment.

An examination of these formulæ will show that in many cases the combination of strains is a matter of importance.

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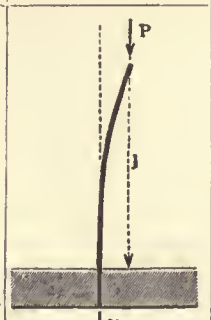
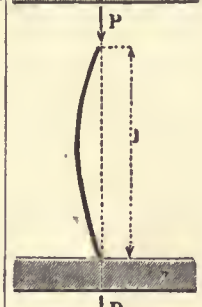
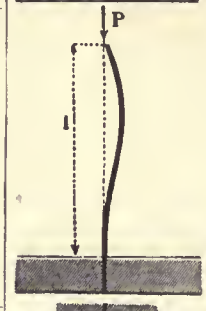
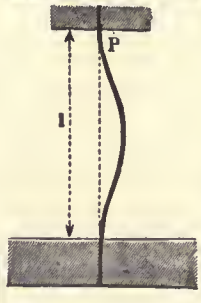
BUCKLING STRAINS.

No.	Application.	Theoretical Supporting Power.	Remarks.	The column is to be considered under compression, when: For circular section $\frac{l}{d}$ is less than	For rectangular section $\frac{l}{b}$ (b = lesser side).	Material.
I.		$P = \frac{\pi^2}{4} \frac{JE}{l^2}$	Freely loaded post. The lower end fixed. Weakest section at the point of attachment.	5	5¾	Cast iron.
					14	Wrought iron.
				6	8	Wood.
II.		$P = \pi^2 \frac{JE}{l^2}$	Post free at both ends, but kept in the line of the axis. Weakest section in the middle.	10	11½	Cast iron
				24	28	Wrought iron.
				11½	13½	Wood.
III.		$P = 2\pi^2 \frac{JE}{l^2}$	Post fixed below, and held in the line of the axis above.	14	16	Cast iron.
				33	38	Wrought iron.
				16	19	Wood.
IV.		$P = 4\pi^2 \frac{JE}{l^2}$	Post fixed at both ends in the line of the axis. Weak points at the ends and in the middle.	20	23	Cast iron.
				48	56	Wrought iron.
				23	27	Wood.

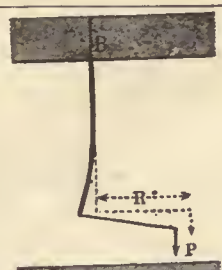
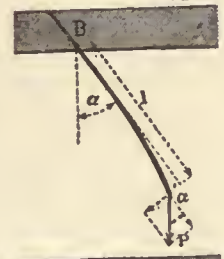
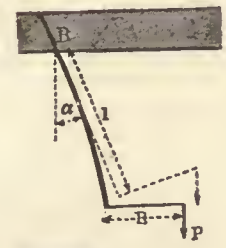
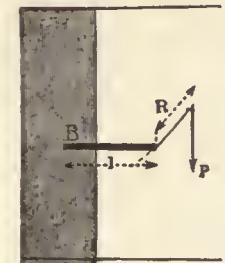
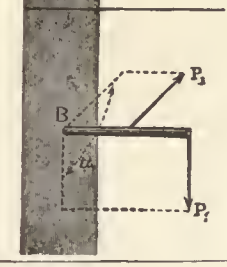
For example, in case I., if $R = \frac{h}{2}$, *i. e.*, if the load is hung at the edge of the section, $P = \frac{Sbh}{4}$, and hence is only one-fourth as great as it would be if applied centrally. If the section is circular (d), we have $P = \frac{S\frac{\pi}{4}d^2}{1 + 8\frac{R}{d}}$, making $R = \frac{d}{2}$; $P = \frac{S}{5}\frac{\pi}{4}d^2$, and the sustaining power is still less than with a rectangular section. Case III. is derived from I. and II., and may be changed into either by making a, or $R = o$.

The so-called ideal moments are especially useful in these calculations. It will be noticed that in the case of elliptical and rectangular sections, h is taken in the plane of beuding. These dimensions being known in advance, since the choice of profile is frequently permitted, it is possible by the use of the ideal moments to consider the question of combined strains, since the quantity in the parenthesis to the right is the expression for the lever arm of the force P for each case. This can generally be readily determined graphically, and so determined just like any case of ordinary bending.

For example, in case II., if $a = 45°$, we have $cos\ a = sin\ a = 0.707$ for the value of h, and the section at B is to be calculated as if acted upon by a force P, with a lever arm $0.707\ l$ (the projection of l on the plane of attachment) $+ 0.707\ \frac{h}{6}$.

In case I., making $R = o$, for a circular section $(M_b)_i = P\frac{d}{8}$, and substituting $S\frac{\pi}{32}d^3$, we get $P = S\frac{\pi}{4}d^2$, as we should since the stress is now purely tensile. In this case $\frac{d}{8}$ is the lever arm which, if acting with a bending force P, would produce a strain of the same amount as that in the line of the axis. This is only rigidly exact when the shearing action which occurs in bending is neglected. Many useful applications of cases IV. and V. are found in discussing axles aud shafts.

COMPOUND STRAINS.

No.	Application.	Sustaining Power.	Ideal Moments.
I.		$P = \frac{SF}{1 + R\frac{F}{Z}}$ for rectangular section, (bh) $P = \frac{Sbh}{1 + 6\frac{R}{h}}$	Ideal bending moment for stress S: $(M_b)_i = P\left(R + \frac{Z}{F}\right)$. For circular section (d): $(M_b)_i = P\left(R + \frac{d}{8}\right)$ | For elliptical section (bh): $(M_b)_i = P\left(R + \frac{h}{8}\right)$ | For rectangular section (bh): $(M_b)_i = P\left(R + \frac{h}{6}\right)$
II.		$P = \frac{SF}{\cos a + \frac{F}{Z} l \sin a}$ for rectangular section, (bh) $P = \frac{Sbh}{\cos a + 6\frac{l}{h}\sin a}$	Ideal bending moment for stress S: $(M_b) = P\left(l \sin a + \frac{Z}{F}\cos a\right)$. For circular section (d): $(M_b)_i = P\left(l \sin a + \frac{d}{8}\cos a\right)$ | For elliptical section (bh): $(M_b)_i = P\left(l \sin a + \frac{h}{8}\cos a\right)$ | For rectangular section (bh): $(M_b)_i = P\left(l \sin a + \frac{h}{6}\cos a\right)$
III.		$P = \frac{SF}{\cos a + \frac{F}{Z}(l \sin a + R \cos a)}$ for rectangular section, (bh) $P = \frac{Sbh}{\cos a + 6\frac{l}{h}(\sin a + \frac{R}{l}\cos a)}$	Ideal bending moment: $(M_b)_i = P\left(R\cos a + l \sin a + \frac{Z}{F}\cos a\right)$. For circular section (d): $(M_b)_i = P\left(R\cos a + l\sin a + \frac{d}{8}\cos a\right)$ | For elliptical section (bh): $(M_b)_i = P\left(R\cos a + l\sin a + \frac{h}{8}\cos a\right)$ | For rectangular section (bh): $(M_b)_i = P\left(R\cos a + l\sin a + \frac{h}{6}\cos a\right)$
IV.		$P = \frac{SZ}{\frac{3}{8}l + \frac{5}{8}\sqrt{l^2 + R^2}}$ Pl is a bending moment Mb, PR is a twisting moment Md.	Ideal bending moment for the stress S: $(M_b)_i = \frac{3}{8}M_b + \frac{5}{8}\sqrt{M_b^2 + M_d^2}$ Ideal twisting moment: $(M_d)_i = \frac{3}{5}M_b + \sqrt{M_b^2 + M_d^2}$
V.		$P = \frac{SZ}{\sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos a}}$ in which M_1 is the bending moment of P_1, M_2 that of P_2.	Ideal bending moment for the stress S: $(M_b)_i = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos a}$

In cases IV. and V. it is supposed that the section is arranged in four symmetrical portions about two lines of gravity, perpendicular to each other.

§ 19.

Resistance of Walls of Vessels.

Boilers, Cylinders, Etc.

The following table will serve to determine the resistance of the walls of cylindrical vessels subjected to pressure for the cases which usually occur in practice.

The theory of resistance under these conditions is not fully settled, especially in the case of comparatively thin shells subjected to external pressure, for which the corresponding formulæ do not give satisfactory results.

In the following cases, let:

p = the unbalanced pressure upon the walls of the vessel;
S = the maximum stress for the material used;
E = the modulus of elasticity of the material:
r = the radius of the vessel;
δ = the thickness of the walls.

Although only approximate, the formulæ for cases I. and II. hold good up to the limit of rupture.

Examples: 1. Given a wrought iron cylinder, 40 inches diameter, ⅜" thick, with a stress upon the material of 11,500 lbs. Under conditions of case I., the internal pressure permissible would be

$$P = 11{,}500\left(\sqrt{\frac{20.75}{20}} - 1\right)$$

$$= 11{,}500 \times 0.0185 = 212 \text{ lbs.}$$

2. A spherical vessel of the diameter and thickness given above, according to case II., would have a safe resistance

$$P = 23{,}000\,\frac{0.375}{20} = 431 \text{ lbs.}$$

3. A plate held as in case IV., 40" dia., ~~⅜" thick~~, and a pressure of 212 lbs., with a maximum stress $S = 11{,}500$, would have a thickness

$$\delta = 20\sqrt{\frac{2}{3}}\sqrt{\frac{212}{11{,}500}} = 20 \times 0.816 \times 0.136 = 2.22''$$

or about 2¼ inches.

The deflection f, which a circular plate gives under a force p, may be determined, according to Grashof, by the formula for case III.:

$$\frac{f}{\delta} = \frac{5}{6}\left(\frac{r}{\delta}\right)^4 \frac{p}{E} \qquad (24)$$

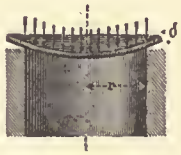
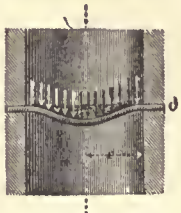
and for case IV.:

$$\frac{f}{\delta} = \frac{1}{6}\left(\frac{r}{\delta}\right)^4 \frac{p}{E} \qquad (25)$$

Example: The plate of Example 3 preceding, with a value of $E = 28{,}400{,}000$, would have a deflection of

$$f = \frac{2.25}{6} \cdot \left(\frac{20}{2.25}\right)^4 \cdot \frac{212}{28{,}400{,}000} = 0.0175''$$

RESISTANCE TO PRESSURE.

No.	Application.	Pressure p.	Thickness δ.
I. Cylinder.		$p = S\left(\sqrt{1+\frac{2\delta}{r}}-1\right)$	$\frac{\delta}{r} = \frac{p}{S}\left(1+\frac{p}{S}\right)$
II. Sphere.		$p = 2S\frac{\delta}{r}$	$\frac{\delta}{r} = \frac{p}{2S}$
III. Round Plate.		$p = S\left(\frac{\delta}{r}\right)^2$	$\frac{\delta}{r} = \sqrt{\frac{p}{S}}$
IV. Round Plate.		$p = \frac{3}{2}S\left(\frac{\delta}{r}\right)^2$	$\frac{\delta}{r} = \sqrt{\frac{2}{3}}\sqrt{\frac{p}{S}}$

For vessels whose walls are required to be made very thick, as in the case of the cylinders and pumps of hydraulic presses or for cannon, etc., the preceding formulæ do not apply. Under these conditions the relative radial distances of the various portions of the thickuess of the metal vary greatly, and their relation has an important influence upon the resistance. It is the relation which exists between the various stresses at different points which governs the various formulæ for the thickness of the walls, which are given below. Brix calculates the stresses at different points on the radius upon the supposition that the internal diameter is not altered by the pressure; Barlow admits such an alteration by pressure that the area of the annular section of metal is not reduced; Lamé makes neither of these assumptions, but calculates very closely the changes in the various stresses which are caused by the internal pressure at each point, and iu this way has obtained the most reliable data as to the real behavior of the particles of the material, according to the modern theory. The results of the three theories are given in the following table:

Quantities.		Brix.	Barlow.	Lamé.
Cylinder.	$p =$	$S \log \text{nat } e^{\frac{p}{S}} - 1$	$\frac{S}{1+\frac{r}{\delta}\frac{p}{S-p}}$	$S\frac{(r+\delta)^2-r^2}{(r+\delta)^2+r^2}$
	$\frac{\delta}{r} =$	$e^{\frac{p}{S}} - 1$		$\sqrt{\frac{S+p}{S-p}} - 1$
Sphere.	$p =$	$2S\frac{\delta}{r}$	$\frac{2S}{1+\frac{r}{\delta}\frac{p}{2S-p}}$	$2S\frac{(r+\delta)^3-r^3}{(r+\delta)^3+2r^3}$
	$\frac{\delta}{r} =$	$\frac{1}{2}\frac{p}{S}$		$\sqrt[3]{\frac{2(S+p)}{2S-p}} - 1$

For the stress S' iu an annular ring lying between the radii r' aud r, Lamé gives

$$S' = \frac{S}{2}\left[1+\left(\frac{r}{r'}\right)^2\right] - \frac{p}{2}\left[1-\left(\frac{r}{r'}\right)^2\right]$$

If r' is the external radius of the vessel, so that $r' = r + \delta$, we have:

$$S' = \frac{S}{2}\left[\frac{\left(1+\frac{\delta}{r}\right)^2+1}{\left(1+\frac{\delta}{r}\right)^2}\right] - \frac{p}{2}\frac{\left(1+\frac{\delta}{r}\right)^2-1}{\left(1+\frac{\delta}{r}\right)^2} \quad (26)$$

or if we put $\left(1+\frac{\delta}{r}\right) = \mu$

$$S' = \frac{S}{2}\frac{\mu^2+1}{\mu^2} - \frac{p}{2}\frac{\mu^2-1}{\mu^2}$$

Example: If $\delta = r$, that is, $\mu = 2$, $S' = \frac{5}{8}S - \frac{3}{8}p$, and as in the preceding formulæ, taking $p = \frac{3}{5}S$, we have

$$S' = \frac{5}{8}S - \frac{9}{40}S = \frac{2}{5}S.$$

This shows that the material is not used in an economical manner in vessels with excessively thick walls.

All three theories admit that the inner portion of the wall is strained the most, and hence it is for the inner wall that S should be chosen. The formulæ of Lamé, as well as those of Barlow, show that beyoud certain limits an increase in the thickness is not atteuded with any increase of strength. With a giveu resisting power S, this limit will be reached when $p = S$; the theoretical resistance will be attaiued when $p =$ the modulus of resistance of the material. At this point the internal pressure begius to stretch the inner fibres of the walls, and any increase in strain will cause rupture. The theoretical limit in this case is reached when $p = T$, which is

For Cast Iron = 10,650 lbs.
" Wrought Iron = 21,300 "
" Cast Steel = 36,000 "

Lack of homogeneity in the material may cause the danger pressure to be reached far within these limits, the material breaking without previously stretching.

Since stresses exceeding 36,000 pouuds are reached in guns of large calibre, it is evident that ordinary bronze is unsuitable for such couditions, aud even homogeneous steel is often unequal to the pressure. The erosion of the chamber in the case of ordinary bronze cannon also acts to weaken the inner riug of material, and must be considered as a chemical deteriorating action.

Various methods have been devised for strengthening guns by giving the various layers different tensions. Of these methods the principal is that of hooping. The priucipal result of this construction is to produce a compression in the iuner layer. The pressure of the gases of explosion must then first overcome this compression and restore the normal condition before it can produce any extension of the fibres, and as a result a much higher degree of resistance is secured than when the metal is left iu its normal condition.

The calculations of the resistance of hooped guns offer many difficulties. If we have not only the iuner pressure, but also the outer pressure, p', to consider, we may take the followiug formula, after Lamé:

$$\left(1+\frac{\delta}{r}\right)^2 = \frac{S+p}{S-p+2p'} \quad (27)$$

Putting $1+\frac{\delta}{r} = \mu$, as before, and solving with regard to p, we have:

$$p = S\frac{\mu^2-1}{\mu^2+1} + 2p'\frac{\mu^2}{\mu^2+1} \quad (28)$$

in which S will become less with regard to p, the greater p' becomes.

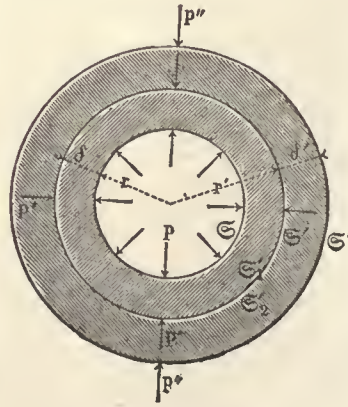


FIG. 6.

In the case of hooped guns p' is not constant aud iuvariable, but depends upon the effect which the iuternal pressure p has through the walls upon the hoops.

Referring to Fig. 6, let it first be considered that uuder normal conditious the inuer riug is under no strain, that is, $p = o$, and also $S'_1 = S''_2 = o$.

Now when the inner pressure p becomes sensible while the external pressure $p'' = o$, or at least may be neglected, then the layer at r' will become extended, and the tensious will be $S_1' = S_2'$. The stress S_2' in the inner side of the hoop reacts with a pres-

sure p', and substituting this in Lamé's formula, making $1+\frac{\delta'}{r'}=\mu'$, will give

$$p'=S_2'\frac{\mu'^2-1}{\mu'^2+1} \qquad (29)$$

Making $S'=S_2'=S_1'$ and substitutiug this value of p' in (28), gives

$$p=S\frac{\mu^2-1}{\mu^2+1}+2S'\frac{\mu'^2-1}{\mu'^2+1}\frac{\mu^2}{\mu^2+1}$$

According to (26), S' is dependent upon p and S, and by substituting and transforming, we get

$$p=S\frac{\mu^2\mu'^2-1}{\mu^2\mu'^2+1}=S\frac{\left(1+\frac{r}{\delta}\right)^2\left(1+\frac{r'}{\delta'}\right)^2-1}{\left(1+\frac{r}{\delta}\right)^2\left(1+\frac{r'}{\delta'}\right)^2+1} \qquad (30)$$

In this case the stress S upon the inner ring is always greater than p, but the ratio approaches much nearer to unity thau before, as the following table shows:

When		We have		And also			
$\frac{\delta}{r}$	$\frac{\delta'}{r'}$	μ	μ'	$\frac{p}{S}$	$\frac{S}{p}$	$\frac{S'}{p}$	$\frac{S'}{S}$
1	0	2	1	0.600	1.667	0.667	0.400
1	0.5	2	1.5	0.800	1.250	0.406	0.325
2	1	3	2	0.905	1.057	0.143	0.135

It will be seen that the mere hooping of a gun with a ring of the same material as the inner tube adds very materially to its strength. If, however, the ring is forced on in any manner so as to produce an initial strain p' upon the tube, a still greater advantage will be the result.

If we insert the value of p' from (29) into formula (28), we have

$$p=S\frac{\mu^2-1}{\mu^2+1}+2S_2'\frac{\mu'^2-1}{\mu'^2+1}\frac{\mu^2}{\mu^2+1} \qquad (31)$$

In this formula S_2' is partially a function of p, and also depends partly upou the extent to which the tube reacts. This latter condition exerts a most importaut influence upon the strength, as we shall see hereafter.

If we assume that the hoop is under such an initial strain that, for the maximum value of p, the value of $S_2'=S$ (which is doubtless the most desirable condition), we shall theu obtain from (31)

$$\frac{p}{S}=\frac{\mu^2-1}{\mu^2+1}+2\frac{\mu'^2-1}{\mu'^2+1}\frac{\mu^2}{\mu^2+1} \qquad (32)$$

If $\delta=r$, $\delta'=\delta$, $=\frac{1}{2}r'$, then we have $\mu=2$, $\mu'=\frac{3}{2}$, and this gives

$$\frac{p}{S}=\frac{3}{5}+\frac{10}{13}\cdot\frac{4}{5}=\frac{79}{65}$$

This shows S to be less iu value than p, or in other words, it is possible to permit the internal pressure p to exceed the modulus of resistance without overstraining the material. It is also evident that by encircling the hoops with additional hoops, this principle may be extended still further, and the ratio betweeu p and S still further increased.

If the material of the gun be takeu as ordinary cast steel, with a modulus of resistance of 36,000 lbs., the pressure of the gases of explosion could not be permitted to exceed 43,000 lbs., without causing a permanent deformation of the bore. Receut experiments, however, have shown somewhat greater figures than the above.

Some of the later tests in England have shown pressures of 25.8 tons on the square inch, although this pressure is considered by some engineers to be rather too high to be safe. It is quite possible that in this case the modulus of resistance of the material exceeded that given above; or the interior tube may have been hardened, which, if properly done, is decidedly advantageous.

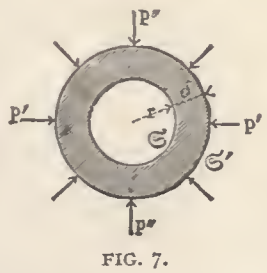


FIG. 7.

The compression exerted upon a cylindrical tube by an external pressure, as in Fig. 7, may be determined by an application of formula (27). If we assume the internal pressure p, to be $=o$, or at least so small as to be neglected, we get:

$$(S+2p')\mu^2=S$$

from which:

$$p'=\frac{S}{2}\frac{1-\mu^2}{\mu^2}$$

or:

$$S=-p'\frac{2\mu^2}{\mu^2-1}=-p'\frac{2\left(1+\frac{\delta}{r}\right)^2}{\left(1+\frac{\delta}{r}\right)^2-1} \qquad (33)$$

The minus sign indicates the change from tension to compression. When the internal pressure $=o$, the stress in the external fibres is:

$$S'=\frac{S}{2}\frac{\mu^2+1}{\mu^2}$$

which gives:

$$S'=-p'\frac{\mu^2+1}{\mu^2-1}=-p'\frac{\left(1+\frac{\delta}{r}\right)^2+1}{\left(1+\frac{\delta}{r}\right)^2-1} \qquad (34)$$

This value is less than the preceding; for by division we obtain the ratio:

$$\frac{S'}{S}=\frac{1}{2}\frac{\mu^2+1}{\mu^2}=\frac{1}{2}\frac{\left(1+\frac{\delta}{r}\right)^2+1}{\left(1+\frac{\delta}{r}\right)^2} \qquad (35)$$

which can only $=1$, when $\delta=o$. Hence for external pressure the greatest stress is always on the innermost fibres.

If, for example, $\delta=r$, and hence $\mu=2$, then the stress in the inner wall of the tube will be $S=-\frac{8}{3}p'$ and $S'=-\frac{5}{3}p'$, so that $S'=\frac{5}{8}S$. This is a greater proportion than when the pressure is from within, as under these circumstances according to formula (30) $S=\frac{5}{3}p$, only.

It is not uncommon in machine construction to strengthen hubs and other parts of machinery by forcing on hoops or rings, and the calculations relating to such construction are closely allied to the preceding. The following case will serve to illustrate.

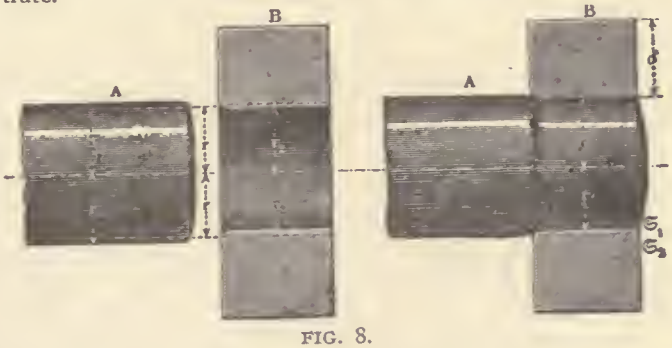


FIG. 8.

In Fig. 8 is a ring B, which is to be forced on to the cylindrical shaft A. The following applies either to shrinking, or to cold forcing. Before the operation the radius of the shaft is r_1, and the radius of the hole in the ring r_2, while afterwards they both have the same radius r.

Under these conditions the shaft B will be subjected to a uniformly distributed compression S_1, while the inner surface of the ring will be under a similar tension S_2. Taking the corresponding moduli of elasticity E_1, and E_2, we have from formula (2):

$$\frac{S_1}{E_1}=\frac{r_1-r}{r_1}, \qquad \frac{S_2}{E_2}=\frac{r-r_2}{r_2}$$

Adding these together, we get:

$$r_1\frac{S_1}{E_1}+r_2\frac{S_2}{E_2}=r_1-r_2$$

It is most important for the designer to know the best values for r_1 and r_2.

If we call $\psi=\frac{r_1-r_2}{r_2}$, we have

$$\psi=\frac{r_1}{r_2}\frac{S_1}{E_1}+\frac{S_2}{E_2}=\frac{\frac{S_1}{E_1}+\frac{S_2}{E_2}}{1-\frac{S_1}{E_1}} \qquad (36)$$

S_1 and S_2 are dependent upon each other, and their relation is expressed by Lamé's formula:

$$S_1 = S_2 \frac{\mu^2 - 1}{\mu^2 + 1} = S_2 \frac{\left(1 + \frac{\delta}{r}\right)^2 - 1}{\left(1 + \frac{\delta}{r}\right)^2 + 1}$$

which may be abbreviated by putting

$$S_1 = S_2 \rho$$

This gives:

$$\psi = \frac{\frac{S_1}{E_1} + \frac{S_1}{\rho E_2}}{1 - \frac{S_1}{E_1}} = \frac{\frac{S_2}{E_2} + \frac{S_2 \rho}{E_1}}{1 - \frac{S_2 \rho}{E_1}} \qquad (37)$$

The difference between the value of the denominator and unity is so slight that in practice it may be neglected, and for a practical and useful formula we have

$$\psi = \frac{S_1}{E_1} + \frac{S_1}{\rho E_2} = \frac{S_2}{E_2} + \frac{S_2 \rho}{E_1} \qquad (38)$$

In this formula we have for the following:

$\frac{\delta}{r}$ =	0.5	0.6	0.7	0.8	1.0	1.5	2.0	3.0
ρ =	0.385	0.438	0.486	0.528	0.600	0.724	0.800	0.882

We also have from equation (38):

$$S_1 = \frac{\psi E_1}{1 + \frac{E_1}{E_2}\frac{1}{\rho}} \text{ and } S_2 = \frac{\psi E_2}{1 + \frac{E_2 \rho}{E_1}} \qquad (39)$$

This value of ψ is generally so small that great care is necessary, in turning and boring, to secure the correct sizes for r_1 and r_2.

Example: With a wrought iron shaft and a cast iron hub we have

$$E_1 = 28{,}400{,}000;\ E_2 = 14{,}200{,}000.$$

If $\delta = 2r$, then $\rho = 0.8$ by the table above; and we may also assume that the stress, S_2, in the interior of the ring due to the forcing should not exceed 7200 lbs. This gives from equation (38)

$$\psi = \frac{7200}{14{,}200{,}000} + \frac{7200 \times 0.8}{28{,}400{,}000} = \frac{1}{1408} = .00071,$$

or, in other words, the increased diameter of the shaft over that of the hole must be 0.00071 times its own diameter.

If we make $\psi = \frac{1}{600}$, we shall have a stress in the ring of

$$= \frac{\frac{1}{600} \times 14{,}200{,}000}{1 + \frac{14{,}200{,}000 \times 0.8}{28{,}400{,}000}} = 16{,}950,$$

or nearly 17,000 pounds, which would be too great for the ring to stand.

§ 20.

The Calculation of Springs.

The materials used in machine construction are all more or less elastic and yielding, so that it is only by a judicious disposition and proportioning that we are able to avoid an injurious deformation of their parts when subjected to the action of external forces. Indeed, it is the principal aim of the constructive engineer to keep the various forms of distortion, such as extension, compression, bending and twisting, within as narrow limits as possible. In the case of springs, however, it is sought to utilize this property of elasticity for a variety of purposes; such as to modify shocks, as in the case of buffers and car springs, or as a source of motive power in clocks and watches; or in cushions, mattresses, etc.

All bodies which will permit great alterations of form within the elastic limit may properly come under the designation of springs.

The only substances which are of service for springs under the action of tension and compression are those which are soft and readily compressible, such as rubber; while the more rigid materials, such as wood and the metals, are used in flexure, or in torsion.

In the following table is given a number of forms of the most usual springs, both for bending and torsion, with their respective properties.

Next to elasticity, the property of a spring to be considered is the economy of material, both on account of cost and space occupied. In order to make it possible to compare different springs in this respect, the relative volume is given in the last column of the table, for the same load and application in the different cases, the volume of the triangular spring being taken as unity.

In all the formulæ of the table we have

E = the modulus of elasticity,

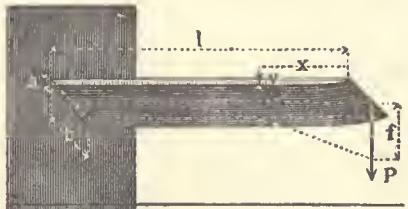
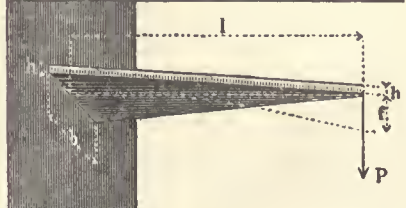
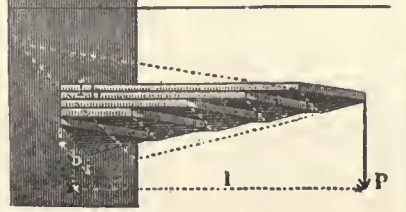
G = the modulus of torsion $= \frac{2}{5} E$, (see § 13).

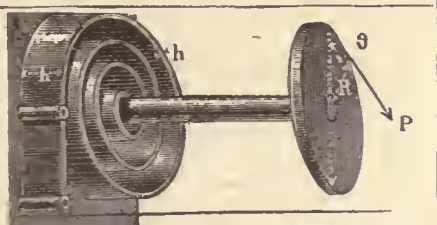
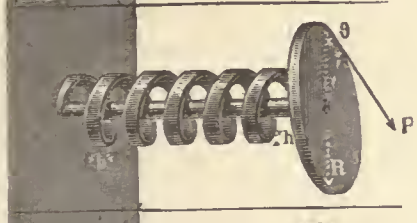
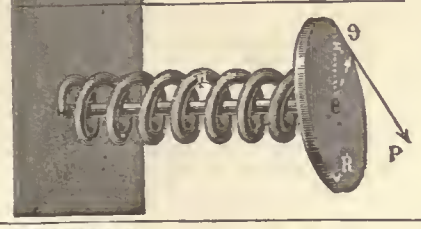
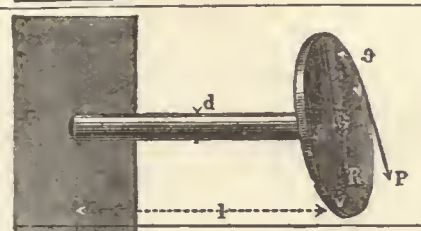
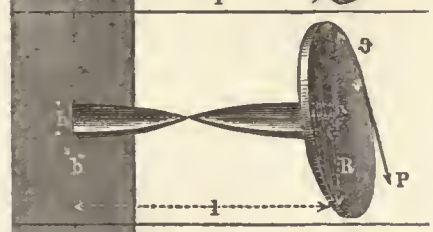
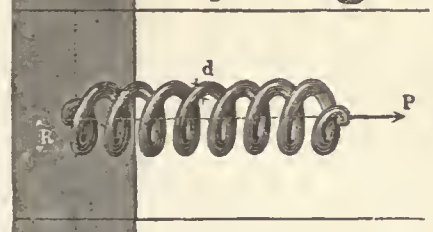
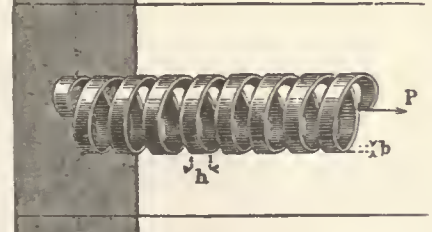
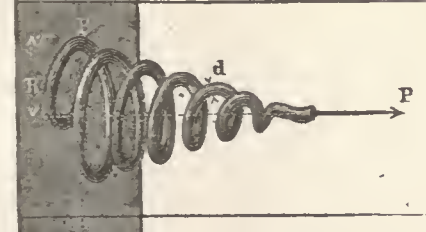
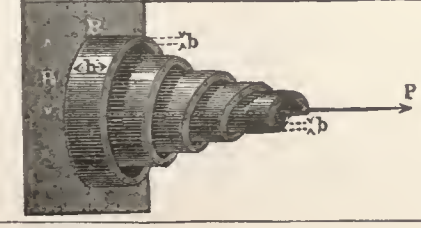
The coefficients for the resistance of the materials used in springs will be found in § 2. It must not be forgotten that for materials used in torsion, to obtain the same security as when used in flexure, the permissible stress S should be $\frac{4}{5}$ its usual value (see § 5). The formulæ are intended only to be used when the force P is applied as shown in the figures.

The volume V of any form of spring is according to the formula:

$$V = C.(P.f)\frac{E}{S^2} \qquad (40)$$

in which C is a constant depending upon the form of the spring; while Pf is the product of the load into the deflection, or the so-called work of the spring. This shows the interesting fact that springs of the same general form and same material are always of the same weight for the same work, without regard to the actual length or proportion of dimensions.

No.	Form.	Name.	Supporting Power.	Deflection.	Elasticity.	Relative Volume.	Remarks.
I.		Rectangular Spring.	$P = \frac{S}{6}\frac{bh^2}{l}$	$f = 6\frac{Pl^3}{Ebh^3}$	$\frac{f}{l} = \frac{S}{E}\frac{l}{h}$	$\frac{3}{2}$	An approximation to $\frac{y}{h} = \sqrt{\frac{x}{l}}$ will be secured by making the end $= \frac{2}{3} h$.
II.		Simple Triangular Spring.	$P = \frac{S}{6}\frac{bh^2}{l}$	$f = 6\frac{Pl^3}{Ebh^3}$	$\frac{f}{l} = \frac{S}{E}\frac{l}{h}$	1	Body of uniform resistance to bending. In practice the end is made somewhat thicker.
III		Compound Triangular Spring.	$P = \frac{S}{6}\frac{ibh^2}{l}$ i = No. of plates.	$f = 6\frac{Pl^3}{Eibh^3}$	$\frac{f}{l} = \frac{S}{E}\frac{l}{h}$	1	This is equivalent to a simple triangular spring, with a base $= i\,b$, as shown by the dotted lines.

No.	Form.	Name.	Supporting Power.	Deflection.	Elasticity.	Relative Volume	Remarks.
IV.		Flat Spiral Spring.	$P = \frac{S}{6}\,\frac{bh^2}{R}$	$f = R\vartheta = 12\,\frac{PlR^2}{Ebh^3}$	$\frac{f}{R} = 2\,\frac{S}{E}\,\frac{l}{h}$	1	l = the developed length of the spiral.
V.		Flat Helical Spring.	$P = \frac{S}{6}\,\frac{bh^2}{R}$	$f = R\vartheta = 12\,\frac{PlR^2}{Ebh^3}$	$\frac{f}{R} = 2\,\frac{S}{E}\,\frac{l}{h}$	1	All three forms of uniform resistance. The value $\frac{f}{R}$ is the angle of rotation ϑ, produced by the load P.
VI.		Round Helical Spring.	$P = \frac{S\pi}{32}\,\frac{a^3}{R}$	$f = R\vartheta = \frac{64}{\pi}\,\frac{Pl}{E}\,\frac{R^2}{d^4}$	$\frac{f}{R} = 2\,\frac{S}{E}\,\frac{l}{d}$	$\frac{4}{3}$	
VII.		Simple Round Torsion Spring.	$P = S\,\frac{\pi}{16}\,\frac{d^3}{R}$	$f = R\vartheta = \frac{32}{\pi}\,\frac{P}{G}\,\frac{R^2 l}{d^4}$	$\frac{f}{R} = 2\,\frac{S}{G}\,\frac{l}{d}$	$\frac{5}{12}$	Cases VII. to X. are bodies of uniform resistance to torsion.
VIII.		Simple Flat Torsion Spring.	$P = \frac{S}{3R}\,\frac{b^2h^2}{\sqrt{b^2+h^2}}$ Approximately when $h > b$, $P = \frac{S}{R}\,\frac{b^2h^2}{3(0.4b + 0.96h)}$	$f = R\vartheta = 3\,\frac{PR^2l}{G}\cdot\frac{b^2+h^2}{b^3h^3}$	$\frac{f}{R} = \frac{S}{G}\cdot\frac{l\sqrt{b^2+h^2}}{bh}$	$\frac{5}{8}$	Springs of the form of VII. and VIII. may also be combined into compound forms.
IX.		Helical Spring of Round Wire	$P = S\,\frac{\pi}{16}\,\frac{d^3}{R}$	$f = \frac{32}{\pi}\,\frac{PR^2l}{Gd^4}$	$\frac{f}{R} = 2\,\frac{S}{G}\,\frac{l}{d}$	$\frac{5}{12}$	In cases IX. to XII. l is always the developed length of the spring.
X.		Helical Spring of Flat Wire.	$P = \frac{S}{3R}\,\frac{b^2h^2}{\sqrt{b^2+h^2}}$ Approximately when $h > b$, $P = \frac{S}{R}\,\frac{b^2h^2}{3(0.4b + 0.96h)}$	$f = 3\,\frac{PR^2l}{G}\cdot\frac{b^2+h^2}{b^3h^3}$	$\frac{f}{R} = \frac{S}{G}\cdot\frac{l\sqrt{b^2+h^2}}{bh}$	$\frac{5}{8}$	It is immaterial whether the breadth of the plate is parallel, normal, or oblique to the axis.
XI.		Conical Spring of Round Wire	$P = S\,\frac{\pi}{16}\,\frac{a^3}{R}$	Approximately $f = \frac{16}{\pi}\,\frac{PR^2l}{Gd^4}$	$\frac{f}{R} = \frac{S}{G}\,\frac{l}{d}$	$\frac{5}{6}$	Here, as in case XII., also the spring is measured to the apex of the cone. The weakest point is at B.
XII.		Flat Volute Spring.	$P = \frac{S}{3R}\,\frac{b^2h^2}{\sqrt{b^2+h^2}}$ Approximately when $h > b$, $P = \frac{S}{R}\,\frac{b^2h^2}{3(0.4b + 0.96h)}$	Approximately $f = \frac{3}{2}\,\frac{PR^2l}{G}\cdot\frac{b^2+h^2}{b^3h^3}$	$\frac{f}{R} = \frac{1}{2}\,\frac{S}{G}\cdot\frac{l\sqrt{b^2+h^2}}{bh}$	$\frac{5}{4}$	By making a gradual reduction in the value of h, from B to the end, this may be made a form of uniform resistance.

The quotient $\frac{E}{S^2}$ shows that a small modulus of elasticity, when combined with a high modulus of resistance, indicates the best material for the construction of springs. According to the table in § 2, we have:

Hardened and tempered steel	$\frac{E}{T^2} = \frac{42,600,000}{(90,000)^2}$	$= .00052$
Ordinary steel (not hardened)	$= \frac{28,400,000}{(35,500)^2}$	$= .00223$
Brass	$= \frac{9,230,000}{(6,816)^2}$	$= .01986$
Wood	$= \frac{1,562,000}{(2,840)^2}$	$= .01936$

This shows that hardened and tempered steel is theoretically the best material of springs. It is also worthy of note that in all the examples given, the deflection is proportioned to the load. It follows from this fact that the time of vibration which any of these loaded springs possesses, is of the so-called "simple" character, of the same nature as that of a pendulum. Neglecting the weight of the spring itself, we have for the vibration of a loaded spring the same rate as that of a simple mathematical pendulum of a length equal to the deflection of the spring f, which is

$$t = \pi \sqrt{\frac{f}{g}} \qquad (41)$$

in which g is the acceleration of gravity = 32.2 ft.

Examples on the theory of springs: 1. Given a simple triangular spring, as in case II., for a load $P = 110$ lbs., and a deflection $f = 0.78''$. Taking the material as cast steel, with $E = 42,600,000$, and making S the greatest permissible stress, = 56,800 lbs., and also taking the length $l = 15.75''$, we then have

$$\frac{f}{l} = \frac{S}{E} \cdot \frac{l}{h} \quad \text{or} \quad \frac{0.78}{15.75} = \frac{56,800}{42,600,000} \cdot \frac{15.75}{h}$$

from which

$$h = \frac{56,800 \times 15.75 \times 15.75}{0.78 \times 42,600,000} = 0.424''$$

Substituting this in the formula

$$f = 6 \frac{Pl^3}{Ebh^3} \text{ or } b = 6 \frac{Pl^3}{Efh^3}$$

we get:

$$b = \frac{6 \times 110 \times (15.75)^3}{42,600,000 \times 0.78 \times (0.424)^3} = 1.018''$$

The volume $V = \frac{bhl}{2} = \frac{1.018 \times 0.424 \times 15.75}{2} = 3.4$ cu. in.

Example 2: If we keep the same conditions, but make the length 11.8", we shall have

$$h = \frac{56,800 \times 11.8 \times 11.8}{0.78 \times 42,600,000} = 0.238''$$

$$b = \frac{6 \times 110 \times (11.8)^3}{42,600,000 \times 0.78 \times (0.238)^3} = 2.42''$$

The volume in this case $= V = \frac{bhl}{2} = \frac{2.42 \times 0.238 \times 11.8}{2} = 3.39$ cu. in., thus confirming the remarks on formula (40) by showing that the volume depends upon the load and the deflection, and is independent of the proportional dimensions.

Example 3: Let us now suppose the same conditions to be applied to a helical spring such as No. IX., also made of cast steel. Since this is a torsion spring, in order to obtain the same security we must make $S = \frac{4}{5}$ of its preceding value, or $\frac{4}{5} \cdot 56,800 = 45,440$; and the wire may be taken as 0.24 in diameter.

We then have from the table

$$P = S \frac{\pi}{16} \frac{d^3}{R}, \text{ or } 110 = \frac{45,440}{16} \pi \frac{(0.24)^3}{R}$$

from which we get

$$R = \frac{45,440 \times 3.1416 \times (0.24)^3}{16 \times 110} = 1.121''$$

The length l is obtained from column (6).

$$\frac{f}{R} = 2 \frac{S}{G} \frac{l}{d}; \text{ or } l = \frac{fGd}{2RS}$$

in which $G = \frac{2}{5} E = 17,040,000$.

$$l = \frac{0.78 \times 17,040,000 \times 0.24}{2 \times 1.121 \times 45,440} = 31.3''$$

This would make the number of coils

$$i = \frac{l}{2\pi R} = \frac{31.3}{2 \times \pi \times 1.121} = 4.4,$$

or about 4½ coils. If more coils are preferred, the diameter, d, of the wire must be reduced and the calculation repeated.

The volume $V_1 = l \frac{\pi}{4} d^2 = 31.3 \times 0.7854 \times (0.24)^2 = 1.416$ cu. in.

This gives the ratio $\frac{V_1}{V} = \frac{1.416}{3.4} = 0.416$ or $\frac{5}{12}$, as given in the table.

Example 4: Torsion springs have recently been applied to railway cars in the form shown in Fig. 9, which is the design of an American, Mr. Dudley. The U shaped spring is bent at the ends into two elbows, A B, which are attached by bolts to a block which rests on the axle box. A saddle, A, transmits the load of the car to the spring, while the other end is supported at C by a hook.

In order to determine the stress S, in one branch A C, of such a spring, let us take the diameter $d = 1.14''$, and the lever arm, R, which is the horizontal projection of A B, as 4". The load on the spring is one-fourth the load on the car, 22,000 lbs. + one-fourth the weight of the car itself, 18,000, and one-half of this is borne by each branch of the spring, making the load at the end of the lever R in this case to be 5,000 lbs.

In the preceding table, under case VII., column 4,

$$S = \frac{16}{\pi} \frac{PR}{d^3} = \frac{16}{\pi} \frac{5000 \times 4}{(1.14)^3} = 68,750$$

If this spring is made of Sheffield steel which has a modulus of elasticity $E = 24,140,000$, then the modulus of torsion $G = \frac{2}{5} E = 9,656,000$.

If the length $l = 33.5''$, the deflection, according to column 6 in the table, will be

$$f = \frac{2RSl}{Gd} = \frac{2 \times 4 \times 68,750 \times 33.5}{9,656,000 \times 1.14} = 1.67''.$$

The above described spring weighs 24.2 pounds, which is about $\frac{1}{113}$ of its gross load, or about $\frac{1}{137}$ of its net load.

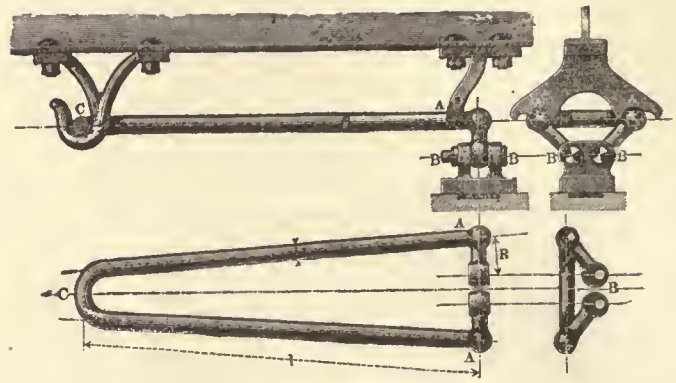


FIG. 9.

A double armed plate spring of the form No. III., to have the same supporting power would weigh about a hundred pounds, or $\frac{1}{100}$ its gross load and $\frac{1}{85}$ its net load.

As long ago as 1857 I called attention to the superior economy of torsion springs over plate springs for railway use. The principal reason for the tardiness of railway men in appreciating this fact may have been partly due to the difficulty of securing a proper temper in the round steel, although this seems to have been entirely overcome in the case of the Dudley Spring.

In the little pamphlet on "The Construction and Calculations of Springs," which I published at that date, the comparative weight of the torsion springs VII. and IX., and the triangular plate springs II. and III., is given as $\frac{1}{75}$, instead of $\frac{1}{85}$ as in the preceding table, but the latter is shown to be more nearly correct in practice.

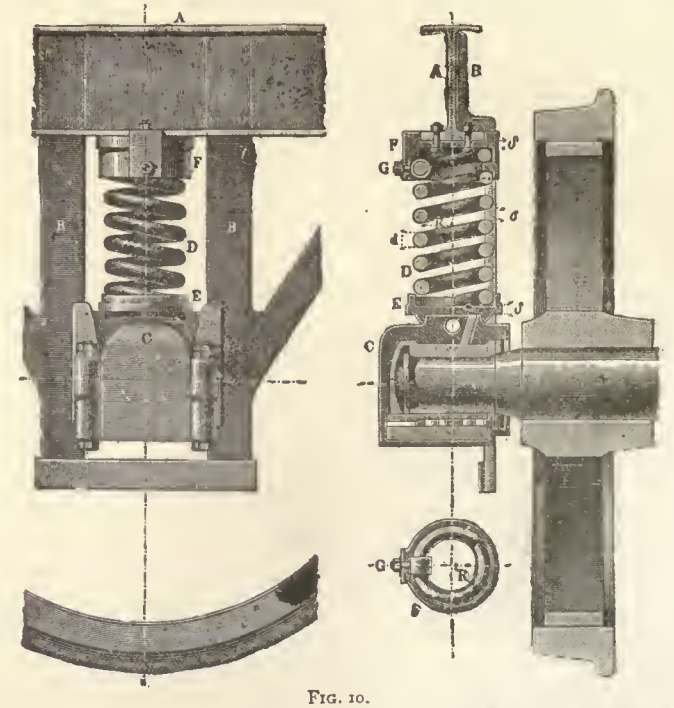


FIG. 10.

In Figure 10 is shown the manner in which a helical spring may be applied to the bearing of a goods wagon in the place of a plate spring. The box is guided in the frame B, B, and the spring D is interposed between the sill A of the wagon and the journal box C. The form of the spring is a single helix with the lower end flattened for about ¾ of a turn in order to give a fair bearing in the cap E of the box. The upper end screws for about 1½ turns into the cap F, where it is clamped by the screw G after the load has been equalized, and in this way any desired adjustment may be secured.

An example will probably be the best method of showing the manner of calculating such a spring.

An ordinary German four wheeled goods wagon weighs about 11,000 pounds, and carries about 22,000 pounds load. This gives about 8,250 pounds to be supported by each spring. We will assume a deflection of 1¾", with a permissible fibre stress S of 68,000 lbs., and take $G = 9,656,000$, as before.

Since it is desirable to use such diameters of spring steel as correspond to commercial sizes, it is better to select a diameter d for the steel, and deduce a corresponding radius R for the helix, according to the formula for case IX., col. 4, page 64,

$$R = S \frac{\pi}{16} \frac{d^3}{P}$$

We will take the successive cases in which the diameter d of the steel is 1", $1\frac{1}{16}$", $1\frac{1}{8}$" and $1\frac{3}{16}$".

Now for these respective values we must select such a number of coils, n, that with a load $P = 8{,}250$ lbs., we shall get a compression $f = 1.75$".

We have for n the fact that $2\pi Rn$ = the length l of the uncoiled spring. Substituting this in the formula for f, case IX., we get

$$f = 2\frac{S}{G}\frac{2\pi R^2 n}{d}$$

from which

$$n = \frac{fG}{4\pi S}\cdot\frac{d}{R^2}$$

Now the least possible distance between the cap F and the socket E is $nd + f$, and we must also provide for a space σ between the coils, say 0.3". This gives the distance s from centre to centre of coils of the unloaded spring

$$s = \frac{nd + f + n\sigma}{n}$$

The total height of the spring, however, will be greater than ns by $1.5s + d$, since 1½ coils enter into the cap F, and one-half the diamater of the steel in the last coil must be added both top and bottom. Adding to this the thickness δ of the cap and the socket, as ½", we obtain the entire height occupied by the spring and its fittings. Of course this height is limited by the space at our disposal between A and C, and in this case it may be taken at 14 inches, and in any case it will generally be dependent upon other circumstances. Substituting these values in the several formulæ, as given above, and tabulating the results, we have:

d =	1"	$1\frac{1}{16}$"	$1\frac{1}{8}$"	$1\frac{3}{16}$"
R =	1.619	1.94	2.30	2.71
or say =	$1\frac{5}{8}$	2.00	$2\frac{5}{16}$	$2\frac{3}{4}$
h =	7.5	5.6	4.18	3.19
nd =	7.5	5.95	4.70	3.78
$n\sigma$ =	2.25	1.68	1.25	0.96
f =	1.75	1.75	1.75	1.75
$n(d+\sigma)+f$ =	11.50	9.38	7.70	6.49
s =	1.53	1.67	1.84	2.03
or say =	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{13}{16}$	2.00
$1.5s$ =	$2\frac{1}{4}$	$2\frac{1}{16}$	$2\frac{3}{4}$	3.00
2δ =	1.00	1.00	1.00	1.00
Total height =	15.75	13.82	12.45	11.49
or about =	$15\frac{3}{4}$	$13\frac{7}{8}$	$12\frac{1}{2}$	$11\frac{1}{2}$

It will be seen that the first size is too high for the space at our disposal, but that the others may be used as circumstances may dictate, all four springs having the same compression and stress upon the material. The simplicity of this construction is noteworthy, and the economy of material is noticeable. For passenger coaches, where more elastic springs are desirable, f may be made from 2 to 3 inches, and a combination of several springs may be used.

The use of vulcanized rubber for springs and buffers is now quite general, usually in the shape of rings or collars between plates of iron. The resistance of rubber to distortion has not yet been fully investigated in an experimental manner, but the following examination of buffer springs may be of service; the data being from the valuable researches of Chief Engineer Werder, of the establishment of Klett & Co., of Nuremberg.

The usual sections of buffer rings are shown in Figs. 11 and 12, in which one of the plates carries an annular projection, and the other a corresponding depression, between which the springs are held, and lateral motion in the buffer case prevented.

When the ring of rubber is under compression its volume is unchanged; the cross section is reduced, but the diameter of the ring is increased proportionally. The elements which are subjected to the greatest strain lie at the circumference E, and are under tension, as is proved by the cracks which appear when the limit of elasticity is exceeded.

The limit of elasticity will be reached by a load of about 700 pounds to the square inch of *original* cross section normal to the axis; or, in other words, a modulus of compression $T = 700$ may be taken. This modulus is slightly higher for rubber of the lightest specific gravity (about 780 lbs.), and less for a heavier specific gravity (about 640 lbs.). The specific gravity γ, which depends upon the proportion of sulphur, varies from 1 for the lightest, to 1.15, or even 1.32, for the heaviest.

When the elastic limit is reached, the middle section $E\,F$ of the ring is double its original area, while the periphery is $\frac{4}{5}$ the original periphery $A\,B\,C\,D$.

The compressibility within the elastic limit is dependent upon the quality of the rubber, and may be approximately determined by the following empirical formula:

$$\lambda = 0.026\,\frac{l}{\gamma}\sqrt{\frac{P}{q}}; \qquad (42)$$

in which: λ = the extent to which the spring is compressed by a load P; l = the original thickness of the spring; q = the original cross section in a plane normal to the axis; and γ = the specific gravity of the material.

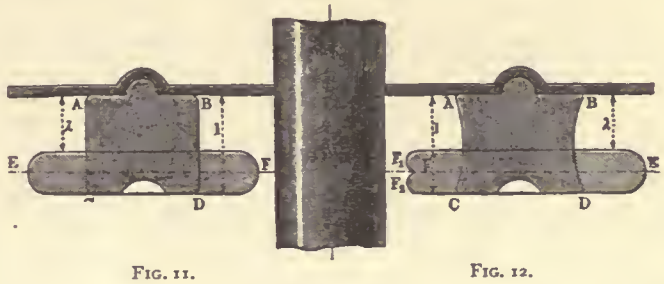


FIG. 11.

FIG. 12.

Example: A buffer spring similar in shape to Fig. 11 has an outside diameter of $5\frac{5}{8}$", and an inside diameter of $2\frac{13}{16}$", which gives an area of cross section q of 18.07 square inches. The thickness of the uncompressed ring $l = 1\frac{3}{16}$", and its specific gravity $\gamma = 1$. The load to be supported is 5500 pounds, which corresponds to a pressure per square inch $\frac{P}{q} = 305$ pounds, which is well within the elastic limit.

According to (42) the compression will be:

$$\lambda = 0.026 \times 1.375\sqrt{305} = 0.624, \text{ say } \tfrac{5}{8}".$$

The Belgian engineer, Stévart, has also made extensive researches upon the subject of the resistance of rubber. These experiments appear to confirm the opinion that any change of shape is unaccompanied by any change of volume, and that rubber is practically as incompressible as water. The experiments on tension gave a modulus of elasticity of 119.28 lbs. In regard to compression Stévart deduced a formula similar to the preceding:

$$\frac{l}{l-\lambda} = \sqrt{a\,P + 1};$$

in which a is a coefficient dependent upon the form of the spring, and determined experimentally. In the case of locomotive buffers, which are composed of several rings, the compression of each ring should be computed separately, and their sum taken.

Rubber springs are used very extensively, but the principal objection to them is that the material gradually loses its elasticity and becomes a hard, unyielding mass. It has been found that this is largely due to friction between the rings and their cases, and great care should be taken that boxes for rubber springs should have ample allowance made for the increased diameter of the spring when compressed.

It is a matter of importance to choose such a shape for a rubber spring that it shall not have a tendency to form puckers in the edge when it is under pressure. This is shown in Fig. 12, where the slight concavity in the edge would soon develop a crack when compressed, as at $F_1\,F_2$, while the shape in Fig. 11 has no such tendency.

SECTION II.

THE ELEMENTS OF GRAPHOSTATICS.

§ 21.

Introductory.

The equilibrium of forces may be very clearly shown by the graphical method, since it is possible to show the direction, extent and position of any force by a right line. The direction of a force is determined by the angle which its representative line makes with the horizontal axis of co-ordinates; the length of the line gives the absolute amount of force exerted, the algebraic character of the force (plus or minus) being indicated by arrow-heads; while the position of the line in the system of co-ordinates, makes it possible to show any constants which may occur in the equation of any right line. This representation of forces by means of geometric magnitudes makes it possible to solve problems in statics entirely by means of geometrical constructions, and in many cases this will be found simpler and more convenient than the use of algebraic analysis, especially in those cases in which the values to be determined are themselves geometrical quantities, and therefore are to be drawn when found. The various details of the method have been arranged and collected into a system which has been called Graphical Statics, or, as we have here termed it, Graphostatics.* This method is of especial value in the study of Machine Design, and in the following sections of this work many applications of it will be found. It is for this reason that the following brief exposition of the leading principles of the method have been here grouped together.

There is a distinction to be made between Graphostatics, properly so-called, and the mere graphical calculations of simple values, considered merely as magnitudes. This is more properly to be considered as graphical arithmetic, or Arithmography.†

In the following pages this branch of the subject is not very fully discussed, only an outline of its application to pure arithmetic being given. It will be found, however, to be a subject of much use to the mechanic, as many examples of its application in future pages will show.

§ 22.

Multiplication by Lines.

In graphical calculations dimensions are taken with the dividers and scale, and any convenient unit may be selected, such as the inch, millimetre, decimeter, square foot, cubic foot, unit of velocity, unit of money, etc, etc. It is readily apparent that the operations of addition and subtraction may be performed by simply marking off the various values upon any line. The operation of multiplication is not quite so simple, and a brief explanation may be necessary.

In all cases it is of course necessary that the same unit must be chosen for all the quantities involved, and this holds good for multiplication as well, and the same unit must be used to measure the result as has been chosen to express the original quantities. If, now, we wish to multiply two lines, a and b, together, or, more correctly, to multiply a line of the length a by a line of the length b, we must find a line x, which will contain our chosen unit, $a \times b$ times. This is a simple operation, and may be performed in several ways by means of similar triangles.

I. Draw OE, Fig. 13, horizontal, making its length equal to unity; erect at E a perpendicular, and intersect this from O with $OB = b$. Lay off $OA = a$, and from A draw a parallel to EB, intersecting OB produced at C. Then OC will be the desired product x. That is to say, $\frac{OC}{AO} = \frac{OB}{OE}$, and since $OE = 1$, we have $x = \frac{ab}{1}$. This solution requires that one factor, b, shall be greater than unity.

II. Fig. 14. A modification of the preceding may be made by drawing EB inclined, instead of perpendicular to OE, in which case both factors may be less than unity.

III. We may make, as in Fig. 15, OE and OB as before, produce $OA = a$, and draw AC, so that the angle $OAC = OEB$, so that AC will be the *anti-parallel* to EB. Then OC

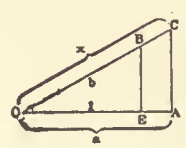


Fig. 13.

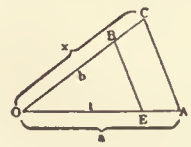


Fig. 14.

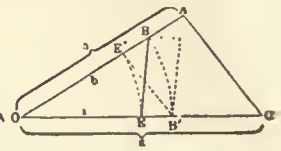


Fig. 15.

will be the desired product x, since the triangles OEB and OAC are similar. This *anti-parallelism* is shown by the fact that $OE' = OE$, $OB' = OB$, and AC is parallel to $E'B'$. If the triangle $BE'B'$ is rotated to the right about an axis, passing through BB', the two triangles, $BB'E'$ and $BB'E$, will form a parallelogram; hence the term anti-parallel. This construction is most convenient when EB is perpendicular to OE, which can only occur when b is greater than 1.

IV. We may make, as in Fig. 16, $OE =$ unity, lay out on OE the factor $OA = a$, and erect a perpendicular or inclined line at E, in which $EB = b$; then draw through A a parallel to EB, and this latter line will intersect OB, prolonged so that

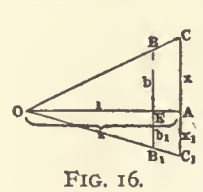


Fig. 16.

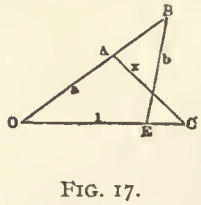


Fig. 17.

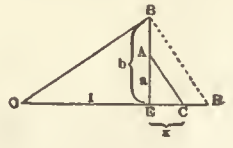


Fig. 18.

$AC = x$, since $CA : OA = BE : OE$, or $x : a = b : 1$; a and b being either greater or less than 1. Now make $EB_1 = b_1$, and draw OB_1 to intersect CA, prolonged at C_1, then $AC_1 = x_1$, the product of a and b_1, and CC_1 will be the product of a into BB_1, or:

$$x + x_1 = a(b + b_1).$$

Of course, the factor b, which is to be multiplied by a, may extend on both sides of the base line OE, and the desired product, $ab = x$, will then be the distance on the parallel to b, which is included between the two lines drawn from O through the extremities of b.

V. In Fig. 17 $OE =$ unity, $EB =$ the factor b, and OB any value, so that $OB < OE + EB$. Lay out OA on OB, making it = the factor a, and draw from A an anti-parallel to EB (see III.), then AC will $= x$. For $AC : OA = BE : OE$, or $x : a = b : 1$, and a and b may both be less than 1.

VI. Again, we may make Fig. 18 $OE = 1$, erect a perpendicular at E, make $EA = a$, $EB = b$, join O with B, draw BB' normal to OB, and draw from A a parallel to BB', then will $EC =$ the desired product x. For we have $EC : EA = BE : OE$ or $x : a = b : 1$.

It often occurs in designing that we have already a diagram drawn which may serve for a portion of the construction, and in such cases the following methods may be found convenient.

VII. Fig. 19. $OA = a$ and $B'B = b$ are either at right angles or inclined to each other, so that B' falls between O and A. Lay out on OA the unit OE, join B with E, and draw from A a parallel to BE, and from the point C, where it inter-

* See Culmann, "Graphical Statics," Zürich, 1866.

† See "Principles of Graphical Arithmetic," by Dr. Eggers, Schaffhausen, 1865; also Schlesinger, "Power Curves," in Journal of the Austrian Society of Engineers and Architects, 1866; also E. Stamm, "Graphical Calculus," Proc. Royal Inst., Lombardy, Vol. VI.

sects $O C$, draw $C C'$ parallel to $B B'$, then $C C'$ will $= x$, for we have $C C' : O A = B B' : O E$ or $x : a = b : 1$.

VIII. Fig. 20. Given as before, $O A = a$ and $B B' = b$, either perpendicular or inclined to $O A$. Draw $O E$ parallel to $B B'$, and equal in length to unity; join E to A, and draw from B a

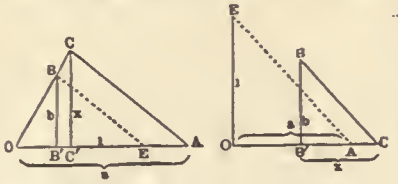


FIG. 19.

FIG. 20.

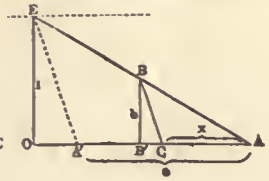


FIG. 21.

parallel to $E A$. This will cut off on $O A$ prolonged the distance $B' C = x$, for $B' C : B' B = O A : O E$ or $x : b = a : 1$.

IX. Fig. 21. Given $A A' = a$ and $B B' = b$ perpendicular. Draw $A B$, and prolong it until it intersects at E a line drawn parallel to $A A'$ at a distance $O E = 1$. Join $E A'$, and draw from B a parallel to it, cutting $A A'$ at C, then will $A C = x$; for $A C : C B = A A' : A' E$, and $A C : B' B = A A' : E O$, or $x : b = a : 1$.

X. Fig. 22. Given $A A' = a$ and $B O = b$, perpendicular to $A A'$. Open the dividers to $O E = 1$, and intersect $A A'$ at E. Draw from A' a parallel to $O E$, and from A a normal, the

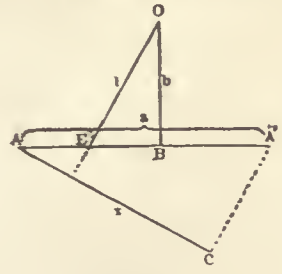


FIG. 22.

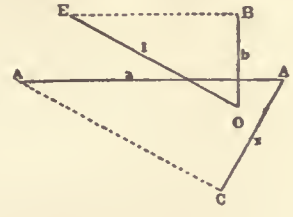


FIG. 23.

two lines intersecting at C, then $A C =$ the desired product x. For, since the angle $C A A' = B O E$, we have $A C : A A' = O B : O E$, or $x : a = b : 1$. The line $A A'$ is in this case projected upon a perpendicular to $O E$, or is what is called the *anti-projection* of $A A'$ to $O E$.*

XI. Fig. 23. When a and b intersect each other at right angles, as in the figure where $A A' = a$ and $B O = b$, then

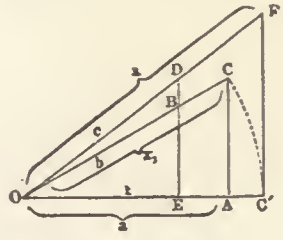


FIG. 24.

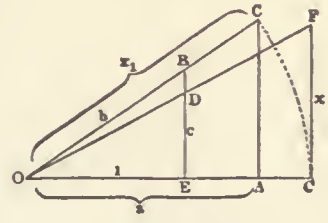


FIG. 25.

draw from B a parallel to $A A'$, and mark off with the dividers from O, $O E = 1$. Draw $A' C$ parallel to $O E$, and $A C$ normal to $A' C$, then $A C = x$, for since the angles at E and A' are equal, we have $A C : A A' = O B : O E$, or $x : a = b : 1$.

The continuous multiplication of several factors may be accomplished by combining the preceding methods in various ways.

Suppose we desire to obtain the product of three lines, a, b, c, we may first find, according to I, the product $x_1 = a b$ (Fig. 24), transfer $O C = a b$ down to $O C'$ upon $O A$, draw from O the line $O D = c$, erect from C' a perpendicular, and prolong $O D$ to F, and $O F$ will be the desired product, $x = a b c$.

Or we may make, as in Fig. 25, after having found $O C' = a b$, draw $E D = c$ (Case IV.), and prolong $O D$ until it intersects at F a perpendicular from C', when $C' F = x$.

§ 23.

DIVISION BY LINES.

Division may readily be accomplished by reversing the methods employed for multiplication. To divide a line a by a line b, we must find a third line x, which must contain the unit

* See Culmann's "Graphical Statics."

of a and b, $\frac{a}{b}$ times. From the previous examples we may derive the following methods of division.

I. Fig. 26. Make $O E =$ unity, erect at E a perpendicular or inclined line, intersect it with the divisor $O B = b$, prolong

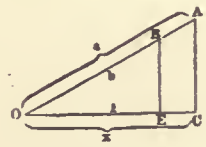


FIG. 26.

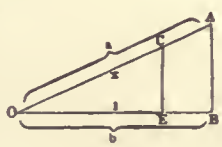


FIG. 27.

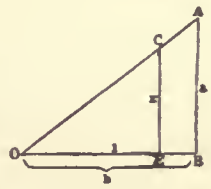


FIG. 28.

$O B$, and make $O A =$ the dividend a. Draw from A a parallel to $B E$, and its intersection with $O E$ prolonged will give the quotient x. For we have $O C : O E = O A : O B$, that is, $x : 1 = a : b$, or $x = \frac{a}{b}$.

II. Fig. 27. Make $O E =$ unity, also lay off on $O E$ the distance $O B =$ the divisor b, erect at B a perpendicular, and intersect it from O with $O A =$ the dividend a. A perpendicular, erected from E, will then intersect $O A$ at C, and $O C = x$, for we have again $O C : O E = O A : O B$ or $x : 1 = a : b$.

III. Fig. 28. Make $O B =$ the divisor b; on $O B$ lay off $O E = 1$; at B erect a perpendicular $A B =$ the dividend a; join $O A$. Erect at E a perpendicular, and it will intersect $O A$ at C. Then $E C = x$, for $E C : O E = A B : O B$, or $x : 1 = a : b$.

§ 24.

MULTIPLICATION AND DIVISION COMBINED.

When it is desired to multiply a number a into a fraction $\frac{b}{c}$, the operation really consists in multiplying a by b, and dividing the product $a \times b$ by c, in order to obtain the result x. If we recollect that for $x = \frac{a\ b}{c}$, we may write $x : a = b : c$, we will see at once how the combined operation may be performed by making the distance $O E$ equal to the denominator c, instead of unity, as heretofore. We will then be multiplying the line a by the ratio $\frac{b}{c}$, instead of $\frac{b}{1}$. The following illustrations will make the operation clear.

I. In order to multiply a quantity a by a fraction $\frac{b}{c}$, we make, in Fig. 29, $O A = a$, lay off on $O A$, $O E = c$, erect at E a perpendicular, and intersect it at B, with a distance from O

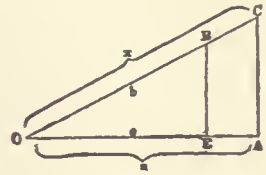


FIG. 29.

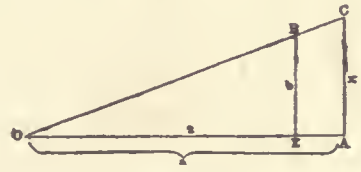


FIG. 30.

equal to b; then prolong $O B$ until it intersects at C a line drawn from A, parallel to $E B$. Then $O C$ will equal x, for we have $O C : O B = O A : O E$, or $x : b = a : c$, or $x = \frac{a\ b}{c}$.

II. If we wish to find the product $\frac{a\ b}{2}$, we make, Fig. 30, $O A = a$, and make the distance $O E =$ twice the unit of measurement, draw $E B = b$ perpendicular to $O E$; draw a line from A parallel to $E B$, and prolong $O B$ until it intersects this last line at C. Then $A C$ will be the desired product x, for $A C : O A = B E : O E$, or $x : a = b : 2$, or $x = \frac{a\ b}{2}$.

These methods, which may be extended much in the same manner as the various methods of multiplication given in § 22, will be found of great service in the graphical calculations of areas, as we shall see.

§ 25.

AREA OF TRIANGLES.

Since the area of a triangle is equal to the half-product of its base and altitude, it is readily calculated by the method given in the preceding section.

I. Fig. 31. Selecting the side $O B = b$ of the given triangle $O A B$ as a base, which gives the perpendicular $A A' =$ the

height h, although this line need not be drawn, we mark off the distance $O\,E = 2$ units (inches, decimeters, etc.), and draw

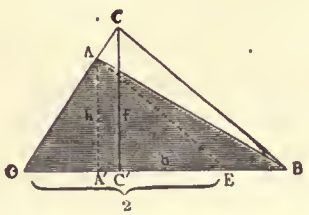


FIG. 31.

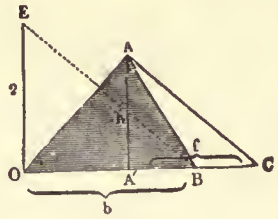


FIG. 32.

from B a line $B\,C$, parallel to an imaginary line $A\,E$. This line $B\,C$ will intersect the side $O\,A$ prolonged at C, and a perpendicular dropped from C to $O\,B$, will give $C\,C' = \frac{h\,b}{2}$ = the desired area f (see VII., § 22, and II., § 24).

II. Fig. 32. From the end of the base line $O\,B$ draw the perpendicular $O\,E = 2$ units, draw the altitude $A\,A'$; also draw from A the line $A\,C$ parallel to $E\,B$. This will cut off on the base line the distance $A'\,C$, which is the product $f = \frac{h\,b}{2}$. (§ 22, VIII., and § 24, II.)

III. Fig. 33. Prolong the base line $B\,C$ and the side $B\,A$ until the vertical distance between them $O\,E = 2$ units. Join

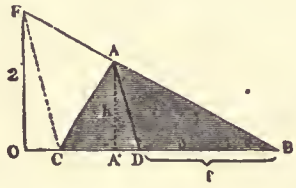


FIG. 33.

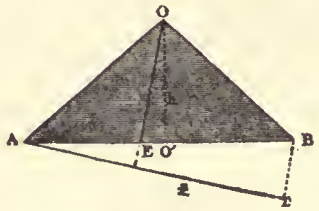


FIG. 34.

E to C, and draw from A a line parallel to $E\,C$, intersecting the base at D, and $B\,D = \frac{h\,b}{2} = f$. (§ 22, IX., and § 24, II.)

IV. Fig. 34. From the vertex O, with the dividers open a distance equal to 2 units, intersect the base at E, and make the anti-projection of the base $A\,B$ by drawing $B\,C$ parallel to $O\,E$, and $A\,C$ normal to $B\,C$. Then $A\,C$ = the product of the base b, and one-half the altitude $O\,O' = h$, and hence is the desired area f of the triangle. (§ 22, X., and § 24, II.)

If the unit is taken as one inch, the value of the area f will be given in square inches, or if a decimeter is taken as the unit, the area will be in square decimeters, etc.

If we find $f = \frac{7}{8}''$, the area of the triangle is seven-eighths of a square inch; or if it measures 72 millimeters, the area would be 0.72 square decimeters, or $0.70 \times 10{,}000 = 7200$ sq. mm.

§ 26.

AREA OF QUADRILATERAL FIGURES.

In determining the area of a quadrilateral figure, it is either obtained directly, as in the case of a parallelogram; or it may

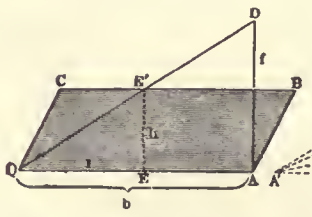


FIG. 35.

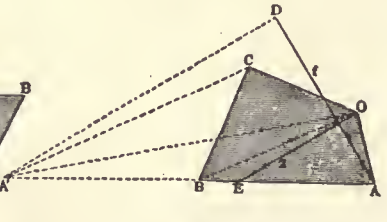


FIG. 36.

be separated into triangles, which are measured separately; or the figure may be reduced to its equivalent triangle.

I. Required the area of the parallelogram $A\,B\,C\,O$, Fig. 35. Taking the side $O\,A$ as a base line, lay off $O\,E$ = unity, and erect the perpendicular $E\,E' = h$. Prolong $O\,E$ until it intersects a perpendicular from A at D, and the distance $A\,D$ will be the area of $f = b\,h$. (§ 22, IV.)

II. The quadrilateral figure $A\,B\,C\,O$, Fig. 36, may readily be replaced by a triangle of equal area by drawing the line $O\,A'$ parallel to the diagonal $O\,B$, for since the triangle $O\,A'\,B$ is equal in area to $O\,B\,C$, we have the area of the triangle $O\,A'\,A$ is equal to the area of the figure $A\,B\,C\,O$. Now, according to IV., § 25, we make $O\,E = 2$, and draw $A\,D$, the anti-projection of $A\,A'$ and $A\,D = f$, the desired area.

III. Fig. 37. The diagonal $A\,C = b$ divides the figure $A\,B\,C\,O$ into two triangles, the sum of whose heights = $O\,O'$, which is

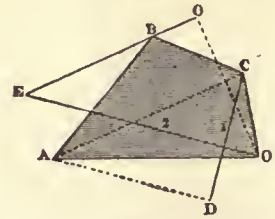


FIG. 37.

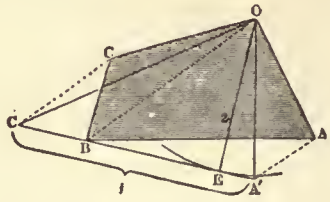


FIG. 38.

the anti-projection which $O\,B$ makes on $A\,C$. The multiplication of $O\,O'$ by $\frac{b}{2}$ may be made according to XI., § 22, and II., § 24. Draw $O'\,B\,E$ parallel to $A\,C$, making $O\,E = 2$, also draw $A\,D$ parallel to $E\,O$, and $C\,D$ normal to $A\,D$, then $C\,D = f$ = the area of $A\,B\,C\,O$.

IV. Fig. 38. The figure $A\,B\,C\,O$ may be converted into a triangle whose altitude = 2, when the base will be equal to the product $\frac{h\,b}{2}$. From O describe an arc with a radius $O\,E = 2$, and draw a tangent passing through an angle of the figure at B, opposite the angle O. From the other two angles, A and C, draw lines parallel to the diagonal $O\,B$, intersecting the tangent at A' and C'. $A'\,C'$ will then be the base of a triangle whose altitude = 2, and whose area is the same as the figure $A\,B\,C\,O$, and the area $f = A'\,C'$. Many similar methods may be deduced from the preceding examples.

§ 27.

AREA OF POLYGONS.

The area of a polygon is measured by reducing it to its equivalent triangle. This may be done in the following manner: From the angle O of the polygon $O\,A\,B\,C\,D\,E$, Fig. 39, draw a diagonal $O\,B$ to the next angle but one, and then from the

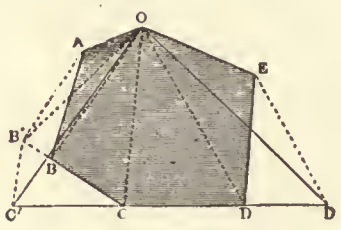


FIG. 39.

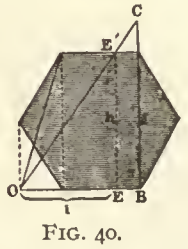


FIG. 40.

intermediate angle A draw $A\,B'$ parallel to $O\,B$, prolonging the third side $B\,C$ to B'. If we join $O\,B'$, we have the triangle $O\,B\,B' = O\,B\,A$, and hence the figure $O\,B'\,C\,D\,E$ will have the same area as the original figure, but will have one less side. Then join $O\,C$, and draw $B'\,C'$ parallel to $O\,C$, and so we may proceed until we have obtained a triangle $O\,C'\,D'$ of equivalent area to the original figure, and whose area may be determined by any of the preceding methods.

Regular polygons, such as the hexagon, Fig. 40, only require half the operation to be performed, and then the area measured as a parallelogram.

§ 28.

GRAPHICAL CALCULATION OF POWERS.

A line a, raised to the nth power, really means the determination of a line x whose length shall contain the unit of measurement a^n times. The following methods are applicable when a is a positive or negative whole number, and the process is really a repeated application of the multiplication of a by a. As in the previous cases, this operation may be performed in various ways.

I. (See § 22, I.) In Fig. 41 make $O\,E$ = unity, erect at E a perpendicular, and intersect it at A_1 with the distance $O\,A_1 = a$, the original factor. Carrying this distance $O\,A_1$ down to B_1, and erecting a perpendicular at B_1, we get $O\,A_2 = a^2$ (see I., § 22). This again carried down to B_2, and a perpendicular erected at B_2, gives $O\,A_3 = a^3$, and so $O\,A_4 = a^4$, $O\,A_5 = a^5$, etc.

If we lay off $O\,B_m$, equal to any power of a, say a^m, and erect perpendicular at B_m, the intersection with $O\,A_1$ prolonged will give the value of a^{m+1}. Again, if we drop a perpendicular from the end point A_{m+1} of any power of a to the axis $O\,E$, it will cut off a distance $O\,B_m$, which will be the next lesser power of a (see I., § 23).

The perpendicular $A_1 E$, from A_1 upon $O E$, gives the first power a^1. If we now make $O A_0 = O E$, and drop the perpendicular $A_0 B_{-1}$, we have $O B_{-1} = a^{-1}$, which $= \frac{1}{a}$, which is the reciprocal of $O A_1$; in the same manner we get $O B_{-2} = \frac{1}{a^2}$; $O B_{-3} = \frac{1}{a^3}$.

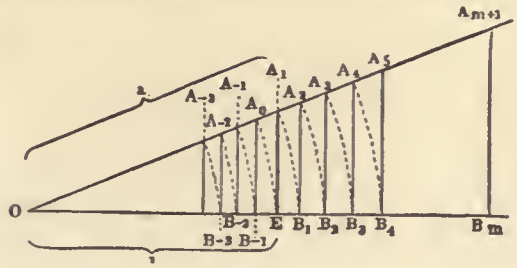


FIG. 41.

II. By combiuing the methods of multiplication I. and III. of § 22, the following method for powers is derived. In Fig. 42, make $O E = 1$, $O A_1 = a$, $E A_1$ perpendicular to $O E$, and draw from A_1 a perpendicular to $O A_1$, cutting $O E$ at A_2; then $O A_2 = a^2$. From A_2 a perpendicular to the base will give A_3 and $O A_3 = a^3$; another perpendicular to $O A_3$ gives A_4 and $O A_4 = a^4$, and this may be continued indefinitely for positive powers of a. By working backward from E, we get $O A_{-1}$ as the reciprocal of a, $O A_{-2} = \frac{1}{a^2}$, and so on for negative powers of a.

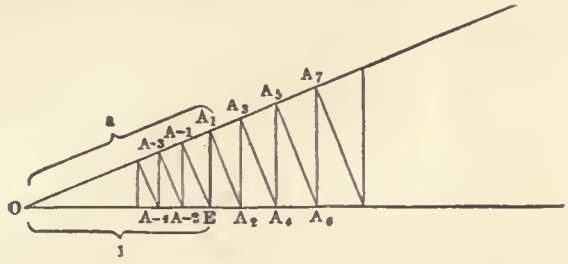


FIG. 42.

Both the preceding methods assume that a is greater than 1; the following may be used when a is less than 1:

III. In Fig. 43 make $O E = 1$, and draw $O A = a$ at such an angle that $A E$ is perpendicular to $O A$. Erect the perpendicular $E 1$, and continue with the alternate perpendiculars 1 2, 2 3, 3 4, etc., and we have: $O 1 = \frac{1}{a}$, $O 2 = \frac{1}{a^2}$, $O 3 = \frac{1}{a^3}$, etc.

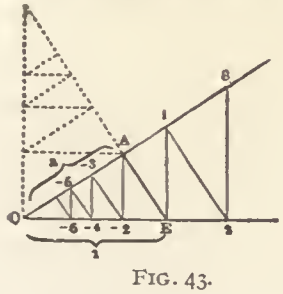


FIG. 43.

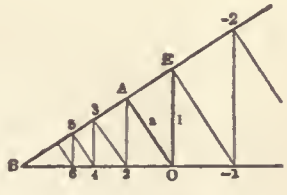


FIG. 44.

Working to the left from E in a similar manner, we get $O - 2 = a^2$, $O - 3 = a^3$, $O - 4 = a^4$, etc., the positive powers being to the left, and the negative powers to the right.

The zigzag lines which are thus drawn back and forth between the two axes have a relation to the powers of a which may be utilized in the following manner:

IV. Make, in Fig. 44, $O E = 1$, $O A = a$, and the angle $O A E = 90°$; also $O B$ at right angles to $O E$, and prolong $E A$ to B. Now draw the alternate perpendiculars as before, and we have the following values: $O A = a$, $A 2 = a^2$, $2 3 = a^3$, etc., also $O E = a^0$, $E - 1 = a^{-1} = \frac{1}{a}$, $- 1 - 2 = \frac{1}{a^2}$, etc.

V. Fig. 45. Make $O E = 1$, and describe upon it as a diameter a semicircle, make $O 1 = a$, and from 1 drop a perpendicular 1 2 upon $O E$, then $O 2 = a^2$ (see Problem III. of this section). With $O 2$ as a radius from O, describe an arc, and from its intersection with the circumference drop the perpendicular 2 4, and $O 4 = a^4$, and by continuing in the same manner, we get $O 8 = a^8$, $O 16 = a^{16}$, etc. The intersection 3 of the radius $O 1$ with the perpendicular 2 4 is, at a distance from O, equal to a^3. For we have: $O 3 : O 1 = O 4 : O 2$; or, $O 3 \cdot a = a^4 : a^2$, that is, $O 3 = a^3$.

In this way we may prove that each line drawn from O to the upper extremity of the successive perpendiculars on $O E$, intersects the following perpendicular at a distance from O equal to the next less power of a. This provides a method of obtaining the intermediate powers of a by merely drawing radii and perpendiculars. Each newly-found power gives a radius for a succeeding one, and the operation may be continued indefinitely, as shown in the diagram.

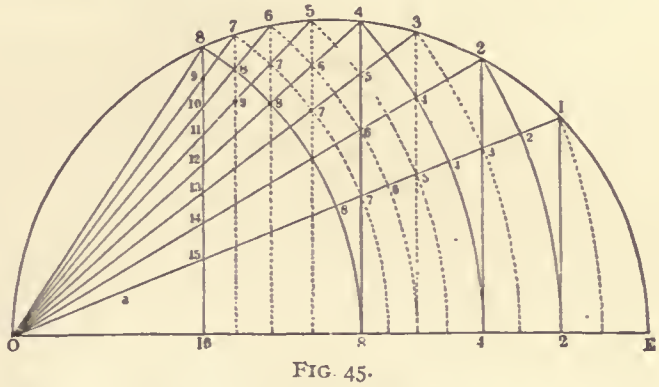


FIG. 45.

VI. The following method is suitable for any given value of a, whether greater or less than 1. In Fig. 46 make $O E = 1$ on the axis $X O X$, erect a perpendicular at O, $Y O Y$, and mark off $O A = a$. Join $A E$, and draw $A 2$ normal to $E A$, and it will cut off on the axis of X, a distance $O 2 = a^2$; then draw 2 3 at right angles to $A 2$, and we get on the axis of Y, $O 3 = a^3$, and on the axis of X, $O 4 = a^4$, thus getting the even, positive powers of a on the axis of X, and the odd powers on the axis of Y. By carrying the spiral in the other direction we get the negative powers in a similar manner. Joining $A E$, we have $O E = a^0 = 1$; from that we get $O - 1 = \frac{1}{a}$, and in a similar manner $\frac{1}{a^2}$, $\frac{1}{a^3}$, etc. (See § 22, VI.) This method is very suitable for showing a succession of powers in a single diagram.

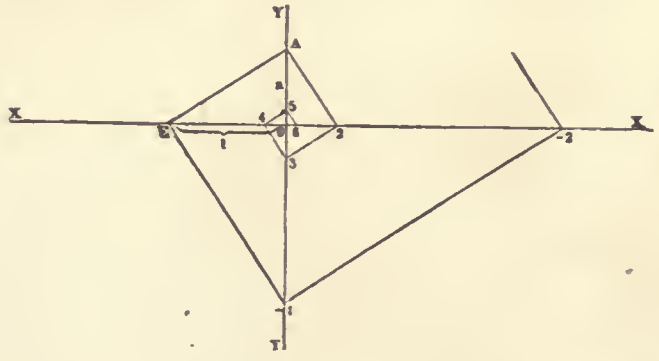


FIG. 46.

§ 29.

POWERS OF THE TRIGONOMETRICAL FUNCTIONS.

The methods already given for the determination of the powers of numbers are also applicable to the powers of the trigonometrical functions with but slight modifications.

I. Powers of Sines and Cosines. Fig. 47. Make $O E = 1$; the angle $E O A = \phi$, the angle the powers of whose functions are to be determined, $E A$ being at right angles to $O A$. Draw also the alternate perpendiculars $A 2$, 2 3, 3 4, etc., and $E - 1$, $- 1 - 2$, etc. Then $O A = \cos \phi$, $O 2 = \cos^2 \phi$, $O 3 = \cos^3 \phi$, $O 4 = \cos^4 \phi$; $O - 1 = \frac{1}{\cos \phi}$, $O - 2 = \frac{1}{\cos^2 \phi}$, etc.

By drawing the alternate perpendiculars $A II.$, $II. III.$, $III. IV.$; $O - I$, $- I$, $- II$, etc., we also get $A E = \sin \phi$, $A II = \sin^2 \phi$, $II, III = \sin^3 \phi$, $III, IV = \sin^4 \phi$, $O - I = \frac{1}{\sin \phi}$, $- I - II = \frac{1}{\sin^2 \phi}$, etc.

II. Powers of Tangents and Cotangents. Fig. 48. Make $E O = 1$, and $O E A = \phi$. Draw from A the spiral of perpendiculars as in V., § 28, and we get the following values: $O A = \tan \phi$, $O 1 = \tan^2 \phi$, $A 3 = \tan^3 \phi$, etc. $O E = 1 = \tan^0 \phi$,

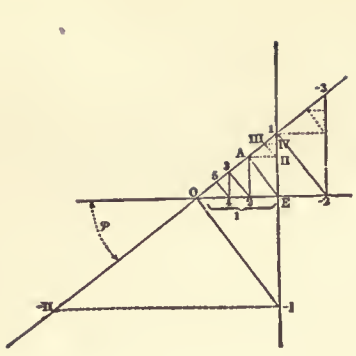


FIG. 47.

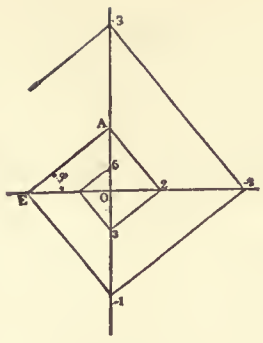


FIG. 48.

$O - 1 = \cot \phi$, $O - 2 = \cot^2 \phi$, etc. This method shows very clearly the convergence and divergence according to the sign of the power under consideration.

§ 30.

EXTRACTION OF ROOTS.

The extraction of the square root is readily performed by the graphical method, as will be seen at once when it is remembered that $\sqrt{a}$ is a mean proportional between a and 1. The previously described methods for powers also suggest methods for

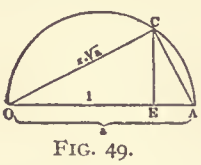


FIG. 49.

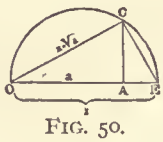


FIG. 50.

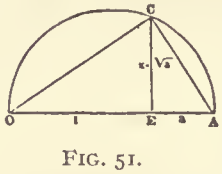


FIG. 51.

the extraction of roots, and the three following cases will suffice:

I. In Fig. 49 make $O E = 1$, $O A = a$, describe a semicircle on $O A$, erect a perpendicular at E, intersecting the circumference at C, and join $O C$, then $O C = x = \sqrt{a}$ (see § 28). In this case $a > 1$, but in the following case $a < 1$.

II. Fig. 50. Make $O E = 1$, $O A = a$, describe a semicircle on $O E$, erect a perpendicular at A, and join $O C$, then will $O C = x = \sqrt{a}$.

III. Fig. 51. Make $O E = 1$, and mark off on $O E$ prolonged $E A = a$, draw on $O A$ a semicircle, and erect a perpendicular at E, intersecting the circumference at C; then will $E C = x = \sqrt{a}$.

The extraction of the fourth root may be performed by repeating the method for square root. The graphical extraction of the cube root, fifth root, etc., is not so simple. Culmann uses for this purpose the logarithmic spiral, and Schlesinger constructs a curve according to the method in § 28, but the advantages are not sufficient to warrant a further examination of the subject at this point.

§ 31.

ADDITION AND SUBTRACTION OF FORCES.

In all the preceding operations we have only considered the lines to represent absolute quantities, and paid little or no attention to their direction or position in the plane of the diagram. The principal advantages of the graphical method are those which are connected with problems relating to the equilibrium of forces, and it is the application of the preceding methods of graphical arithmetic to the calculation of forces which really constitutes the method of graphostatics.

When several forces are acting upon the same point, their resultant may be obtained by the addition of the lines representing the forces when projected upon the co-ordinate axes. This addition of the projection of forces is known as graphical addition. This addition is performed by placing the lines representing the forces end to end, forming a polygon, care being taken to avoid repeating any of the lines. If the forces, 1, 2, 3, 4, 5, 6, Fig. 52, acting at O are in equilibrium, the sum of their projections will equal zero, and the polygon formed by the lines, as shown in Fig. 56, will close. The figure thus constructed is called a force polygon. It is immaterial as to the order in which the lines are taken, as in Fig. 53 the result is the same whether taken in the order, 1, 2, 3, 4, 5, 6, or 1, 3, 4, 6, 5, 2, although the shape of the polygon will be different. As in arithmetic, the graphical subtraction of forces is the reverse of addition, and practically amounts to a separation of the sides of the force polygon into their respective forces. In graphostatics the forces are all taken in one plane, by projecting upon the plane of the diagram those forces which may be without it. The

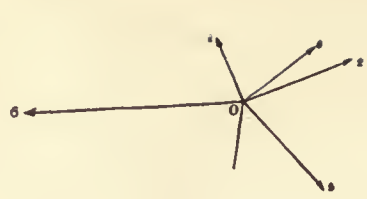


FIG. 52.

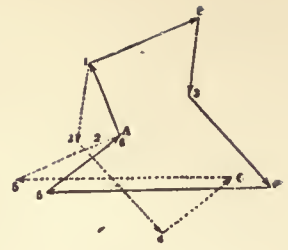


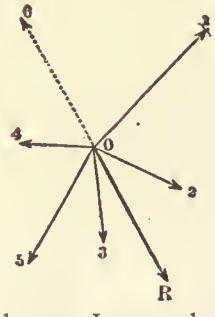
FIG. 53.

preceding method of addition and subtraction of lines, which here represent forces, but which may be taken to represent anything, is called geometrical addition and subtraction. They bear the same relation to geometrical multiplication and division as the corresponding arithmographical methods do to each other. These little used methods, which are of the greatest interest to geometers, we cannot discuss here.

§ 32.

RESULTANT OF SEVERAL FORCES.

In the preceding section we assumed that the given forces held each other in equilibrium, from which it followed that the diagram formed by the lines representing the forces returned to the starting-point and formed a closed polygon. If, however, the force polygon for a group of forces, such, for example, as the forces 1 to 5, Fig. 54, does not close, it follows that equilibrium does not exist at the point O. In order to obtain equilibrium it is necessary to apply a force 6 to the same point, whose direction and extent correspond to the line 5 6 of the polygon. This is the force necessary to bring the other forces into a state of equilibrium, and from it we also obtain a resultant force R, which is given in direction and absolute extent by the closing line of the polygon, but acts as an expression of the algebraic sum of the other forces, as shown by the arrow-head. From this it follows that in every closed force polygon each single force represents the resultant of all the others in absolute extent and direction, except that the resultant tends to produce motion in an opposite direction from the corresponding force in



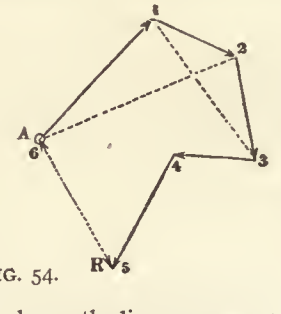


FIG. 54.

the polygon. In an unclosed polygon the line necessary to close the figure gives the direction and extent of the resultant of the other forces, always tending to produce motion opposed to the closing force.

For example, in Fig. 54, A^2 is the resultant for 1 and 2, and in a similar manner the resultant for any of the other forces in combination may be found.

The method of representing the properties of forces by lines is also applicable to other quantities which possess the attributes of magnitude and direction, such as velocities; also to the determination of the path of the line which passes through the centres of gravity of the stones of a vault, for instance; and in a figurative sense it may be applied to scientific discussions, in which the final result acts as a closing line to the force polygon of argument.

§ 33.

ISOLATED FORCES IN ONE PLANE. CORD POLYGON.

If lines which represent forces, and hold a body in equilibrium, do not intersect in one point, a condition which frequently occurs, but have a number of intersections from n to $\frac{n}{2}(n-1)$ in number, the foregoing solution can no longer be used; but at the same time this more complicated case may readily be reduced to the simpler form.

For this purpose we assume the existence of a system of rigid, straight lines which, extending from each force to the next, form a polygon capable of resisting both tension and compression in the direction of its sides, and in which each single force is in equilibrium with the two forces which act along the sides intersecting it. A polygon formed in this manner is called a

Cord Polygon, or in arch construction a thrust line, because all the sides are in compressiou, and in general such a figure may be called a link polygon.

The angles of the cord polygon are called "knots." The link polygon may be used for the investigation of forces according to the preceding methods, when at each knot there exists

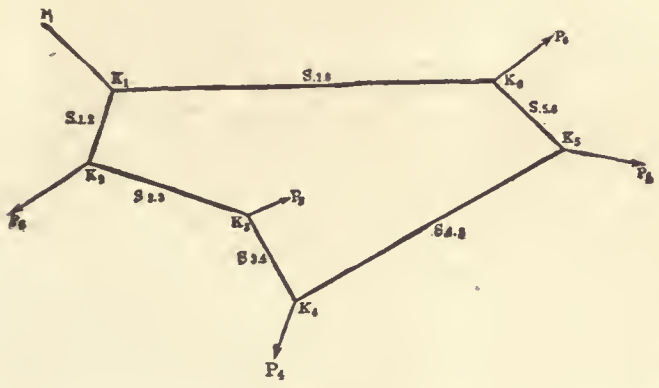


FIG. 55.

an equilibrium between the external forces and the stresses in the sides of the polygon; for example, when the forces $S_{1\cdot2}$ and $S_{2\cdot3}$, at the knot K_2, have a resultant equal and opposed to P_2, in extent and direction. The forces in the sides of the polygon may be called the internal forces of the link polygon. We have, then, for any given case two sets of forces to investigate:

(1) the external forces, (2) the internal forces,

since for each set there exists an equilibrium.

§ 34.

EQUILIBRIUM OF THE EXTERNAL FORCES OF THE CORD POLYGON.

If we take the forces P_1 and P_2, find their resultant, combine this with P_3, find a second resultant, combine it with P_4, etc., we will find that in order to obtain equilibrium, the resultant with the next to the last force P_{n-1}, of the polygon, will be equal and opposed to the closing force P_n. This holds good so long as the direction and extent of the forces remains unchanged. From this it follows that the co-ordinate distances of the point of application of any force may be made equal to zero, without affecting the equilibrium of the external forces. The combiuation of these latter forces may then be effected in the same manner as if they acted at a single point. In this way the force

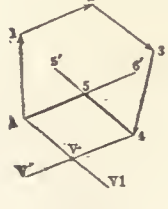


FIG. 56.

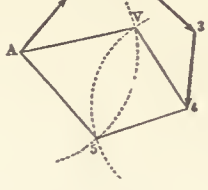


FIG. 57.

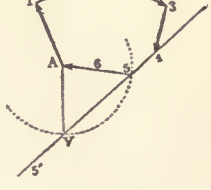


FIG. 58.

polygon can be used to determine the equilibrium of several independently-acting forces. If equilibrium exists, the polygon closes, and if it does not close, it shows the extent and direction of the force necessary to maintain equilibrium. It is practicable in this way to determine *two* unknown quantities in a force polygon. These may also refer to two forces, and may be either direction or extent, or, as sometimes occurs in practice, the direction of one force and the extent of the other.

The following cases will serve to illustrate:

I. Both directions given. In Fig. 56 we have the directions of the force lines 4 5′ and A 6′, and by their intersection at 5, we determine at once their length 4 5 and A 5. If their directions are interchangeable we have two solutions possible, the second giving the directions A VI', and 4 V', and hence the forces A VI and 4 V.

II. The extent of both forces given. Fig. 57. With the distances equal to the extent of the two forces, we describe circular arcs from A and 4, and the intersection of these arcs determines the direction of the forces. Since the arcs intersect at two points, two solutions follow, giving the lines A 5, 4 5, and A V, 4 V.

III. The direction of one force and the extent of the other given. In Fig. 58 let the line 4 5 be the given direction of one force. With a radius A 5, equal to the extent of the other force, describe the arc shown by the dotted curve, and the two intersections give two solutions of the problem, as in cases I. and II. If the arc failed to intersect the line at all, it would prove the case to be impossible.

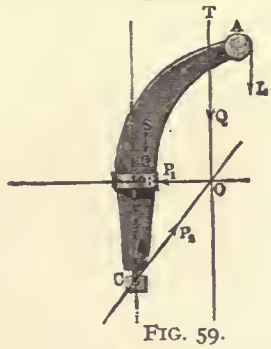


FIG. 59.

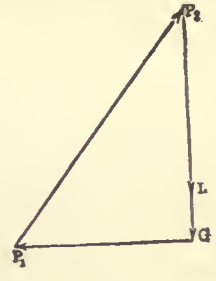


FIG. 60.

The following examples will show the practical applications of the preceding principles:

Example I. A crane A B C, Fig. 59, carries a load L at A; it is of a cylindrical shape at B, and held in position hy a roller bearing, and at C there is also a pivot step. Required the forces P_1 and P_2 at B and C. The centre of gravity of the crane itself is at S, and its weight is equal to G.*

Both L and G act in a vertical direction, and the force at P_1, if the bearing is smooth and we neglect its friction, acts in a horizontal direction. Combining G and L into one force $Q = G + L$, the position of whose resultant is T Q, we have the intersecrion O of a vertical through T Q, with a horizontal through P_1 as a point in the direction of the line of the force P_2. This force must also act through the centre of the pivot C, since this is restrained from lateral motion by its bearing. This gives C O for the direction of the force P_2. We can now draw the force polygon, Fig. 60, drawing $L + G$ vertical, G P_1 parallel to O P_1, and P_2 P_1 parallel to O C. This determines the extent of both P_1 P_2, and hy further analysis the entire load on the pivot C may be found.

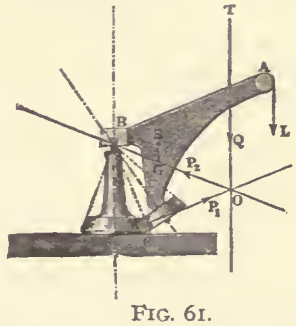


FIG. 61.

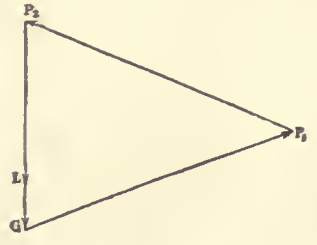


FIG. 62.

Example II. A crane constructed as shown in Fig. 61 carries a similar load to the preceding. It is arranged with a cylindrical bearing at B, and at C there is a conical roller bearing upon a conical surface on the base of the column, the axes of both cones intersecting in the middle of the bearing B.

We have, as before, the mean load $Q = L + G$; we also have the direction of the pressure P_1, as it must be normal to the surface of the cone at the point of contact. The intersection of P_1 and Q determines O, and a line from O through the centre of the bearing B gives the direction of P_2. The force polygon can now be drawn, as shown in Fig. 62, and hy making the vertical equal to Q, and the other two sides parallel to P_1 and P_2, we determine at once the extent of the two latter forces. The vertical component of P_2 will, in this case, be less than the total load Q, while in the previous example they were alike. Hence it follows that the conical roller supports a portion of the load.

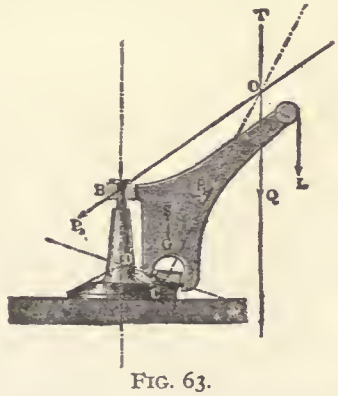


FIG. 63.

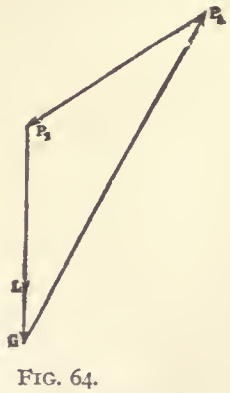


FIG. 64.

Example III. The crane shown in Fig. 63 is similar in construction to the preceding one, except that the axes of the conical rollers intersect at a point D below the bearing B. If we now draw C O normal to the surface of contact C D, to the point O, and construct the force polygon, Fig. 64, we see that the change in the position of the apex of the cone D causes the force P_2 to act from helow instead of from above, as in the previous case. It will therefore be necessary to provide the bearing B with a collar to oppose the upward pressure.†

* In ordinary wharf cranes the value of G, which mainly depends upon the capacity and overhang of the crane, may be taken at ⅛ to ¼ the load.

† This defect may be seen in numerous existing examples of crane construction. In a case which came under the author's observation, a crane intended to have a capacity of thirty tons gave way under a load of only about twenty tons, because the proper provision was not made for the direction of a force upon a bearing.

Example IV. Three forces of 70, 50 and 80 pounds act, as shown in Fig. 65, upon a body $A\ B$ in such a manner that their resultant passes through the point A. Two other forces of 95 and 60 pounds also act upon the point A, and hold the preceding forces in equilibrium. Required the angles which the latter forces make with the former.

Lay out the forces of 70, 50 and 80 pounds, as shown from C to D in Fig. 66, by the heavy lines, then describe from C and D circular arcs with radii of 60 and 95 respectively, and thus obtain the intersections $E\ E'$ or $F\ F'$, which, when joined with C and D, complete the force polygon. Both solutions are given in the diagram.

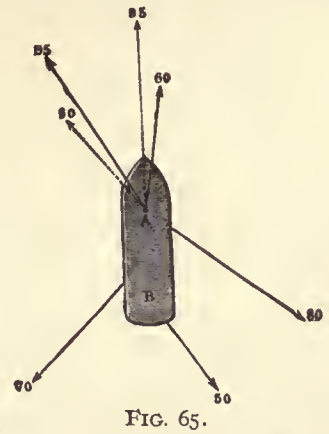


FIG. 65.

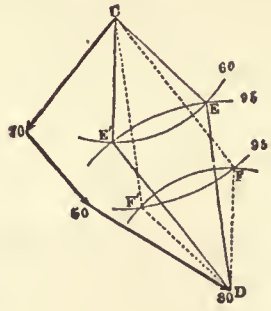


FIG. 66.

Example V. An obelisk is to be raised upon its base, Fig. 69, by turning it about the angle A, the lifting force to be applied in a given direction at the apex B.

Required the direction to be given to a force P_3 of given extent, applied to the point A, in order that the base shall only be subjected to vertical pressure. Draw a vertical line through the centre of gravity S of the obelisk, intersecting the direction of the force P_1 at O. A line from O through A will then give the direction of the resultant of the two forces. This resultant is now to be resolved into a vertical component P_2, and a force P_3 of given extent but undetermined direction. To determine the direction we draw, as in Fig. 68, $C\ Q$ and $Q\ P_1$, and erect a perpendicular

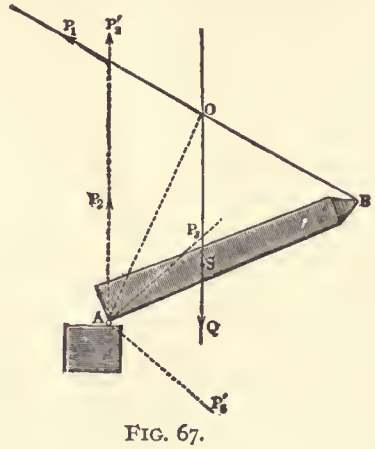


FIG. 67.

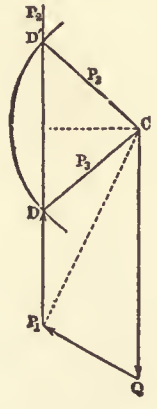


FIG. 68.

through P_1. From C with a radius equivalent to P_3, describe an arc, intersecting the vertical at D and D', showing that two solutions are possible—one giving P_2 the value $P_1\ D$ and P_3, the direction $D\ C$; the other giving P_2 the value $P_1\ D_1$ and P_3, the direction $D_1\ C$.

If P_3 should just equal the perpendicular distance from C to $P_1\ P_2$, then but one solution exists. The two results for the example given are shown in Fig. 67 at $A\ P_3$ and $A\ P_3'$.

Examples of this character seldom occur in actual practice.

§ 35.

EQUILIBRIUM OF INTERNAL FORCES IN THE CORD POLYGON.

As already stated, we mean by the internal forces of the cord or link polygon the tension or compression which may exist in the different sides of the figure, as shown at $S_{1\cdot2}$, $S_{2\cdot3}$, etc., Fig. 69. These forces are of such an extent that they hold each other in equilibrium at the knots $K_1\ K_2\ K_3$, etc. Any two of these, for example, $S_{1\cdot2}$, $S_{2\cdot3}$, may be determined from their resultant P_2, when either their direction, their magnitude, or one direction and one magnitude are given (see § 34). This is done in the following manner: Construct the force polygon, Fig. 70, of the external forces P_1, P_2, P_3, which, if equilibrium exists, will form a closed figure. From the extremities of the sides corresponding to the force P_2 draw two lines parallel to the sides $S_{1\cdot2}$, $S_{2\cdot3}$, intersecting at O; then the length of the lines O_1 and O_2 will represent the magnitude of the stresses in the sides $S_{1\cdot2}$, $S_{2\cdot3}$. In a like manner we may draw lines connecting the several corners of the polygon, Fig. 70, with the pole O, and determine all the internal forces of the link polygon, both in magnitude and direction; so that when the external forces are known, and also the direction of two of the internal forces, the direction and magnitude of the others can be determined. This assists greatly in the construction of the link polygon, for by selecting one knot and determining the pole O, the sides of the link polygon can be drawn parallel to the respective rays.

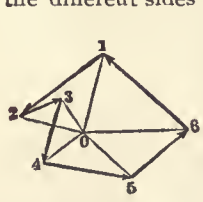


FIG. 70.

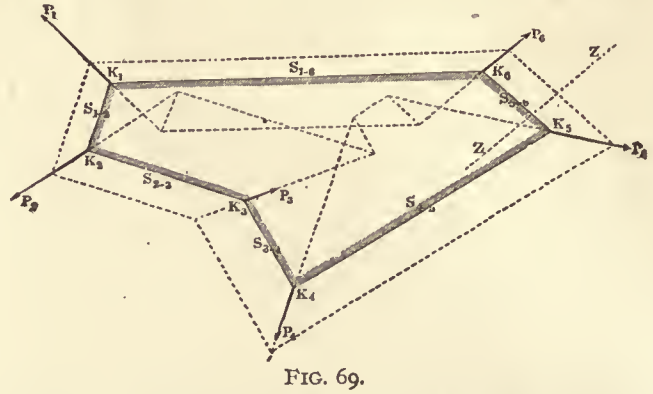


FIG. 69.

The actual lengths of the sides of the link polygon are determined by the positions of the lines of the external forces, from which the positions of the internal forces are also determined.

The cord polygon will vary in its form according to the choice of a starting-point from which it is drawn. In Fig. 69 two forms are shown in dotted lines within the cross-hatched figure, their sides being parallel to those of the first polygon. Another solution of the same problem (the combination of the external

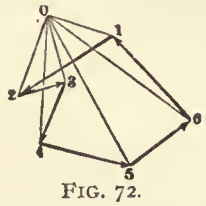


FIG. 72.

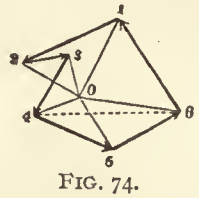


FIG. 74.

forces into a link polygon) may be obtained by an application of the double solution of Case I., § 34.

In Fig. 72 we have the directions $S_{1\cdot2}$ and $S_{2\cdot3}$ drawn from the extremity of the force P_2, giving a new cord polygon, Fig. 71, of a very different form from the preceding one, which is also included in Fig. 71 for purposes of comparison. With the exception of the first two sides, we have an entirely different

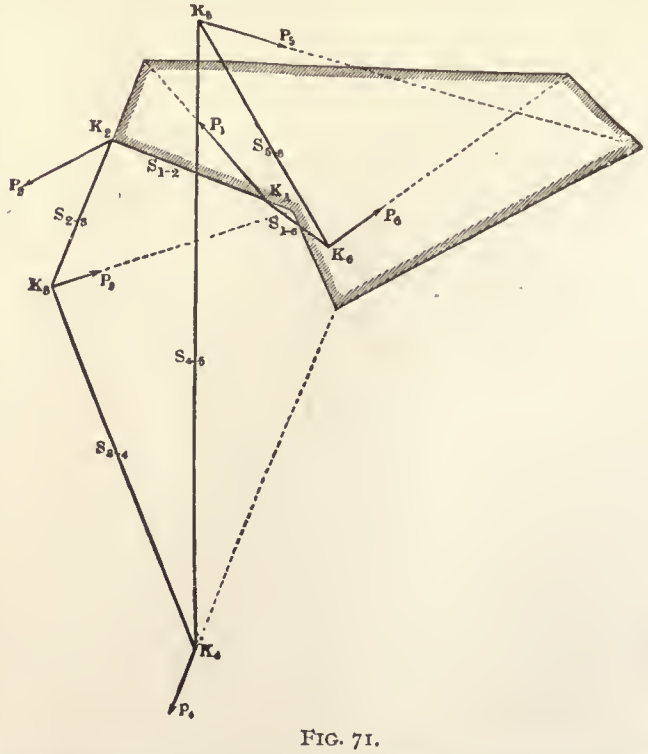


FIG. 71.

figure, showing the variety of polygons which may be constructed from a given set of forces.

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The cord or link polygon, when taken in connection with the force polygon, forms what has been termed the graphical plan of forces. In most cases the entire subject can be discussed by the construction of one figure which may then be called the Force-pan, and of which examples are given in § 48.

§ 36.

RESULTANT OF ISOLATED FORCES IN ONE PLANE.

If we assume two of the sides of a cord polygon to be divided, and insert at the points of division forces corresponding to the stresses in the divided sides, the equilibrium will remain undisturbed, as, for instance, in Fig. 73, the sides $K_1 K_6$, and $K_4 K_5$, are cut and sustained. It will then be evident that the resultant of the forces, $S_{1\cdot6}$ and $S_{4.5}$, either on the right or the left will hold the remainder of the forces of the polygon in equilibrium. The position of this resultant force is determined by prolonging the sides until they intersect at M. The direction and extent of this resultant is determined in the force polygon, Fig. 74, by the diagonal 4.6, which is the closing line of the forces $S_{1\cdot6} = O_6$, and $S_{4.5} = O_4$. This force is also on the one side the resultant of the forces P_5 and P_6, and on the other side, of the forces P_1, P_2, P_3 and P_4.

In general it may be stated that the point of intersection of any two prolonged sides of the polygon is a point of the resultant of all the external forces beyond these sides, from which the direction and extent of said resultant may be determined.

This principle is of great utility, as many examples will hereafter illustrate. By reversing the above rule, the cord and force

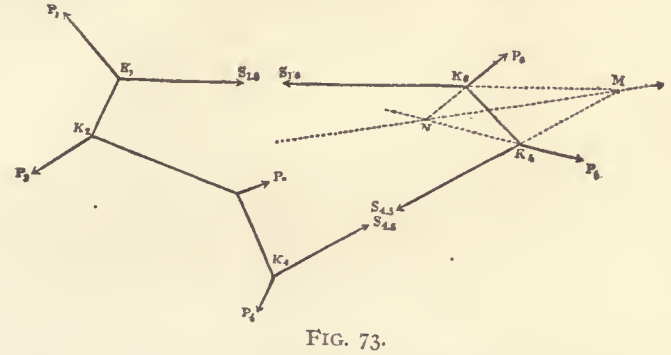

FIG. 73.

polygons may also be used for the decomposition of forces, as well as their resolution. For instance, if it is desired to decompose the force 4.6 into two others, P_5 and P_6, of giveu direction, draw one of them (for example, P_6) in the cord polygon until it intersects 4.6 in the point N, and through this point draw P_5, parallel to the side 4.5 of the force polygon. The first chosen line, $K_6 N$, may be drawn either forwards or backwards on $M N$, without disturbing the equilibrium.

§ 37.

CONDITIONS OF EQUILIBRIUM FOR ISOLATED FORCES IN ONE PLANE.

In the preceding discussions it has beeu assumed that the forces whose equilibrium has been investigated were so situated that equilibrium really existed, so that according to the rule in the preceding paragraph it would be possible to reduce them to

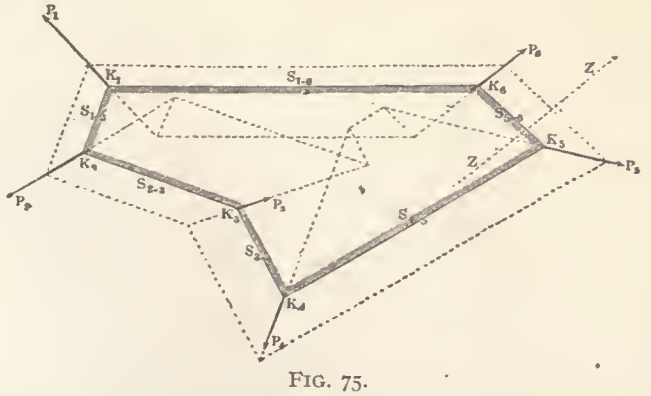

FIG. 75.

two equal and opposing forces. This is, however, not necessarily the case when the force polygon is a closed figure, but it must follow when the cord polygon is also a closed figure, *i. e.*, the actual positions of the forces must also be taken into account. If the positions are not correctly taken, the cord polygon will show what modification must be made in order to secure equilibrium and avoid the formation of rotating couples; which will be discussed in the next section. For this purpose one of the forces should be left to be determined in position a the last.

I. Let this force be P_6, Fig. 75. Its magnitude is known, and its direction is parallel to the given line $Z Z$. After constructing the force polygon, Fig. 76, choose a pole O, and draw the rays to the angles from 1 to 6, so that $K_1 K_2$ is parallel to 1 O, $K_2 K_3$ to 2 O, $K_3 K_4$ to 3 O, etc., until $K_5 K_6$ is reached. Then the closing line of the cord polygon must have the direction 6 O, and must also pass through K_1. This determines its position entirely, and its intersection K_6 with $K_5 K_6$ is a point of the force P_6, which is now drawn parallel to 5.6.

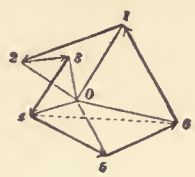

FIG. 76.

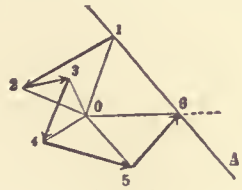

FIG. 78.

If the final force is not given either in direction or magnitude, it may be determined from the direction and position of the other forces as follows:

II. Let the yet indeterminate force be P_6, Fig. 77, while we have given the direction of the force P_1, which is $K_1 P_1$, and its position K_1. We can draw the force polygon from the points 1 to 5, while from the point 1 we have only given the direction A 1. The cord polygon may also be commenced by starting from K_1, and continuing through the points K_2, K_3, K_4, K_5 and

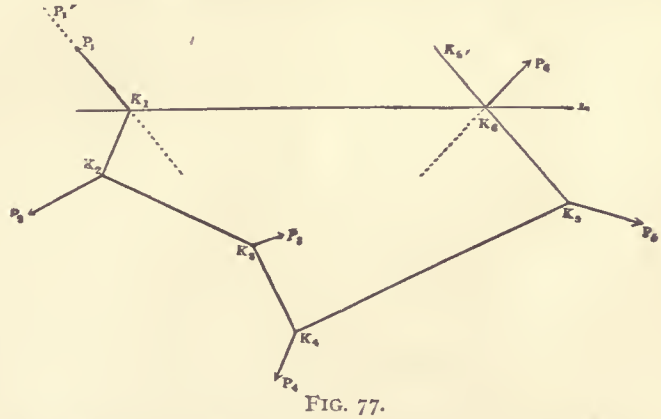

FIG. 77.

K'_5. We may then select any direction for its closing side $K_1 L$, and its intersection K_6 with $K_5 K'_5$ will be a point in the line of the desired force P_6. In order to determine its magnitude and direction, draw, in Fig. 78, O 6 parallel to $K_1 L$, and join the point 5 with the point 6, when the line 5.6 will give the desired magnitude and direction of the force P_1.

§ 38.

FORCE COUPLES.

When a plane figure is subjected to the action of forces in conples, acting in its plane in such a manner that, while equal in magnitude and opposite in dirccion, they fall upon parallel lines, and do not oppose each other in the same straight line,

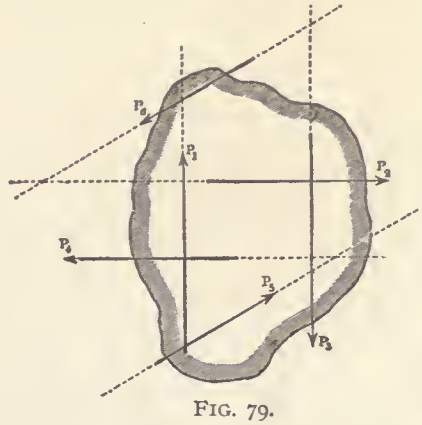

FIG. 79.

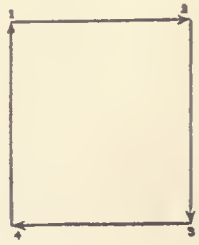

FIG. 80.

the force polygon will close without necessarily proving the existence of equilibrium iu the figure.

The conditions which obtain under these circumstances may be examined as follows:

The forces P_1 P_3 and P_2 P_4, Fig. 79, form a closed force polygon 1, 2, 3, 4, Fig. 80, but at the same time equilibrium does not

exist in the figure, but instead, a tendency to rotate about a common point with a statical moment which is equal to the sum of the moments of the couples $(P_1 - P_3)$ and $(P_2 - P_4)$. In order to secure equilibrium it is necessary to introduce an additional couple $(P_5 - P_6)$, whose tendency shall be to cause a rotation in the opposite direction, and whose moment shall be equal to the combined moments of the previous couples, and whose direction shall be parallel to the lines *V V* and *VI VI*, Fig. 82.

Let us take, Fig. 81, the force polygon *A* 1, 2, 3, 4. This is not yet complete, for we still lack the forces 5 and 6. We know that they must act through *A*, in opposition to the other couples, but their magnitude is not yet determined.

As already said, the two forces must be equal and parallel in order to be in equilibrium with the other couples, and only two forces can fulfill the conditions. Their direction is given, and can be laid off as at *A Z*. We choose any pole *O*, and join the rays *O A*, *O* 1, *O* 2, *O* 3, *O* 4 (= *O A*), and can then proceed to construct the cord polygon, Fig. 82.

For this we have lines of direction *I I*, *II II*, etc., up to *VI VI*, given from Fig. 79. Starting from any point K_1 on *II*, we draw lines parallel to the rays *O A* and *O* 1 (their resultant being the force P_1) until they intersect *VI VI* in K_6, and *II II* in K_2; then draw $K_2 K_3$ parallel to *O A*, intersecting *III III* at K_3, $K_3 K_4$ parallel to *O* 3 until it intersects *IV IV*, and $K_4 K_5$ parallel to *O* 4, intersecting *V V* at K_5. Only the closing line

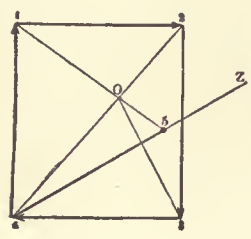

FIG. 81.

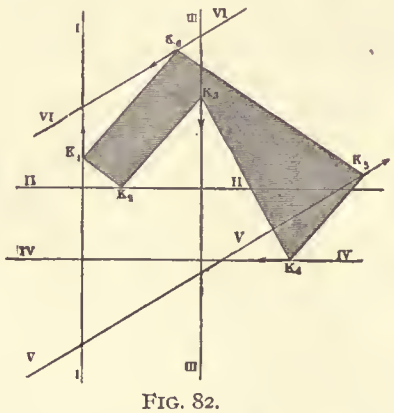

FIG. 82.

of the cord polygon is now lacking, as it is the line joining K_5 with K_6, which latter point has already been determined. We can now (see § 37, II.) draw the ray O_5 parallel to $K_5 K_6$, completing the force polygon and the line *A* 5, will give the magnitudes of P_5 and P_6. The path around the force polygon may be taken as *A* 1, 2, 3, 4, 5 *A*, the sides 4-5 and 5 *A* being supposed to make an infinitely small angle with each other.

The previous examples upon the force and cord polygon serve to show how geometrical addition and subtraction may be used to determine the equilibrium of diverging forces in one plane. Forces acting in intersecting or parallel planes may be examined in the same manner, and in many cases without a great degree of complication, as some following examples will illustrate. It is not intended, however, to undertake a general discussion of the subject here, but rather proceed at once to practical applications of the special case of parallel forces.

§ 39.

Equilibrium Between Three Parallel Forces.

In discussing the equilibrium between parallel forces, we may use purely arithmetical methods, or use geometrical addition and subtraction (force and cord polygons), as may be found most convenient.

The present problem may be stated as that in which a force *Q* acts upon a body, and is to be held in equilibrium by two un-

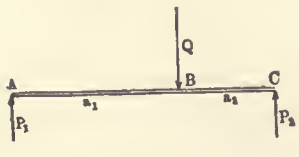

FIG. 83.

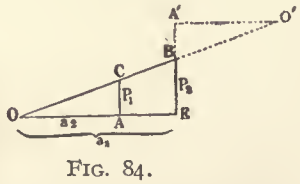

FIG. 84.

known forces, P_1 and P_2, acting parallel to it and to each other.

Drawing the line *A B C*, Fig. 83, normal to the given direction of the forces, we must have, in the existence of equilibrium, $P_1 . \overline{A B} = P_2 . \overline{B C}$, or $P_1 a_1 = P_2 a_2$, and also $P_1 + P_2 = Q$.

In order to determine $P_1 = P_2 \frac{a_2}{a_1}$ graphically, we may follow the method in § 24, and in Fig. 84 make *O E* = the divisor a_1, *O A* = the factor a_2, and taking *E B* to represent temporarily the force P_2, draw *A C* parallel to *E B*, which gives the proportional value of P_1. By placing the triangle *C A O* in the dotted position *O′ B A′*, we have $A' E = P_1 + P_2 = Q$. This gives a figure in a form well suited for application to Fig. 83, as will be shown in the following examples:

I. In Fig. 85 draw *A D* equal in value to *Q*, join *D* with the third point of application *C*, and prolong *Q* until it intersects

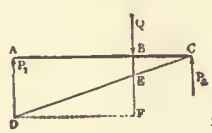

FIG. 85.

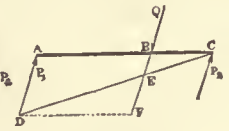

FIG. 86.

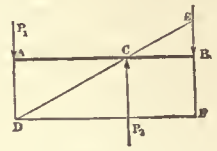

FIG. 87.

at *F* a line drawn through *D* parallel to *A C*. Then will we have the following relations, $B E = P_1$, $E F = P_2$. In Fig. 86 is shown a similar case, but with *Q* inclined to *A B C*, and in Fig. 87 *Q* is beyond *A C*.

II. By resolving the force *Q* into two components applied at the points *A* and *C*, Figs. 88, 89, 90, we obtain inclined forces

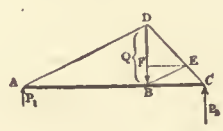

FIG. 88.

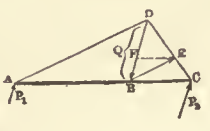

FIG. 89.

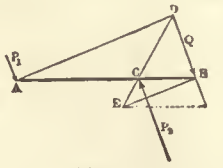

FIG. 90.

whose components parallel to *Q* are the desired values for P_1 and P_2, while the components which are parallel to *A B C* neutralize each other. In all three figures $B F = P_1$ and $F D = P_2$.

III. By constructing the force polygon, making *A D* = *Q*, and using any pole *O*, Figs. 91, 92, 93, and drawing the sides of the link polygon, so that *a b* is parallel to *A O*, *b c* parallel to

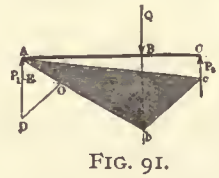

FIG. 91.

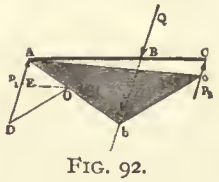

FIG. 92.

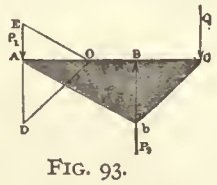

FIG. 93.

D O, and joining the closing line *c A*, the parallel to the latter in the force polygon *O E* will give $E A = P_1$, and $D E = P_2$.

If it is desired to make the closing line fall upon *A B C*, or lie parallel to it, the cord polygon *A b C* must be first drawn, and the pole *O*, determined by the intersection with *A B* of a line *D O* parallel to *b C*, *D A* having first been drawn equal to *Q*, *O E* may then be drawn parallel to *A b*, and we have $E A = P_1$, and $E D = P_2$.

In these cases *Q* is equal in magnitude to the resultant of P_1 and P_2, and opposed to them in direction. If *Q* is to be determined when P_1 and P_2 are given, similar methods to the foregoing are to be followed.

Returning to the diagram *O E A C B*, Fig. 94, which we have already used in case *I*, we construct the triangles *C A O* and

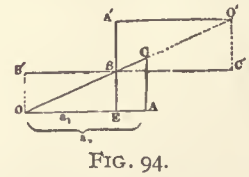

FIG. 94.

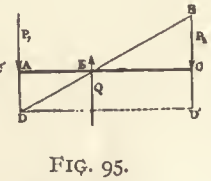

FIG. 95.

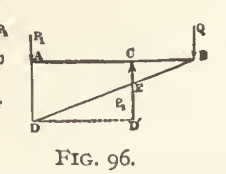

FIG. 96.

B A′ O′, and draw *B′ C′* parallel to *O A*; *O′ C′* and *O B′* parallel to *A′ B*, giving $B' B = a_1$, $B C' = a_2$, $B' O = P_2$, $O' C' = P_1$.

From this we obtain the following solutions:

IV. Transfer one of the forces to the opposite side of *A C*,

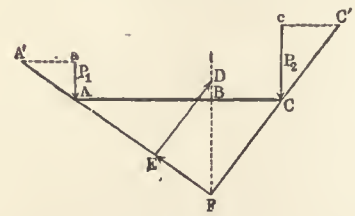

FIG. 97.

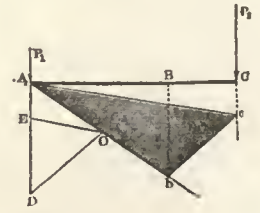

FIG. 98.

Figs. 95, 96, so that $A D = P_2$ and $E C = P_1$, join *D* to *E*, and

the line $D\,E$ will intersect $A\,C$ at B, which will be the point of application of the resultant Q, whose magnitude $= E\,D' = P_1 + P_2$, since $D\,D'$ is drawn parallel to $A\,C$.

In Fig. 96 P_1 and P_2 act in opposite directions, and their algebraic sum $D'\,E$ must be taken, and, as shown, the resultant Q acts beyond $A\,C$.

V. The method shown in Fig. 97 follows from (II):

From the extremity a of $a\,A = P_1$ draw a line $A'\,a$ of any length, making it parallel to $A\,C$. In a similar manner draw $c\,C'$ from the extremity of $c\,C = P_2$. Draw $A'\,A$ and $C'\,C$, prolonging them until they meet at F, which latter will be a point in the line of the resultant $F\,B$, and the value of Q will be $P_1 + P_2$, which is also the resultant of $D\,E = C'\,C$ and $E\,F = A'\,A$.

VI. Following the method in (III), we may proceed as follows, Fig. 98: Make $D\,E = P_2$, $E\,A = P_1$, choose a pole O, and join the closing line $O\,E$ of the force polygon. Draw $A\,c$ parallel to $E\,O$, $c\,b$ parallel to $O\,D$, and $A\,b$ parallel to (or, as in this case, the prolongation of) $A\,O$, and the intersection b will be a point in the line of the resultant Q, whose magnitude $= D\,A$.

§ 40.

Resultant of Several Parallel Forces.

When we have a number of parallel forces Q_1, Q_2, Q_3, Q_4, acting upon a body in given positions in one plane, we can determine their resultant by a combination of the preceding methods, resolving them in pairs until all are combined.

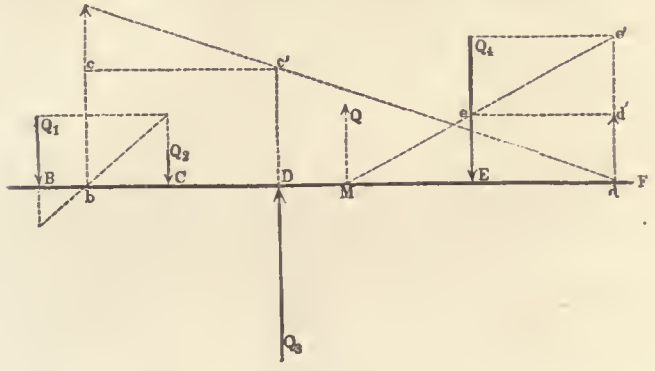

Fig. 99.

I. In order to combine the forces Q_1 to Q_4, intersecting a common normal $A\,F$, Fig. 99, we first combine Q_1 and Q_2 by transposition, as in Fig. 96, and obtain the resultant, $Q_1 + Q_2 = b\,c$. This may then be combined with Q_3, giving $d\,d' = Q_1 + Q_2 + Q_3$, and this result with Q_4, which finally gives the resultant, $Q = Q_1 + Q_2 + Q_3 + Q_4$ passing through M. This solution is one which is sometimes desirable in machine construction, as, for example, in the distribution of the weight of a locomotive engine upon the various axles. The method of determining the resultant of several parallel forces in this way by the successive combination of pairs is very tedious and of limited application, and the method given below of using the force and cord polygons is much simpler.

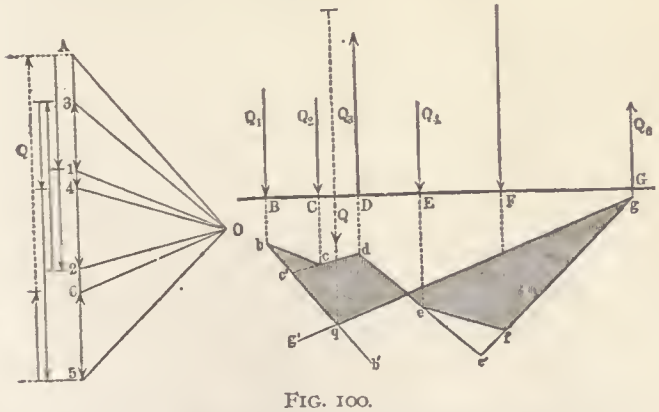

Fig. 100.

II. Fig. 100. Form the force polygon of the given forces Q_1 to Q_6, by laying off lines successively from A, equal in length to the magnitudes of the several forces A 1, 2, 3, 4, 5, 6, as shown in the left of the figure. The magnitude of the resultant will then be equal to the length of the closing line 6 A.

To determine its position, proceed as follows: Select any point beside the line A 5, as a pole O, and join the rays $O\,A$, O 1, O 2, O 3, etc. Starting from a point b under Q_1, draw $b\,b'$ parallel to $A\,O$, and $b\,c$ parallel to 1 O, and continue by drawing $c\,d$ parallel to 2 O, $d\,e$ parallel to 3 O, etc., and finally reaching the closing line of the polygon $g\,g'$ parallel to O 6, intersecting $b\,b'$ at q, which determines the position of the resultant Q (see § 35).

The method shown in (§ 36) may also easily be applied to the resolution of such forces, as in Fig. 100, the intersection of $d\,c$, prolonged to c', gives the position of the resultant of Q_1 and Q_2, and its magnitude is shown at $A\,.\,_2$ in the force polygon, and in a like manner e' is the position of the resultant of Q_4 and Q_5.

§ 41.

Decomposition of Forces into Two or More Parallel Forces.

The methods of resolving forces by means of the cord polygon will also serve for their decomposition.

If, for example, in any portion of a cord polygon $a\,q\,b\,c\,d$, Fig. 101, it is desired to substitute for a force Q, two forces Q_1 and Q_2 passing through e and f, we have only to join the points e and f to obtain the form of cord polygon for the new forces,

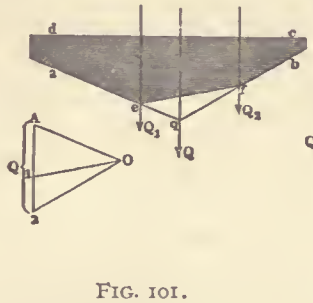

Fig. 101.

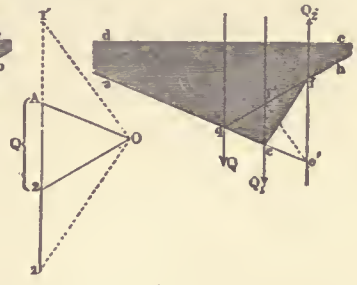

Fig. 102.

and determine their relative magnitudes by drawing O 1 parallel to $e\,f$ in the force polygon below. If the required force Q_1 and Q_2 both lie on the same side of Q, Fig. 102, the solution is similar. We now prolong $a\,q$ to its intersection e with Q_1, and join $e\,f$. Also mark the intersection of Q_1 with $q\,b$, and Q_2 with $q\,a$. In the force polygon below we have $Q_1 = A$ 1, $Q_2 = 1\,.\,2$, or A 1′ $= Q_2$ and 1′ . 2 $= Q_1$.

If we have a beam $A\,G$ loaded with parallel forces Q_1 to Q_5,

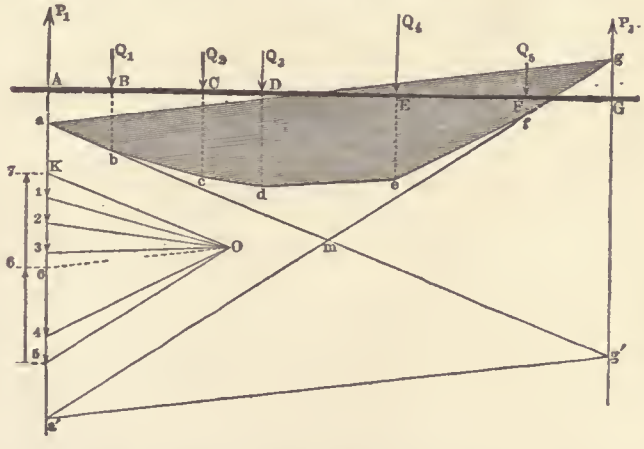

Fig. 103.

Fig. 103, whose load is to be opposed by reactions P_1 and P_2 at A and G, we may first determine a resultant Q of all the forces, as in (§ 40), and then decompose this into values for P_1 and P_2 by the method just given. We also omit the determination of Q altogether, and proceed to determine P_1 and P_2 directly as follows:

Choose any pole O, and form the force polygon K 1 . 2 . . . . 5 O, and construct the cord polygon, making its sides parallel to their respective rays, and draw $b\,a$ parallel to $K\,O$ and $f\,g$, parallel to O 5, their intersections with the lines of the forces P_1 and P_2 being a and g. Join $a\,g$, which will be the closing line of the polygon, and its parallel O 6 in the force polygon gives $P_2 =$ 5. 6 and $P_1 =$ 6 . 7. If the sides $a\,b$ and $f\,g$ of the cord polygon are prolonged in the other direction we obtain a' and g', giving, however, the same result, since $a'\,g'$ is parallel to $a\,g$. The cord polygon would then be the figure $a'\,g'\,m\,b\,d\,c\,e\,f\,m\,a'$, and m indicates the position of the resultant of the forces Q_1 to Q_5, or of P_1 and P_2.

When a loaded beam is supported by three or more bearings it is necessary to take into account the resistance of the beam itself with some degree of accuracy, or else the problem becomes indeterminate. This indeterminate character may, however, be eliminated by the introduction of an equalizing lever.

Suppose we have, Fig. 104, a beam $B\,C\,D$, the resultant Q of whose entire load acts at M, and is opposed by the reactions of three supports at P_1, P_2, P_3, at right angles through the points B, C and D.

We may now assume, temporarily, an approximate ratio between two of the forces, *e.g.*, P_1 and P_2, and permit them to act at the extremities of an equalizing beam $B_1\,C_1$, which in turn supports the main beam at $E\,E_1$; making the ratio of $E_1\,C_1 : E_1\,B_1$ the same as has been chosen for $P_1 : P_2$. Now decompose Q into the components acting at E and D by means of the cord and

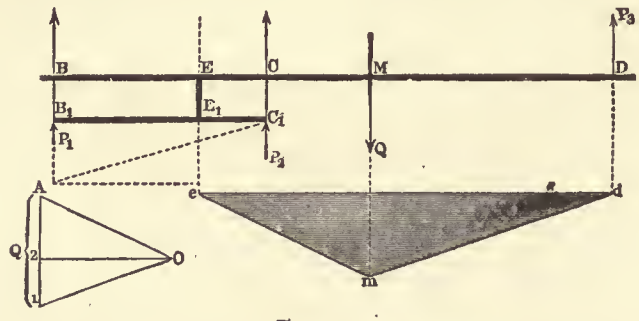

FIG. 104.

force polygons $e\,m\,d$ and $A\,O\,1\,2$. This gives $A\,1 = Q$, $1\,.\,2 = P_3$, $2\,A = P_1 + P_2$, which last sum may be then divided between B_1 and C_1 by any of the above methods.

Each different approximation of the ratio $\frac{P_1}{P_2}$, will give a different value for P_3. If P_1 and P_2 are made equal to each other, E will be in the middle of $B\,C$, and the equalizing lever will be of equal arms. The distribution of the load of locomotives and cars upon their spring is usually made with such equalizing levers.

If the load is to be supported upon more than three or four points it will be necessary to use several equalizing levers, and

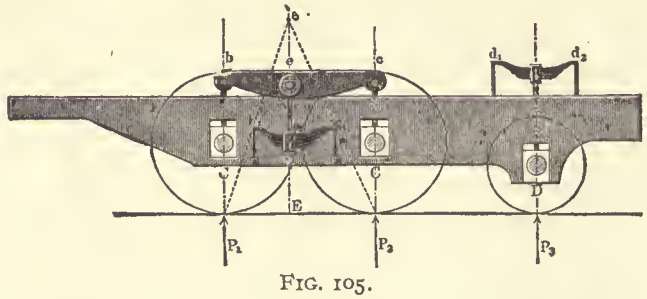

FIG. 105.

examples of this will be found in some locomotives. If, for example, we suppose M, Fig. 105, to be the point of application of the total load Q of a locomotive, supported upon three axles $B\,C\,D$ in such a manner that the weight shall be transmitted to the axles through the springs as shown, and also that the weights upon the wheels C and D shall bear a determinate relation to each other. This can be accomplished by the use of three springs and one equalizing lever upon each frame of the locomotive, the whole weight being thus supported upon eight points. Taking the relation between the forces P_2 and $P_3 = p : q$, we erect a perpendicular $E\,e$, whose distance from the axle C and D is in the proportion $q : p$. From any point e' on this

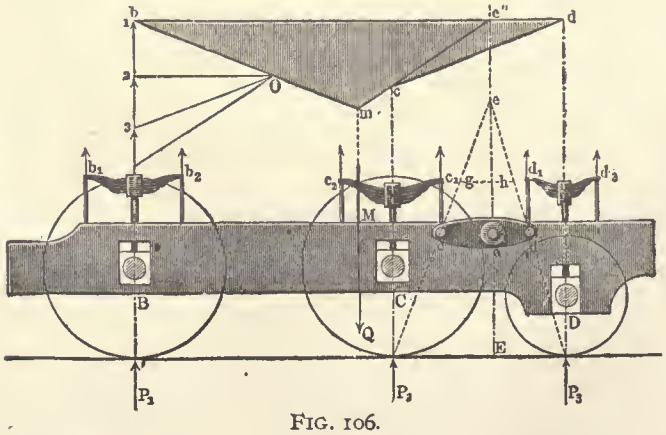

FIG. 106.

line draw lines to the bearing points of the wheels upon the rails, and any horizontal line will intersect these inclosed lines in points which will give the proportional length of arms $c\,e\,d$ for the equalizing lever. The distances of the points c and d from the verticals through C and D give the length of the arms of the springs $c_1\,c_2$ and $d_1\,d_2$. These springs are made with arms of equal stiffness, since they are to support equal loads at both ends. For any chosen ratio $p : q$, and given distance between the axles, the actual length of the equalizing lever will not affect the ratio of P_1 to the sum $P_2 + P_3$, as an inspection of the cord polygon $b\,m\,c\,d$ will show.

The springs which are attached to the ends of the equalizing lever must, of course, be made of sufficient stiffness to support the load which is thrown upon them, and the length of the supports and their proportions chosen according to the previously determined distribution of the weight.

Many similar examples to the preceding might be given, as they are of frequent occurrence in practice. The two springs which are attached to the equalizing lever may be replaced by a single spring, as in Fig. 106. In this case the axes $C\,C$ are connected rigidly to the lever $b\,e\,c$, and the lever itself rests upon a spring $b_1\,e_1\,c_1$, whose extremities are fastened to the frame.

The arms $b_1\,e_1$ and $c_1\,e_1$ of the spring are of unequal length, and have the same relation $p : q$ as that which exists between the arms of the lever $b\,e\,c$. If the arms of the lever are not properly proportioned, or if any error has been made in the distribution of the load, it will be made apparent by the inclined position which will be assumed by the equalizing lever.

§ 42.

UNIFORMLY DISTRIBUTED PARALLEL FORCES.

When a beam is subjected to a uniformly distributed load, the force and cord polygons cannot be determined by the preceding methods, since in such cases the cord polygon becomes a figure

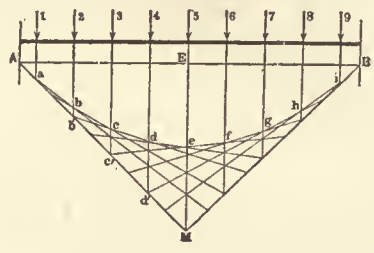

FIG. 107.

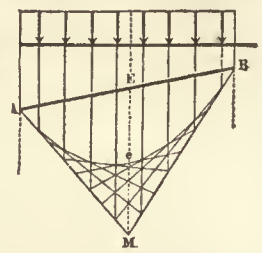

FIG. 108.

of curved outline. The character of the curve may be determined in the following manner: If we assume the load to be concentrated at a number of equidistant points, as in 1, 2, - - - - 9, Fig. 107, and construct the cord polygon for these conditions, it will be evident that the sides $a\,M$ and $b\,c$ will intersect midway between 1 a and 2 b, and also midway between $a\,b'$, since the forces 1 and 2 are equal to each other. In the same manner $c\,d$ and $a\,M$ intersect midway between 3 c and 1 a, which is also in the line of 2 b, that is, at b', and likewise $d\,e$ and $a\,M$ intersect midway between b' and c'. In this way it may be shown that the intersections of the prolonged sides of the polygon from $a\,M$ to $i\,M$ are at equal distances from each other. This indicates a known property of the parabola whose vertex lies on line $E\,M$, and whose abscissa $e\,E = \frac{E\,M}{2}$. This parabola is the form assumed by the cord polygon when the load is uniformly distributed, as was previously assumed. If we note that the triangle $A\,M\,B$ represents the entire load collected at E, it will readily be seen how the curve may be drawn in any case. If the chord $A\,E\,B$ is inclined, as shown in Fig. 108, the divisions of $A\,M$ and $M\,B$ will be equal in number, but the divisions of $A\,M$ will be of different size from those of $M\,B$. The point e lies in the middle of $E\,M$, but is not the vertex of the parabola.

Link polygons which assume the form of curves may also be used to show the effect of moving loads, and are then the figures which are contained within the successive sides of a regular polygon. Many examples are to be found in the case of railway bridges, traveling cranes, engine guide bars, etc.

§ 43.

The investigation of the action of parallel forces, such as Q_1 to Q_4 and $P_1\,P_2$, Fig. 109, whose direction is normal to a beam, requires a knowledge of the statical moments of the external forces. These can best be obtained by use of the force and cord polygons. After constructing the force polygon $A\,O\,4$, and cord polygon $a\,b\,c\,d\,e\,f$, let it be required to find the statical moment for any point S upon the beam. This moment is the product of the resultant of all the forces upon one side or the other of the line $S\,S_1$ into the lever arm l of this resultant from $S\,S_1$.

The magnitude of this resultant is obtained from the distance $h\,i = 1\,.\,5$ in the force polygon, cut off by the rays $O\,1$ and $O\,5$,

which are parallel to $b\,c$ and $f\,a$, and its point of application is determined by prolonging these sides until they intersect at g. By drawing the perpendicular $g\,g_o$, the lever arm l of the resultant $P = h\,i$ is determined, for the force acting at the point S, and hence we have $M = P\,l$.

This multiplication may also be performed graphically. By drawing the perpendicular $O\,k$ in the force polygon, we obtain the altitude of the triangle $O\,h\,i$ from the base $h\,i$, and this triangle is similar to the triangle $g\,s\,s_o$, whose altitude is t. Call in $O\,k = H$ and $s\,s_o = t$, we have

$$P : H = t : l,$$
$$\text{or} \quad M = P\,t = H\,t.$$

This proves that the statical moment at any point in a beam is proportional to the corresponding ordinate of the cord polygon, parallel to the direction of the forces, since H is a constant. By making H equal to unity the conditions become similar to Case I., § 22, in graphical multiplication, and the moment M becomes equal to the ordinate t. It is not necessary to determine the position of the point of application g of the resultant,

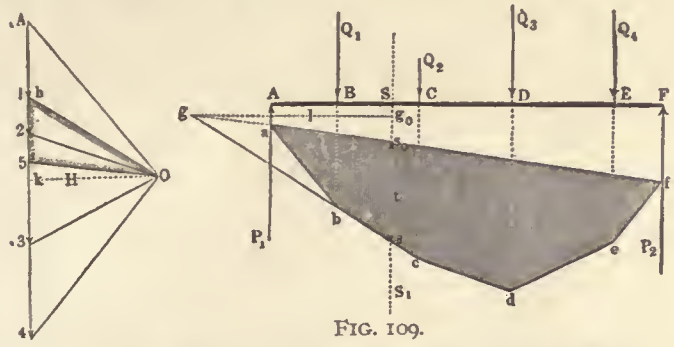

FIG. 109.

since it is the relation between the statical moments which is most desirable, whether H be chosen as a unit or not. This property of the cord polygon for parallel forces is most useful, and an example may be found in the case of axles.

For such cases as for many others, it is most useful, since no modification of the diagram is necessary, the moments being found by the same construction which is required for the determination of the forces. It is often convenient in practice to cover the figure containing the moneut ordinates with section lining or with a light tint of color.

§ 44.

COMPOSITION AND DECOMPOSITION OF STATICAL MOMENTS.

As shown iu the preceding section, statical moments may be shown by means of lines of definite length and position in the same manner as simple forces. When two statical moments act iu the same or in different directions, they may be combined by means of graphical addition in the same manner as has already

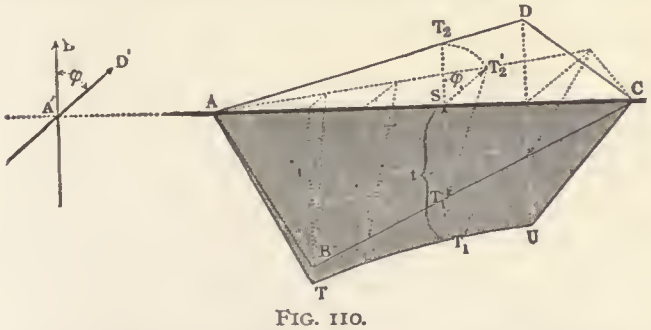

FIG. 110.

been shown in § 31. If $A\,B\,C$ and $A\,D\,C$, Fig. 110, represent the cord polygons for two sets of parallel forces which act normal to the axis of a revolving body $A\,C$, in the directions $A'\,B'$ and $A'\,D'$ we have the following method: For a point S on the axis of the body we have the triangle $T_1\,S\,T_2'$, in which the angle $\phi = B'\,A'\,D'$ and $T_1\,T_2' = S\,T_1' = t$ for the desired moment. The combination of the cord polygons $A\,B\,C$ and $A\,D\,C$, which may be called the moment surfaces, will give the resultant moment surface $A\,T\,U\,C$. The sides $A\,T$ and $C\,U$ are here straight lines, while $T\,U'$ is a curve, in most cases a hyperbola. In actual practice the straight line joining T and U may be used with but little error, and its detailed construction is unnecessary.

By a reversal of the above construction it is possible to decompose any given statical moment t into two others, t_1 and t_2, if their directions be given.

§ 45.

TWISTING MOMENTS AND THEIR GRAPHICAL COMBINATION WITH BENDING MOMENTS.

Next in importance to bending moments, and often acting in combination with them, are twisting moments. In Fig. 111 let $A\,B\,C\,D$ be the axis of a rotating body, subjected to bending forces at $B\,C$, and supported at A and D; the force polygon being represented at $A\,O\,2$ and the moment-surface at $A\,b\,c\,D$, and let the portion between B and C be subjected to a twisting moment $P.R$, and the moment-surface of the latter be required.

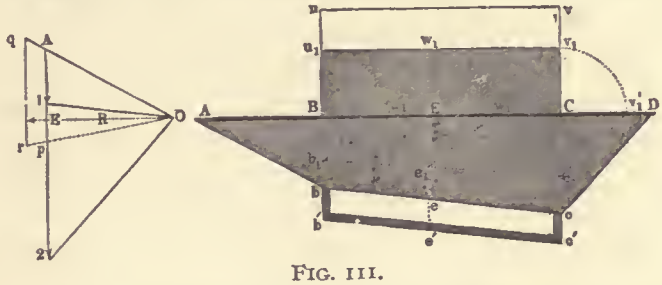

FIG. 111.

According to § 43, and the method of multiplication given in Rule I., § 22, we find a line corresponding in value to $P\,R$ by laying off in the force polygon $A\,p = P$, joining the ray $O\,p$, prolonging $O\,A$ and $O\,p$, and drawing $q\,r$ parallel to $A\,p$ at a distance equal to R, giving a leugth $q\,r$ equal to $P\,R$ upon the same scale used for the polygon $A\,b\,c\,D$. The moment-surface for the twisting between B and C will then be included in the rectangle $B\,C\,v\,u$, whose altitude $B\,u = C\,v = q\,r$. In common practice it is desirable to convert this torsion surface into one representing equivalent bending moment. This may be done by taking a proportional value which shall give the same security as the bending moment. It has been shown in § 18 that the latter is equal to $\frac{5}{8}$ the twisting moment. We may them make $B\,u_1 = C\,v_1 = \frac{5}{8}\,B\,u$, in order to obtain the moment-surface of the bending moment between B and C, which may be measured upon the same scale as $A\,b\,c\,d$.

If we wish to combine this with the given bending moment we may do so graphically by first using the formula IV. of the table of § 18, p. 60, in which the ideal bending moment for the combined action of a twisting moment Md and a bending moment Mb is:

$$Mi = \frac{3}{8}\,Mb + \frac{5}{8}\,\sqrt{M\,b^2 + M\,d^2}.$$

In this case we make $B\,b_1 = \frac{5}{8}\,B\,b$, $C\,c_1 = \frac{5}{8}\,C\,c$, $E\,e_1 = \frac{5}{8}\,E\,e$, etc., rotate $B\,u_1$, $C\,v_1$ and $E\,w_1$, down upon $A\,D$, and add the hypoteneuses $b_1\,u_1'$, $c_1\,v_1'$, $e_1\,w_1'$ to the lines $b\,b_1$, $c\,c_1$, $e\,e_1$. The combined length of these lines gives the length for the ordinates at B, C and D, from which the resulting ideal cord polygon $B\,b\,b'\,e'\,c'\,c\,D$ may be constructed.

§ 46.

DETERMINATION OF THE CENTRE OF GRAVITY BY MEANS OF THE FORCE PLAN.

The position of the centre of gravity of a plane figure may

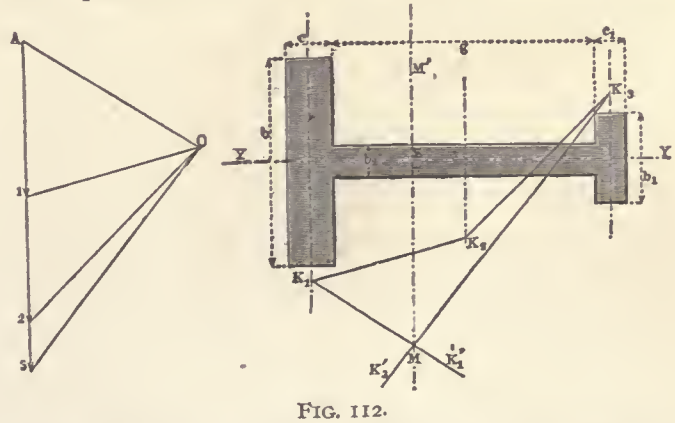

FIG. 112.

often be very conveniently determined by means of the force plan. This may be done by dividing the figure into a number of strips of uniform width, such that their area may be considered as proportional to their middle ordinate, constructing

the force and cord polygons, and taking the line of the resultant as a line of gravity. If the figure is not symmetrical, it will be necessary to divide the figure again in another direction aud determine another line of gravity, wheu the position of the centre of gravity will be found at the intersection of the two lines. For figures of simple form larger determinate sections may be taken instead of strips, their area determined in any convenient manner, and the diagram constructed accordingly.

Suppose, for example, that it is required to determine the position of the centre of gravity of the T-shaped section shown in Fig. 112. The figure is symmetrical about the axis $Y\ Y$, so that the centre of gravity must lie somewhere in that line. We may divide the figure into the rectangular $b \times c$, $b_1 \times c_1$ and $b_2 \times g$, which we will call respectively the areas 1, 2 and 3.

We have also given $c = 1.5\ b_2$ and $c_1 = b_2$. This gives the three forces as $1.5\frac{b}{2}$, $\frac{g}{2}$, and $\frac{b_1}{2}$, which are then laid off at A 1 2 3, a pole O selected, and $K_1' K_1$ drawn parallel to $O\ A$, $K_1 K_2$ parallel to O 1, $K_2 K_3$ parallel to O 2, $K_3 K_3'$ parallel to O 3, when the intersection of the sides $K_1 K_1'$ and $K_3 K_3'$ at M gives a point on the line of gravity $M M'$, whose intersection S with the axis $Y\ Y$ is the centre of gravity of the figure.

§ 47.

RESULTANT OF THE LOAD ON A WATER WHEEL.

It is very important in designing a water wheel to be able to determine the position of the resultant of the water acting upon it, and the method of doing so will furnish an excellent illustratiou of the application of the principles of the preceding sections.

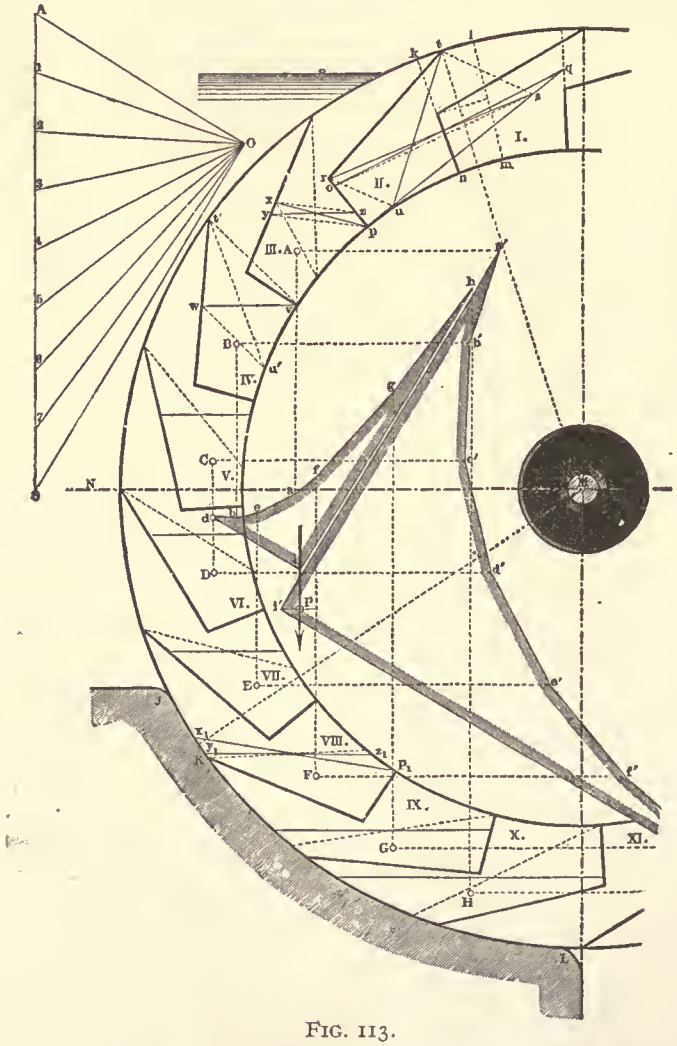

FIG. 113.

In the breast wheel, which is shown in Fig. 113, there are ten buckets in the half section, the third from the top being the first to receive a charge of water, the amount being estimated from a previously determined coefficient. The level of the water in the succeeding buckets may be considered as horizontal, and the discharge from the buckets is prevented by the culvert $K\ L$, so that if we neglect the leakage around the edges of the culvert we may count that all the buckets from No. III. to No. X. contain the same load of water, acting in each case as if its weight were concentrated at the ceutre of gravity of each of the respective prisms of water. Bucket No. XI. we may consider as entirely empty.

I. Determination of the culvert arc $K\ L$. The contents of a bucket section are determined by the cross section contained between two adjoiniug bucket divisions prolonged, as governed by the coefficient of charge, = 0.04. Now, in bucket I. lay off $k\ l = 0.4$ of the bucket spacing, and draw $l\ m$ radial; then the section $k\ l\ m\ n$ will represent a bucket charge. In bucket II. its figure assumes the shape $r\ p\ u\ t$, in which the angle t is the beginning of the scoop of the bucket $r\ l$, and $k\ t\ u$ will be equal to the desired culvert angle $K\ M\ L$, and $u\ t\ M$ will be equal to the complement $N\ M\ K$.

The construction is as follows: Iu the right augled triaugle $o\ p\ q$ make $o\ p$ = the middle breadth of the figure $k\ l\ m\ n$, and also make $p\ q = 2\ .\ l\ m$; then transform this triangle into one of equal area, $r\ p\ s$ (by drawing $o\ s$ parallel to $r\ q$, and joining $r\ s$, see § 25. Join $s\ t$, and draw $r\ u$ parallel to it, and join u and t, and if we neglect the curvature of $p\ u$, we may consider the quadrilateral $r\ p\ u\ t$ as the form of a filled bucket just at the moment of discharge. This requires the augle $K\ M\ N = u\ t\ M$, but owing to the splashing of the water, the culvert is raised as high as f.

II. Determination of the water level in the various buckets. We will begin with bucket IV. Here the figure $r\ p\ t\ u$ is again drawn, and the line $t'\ v$, and its parallel $w\ u'$, determined experimentally, so that the diagonal $w\ v$ shall be horizontal, which may readily be done after a few trials. Proceed in the same manner with buckets V., VI. and VII.

In bucket III. the figure $r\ p\ u\ l$ is first converted into the quadrilateral with the upper line $p\ x$, and this then into the pentagonal figure, with the level upper line $y\ z$.

In bucket VIII. we first get the figure with the upper line $p_1\ x_1$, and then from this the figure with the level upper liue $y_1\ z_1$. Proceed in the same manner for buckets IX. and X.

III. Force plan for the water load. Now determine the centre of gravity for each loaded bucket, and also lay off the force polygon $A\ O$ 8 for these eight forces. From this constant the link polygon $d\ b\ e\ a\ f\ g\ h\ i$, according to the methods previously given, and i, will be a point in the resultant of all the forces. It is to be noticed that the centres C and D fall so nearly in the same line that their forces have been united, so that $i\ d$ is parallel to $A\ O$, $d\ b$ to O 2, and the intermediate paraliel to O 1 omitted.

Suppose, now, that it is desired to determine completely the position of the centre of gravity P, of all the prisms of water. Draw through A, B, C, etc., horizontal lines, assume the force polygon $A\ O$ 8, to be turned around 90°, and draw a second cord polygon; or, what is shorter, draw the second polygon with its sides normal to the rays of the force polygon, giving the figure $a'b'c'd'e'f'^1 - - - i'$. A horizoutal through i will then intersect the vertical which was previously determined, and so fix the position P, of the centre of gravity of the entire mass of water. By taking the buckets in a different position, a slight difference in the position of $i\ P$, may be found; but in most cases the deviation will be very slight.

§ 48.

FORCE PLANS FOR FRAMED STRUCTURES.

Framed structures are of very general application wherever loads are to be supported, and their discussion may be classified as a system by itself, while their use extends from the simple trussed beam to the bridge and roof truss; also for walking beams and many other uses.

The tensile and compressive stresses in these various forms may readily be examined by means of the force plan, which consists of both the force and cord polygons and their modifications. The subsequent examples will serve to illustrate the principal cases. In all of these cases it is assumed that at the knots – *i. e.*, at the points where several members meet,—a joint is supposed to exist; or at least no account is taken of the resistance to bending at the knots.

In order to form such a plan for any given construction, it is necessary first to determine the division and direction of the forces, and then, beginning at one of the external forces and laying off its direction and magnitude to the next knot, combining it there with the external forces at that point, laying off the resultant to the next bend, etc. Upon such combinations of force triangles or quadrangles the force plan is constructed.

If it is desired to determine the directions of the components of a given or determined force, the principles laid down in § 32 must be borne in mind. These may be generally expressed in the following rules:

If one force is to be separated into two or more forces, its direction is to be reversed and it is to be made the closing line S' in the paths of the other forces.

If two or more given forces are to be conbined with two or more other forces, the force polygon will consist of the given forces and their closing line S.

The first rule is only a special case under the second or general rule, since the single force may be considered as an unclosed force polygon whose closing line passes backward over the same path to the starting point.

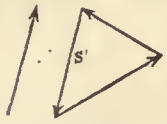

FIG. 114.

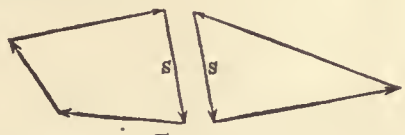

FIG. 115.

In the investigation of each member in a frame without error, it is best to assume the member to be cut, and to determine the external forces at each section which oppose the internal forces; the direction of the forces may then also be determined with precision.

§ 49.

FORCE PLANS FOR FRAMED STRUCTURES.

I. Simple Trussed Beams. Fig. 116. The beam $A\ B\ C$ is supposed to carry at B a load equal to 2 P, acting in a direction normal to $A\ C$, and to be supported at A and C. Since $A\ B = B\ C$, the reaction at each support is equal to P. It is then required to determine the stresses upon the various members from 1 to 5, as marked in the figure.

Referring to the diagram marked a, let $a\ b$ be the reaction P, which acts upward at A. We now have to construct a diagram

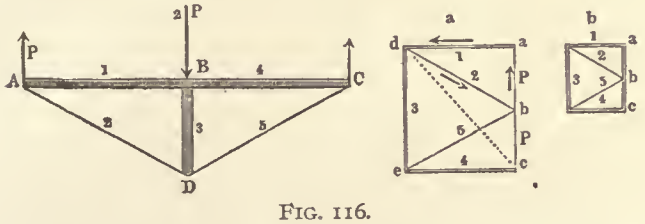

FIG. 116.

of the internal forces acting in $A\ B$ and $A\ D$. To simplify matters, we will give these forces the same numbers as their corresponding members; drawing 1 parallel to $A\ B$, and 2 parallel to $A\ D$. The direction of the force P, in the closing line of the force-triangle, determiues the direction in the other two sides, as shown by the arrows, by the lines 1 and 2. (See § 48.) In this case there will be compression in $A\ B$ and tension in $A\ D$.

In order to show this clearly, in all the following strain diagrams the forces acting compressively in struts or posts will be indicated by double lines, while all tension members, links or rods will be shown by single lines.*

Following out this idea, we shall, in the following illustrations, show all struts or compression members in the construction drawings as having a measurable thickness, as if made of wood, while the tension members will be represented by simple lines, although this is not intended to indicate any limit as to the choice of materials.

For the knot at B we make $a\ b\ c = 2\ P$, and, following in the direction $d\ a\ c$ (because the thrust is from A towards B), and join the closing lines 3 and 4, both of which represent compression. The combination of 2 and 3 determines 5, which is tension. This gives an entirely symmetrical plan, which was to be

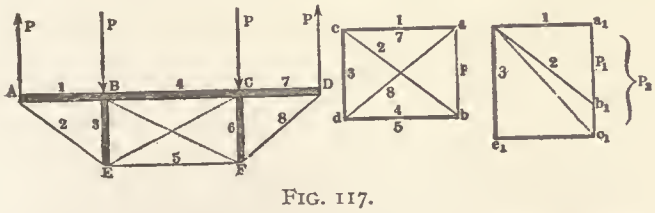

FIG. 117.

expected from the symmetrical form of the structure, and an investigation of one-half is practically sufficient.

If the load 2 P is taken as uniformly distributed over the entire distance $A\ B\ C$, instead of being concentrated at B, the reactions at A and B will each be equal to $\frac{P}{2}$, and the load at $B = P$, so that ½ of the load on $A\ B$ and $B\ C$ is referred to the knots A, B and C. From these conditions we obtain the force plan b, which is geometrically similar to the other, but only half as large.

* This distinction has been suggested by Culmann.

II. Double-trussed Beam (much used for constructions of all sizes). Fig. 117. In this case take vertical forces P, at B and C, and corresponding vertical reactions at A and D. In the first force plan, a is drawn equal to P, and 1 and 2 parallel respectively to $A\ B$ and $A\ E$, thus determining the forces 1 and 2; 1, being compression and 2, tension. Lines now drawn parallel to $B\ E$ and $E\ F$, determine the compression in 3, and the tension in 5, while the compression at 4 is the closing line of 3, 1, and P; and the other half of the diagram is similar. If the vertical forces at A and B are uot of the same magnitude, which is often the case in practice, the structure should be strengthened by the introduction of the diagonals $E\ C$ aud $B\ F$.

The second diagram shows the construction in this case. Let $P_1 = a_1\ b_1$ be the force acting at A, and $P_2 = a_2\ c_2$ at B. Draw a vertical line from 1 to a horizontal through C_1, which gives the length 3, of the vertical force at B, and by drawing the

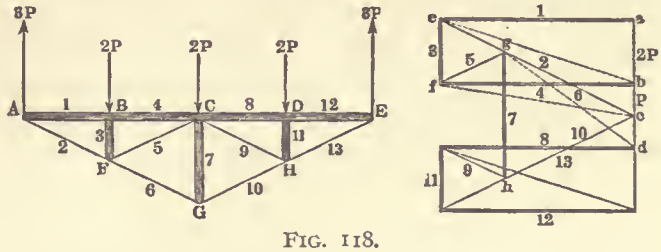

FIG. 118.

dotted diagonal line their resultant is found. If any of the tension members are omitted the framework will tend to take an inclined position until the various parts are at such an angle with each other that both constructions will give the same value for 3. For this reason it is best in nearly every case to use the diagonal counterbraces.

III. Triple Trussed Beam. Fig. 118. The uniformly distributed load upon the framework gives the following distribution of forces. The force 3 $P = a\ b\ c$ is first decomposed in 2 and 1, or $c\ e$ and $e\ a$; then 1, is connected to $a\ b = 2\ P$, by the line $b\ e$, and this latter decomposed into 3 and 4 or $e\ f$, and $f\ b$; 2 and 3 are now joined by $f\ c$, and the components at 5 and 6 or $f\ g$ and $g\ c$ found. Since 6 and 10 are equal to each other, we may draw $c\ h$ parallel to $G\ H$, and equal to $c\ g$, which gives $g\ h = 7$; the rest of the force plan is similar to the first half.

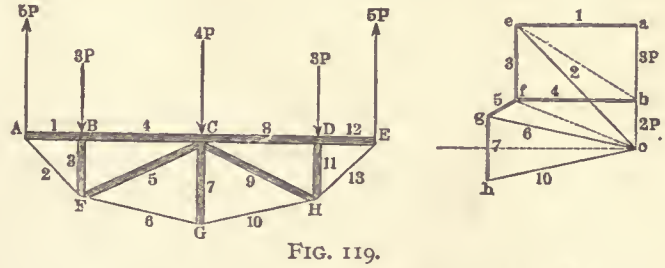

FIG. 119.

IV. "Another form of Triple Trussed Beam is shown in Fig. 119. The space between B and C is twice as great as between A and B, and the uniformly distributed load is equal to 12 P, acting at the various knots as shown in the figure.

In the force plan, make $a\ b\ c = 5\ P$, and draw parallel to 1 and 2, the lines $a\ e$ and $e\ c$; then join 1 with 3 p (for the knot at B), and decompose into 3 and 4, or $e\ f$ and $f\ b$. Now combine 2 with 3, giving $c\ f$, and draw 5 and 6 parallel to $F\ C$ and $F\ G$, respectively. This case differs from the preceding, in that 5 is now compression instead of tension. The equality of the forces 6 and 10 gives $g\ h = 7$, and the similar half of the diagram need not be drawn.

V. Multiple Trussed Beam. Fig. 120. The beam $A\ J$ is divided into eight equal parts, which are represented as being uniformly loaded, the load at each knot being shown in the figure. In constructing the force plan we make $a\ e = 7\ P$, and by drawing the lines parallel to 1 and 2, we obtain $a\ f$ and $f\ e$; then lay off $a\ b = 2\ P$, and join the resultant $b\ f$. This decomposes into 3 and 4, or $f\ g$ and $g\ b$. The forces 2 and 3 combine to give the resultant $g\ e$, which, by drawing lines parallel to $K\ C$ and $K\ L$, gives $g\ h$ and $h\ e$ for the values of 5 and 6. We now find that to proceed further we have three forces of given direction only, and since this is indeterminate, we must obtain one magnitude as well. This, for example, may be done for the force 7 as follows: The strut $C\ L$ sustains the vertical components of 5 and 9, as well as its own direct load 2 P. Now 5 and 9 are equal to each other, since they are placed symmetrically, and carry equal loads from the struts $B\ K$ and $K\ M$. Hence in the force plan we may make $h\ i$, which represents the

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force 7, equal to twice the projection of 5 upon the vertical + 2 *P*. This we can now combine with 6 = *h e*, giving *i e*, which in turn decomposes into *i m* and *m e*, or 10 and 11. Returning to the knot *C*, we may now take the line *h i*, and by drawing parallels to *C L*, *C M* and *C D*, obtain the figure *h i k c*.

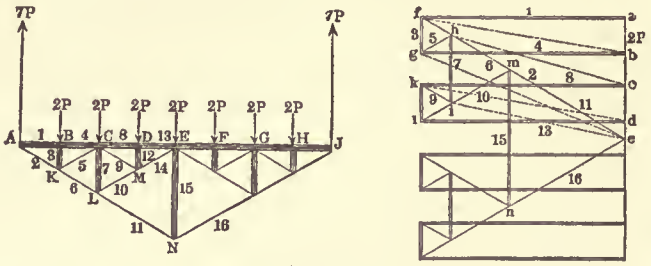

FIG. 120.

which determines the forces 8 and 9. In the same manner proceed from 12 to 15, which will complete the half plan. It may be noted that the principal beam *A J* is subjected to a uniform compression throughout its entire length.

The force plan will, of course, be modified by various distributions of the load, as in the case of simple beams, as shown in cases XII. and XIII., § 6.

§ 50.

FORCE PLANS FOR ROOF TRUSSES.

Roof trusses furnish many and varied examples of framework.* In the following examples a uniformly distributed

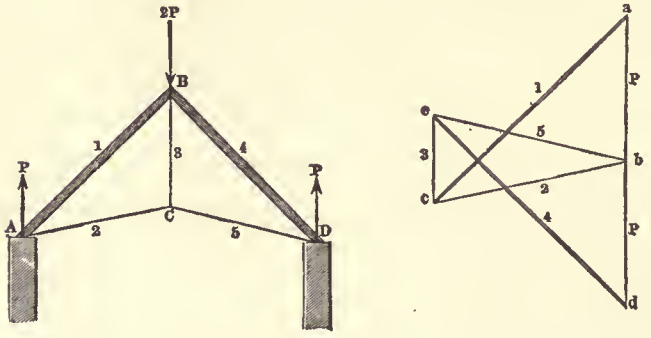

FIG. 121.

vertical load is assumed, so that the burden upon any portion of a rafter may be considered as proportional to the length of that portion.

I. Roof with Simple Principals. Fig. 121. A uniform load 2 *P* upon each half gives as the external forces *P*, 2 *P* and *P* at *A*, *B* and *C*. Lay off in the force plan *a b* = *P*, and draw *a c* and *b c* parallel to *A B* and *A C*, determining the forces 1 and 2; 1 being compression and 2 tension. Then draw the vertical *c e*, and also draw *b e* parallel to *C D*, thus giving both 3 and 5, and the diagram is completed by drawing *d e*.

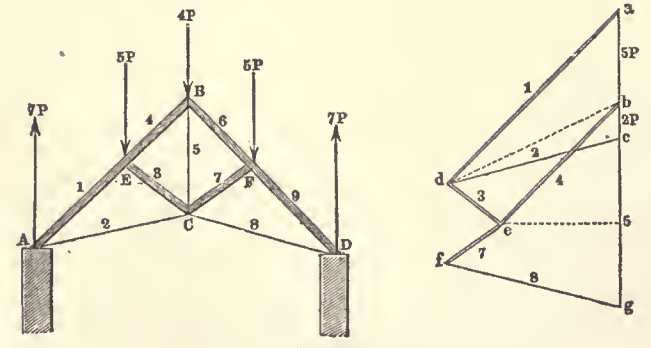

FIG. 122.

II. Roof with Single-Trussed Principals. Fig. 122. This form is similar to the preceding, with the addition of the struts *C E* and *C F*. The distance *A E* is to *E B*, as 3 is to 2; and the loads upon the respective portions are 6 *P* and 4 *P*, which give the forces at the various knots as shown in the figure. Make *a c* in the force plan equal to 7 *P*, and by drawing lines parallel to *A E* and *A C*, obtain the forces 1 and 2, or *a d* and *d c*; then combine 1 with 5 *P* = *a b*, and decompose the dotted resultant into *d e* and *e b* respectively parallel to *E C* and *E B*, giving the forces 3 and 4, both being compression. By repeating 2 and 3, in drawing 7 and 8, we obtain the figure *c d e f g*, in which *c g* gives 5. This latter force might also have been

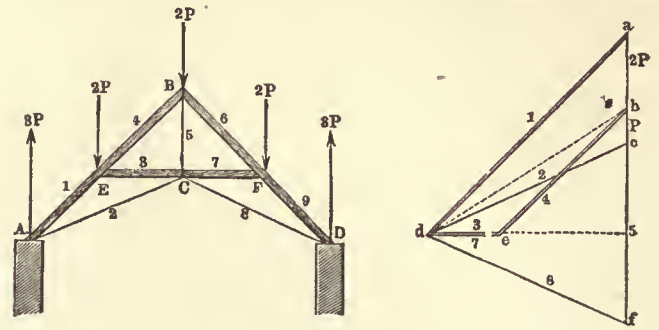

FIG. 123.

found by combining 4 and 4 *P*, and decomposing the resultant by lines parallel to *B C* and *B F*, an illustration of the various methods in which the force plan may be used.

III. Another form, with Single Trussed Principals. Fig. 123. This roof is similar to the preceding except that the struts *E C* and *C F* are placed horizontally. In this case *A E* = *E B*, and the external forces at *A* and *D* are both equal to 3 *P*.

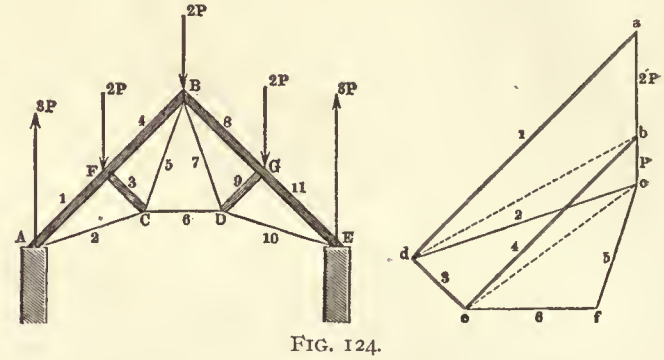

FIG. 124.

The forces from *a* to *c* in the force plan are determined as before, giving *d a* and *c d* for the forces 1 and 2, and the combination of 1 with 2 *P* gives the resultant *d b*, from which the thrusts 3 and 4, or *d e* and *e b*, are obtained. The value of 1 is the same as 3, and 8 is the same as 2; while 5 is the closing line of *c d e d f*, or of *c d f*. The force 5 must also be the combination of the equal forces 4 and 6 with 2 *P*, which the diagram shows to be the case. If the rod *C B* is omitted, as is frequently done, the strut *E C F*, if there is no joint at *C*, will, oppose its resistance to bending to the force 5; but there will be a tendency to rise at the apex *B*, if the fastening be not made sufficiently strong.

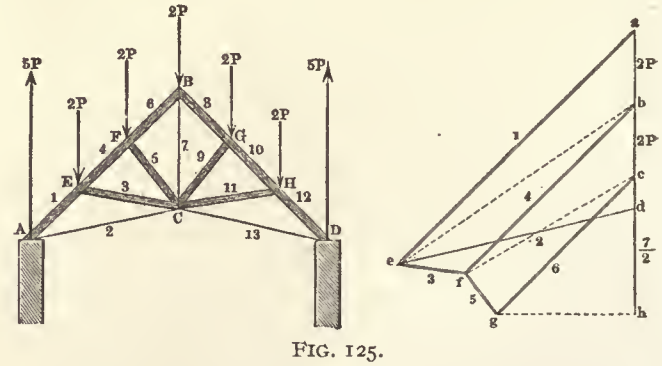

FIG. 125.

IV. Third Roof with Single Trussed Principals. Fig. 124. In this form of truss, frequently known as the Belgian or French truss, the single vertical rod of the preceding form is replaced by a triangle *B C D*. The struts are placed in the middle of the rafters and the external forces are distributed as shown in the figure. In the force plan *a b c* = 3 *P*, and 1, and 2 are determined as before. By the decomposition of the resultant of 1 and 2 *P*, we obtain the forces 3 and 4, or *d e* and *b e*, and from the resultant *e c*, of the forces 2 and 3, we get the tensions 5 and 6, in *c f* and *e f*. The second half of the diagram is the symmetrical counterpart of the first.

V. Roof Truss with Double Trussed Principals. Fig. 125. This construction does not differ greatly from that shown in Fig. 124, except that the struts employed to strengthen the

* Many subjects for Graphical Analysis may be found in Ritter's "Roof and Bridge Construction," Hannover, 1863, in which the forces in the various members will also be found carefully determined numerically, thus affording convenient proof.

rafters are divided into two. The spaces are equal to each other and the load uniformly distributed. As shown in the figure this gives a reaction of 5 P, or A and D. In the force plan $a\,d = 5\,P$, and lines parallel to $A\,E$ and $A\,C$ drawn, determining the forces 1 and 2, or $d\,e$ and $e\,a$. We then combine $e\,a$ with $a\,b = 2\,P$, and decompose the dotted resultant $e\,b$, into the thrusts $e\,f$ and $f\,b$, or 3 and 4, by drawing these lines parallel to $E\,C$ and $E\,F$. Again we take the resultant of the forces 4 and 2 P, and decompose it into 5 and 6, or $f\,g$ and $g\,c$, which brings us to the middle of the symmetrical figure. The force 7 is the resultant of 6, and its counterpart 8, and the load 2 P, and the half of this force is therefore equal to the projection of 6 upou the vertical, less P, or in the diagram, to $d\,h$.

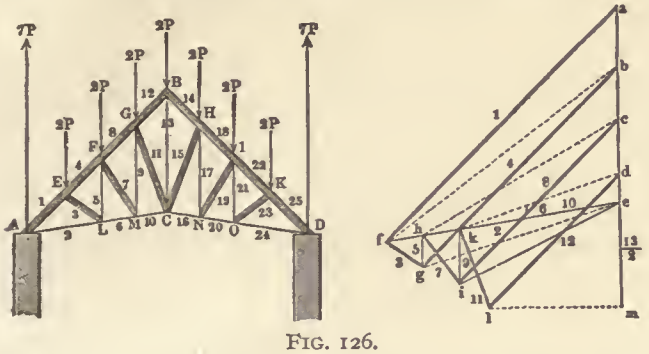

FIG. 126.

VI. English Roof Truss, with Multiple Trussed Principals. Fig. 126. Here we have inclined struts, with vertical tie rods. The load is agaiu uniformly distributed, each space bearing the load of 2 P. The reactions at A and D are each $= 7\,P$. In the force plan we have $a\,b + b\,c + c\,d + d\,e = 3 \times 2\,P + P = 7\,P$, which gives the length of $a\,e$. The forces 1 and 2 are found by drawing $f\,a$ and $e\,f$, parallel to $A\,E$ and $A\,L$. Now consider 1 as combined with $a\,b = 2\,P$, and the resultant $f\,b$, decomposed into $f\,g$ and $g\,b$, giving the forces 3 and 4 ; again, combine 2 and 3, and then decompose the resultant $g\,e$, into 5 and 6, or $g\,h$ and $h\,e$, by drawing these latter parallel to $L\,F$ and $L\,M$. In this manner we continue until we reach 12, or $l\,d$, which we then project upon the vertical. Now taking from $d\,m$, one-half the load $P = d\,e$, we have $m\,e$ for one-half the stress on the middle rod $B\,C$. The remaining half of the force plan is similar.

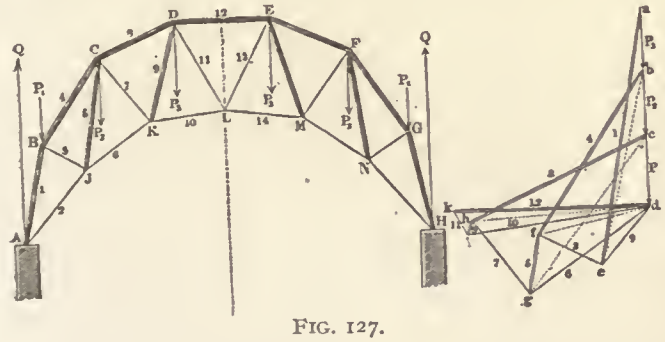

FIG. 127.

VII. Polygonal or Sickel Shaped Roof Truss. Fig. 127. This roof may be considered as a modification of the preceding form, and is used for higher and wider spans. It is hardly proper to assume that the load is here uniformly distributed even if the spaces are equal, for in the case of snow, much less weight would be carried by the steep portions $A\,B$ or $G\,H$, than by the flatter surfaces $C\,D$ or $D\,E$. We must therefore estimate the forces P_1, P_2, P_3, actiug as B, C, D, E, F, G, and make the reactions at A and B equal to $Q = P_1 + P_2 + P_3$.

In the force plan $a\,b = P_1$, $b\,c = P_2$, $c\,d = P_3$, and $a\,d = Q$, which is first decomposed into 1 and 2, by drawing $e\,a$ and $d\,e$ parallel to $A\,B$ and $A\,J$. Then combining 1 with P_1, and decomposing the resultant, as before, we get 3 and 4, or $e\,f$ and $f\,b$. Having 2 and 3, we get in like manner 5 and 6, or $g\,f$ and $d\,g$; then combining 4 and 5 with P_2, and decomposing with parallels to $C\,K$ and $C\,D$, we obtain the forces 8 and 9, and so proceed until we reach 12, which is the middle of the symmetrical figure. The members $K\,L$, $D\,L$, $E\,L$, and $M\,L$ are all subject to tension.

§ 51.

THE GRAPHICAL DETERMINATION OF WIND STRESSES.

In designing large and important roof trusses it is important to investigate the stresses due to wind pressure, as well as those due to the weight of the roof and of snow, and indeed, in some cases, the resistance to wind is the most important of all.

As an illustration of the applicability of the graphical method to the determination of wind stresses, we will take the English roof truss, Fig. 126, whose conditions under a vertical load have already been examined, and consider it as also subjected to a wind stress W, as shown in Fig. 128.

We have first to determine the forces Q_1 and Q_2, acting at the points A and D. The wind pressure will be taken as acting on the surface of the roof from A to B. Let W be the resultant of the entire wind pressure acting normal to AB, and let P be the total vertical load upon that half of the truss. By combining these two forces we obtain the direction FS of their resultant, and also its magnitude, which we then lay off on the force plan at $c\,c_1$. Upon the other half of the truss we have only the vertical load, which may be considered as acting at J, and equal in magnitude to P. By prolonging its direction until it intersects the previously determined line at S, we have at S a point in the resultant of the entire load upon the roof, including wind pressure. By making $c_1\,a_2$ in the force plan equal to P, we have $a\,c$ for the direction of this resultant, which may then be laid off at $S\,T$ in the drawing. In order to determine the forces Q_1 and Q_2 we must recollect that, according to § 34, when we have two

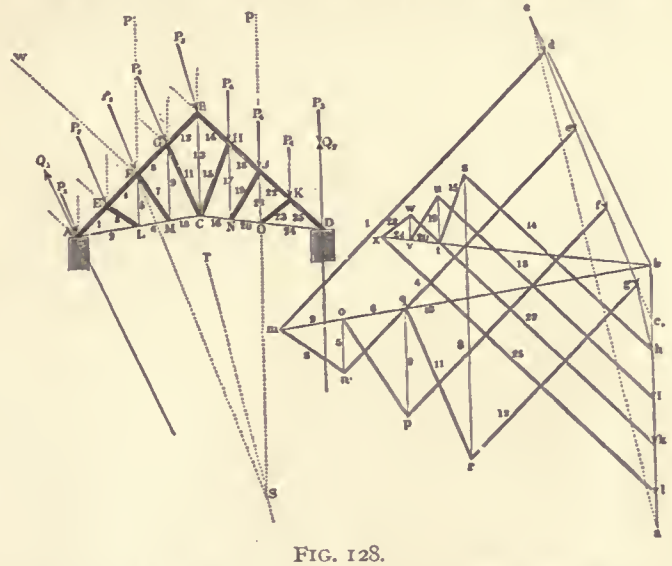

FIG. 128.

closing forces to determine, we must also have at least two conditions given. In this case, then, we must first find the direction of Q_1 and Q_2.

The wind pressure produces a horizontal thrust which must be met by the stability of the walls or columns upon which the roof rests. In each case it must be determined whether this horizontal thrust is borne equally or unequally by both supports, and in what proportion it is divided. To this end we first find (according to § 39) the proportion of the vertical component of the force $a\,c$, which comes upon each support (as found by the intersection of $S\,T$, prolonged with $A\,D$), and then combine these vertical forces with their respective horizontal components. It often happens that all the horizontal thrust is borne by one of the supports, which it must of course be prepared to resist. This often occurs in the case of railway stations, and under such circumstances the direction of each force must be determined separately. First prolong the vertical at D downward until it intersects $S\,T$, and join the intersection with A (the lines are only indicated in the figure). This gives the direction of the force at A. We have now both the direction of the reaction at D and the direction of that at A. We must also consider the distribution of the forces at the various knots between A and B, and between B and D. We have for the points between A and B the resultants between the proportional parts of P and W, while from B to D we have simply the proportional parts of P. This gives at A the force P_1 ; at E, F and G, the force P_2; at the peak, the force P_3; at H, J and K, the force $P_4 = \frac{P}{4}$, and at D, the vertical force $P_5 = \frac{P}{8}$.

Returning now to the force plan, we make $c\,d = P_1$, $d\,e = e\,f = f\,g = P_2$, $g\,h = P_3$, $h\,i = i\,k = k\,l = P_4$, and $l\,a = P_5$. We now have finally the length $b\,l$, for the value of the reaction Q_2 at the point D, and a line (not shown) from b to d, gives the magnitude of the force Q_1 acting at A.

The determination of the stresses in the various members can now readily be made. The decomposition of $b\,d$ by drawing $b\,m$ and $m\,d$ parallel respectively to $A\,E$ and $A\,L$, gives the forces 1 and 2. We thus proceed until we reach the rod $B\,C$, or

No. 13, for which we get the tension $r\,s = 13$, by drawing the vertical $r\,s$ from r, until it intersects the liue $n\,s$, drawn parallel to $B\,D$. We then continue to determine the forces from 15 to 25, as already showu. The force plan shows that uuder these conditions similarly placed struts are subjected to dissimilar stresses. The determination of the stresses might have been made in the reverse order, begiuning with the triangle $x\,b\,l$, which should give the same results, and which may be used to prove the accuracy of the work. A proof is also made by the accuracy with which the line $w\,x$ drawn from w, parallel to $K\,O$, intersects the point x, which was first determined by the intersection of $b\,x$ and $l\,x$. As a matter of fact, it will be found to require careful drawing in order to insure the closing of the diagram.

By comparing the last force plan with that found for the same roof truss in Fig. 126 (the scale beiug the same), it will be seeu how greatly the wind stresses affect the structure. In order to complete the calculation, a second plan should be drawn, assuming the wind to act also upon $B\,D$.

§ 52.

Force Plans for Framed Beams.

Beams of various forms are often framed in various shapes and made both of wrought and cast iron, and have mauy applications, such as walking beams for steam engines, for craues, arms, &c. A few examples will show the method of investigatiou for such cases.

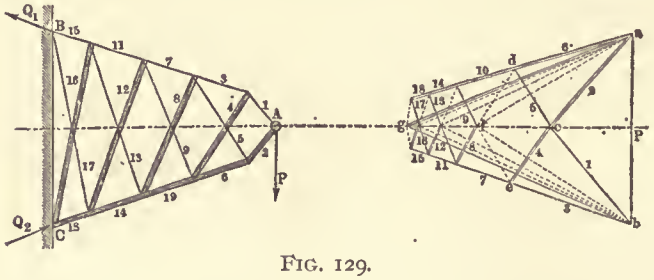

Fig. 129.

I. Projecting Frames with straight members, Fig. 130. The load P acts at A in a directiou normal to the axis of the frame, which is supported at B and C. The force plan is constructed as follows: Draw $a\,b = P$, and from its extremities draw $a\,c$ and $b\,c$ parallel to 1 and 2, which gives the forces in those meu-bers. Each of these is then decomposed into two other forces—1 into 3 and 4, 2 into 5 and 6, giving the triangles $b\,e\,c$ and $a\,d\,c$.

The forces 3 and 5 are then combined and the resultant decomposed into 7 and 8. To do this we transfer $5 = d\,c$ to $f\,e$, and join the resultant $f\,b$, which can readily be separated into 7 and 8. We proceed in this manner for the remaining members, and as the frame is symmetrical about the axis $g\,c$, only

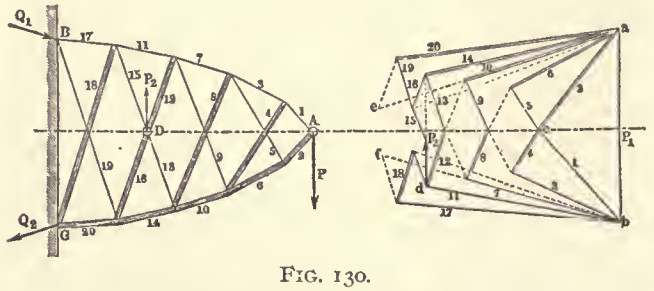

Fig. 130.

one-half of the diagram need be completed. The lines $g\,a$ and $b\,g$, which are the final resultants of 15 with 17, and 16 with 18, are also the external forces at B and C, the points of attachment, provided that their direction be permitted to remain the same.

II. Double Loaded Frame, Fig. 132. In this case we have the force P_1 acting downwards at A, and a force P_2 acting upwards at D, while the points of attachment remain at B and C as before. The members $A\,B$ aud $A\,C$ are polygonal formed. The force plan is drawn just as before, until the force 13 is reached. At D the members are attached to each other at their intersection, so that the force P_2 acts upon both 15 and 16. At this same point we have the action of the forces 12 and 13. Now join the extremities of 12 and 13 by the dotted line shown, and mark off the length of the force P_2, which is subtracted, because its action is upward, thus obtaining the resultant of the three forces. We can then draw 15 and 16 and proceed without interruption to 20. Finally, we draw $b\,f$ and $e\,a$, the external forces at Q_1 and Q_2, which hold the entire frame in equilibrium.

III. Framed Boom for a Crane, Fig. 133. This figure is portion of a framed arch which may be used for the projecting boom of a large crane. At A and D we have the forces P_1 and P_2, and at B and C, the external forces Q_1 and Q_2. The force plan is now required to determine the internal forces acting on the various members of the structure. Before this can be done,

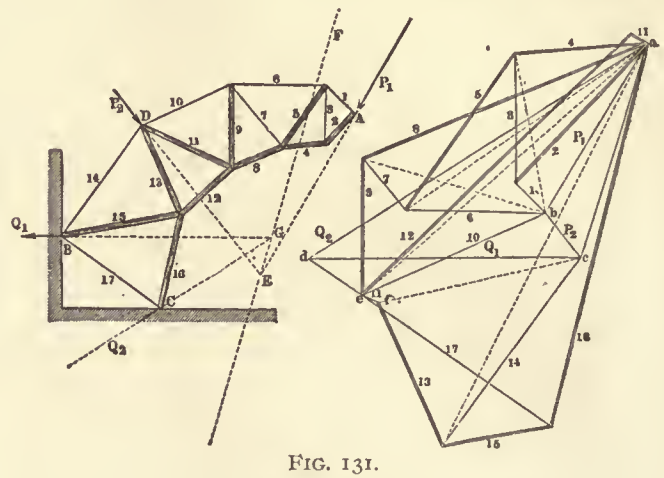

Fig. 131.

we must first determine the as yet uuknown direction of the force Q_2. Prolong P_1 and P_2 to their intersection at E, and by drawing in the force plan, the triangle $a\,b\,c$, determine the direction $F\,E$ of their resultaut; then prolong Q_1 until it intersects $E\,F$ at G, and join $C\,G$, which will be the required directiou of the force Q_2. Completing the figure in the force plan, we have $c\,d = Q_1$ and $d\,a = Q_2$. We now proceed from $P_1 = a\,b$ and lay off the forces 1 and 2, decompose 2 into 3 and 4; combine 3 aud 1 and decompose their resultant, obtaiuing 5 and 6. We thus proceed until we reach 12, which we obtain by combining 9 and 8 and decomposing the resultant into 11 and 12. We now have to combine 10 and 11 with P_2, and decompose the resultant into 13 and 14. We first transfer the force 11 to e, making it equal to $e\,f$, in order to avoid the confusion of lines, which would occur if the construction were made at a. Now drawing the path 11, 10, P_2, we have the closing line $c\,f$, which decomposes into 13 and 14. We then have 15 and 16 from the resultant of 13 and 12, and finally, 17, as the line joining 15 and 16 with d, since 16 and 17 must have the resultant $a\,d = Q_2$. If the work is correctly done, we will find 17 falls parallel to $B\,C$, which affords a convenient and valuable proof for the whole work

§ 53.

Remarks.

The foregoing problems and methods will serve as general examples of the various applications of Arithmography and Graphostatics, and at the same time it must be noted that great care and neatness are most essential in the use of the method.

It may be added that it is desirable to use as few letters and figures as possible in designating the various lines; a common fault of begiuners being the disfigurement of their work in this respect. The necessary marks should be made quite small and iu faint pencil, so that they may be readily erased if so desired.

It is necessary also to be provided with the best grades of pencils, well sharpened, a good drawing-board, reliable protractor scale, dividers, and flexible spline; and it is the author's experience that these canuot be used too carefully. In order to acquire facility in the methods and confidence in the results, the beginner is advised to begin with simple examples which can be thoroughly understood, and practice upon these carefully.

By proceeding in this manner it will be possible to obtain a skill and grasp of the graphical method which will enable the student to use it freely for the solution of a great variety of problems, and extend its scope far beyoud the range of the examples which have been given.

SECTION III.

THE CONSTRUCTION OF MACHINE ELEMENTS.

INTRODUCTORY.

Under the title of "Machine Elements" we may consider those single or grouped parts which are employed to a greater or less extent in all forms of machinery. It is not practicable to determine their number, nor, indeed, is that a matter of importance, since the selection of groups and details is not based upon any positive or generally accepted system. The following selection of the constructive elements of machinery may be found useful and conveuient, which is the principal end to be attained.

In the previous sections a number of general formulæ have been given, while in the cases which follow detailed examples are selected. The dimensions and weights are expressed in inches and pounds, except where otherwise distinctly stated; velocities in feet per second; and rotations in turns per minute. The measure of force is the pound; that of work iu foot pouuds per minute, or for larger quantities in horse-power—(33,000 foot pounds).*

CHAPTER I.

RIVETING.

§ 54.

RIVETS.

Rivets are priucipally used for joining sheet metals or other flat shapes together for the construction of a variety of sheet and framed structures. They may be considered as a fundamental machiue elemeut acting to transform detailed parts into combinations.

In the illustrations various forms of rivets are shown. The common wrought-iron rivet is shown in Fig. 132, with the

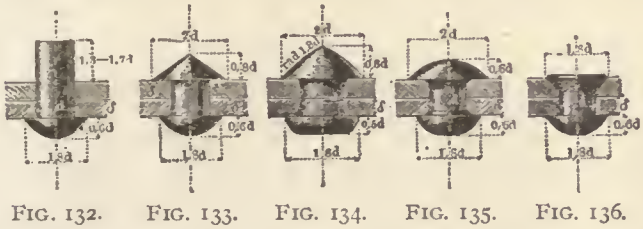

FIG. 132. FIG. 133. FIG. 134. FIG. 135. FIG. 136.

buttou head, while Fig. 133 shows the conical head generally formed by hand riveting. The length of body required to form the head varies from 1.3 to 1.7 times the diameter, according to the completeness with which the rivet fills the hole. When the head is formed by dies instead of the hand hammer, the shape is usually conoidal or spherical, as in Figs. 134–135.

The slight bevel given to each end of the rivet, as shown in Fig. 138, adds materially to its strength. The double conical hole shown in Fig. 137 assists in uniting the plates, and this shape may be produced by using in the punching machine a die slightly larger than the diameter of the punch. This difference has been experimentally determined for wrought-iron plates, and is secured by makiug the hole in the die equal to the diameter of the punch plus ¼ the thickness of the plate.

In Fig. 136 is shown a form of couutersunk rivet used in shipbuilding.

For bridge construction great care should be taken in the choice of proportions. Figs. 137–139 show the proportions adopted for the Dirschauer Bridge after the careful researches of the engineer Krüger. Fig. 137 shows the normal rivet head, and Figs. 138 and 139 the half aud full countersunk heads.

Rivets up to 1 or 1¼ inches iu diameter may readily be closed with hammers of 8 to 10 pounds weight; but if the head is to be formed in a swage or die, a heavier hammer, say 16 pounds weight, is uecessary.

The rate at which this work can be done by skilful riveters per day, according to Molinos and Pronnier, is as follows:

Diameter of Rivet.	No. per Day
5/8″	200 to 250
¾″	180 " 200
⅞″	100 " 125
1″	90 " 100

These figures are for horizontal bridge work; on vertical members about three-fourths these numbers may be taken.

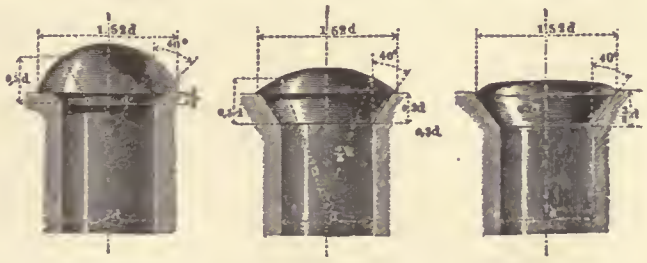

FIG. 137. FIG. 138. FIG. 139.

Much higher rates are shown upon boiler riveting, as may be seen from the following table, based upon observation of eleven days' work at the boiler works at Piedbœuf (Aachen):

Diameter of Rivet.	No. per Day.
19/20	350
5/8	325
¾	300
⅞	280
1	260
1 1/16	240
1 1/8	220
1 3/16	200

In cylindrical shells of more than three feet diameter these rates may be increased ten per cent., while for awkward or difficult work ten per cent. reduction should be made. Each man had the assistance of two strikers, one holder and one boy, sizes less than 5/8″ requiring but one striker.

Hand riveting is now being largely superseded by machine work. These machines possess the advantage of performing the work much more rapidly, thus insuring a stronger joint, besides which they are much more economical. Since their first introduction at the time of the building of the Conway Bridge, they have been extensively used for bridge work, and with the improvements which have been successively made they are rapidly displaciug hand riveting for boiler work.*

* These quantities are all given in metrical units in the original, but have been transformed in the text into English units. It must be remembered that the metrical horse-power (75 kgm.) is slightly smaller than the English horse-power.—TRANS.

* Among modern riveting machines the hydraulic riveter of Tweddell holds the first place. For a description the following references may serve: Polyt. Zeutral bl. 1874, p. 103; Engineering, Jan., 1875, p. 76; Sellers' Improved Tweddell's Machine, Jour. Frank. Inst., 1876, p. 305; Tweddell's Machine on a Crane, Sci. American, 1876, p. 226; Small Tweddell Machine, Revue Indust., 1876, p. 349.

Other forms of steam and hydraulic riveters well suited for boiler work are:

Garforth's Machine, Kronauer's Zeich. III., Johnson's Imp. Cyc., Pl. 42; the excellent steam riveter of Gouin, see Molinos & Pronnier, Ponts Metalliques, p. 180; also the hydraulic riveter at Creusot (giving a pressure of 20 to 80 tons on the rivet, and closing 2 to 25 heads per minute), Revue Indust., 1875, p. 349; also the very heavy machine of Kay & George (giving a pressure of 120 tons on the rivet), Engineering, 1875, p. 223. An apparently very ingenious machine is that of Allen, used especially for boiler riveting. In this machine the frame and post are temporarily held together by a rod operating in a very ingenious manner through one of he open rivet holes. At the Philadelphia Centennial Exhibition this machine closed three or more rivets per minute.

§ 55.

Strength of Riveted Joints.

Riveted joints are intended either to resist direct stresses (as in bridges and similar structures), or to secure a tight joint against moderate internal pressure (as in ships, gasholders, etc.), or in most cases both these conditions are united (as in the case of steam boilers). A distinction may then be made between joints for strength and joints for tightness, the seams of steam boilers standing midway between the two.

Fig. 140. Fig. 141. Fig. 142.

Joints for strength are either simple lap joints, Fig. 140, or butt joints, Fig. 142, the latter coming into general use for bridge work. The joint shown in Fig. 141, called a flap joint, is also somewhat used for vertical tubes, chimney stacks, etc., the flap really being nothing but a narrow plate.

For any given thickness, δ, of plate it is impracticable to make the riveted joint the same strength as the plate itself, but the ratio between the strength of the plate and the strength of the joint can be made a maximum. This will best be attained, with the assumption of a sufficieut margin, when the strength of the rivets and the strength of the remainder of the metal between the rivet holes are equal to each other, *i. e.*, when they reach their limit of elasticity at the same time. If the rivets and plate are of the same material, we have, according to § 5, the stress in the cross section of the rivets as 0.8 that of the plate. From this we derive the following formulæ, in which the friction of the joint is neglected, as being of uncertain value:

Let—

δ = the thickness of the plate,
d = the diameter of rivet,
a = the pitch of rivets, *i. e.*, the distance from centre to centre of adjacent rivets,
n = the number of rows of rivets,
ϕ = the modulus of resistance of the joint, being the ratio of the resistance of the joint to that of the full plate;

then the highest ratio of resistance will be attained when we have for lap joint riveting:

$$\left.\begin{aligned} \frac{a}{\delta} &= n\,\frac{\pi}{5}\left(\frac{d}{\delta}\right)^2 + \frac{d}{\delta} \\ \text{which gives:}\quad \phi &= 1 - \frac{d}{a} = \frac{1}{1 + \frac{1}{n}\,\frac{5}{\pi}\,\frac{\delta}{d}} \end{aligned}\right\} \quad \ldots\ldots\ldots (43)$$

or for butt joint riveting:

$$\left.\begin{aligned} \frac{a}{\delta} &= 2\,n\,\frac{\pi}{5}\left(\frac{d}{\delta}\right)^2 + \frac{d}{\delta} \\ \text{which gives:}\quad \phi &= 1 - \frac{d}{a} = \frac{1}{1 + \frac{1}{2\,n}\,\frac{5}{\pi}\,\frac{\delta}{d}} \end{aligned}\right\} \quad \ldots\ldots\ldots (44)$$

The overlap of the plate is subjected both to shearing and bending. For the former couditions, call the lap b', and for the latter b'', measuring in both cases from the centre of the rivets to the edge of the joint. To obtain the same resistance in the lap as in the perforated portion of the plate we have—

For lap joint riveting:

$$\left.\begin{aligned} \frac{b'}{\delta} &= \frac{5}{8}\,\frac{a-d}{n\,\delta} = \frac{\pi}{8}\left(\frac{d}{\delta}\right)^2 \\ \frac{b''}{\delta} &= \left(0.5 + 0.56\,\sqrt{\frac{d}{\delta}}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots (45)$$

For butt joint riveting:

$$\left.\begin{aligned} \frac{b'}{\delta} &= \frac{5}{8}\,\frac{a-d}{n\,\delta} = \frac{\pi}{4}\left(\frac{d}{\delta}\right) \\ \frac{b''}{\delta} &= \left(0.5 + 0.79\,\sqrt{\frac{d}{\delta}}\right)\frac{d}{\delta} \end{aligned}\right\} \quad \ldots\ldots\ldots (46)$$

In both cases a good value of b, in practice, giving sufficient room for rivet heads, will be secured by making:

$$b = 1.5\,d, \text{ or } \frac{b}{\delta} = 1.5\,\frac{d}{\delta} \quad \ldots\ldots\ldots\ldots (47)$$

A point of interest is the superficial pressure p, which exists between the body of the rivet and the cylindrical surface of the rivet hole. If S_2 is the stress in the punched plate we have—

For lap riveted joints:

$$\frac{p}{S_2} = 0.2\,\pi\,\frac{d}{\delta} \quad \ldots\ldots\ldots\ldots (48)$$

For butt riveted joints:

$$\frac{p}{S_2} = 0.4\,\pi\,\frac{d}{\delta} \quad \ldots\ldots\ldots\ldots (49)$$

The following table and scale will serve to reduce the numerical labor of these calculations:

§ 56.

Table and Proportional Scale.

	$\frac{d}{\delta}$ =	1.0		1.5		2.0		2.5		3.9		4.0	
	n =	1	2	1	2	1	2	1	2	1	2	1	2
Lap Joint.	$\frac{a}{\delta}$ =	1.63	2.22	2.92	4.33	4.52	7.04	6.43	10.37	8.67	14.33	14.07	24.14
	$\frac{b'}{\delta}$ =	0.39	0.39	0.88	0.88	1.57	1.57	2.54	2.54	3.53	3.53	6.28	6.28
	$\frac{b''}{\delta}$ =	1.06	1.06	1.78	1.78	2.58	2.58	3.46	3.46	4.31	4.31	6.48	6.48
	ϕ =	0.39	0.55	0.49	0.65	0.56	0.72	0.61	0.76	0.65	0.79	0.72	0.83
	$\frac{p}{S_2}$	0.63	0.63	0.94	0.94	1.26	1.26	1.57	1.57	1.88	1.88	2.51	2.51
Butt Joint.	$\frac{a}{\delta}$ =	2.26	3.52	4.33	7.15	7.04	12.05	10.37	18.21	14.33	25.61	24.14	44.21
	$\frac{b'}{\delta}$ =	0.79	0.79	0.96	0.96	3.14	3.14	4.91	4.91	7.07	7.07	12.56	12.56
	$\frac{b''}{\delta}$ =	1.29	1.29	2.20	2.20	3.24	3.24	4.37	4.37	5.60	5.60	8.32	8.32
	ϕ =	0.56	0.72	0.65	0.79	0.72	0.83	0.76	0.86	0.79	0.90	0.83	0.9
	$\frac{p}{S_2}$	1.26	1.26	1.88	1.88	2.51	2.51	3.14	3.14	3.77	3.77	5.03	5.03

In the proportional scale, Fig. 143, the principal values are graphically shown. It will be seen that the higher ratios of strength are not very practically obtained, for the large diameter rivets are inconvenient to handle. The advantages of lap joint riveting are also shown. The objection to butt joint riveting, which overrules its advantages, lies in the rapid increase in the value of p, as it will be seen that with a ratio $\frac{d}{\delta} = 3$ the elastic limit of wrought iron is exceeded, the stress reaching 30,000 lbs., while the stress upon the metal between the holes is only 8,600 lbs. This explains the failnre of riveted joints under variable tension loads. If the ratio of $\frac{d}{\delta} = 2$ is used, the excessive stress in the rivet holes cannot occur. Fairbairn, upon whose experimental researches these conclnsions are based, states that for riveted structures the diameter of rivet may best be taken as equal to $\frac{1}{\phi} \times \delta$; but this conclusiou is not fully borne out by experience. The use of the value for the lap $b = 1.5\,d$ is approximately correct at least for lap joint riveting, as the diagram shows it to give a slight margin both over the values of b, for shearing or for bending.

§ 57.

Riveting Disposed in Groups.

If more than two rows of rivets are to be used the efficiency of the joint may be decidedly increased without using inconveniently large rivets by disposing the rivets in groups on either side of a central row, arranging them according to an arithmetical series.

The numbers in adjacent rows may then be placed as follows:

1 : 2 : 1 . . . Total 4
1 : 2 : 3 : 2 : 1 . . . " 9
1 : 2 : 3 : 4 : 3 : 2 : 1 . . . " 16

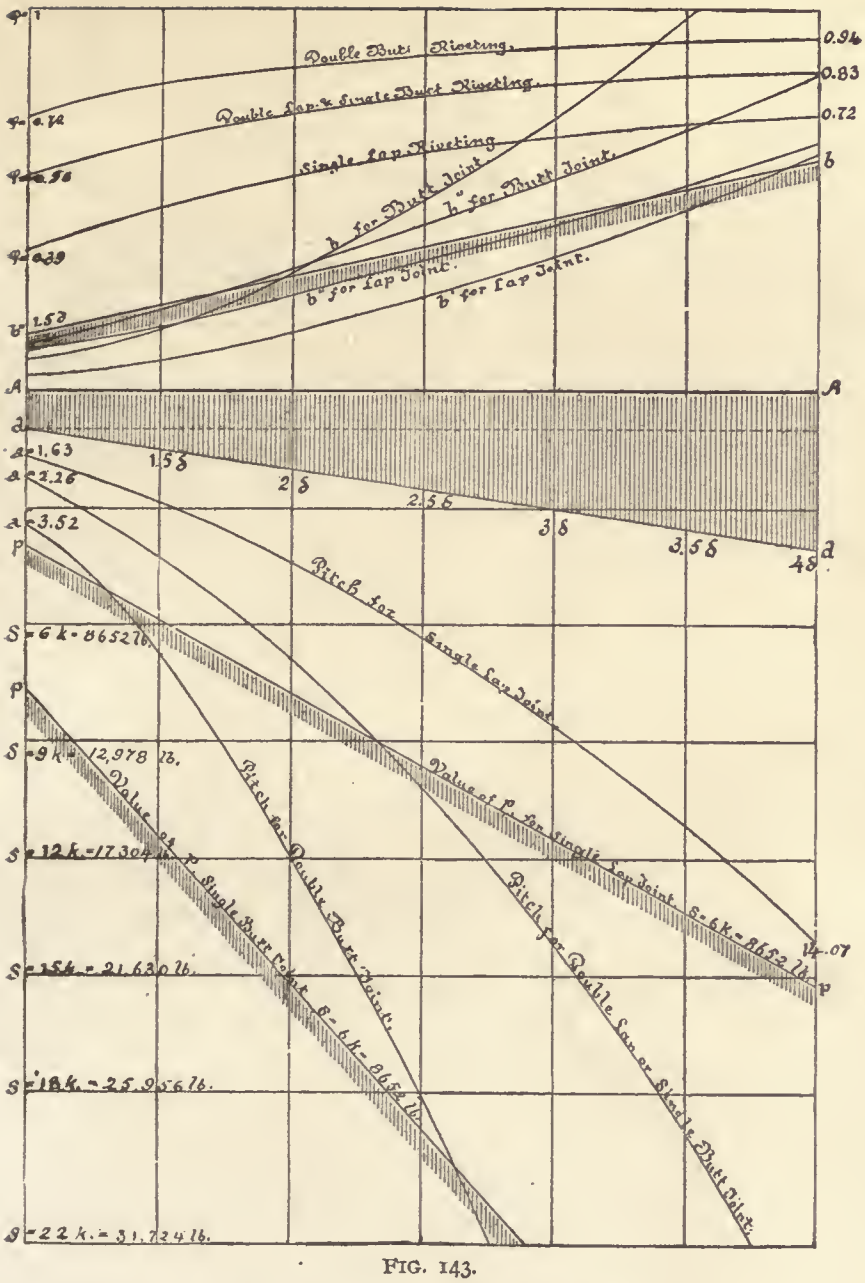

FIG. 143.

The following illustrations show examples of this form of riveting, which may be termed Group Riveting, and is especially adapted to lap joints. The dotted lines show the limit of the area including each group, and the spaces between the

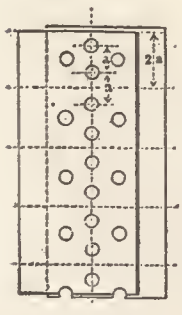

FIG. 144.

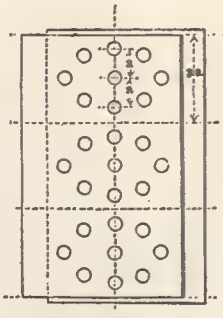

FIG. 145.

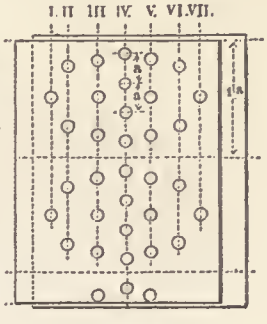

FIG. 146.

rivets in each longitudinal row are uniform. If m be the number of rivets in the middle row of each group, then the total number of rivets in each group is m^2.*

* This follows because, $\frac{1}{2}\left[1+m\right]+\frac{1}{2}\left[1+(m-1)\right](m-1)=m^2$.

If now we assume that the force P, upon each strip between the dotted lines, is equally divided among the rivets, we have for the efficiency of the first row:

$$\left.\begin{aligned}\frac{a}{\delta}&=m\frac{\pi}{5}\left(\frac{d}{\delta}\right)^2+\frac{1}{m}\frac{d}{\delta}\\ \text{or}\qquad\\ \frac{a}{d}&=m\frac{\pi}{5}\left(\frac{d}{\delta}\right)+\frac{1}{m}\end{aligned}\right\}\quad\ldots\ (50)$$

If the stress in the punched plate in the lines I, II, III, IV, etc., Fig. 146 be called S^{I}_2, S^{II}_2 S^{III}_2, S^{IV}_2, etc., we have:

$$\left.\begin{aligned}P&=S^{\mathrm{I}}_2\left(ma-d\right)\delta\\ &=S^{\mathrm{II}}_2\left(ma-2d\right)\frac{m^2}{m^2-1}\delta\\ &=S^{\mathrm{III}}_2\left(ma-3d\right)\frac{m^2}{m^2-3}\delta\\ &=S^{\mathrm{IV}}_2\left(ma-4d\right)\frac{m^2}{m^2-6}\delta\\ &=S^{\mathrm{V}}_2\left(ma-5d\right)\frac{m^2}{m^2-10}\delta\end{aligned}\right\}\quad.\ (51)$$

And from this when $S^{\mathrm{I}}_2=S^{\mathrm{II}}_2$ we have:

$$\frac{a}{d}=\frac{m^2+1}{m}\quad\ldots\ldots\ (52)$$

And upon the same supposition:

$$\frac{S^{\mathrm{III}}_2}{S^{\mathrm{I}}_2}=\frac{m^2-3}{m^2-2};\quad\frac{S^{\mathrm{IV}}_2}{S^{\mathrm{I}}_2}=\frac{m^2-6}{m^2-3};$$

$$\frac{S^{\mathrm{V}}_2}{S^{\mathrm{I}}_2}=\frac{m^2-10}{m^2-4};$$

that is, the stresses at the lines III, IV, V, are smaller than $S^{\mathrm{I}}_2=S^{\mathrm{II}}_2$. The useful application of this fact may be readily seen.

Let us introduce in (50):

$$\frac{d}{\delta}=\frac{5}{\pi}=1.5916,\text{ or say }1.6\ldots\ (53)$$

that is, we make the ratio $d:\delta$ constant and $=1.6$. For the modulus of the efficiency of the joint ϕ, when the stress in the solid plate is S_1, we have:

$$\phi=\frac{S_1}{S_2}=1-\frac{d}{m\,a}=\frac{m^2}{m^2+1}\ldots\ (54)$$

We also have for the pressure p, on the rivets:

$$p=\frac{P}{m^2\,\delta\,d}=S^{\mathrm{I}}_2\quad\ldots\ldots\ldots\ (55)$$

Tabulating the results of the applications of these equations to various groups we have:

m =	2	3	4	5
$\frac{d}{\delta}$ =	1.6	1.6	1.6	1.6
$\frac{a}{d}$ =	2.50	3.33	4.25	5.20
$\frac{a}{\delta}$ =	4.00	5.32	6.80	8.32
ϕ =	0.80	0.90	0.94	0.96

For joining narrow plates the rivets may often be disposed in

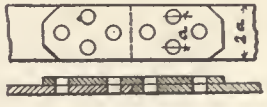

FIG. 147.

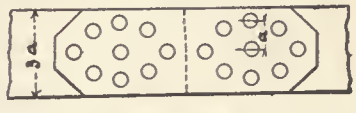

FIG. 148.

double groups, while for the union of several plates, as in the construction of plate girders, a number of groups may be em-

ployed. In Fig. 147 a triple row is shown, and in Fig. 148 a quadruple row, the joint in each case being made with a flap.

Besides the advantage which results from the disposition of the rivets in groups in such cases, there is also a gain in making the flap somewhat thicker than the plates to be joined. The calculation in this case may be derived from the preceding methods; also see the latter part of § 59.

§ 58.

Steam Boiler Riveting.

For the joints of steam boilers parallel riveting is generally used. In this case the question of the tightness of the joint

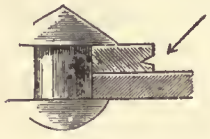

Fig. 149.

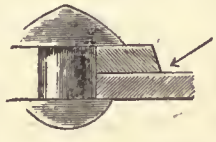

Fig. 150.

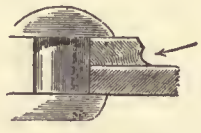

Fig. 151.

preveuts a wide spacing of the rivets. For the same reason the thinner plates require proportionally larger rivets than the heavier plates, and the lap of the plates, and also of the rivet heads, must be greater. The method of caulking is also to be considered. The older method consisted in driving the caulking chisel into the perpendicular edge of the plate and forcing the lower edge of the groove thus made down upon the lower plate, as shown in Fig. 149. The modern method, shown in Fig. 150, requires the plate to be planed on the edge to an angle, which can then be caulked without grooving. The angle of bevel should be about 18½°, or about 1 in 3. The method of the American, Cooney, consists in the use of a round nosed caulking tool, Fig. 151, which is much less liable to injure the lower plate than the sharp chisels of the previous methods; the action extends farther into the plate also.

The consideration of the tightness of the joint compels a modification of the theoretical treatment of the proportions of boiler seams, based upon practical experience.

According to Lemaitre the following proportions are suitable for single riveted joints:

$$\left.\begin{aligned} d &= 1.5\delta + 0.16'' \\ a &= 2d + 0.4'' \\ b &= 1.5d \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (56)$$

Double riveted joints are also much used for steam boilers, especially for the longitudinal seams, while single riveting is used for the circumferential joints, since the stress upon the longitudinal joints is much greater than upon the circumferential joints.

For double riveted joints, that is, for riveting in two parallel rows, we have for the pitch a_2 of the rivets in each row:

$$a_2 = 3d + 0.78'' \quad \ldots\ldots\ldots \quad (57)$$

while the space between the two rows may be taken as equal to the previous value of a, or $2d + 0.4''$. In some cases this value is used for the pitch of both rows (see Fig. 153).

We have previously taken the modulus of efficiency ϕ, so that the rivets and the perforated plate have not the same degree of security. The values of ϕ, for the rivets and for the plate should therefore be determined separately, and the smaller value taken for that of the completed joint. Let:

ϕ' = the modulus for the perforated plate,
ϕ'' = that of the rivets,

then according to the previous formulæ we have:

$$\left.\begin{aligned} \phi' &= \frac{a-d}{a} \\ \phi'' &= \frac{\pi}{5}\, n\, \frac{d^2}{a\delta} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (58)$$

For the pressure p upon the body of the rivet we have finally, both for single and double joints,

$$\frac{P}{S_2} = \frac{\pi}{5}\left(\frac{4}{\delta} + 0.6\right) \quad \ldots\ldots \quad (59)$$

The stress S_2 in the perforated plates of boiler shells is not generally permitted to exceed 5000 to 6000 pounds per square inch of cross section.

The following table contains the collected results of the preceding formulæ for the more commonly occurring proportions:

§ 59.

Table and Proportional Scale for Steam Boiler Riveting.

δ	d	Round Head. Height 0.6d.	Round Head. Dia. 1.8d.	Conical Head. Height 0.8d.	Conical Head. Dia. $2d$.	Length.	a Single Riveting.	a_2 Double Riveting.	Modulus of Efficiency. ϕ'	Modulus of Efficiency. ϕ''	Modulus of Efficiency. ϕ_2''	Weight of 100 Pair of Heads.
1/8	3/8	1/4	5/8	5/16	3/4	7/8	1 1/8	1 7/8	0.66	0.51	0.60	2
8/16	7/16	1/4	3/4	3/8	7/8	1	1 1/4	2 1/8	0.66	0.51	0.60	4 1/2
1/4	9/16	5/16	1.00	7/16	1 1/8	1 1/2	1 1/2	2 3/8	0.63	0.48	0.59	6 3/4
5/16	5/8	3/8	1 1/8	1/2	1 1/4	1 3/4	1 5/8	2 5/8	0.61	0.48	0.59	13 1/4
3/8	11/16	7/16	1 1/4	9/16	1 3/8	2	1 3/4	2 7/8	0.60	0.47	0.59	20
7/16	13/16	1/2	1 1/2	5/8	1 5/8	2 1/4	2	3 1/8	0.60	0.47	0.59	29
1/2	15/16	9/16	1 5/8	3/4	1 7/8	2 1/2	2 1/4	3 1/2	0.59	0.47	0.59	36 1/2
9/16	1	5/8	1 7/8	13/16	2.00	2 3/4	2 3/8	3 3/4	0.58	0.47	0.59	50 1/2
5/8	1 1/16	11/16	2.00	7/8	2 1/8	3	2 1/2	4 1/8	0.58	0.47	0.59	66

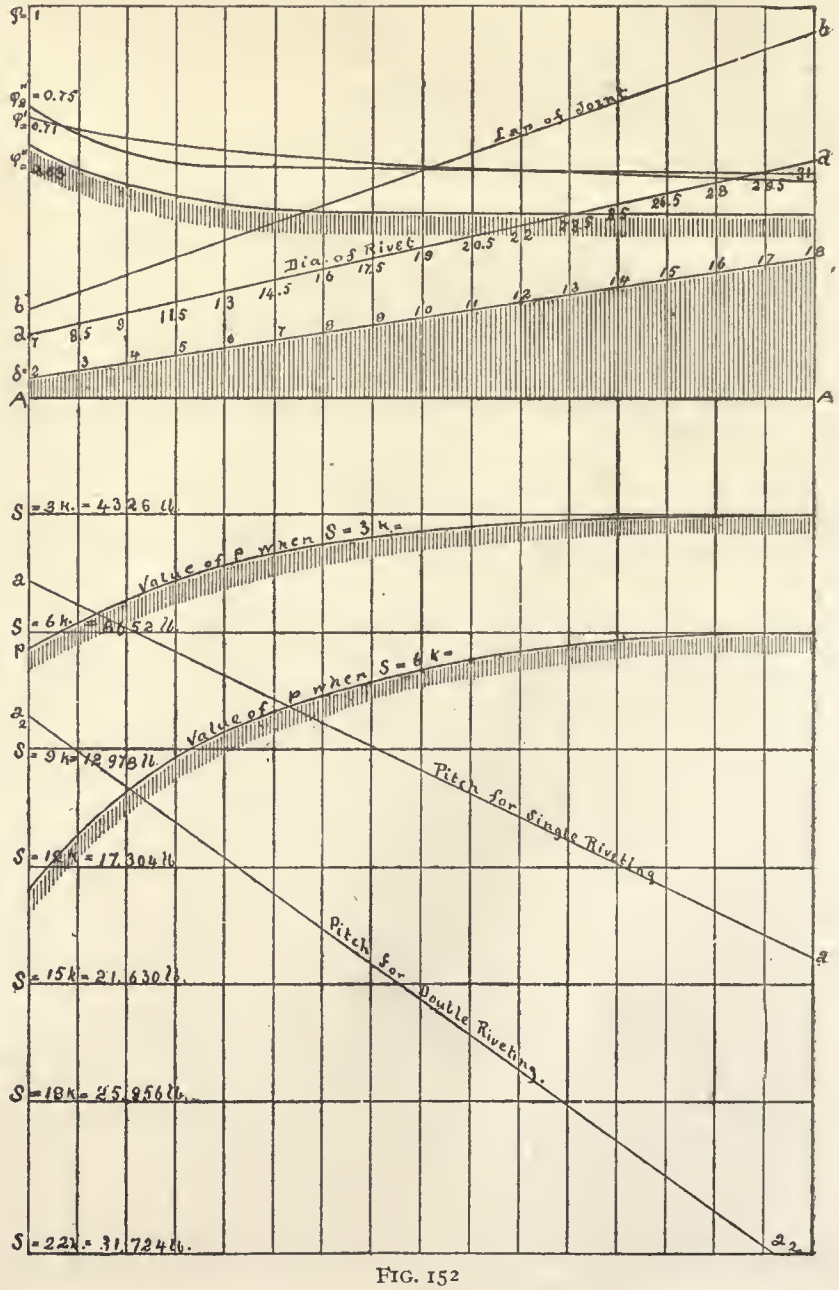

Fig. 152

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The rivet length is $= 2\delta + 1.7d$, upon the assumption that both plates are of the thickness δ, and this length gives an ample allowance for the full clearance of the rivet holes (see § 54). The last column is of service in making estimates of weights.

Fig. 152 is a graphical presentation of the principal results of the preceding formulæ. It will be noticed that for single riveting the modulus ϕ'', for the rivets, is always less than the modulus ϕ' for the perforated plate, and is nearly always less than ½. It follows that for single riveted joints of steam boilers we should never assume a greater strength than one-half that of the solid plate. By the adoption of double riveting, while retaining the same pitch, $a = 2d + 0.4''$, we ought to obtain, according to the formula of § 58, a value of ϕ'', twice as great, which in the case of very light plates would exceed unity. In that case, however, the value of ϕ' is the lesser, and it determines the efficiency of the joint, so that the only gain due to the double riveting in that case is the increase in the value of ϕ'. If, however, we choose for double riveting the pitch a_2, as given from equation (57), both ϕ' and ϕ'' will be increased in value. The lesser of the two moduli is that for the rivets, and its value is obtained from

$$\phi_2'' = 2\,\frac{\pi}{5}\,\frac{d^2}{a\delta} \qquad \ldots\ldots\ldots\ldots\ldots (60)$$

Its value lies between 0.75 and 0.59, and is shown in the table and diagram. The pressure p in all cases remains within practical limits.

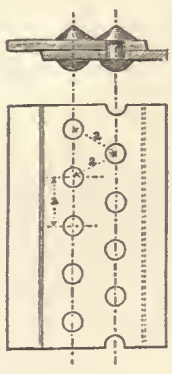

FIG. 153.

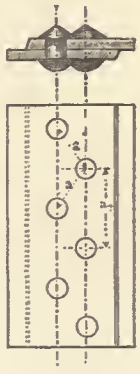

FIG. 154.

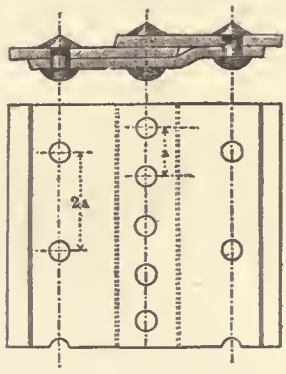

FIG. 155.

In Fig. 153 is shown double riveting in which the pitch kept equal to $2d + 0.4''$, while in Fig. 154 the pitch is made equal to $3d + 0.78''$ for rivets in the same row, while the diagonal distance between rivets of the two rows is the same as for single riveting. For a flap joint such as is shown in Fig. 155, we have a combination of parallel and group riveting. This method is used in Germany for steam boilers, but is little used in America, if at all. The flap is placed on the inside of the boiler shell, and the flap seams have only half as many rivets as the main seam, but of the same diameter. The objection that the inner edge of the main seam is made inaccessible is counterbalanced by the increase in strength. We have

$$\phi' = \frac{2a - d}{2a}$$

$$\phi'' = \frac{0.3\ \pi\ d^2}{a\delta}$$

the lesser of which will be found to exceed the value obtained for ordinary double riveted joints.*

Example. $\delta = \frac{5}{16}''$, $d = \frac{5}{8}''$, $a = 2d + 0.4'' = 1.65''$, say $1\frac{5}{8}''$; $\phi' = \frac{3.3 - 0.625}{3.3} = 0.81$. $\phi'' = \frac{0.3\,\pi \times 0.391}{1.65 \times 0.3125} = 0.71$. It may be remarked that American practice gives wider pitches than are generally used in Europe.†

* The two rivets which lie between any pair of rivets in the main joint each bear a stress of $\frac{1}{3}P$, and the rivets of the main joint also sustain $\frac{1}{3}P$. The flap does not transmit any stress to the rivets of the main seam. For the stress on rivets $P = \frac{3}{4}\pi d^2 S'_3$, for the solid plate $P = S_1\,2a\delta$. The modulus ϕ'', taking S_3 for shearing stress $= \frac{S_1}{\frac{5}{4}S_3}$, is found from $\phi'' = \frac{\frac{4}{5}\cdot\frac{3}{4}\pi d^2}{2a\delta} = \frac{0.3\,\pi d^2}{2ad}$. For the main seam we obtain ϕ', from $P = \frac{2}{3}(2a - 2d)\,\delta\,S'_2 = S_1\,2a\delta$, whence $\phi = \frac{3(a-d)}{2a}$, which is greater than $\frac{2a-d}{2a}$.

† For example, in single riveting ⅜″ plate and 1⅛″ rivets have 1⅞″ pitch (the formula would give about 1¾″); ⅜″ plate and ⅞″ rivets have a pitch of 2⅝″ (the formula would give about 2⅛″); in double riveting, for ⅜″ plates with ¾″ rivets, the pitch would be 3⅜″, while the formula would give about 3″.

Three rows of rivets are used in this form of joint, and the outside rows of wide pitch make this method more troublesome of execution than the group riveting shown in Fig. 144, which has a modulus of 0.80. This is a point which should be borne in mind.

The joints of gasometers exhibit but little variety in plates or rivets. The rivets are usually about ¼″ to $\frac{5}{16}''$ in diameter and 1″ pitch, with a lap at the joint of about ½″, the rivets being closed cold and the joints caulked with red lead.

TABLE OF THE WEIGHT OF SHEET METAL.

Thickness in Inches.	Weight in Pounds per Square Foot.					
	Wro't Iron	Cast Iron.	Brass.	Copper.	Lead.	Zinc.
$\frac{1}{16}$	2.53	2.34	2.73	2.89	3.71	2.34
⅛	5.05	4.69	5.47	5.78	7.42	4.69
$\frac{3}{16}$	7.58	7.03	8.20	8.67	11.13	7.03
¼	10.10	9.38	10.94	11.56	14.83	9.38
$\frac{5}{16}$	12.63	11.72	13.67	14.45	18.54	11.72
⅜	15.16	14.06	16.41	17.34	22.25	14.06
$\frac{7}{16}$	17.68	16.41	19.14	20.23	25.96	16.41
½	20.21	18.75	21.88	23.13	29.67	18.75
$\frac{9}{16}$	22.73	21.09	24.61	26.02	33.38	21.09
⅝	25.27	23.44	27.34	28.91	37.08	23.44
$\frac{11}{16}$	27.79	25.78	30.08	31.83	40.79	25.78
¾	30.31	28.13	32.81	34.69	44.50	28.13
$\frac{13}{16}$	32.84	30.47	35.55	37.58	48.21	30.47
⅞	35.37	32.81	38.28	40.47	51.92	32.81
$\frac{15}{16}$	37.90	35.16	41.02	43.36	55.93	35.16
1	40.42	37.50	43.75	46.25	59.33	37.50

§ 61.

SPECIAL FORMS OF RIVETED JOINTS.

Junction of Several Plates.—In Fig. 156 is shown the junction of three plates. In this case the corner of sheet No. 2 is beveled off and No. 1 worked down over the lap.

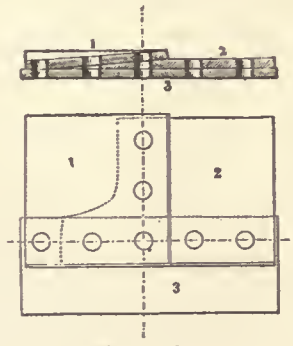

FIG. 156.

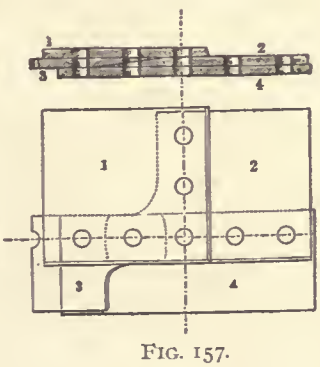

FIG. 157.

In Fig. 157 the junction of four plates is showu. Here the angles of sheets Nos. 2 and 3 are beveled and Nos. 1 and 4 are left unaltered. In the construction of steam boilers the shell may be formed either in cylindrical sections, as shown in Fig. 158, or in sections of a conical shape, the taper of all the sections bearing the same relation to the direction of the flame as shown in Fig. 159. This latter method requires that a slight curvature should be given to the sheets in order to secure the required taper. The determination of the taper and curvature of the sheets and lines for the rivet holes may be made in the following manner :

FIG. 158.

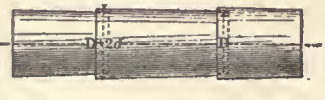

FIG. 159.

Let—

D = the diameter of the shell, as in Fig. 159,
B = the breadth of the sheet, Fig. 160, on a circumferential seam,
L = the length of the sheet between pitch lines of rivets,
f = the versed-sine of the arc B; we then have:

$$\frac{f}{\delta} = \tfrac{1}{4}\,\frac{B}{D}\,\frac{B}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (61)$$

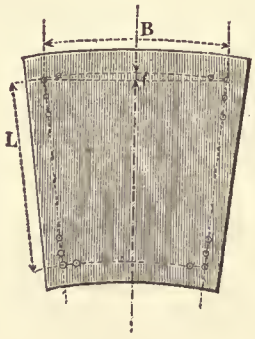

FIG. 160.

Example. In a riveted tube where each section is made of an entire sheet we have $B = \pi D$. If the breadth B is twice the length L, we have

$$\frac{f}{\delta} = 0.7854 \times 2 = 1.5708,$$

so that f will be a little greater than 1½ times the thickness of the plate.

In arranging the junction of sheets when the flap joint is employed, care must be taken to avoid complicated intersections. This is best accomplished by making the flaps on the longitudinal and circumferential seams come on opposite sides of the plates. Where the flaps are both on the same side, they are sometimes let into each other.

Reinforcement of Plates.—This may often be done very readily by the use of angle and T iron. In Fig. 161 is shown

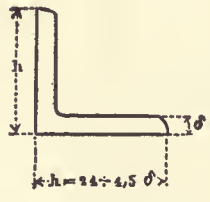

FIG. 161.

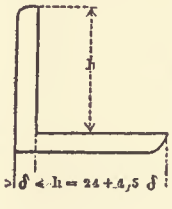

FIG. 162.

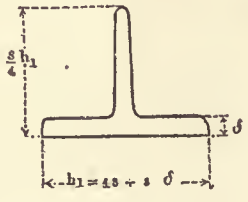

FIG. 163.

an internal angle iron, and in Fig. 162 an external, and in Fig. 163 a simple T iron. The proportions for angle iron given by Redtenbacher are as follows:

h = height of angle arm,
δ = thickness.

$$h = 4.5\,\delta + 1''.$$

For T iron h_1 = the base = $8\,\delta + 2''$, and the height of the rib = $\frac{3}{4}h_1$. In practice a great variety of proportions are made to suit all possible cases, examples of which may be found in the illustrated catalogues of the mills where they are rolled.

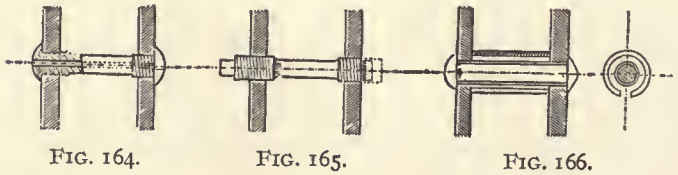

FIG. 164. FIG. 165. FIG. 166.

The strengthening of parallel plates which are near together is best done by the use of staybolts. In Figs. 164 and 165 is shown a copper staybolt after and before riveting, this form being used in locomotive fire boxes and marine boilers. The central hole affords a warning of the corrosion or weakening of the bolt by the escape of steam. It is best to remove the screw thread from the projecting portions before riveting over the heads. Fig. 166 shows a form of iron staybolt for the same purpose. The short piece of tube between the plates prevents them from being drawn out of shape while riveting, and the opening permits a free circulation of water. The bolt is protected from corrosion by being incased in copper. Screw staybolts are now often made of soft wrought iron or mild steel, but copper bolts are still preferred by many.

FIG. 167. FIG. 168. FIG. 169. FIG. 170.

Construction of Angles (Figs. 167–170).—Angle junctions in riveted work are made either by flanging the plate or by the use of angle iron. In Fig. 167 the flange is turned inward, and in Fig. 168 it is turned outward. In these cases h is made the same as for angle iron of the same thickness. Figs. 169 and 170 show the use of internal and external angle iron.

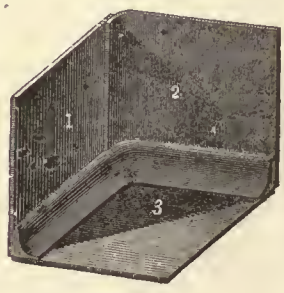

FIG. 171.

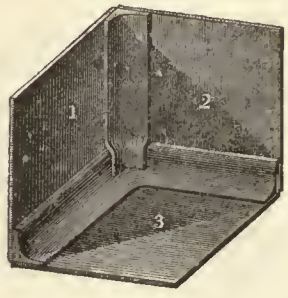

FIG. 172.

Construction of Solid Angles.—These are the most difficult forms of riveted work, and may be made in several manners, the most important being shown in the illustrations. In Fig. 171 the vertical angle is made as in Fig. 167, and the horizontal angles as in Fig. 169, sheet No. 2 being beveled under the angle iron. In Fig. 172 all three angles are made as in Fig. 169, the

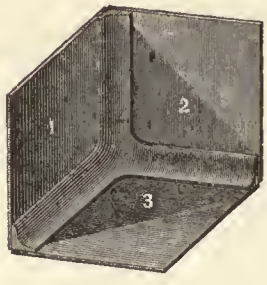

FIG. 173.

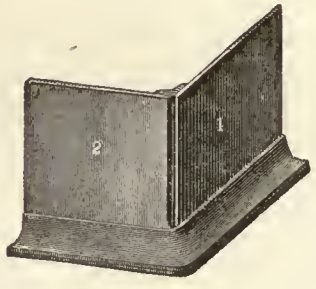

FIG. 174.

vertical angle iron being cut and bent over the horizontal. In Fig. 173 the angles are all made as in Fig. 169, but the angle irons are welded together at their junction. This makes an excellent piece of work, but is difficult and expensive, and requires firm support for the work, and is only applicable for important constructions. In Fig. 174 the vertical angle is made like Fig. 169, while the lower joint is made as in Fig. 170, making simple and substantial corner.

CHAPTER II.

HOOPING.

§ 62.

Hooping by Shrinkage.

The use of hoops or bands is a very efficient method of uniting some combinations of machine elements, and also for strengthening existing combinations. The hoops or bands are arranged so as to encircle the portions to be united, and caused to exert sufficient pressure upon them to create such friction between the surfaces as to prevent any relative motion. It follows that the material in the band is subjected to tension while the parts which are held together are under compression. The bodies to be hooped are nearly always either cylindrical or conical in shape.

The pressure required to secure the hoops may be obtained either by shrinkage, a method formerly used very extensively, or by cold pressure, a modification being described in the latter part of § 64.

The elongation which is produced by elevating the temperature to a red heat may be taken for steel and wrought iron at about $\frac{1}{800}$, while to keep within the limits of elasticity the resistance to contraction should be, for

Cast or wrought iron . . . . . $\frac{1}{1300}$

Cast steel . . . . . $\frac{1}{850}$

Hence the allowance for shrinkage to be made in boring for a cast iron hub to fit over an unyielding centre should not be greater than $\frac{1}{1300}$, and is best made from $\frac{1}{1500}$ to $\frac{1}{1800}$, especially if the centre is very heavy. The ring can then be fitted to its place when at a dull red heat. For wrought iron or steel rings, such as wheel centres, such precautions are not so essential, since these materials permit of a slight extension without injury (see § 2). If the centre possesses but very slight yielding elasticity, there may be danger, however, that the contraction due to excessive cold may overstrain the material.

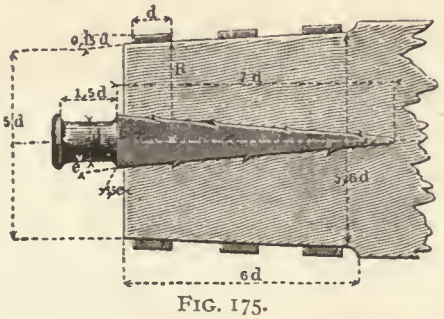

Fig. 175.

When wrought iron bands are to be used to secure iron journals to wooden shafting, as shown in Fig. 175, the end of the shaft is made slightly conical, so that the bands, being raised only to a dull red heat, may be driven on with the hammer. The rings may be forged tapering, but the taper may be also readily produced by Clerk's method by repeated heating and cooling.* The red hot ring is immersed in the cooling tub for

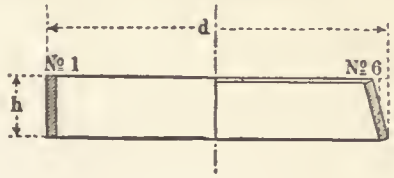

Fig. 176.

about half its axial height. The rapid contraction of one portion of the ring deflects the warmer portion towards the centre, and by repeating the process the taper may be produced to almost any extent which may be required.

The following experiments, made in the Royal Technical Academy, will serve to illustrate the process. The ring shown in Fig. 176 had the following original dimensions:

$$n = 5\tfrac{1}{2}'', \ \delta = \tfrac{9}{32}'', \ D = 8\tfrac{1}{2}''.$$

After the first immersion the contraction was $\frac{3}{80}$

" second " " $\frac{4}{80}$

" third " " $\frac{5}{80}$

" fourth " " $\frac{7}{80}$

" fifth " " $\frac{13}{80}$

" sixth " " $\frac{16}{80}$

After the last immersion the dimensions were found to be $D = 7\frac{15}{16}''$ at the upper edge, and at the lower edge $= 8\frac{9}{16}''$.

A method of connecting two flat bars by shrinking on a hoop is shown in Fig. 177, and has been used at Seraing with good results.

The hubs of gear wheels or revolving cylinders are advantageously strengthened by bands if they are cast in several parts, as in this way they are firmly united into a compact whole.*

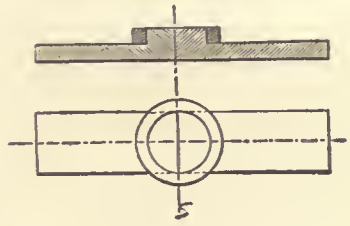

Fig. 177.

§ 63.

Cold Hooping.

In the place of shrinking bands to their places, the more recent method of forcing them on cold has come into use for bands of moderate size, such as for hubs of wheels, cranks and levers. In this case the ring and its seat are both made truly cylindrical, with merely a slight bevel for entrance, and then by means of a press forced together.† The difference in diameter between the ring and hub is very small, and may be calculated as described in § 19.

An investigation of the force required to push a ring on may be found desirable. The force which is necessary to press a cylindrical pin into a hub by continuous motion may be taken as nearly proportional to rate of progress, since it has to overcome the resistance of sliding friction between the surfaces. The pressure p, per unit of surface, is equal to the initial radial stress S_1, which exists upon the pin. If we make r the radius of the pin, l the length of the hole, f the co-efficient of friction, we have for a maximum value of the forcing pressure Q:

$$Q = 2r\pi l S_1 f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Taking $f = 0.2$, as indicated by experience shown in the following cases, we have

$$p = S_1 = \frac{5Q}{2\pi r l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

For the tangential stress S_2, in the metal of the ring, we obtain from formula 37, § 19:

$$\frac{S_2}{S_1} = \frac{1}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

And taking the thickness of the metal of the ring as δ, we get the value of ρ:

$$\rho = \frac{\left(1 + \frac{\delta}{r}\right)^2 - 1}{\left(1 + \frac{\delta}{r}\right)^2 + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

This gives for the following ratios of thickness to radius, corresponding numerical values:

$\frac{\delta}{r}$ =	0.50	0.55	0.60	0.65	0.70	0.75		
ρ =	0.385	0.415	0.438	0.463	0.486	0.508		
$\frac{\delta}{r}$ =	0.80	0.85	0.90	1.00	1.10	1.20		
ρ =	0.528	0.548	0.566	0.600	0.630	0.658		
$\frac{\delta}{r}$ =	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
ρ =	0.682	0.704	0.724	0.744	0.759	0.774	0.787	0.800

The following table shows examples of the practice of many of the leading Continental railways. In the table, $2r$ = diameter, l = length, δ = thickness of hub, Q = total forcing pressure; also, W. I. = wrought iron, C. I. = cast iron, S = steel, C. S. = cast steel, B. S. = Bessemer steel.

* See Procedings of the Royal Society, March, 1873. Civilingenieur, Vol. X., 1864, p. 238.

* For an account of the strengthening of a piston head by shrinking on bands, see the Berliner Verhandlungen, 1876, Sheet XVI.

† Sometimes these surfaces are made slightly conical, such being the case in Nos. 15 to 17 of the following examples.

§ 64.

EXAMPLES OF FORCED CONNECTIONS.

No.	DESCRIPTION.	DIMENSIONS. Inches.			MATERIAL.		Q	REMARKS.
		$2r$	l	δ	Hub.	Axle.	Pounds.	
	EASTERN RAILWAY OF PRUSSIA.							
1	Locomotive Driving and Coupled Wheels,	7½	7⅞	3 15/16	W. I.	C. S.	160,600	With Key. Data as given.
2	Locomotive Trailing Wheels,	7½	6 11/16	3	W. I.	C. S.	160,600	No Key. Data furnished.
3	Tender Wheels,	5½	6 11/16	2 9/16	W. I.	C. S.	118,800	No Key. Data furnished.
4	Car Wheels, with Spokes,	5⅛	9¼	2 5/16	W. I.	C. S.	110,000	No Key. Data furnished.
5	Steel Plate Wheels, for Cars,	5⅛	7⅞	2	C. S.	C. S.	110,000	No Key. Data furnished.
6	Steel Plate Wheels, for Locomotives,	7¼	7	2 1/16	C. S.	C. S.	154,000	No Key. Data furnished.
	UPPER SILESIAN RAILWAY.							
7	Locomotive Driving Wheels,	7¼	6 11/16	3 5/8	W. I.	C. S.	220,000 to 330,000	With Key. Measured Dimensions.
8	Locomotive Trailing Wheels,	6 5/16	7⅛	3⅛	W. I.	C. S.	165,000 to 220,000	With Key. Measured Dimensions.
9	Tender Wheels,	5 11/16	7 5/16	3⅜	W. I.	C. S.	110,000 to 132,000	No Key. Measured Dimensions.
10	Steel Plate Car Wheels,	5⅛	6 11/16	1 9/16	C. S.	C. S.	110,000 to 132,000	No Key. Measured Dimensions.
11	Wrought Iron Spoked Car Wheels,	5⅛	7 5/16	2⅜	W. I.	C. S.	132,000 to 154,000	No Key. Measured Dimensions.
	HANOVERIAN STATE RAILWAY.							
12	Locomotive Driving and Coupled Wheels,	7½	7	3¼	W. I.	C. S.	165,000 to 176,000	With Key. Data furnished.
13	Locomotive Trailing Wheels,	6¾	6¾	2 7/16	W. I.	C. S.	143,000 to 154,000	Without Key. Data furnished.
14	Standard Car Wheel,	5⅛	8	1¾	W. I.	C. S.	88,000 to 110,000	Without Key. Data furnished.
	MAGDEBURG-HALBERSTADT R. R.							
15	Car Wheels,	5⅛	7⅞	2 3/16	W. I.	. . .	110,000 to 132,000	No Key. Measured Dimensions.
16	Car Wheels,	5⅛	8½	4½	C. S.	. . .	110,000 to 132,000	No Key. Measured Dimensions.
17	Locomotive Wheels,	7⅛	6 11/16	3¼	W. I.	. . .	176,000 to 198,000	No Key. Measured Dimensions.
	SAARBRÜCK RAILWAY.							
18	Locomotive Driving and Coupled Wheels,	7	7½	8¼	C. I.	C. S.	137,940	With Key. Measured Dimensions.
19	Locomotive Driving and Coupled Wheels,	7	8	2 9/16	W. I.	C. S.	247,588	With Key. Measured Dimensions.
20	Locomotive Driving and Coupled Wheels,	7 5/8	7 5/16	3¾	W. I.	C. S.	247,588	With Key. Measured Dimensions.
21	Locomotive Trailing Wheels,	6	6	2 11/16	W. I.	C. S.	198,000	No Key. Measured Dimensions.
22	Tender Wheels,	5¼	6⅞	6¼	C. I.	W. I.	136,840	No Key. Measured Dimensions.
23	Tender Wheels,	5 15/16	7⅝	2¼	W. I.	W. I.	168,080	No Key. Measured Dimensions.
24	Tender Wheels,	5 15/16	5 15/16	2	W. I.	C. S.	198,000	No Key. Measured Dimensions.
25	Standard Car Wheels,	5⅛	7⅞	2 9/16	W. I.	C. S.	165,000 to 193,600	No Key. Measured Dimensions.
26	Freight Car Wheels,	5⅛	8½	4⅝	C. I.	W. I.	165,000	No Key. Measured Dimensions.
27	Coal Car Wheels,	5⅛	7¼	2 9/16	W. I.	W. I.	193,600	No Key. Measured Dimensions.
28	Passenger Car Wheels,	4⅛	7½	3⅜	C. I.	W. I.	110,000 to 165,000	No Key. Measured Dimensions.
	RIGA-DÜNABERG RAILWAY.							
29	Driving and Coupled Wheels (Stephenson),	7	7⅜	3¾	W. I.	W. I.	90,200	With Key. Measured Dimensions.
30	Locomotive Trailing Wheels "	6½	6½	2¾	W. I.	W. I.	90,200	With Key. Measured Dimensions.
31	Tender Wheels "	5⅜	7	3	W. I.	W. I.	85,800	With Key. Measured Dimensions.
32	Driving, Coupled & Trailing Wheels (Borsig),	6⅞	6⅛	3¼	W. I.	W. I.	90,200	With Key. Measured Dimensions.
33	Tender Wheels (Borsig),	5 9/16	7	. .	W. I.	W. I.	85,800	With Key. Measured Dimensions.
34	Passenger Car Wheels (Ashbury),	4¾	6½	2¼	W. I.	W. I.	68,200	With Key. Measured Dimensions.
35	Freight Car Wheels (Zypen),	5⅛	10¼	. .	W. I.	W. I.	77,000	No Key. Measured Dimensions.
36	Freight Car Wheels (Zypen),	5⅛	8⅞	. .	W. I.	B. S.	88,000	No Key. Measured Dimensions.
	BORSIG LOCOMOTIVE WORKS.							
37	Locomotive Trailing and Tender Wheels,	6 to 8	6½–7	2½–3	W. I.	W. I. or S.	155,000 to 220,000	With Key. Measured.
38	Locomotive Driving and Coupled Wheels,	6¾–8	7 to 8	3 to 4	W. I.	W. I. or S.	220,000 to 330,000	With Key. Measured.
39	Crank Pius,	4 to 6	7 to 8	2–2½	W. I.	W. I. or S.	110,000 to 165,000	No Key.
	WÖHLER LOCOMOTIVE WORKS.							
40	Locomotive Driving and Coupled Wheels,	7½	7⅞	3¾	W. I.	W. I. or S.	220,000	With Key. Measured.
41	Locomotive Trailing Wheels,	7½	6¾	3	W. I.	W. I. or S.	220,000	No Key. Measured.
42	Tender Wheels,	5½	6¾	2 9/16	W. I.	W. I. or S.	132,000	No Key. Measured.
	NORTHERN RAILWAY OF FRANCE.							
43	Locomotive Wheels (Stephenson),	. .	. .	. .	. . .	. . .	176,000	Data furnished.
44	Locomotive Wheels (Clapeyron),	. .	. .	. .	. . .	. . .	176,000	Data furnished.
45	Tender Wheels, with strong hubs,	. .	. .	. .	. . .	. . .	176,000	Data furnished.
46	Crank Pins,	. .	. .	. .	. . .	. . .	33,000	Data furnished.
	PARIS-LYONS-MEDITERRANEAN R. R.							
47	Locomotive Driving Wheels,	. .	. .	. .	W. I.	W. I.	77,000 to 88,000	With Key.
48	Locomotive Trailing Wheels,	. .	. .	. .	W. I.	W. I.	55,000 to 66,000	No Key.
49	Tender Wheels,	. .	. .	. .	W. I.	W. I.	55,000 to 66,000	No Key.
50	Car Wheels,	. .	. .	. .	W. I.	W. I. or S.	39,600 to 48,400	No Key.
51	Car Wheels,	. .	. .	. .	C. S.	W. I. or S.	22,000.	No Key.
52	Crank Pius,	. .	. .	. .	W. I.	C. S.	66,000	No Key.

From example No. 12 we obtain in formula (63) the value

$$S_1 = \frac{5 \times 176{,}000}{7.5 \times \pi \times 7} = 5{,}336 \text{ lbs.}$$

According to (65) $\rho = 0.53$, and substituting these values in (64) gives $S_2 = 10{,}679$ lbs.

From example No. 10 we have:

$$S_1 = \frac{5 \times 132{,}000}{5.125 \times \pi \times 7.3125} = 5{,}603 \text{ lbs.}$$

also $\rho = 0.44$, and hence $S_2 = 12{,}734$ lbs.

From example No. 37 we have:

$$S_1 = \frac{5 \times 220{,}000}{7.5 \times \pi \times 6.7} = 6{,}970 \text{ lbs.}$$

also $\rho = 0.526$, giving $S_2 = 13{,}250$ lbs.

From example No. 16, taking $Q = 132{,}000$ lbs., we get $S_1 = 4867$ lbs.; $\rho = 0.77$; $S_2 = 6320$ lbs.; and in No. 17, we have $S_1 = 6617$ lbs.; $\rho = 0.569$, and $S_2 = 11{,}629$ lbs., neither of which are excessive.

The force required to force a hub off an axle upon which it has been pressed, is not materially different from the force with which it was pushed on. The bore of such a hub may also be reduced when necessary by forcing rings upon it. Such rings, when used for car wheel hubs, are usually made of rectangular cross sections, the diameter ranging from $2'' \times 1''$, to $1\frac{3}{8}'' \times 1\frac{1}{4}''$, etc.

An inspection of the table will show that there is a tendency towards increasing pressures. For car wheels, where until quite recently, pressures of 60,000 to 90,000 pounds were used, we now find 80,000 to 110,000 pounds not infrequently; while for locomotive wheels, over 200,000 pounds is the rule.

Midway between the methods of shrinkage, and of cold forcing comes the lesser used method of expansion by use of boiling water. This system secures a much more uniform action of the temperature than is practicable with a red heat, and has been used with excellent results upon the Russian railways for fitting tires to plate wheels. The tires are suspended by a crane, in a tank of water which is kept at the boiling temperature by a jet of steam (the allowance for expansion being a little less than $\frac{1}{100}$ of an inch to the foot of diameter. An immersion of 10 to 15 minutes is required to obtain the desired expansion. Three workmen can in this manner fit 12 to 14 tires per day of eleven hours. This method may also be found applicable to the fitting of hubs.

65.

Dimensions of Rings for Cold Forcing.

Since the forms of the various hubs may be taken as cylindrical in nearly every case, the stress may be calculated by the formulæ already given. It is, however, desirable to present these in such form that they may be used to determine the thickness of hub which, when forced on cold, shall resist a determinate force. In (62) instead of the radial stress S_1, substitute the tangential stress S_2, giving $Q = 2\pi r l f S_2 \rho$, which combined with (65) gives:

$$\frac{\delta}{r} = \sqrt{\frac{2\pi r l f S_2 + Q}{2\pi r l f S_2 - Q}} - 1 \quad \ldots\ldots (66)$$

In this, Q is the maximum force which the hub can oppose to turning, at the diameter of the fit. If we take the moment of the force tending to rotate the wheel as PR, we must have $Q r \geqq PR$. $\frac{Qr}{PR}$ will then be the factor of resistance against slipping in any such case. This mode of attachment is then only practicable when $2\pi r l f S_2 > Q$. By choosing different values for S_2, and Q, various thicknesses for the metal of the hub may be obtained.

Example. The following data are taken from Borsig's Express Locomotive at the Vienna Exposition; Two pairs of coupled driving wheels of 38.19″ radius, without keys; bore of cylinders 17″; steam pressure 147 pounds; crank radius $R = 10''$. If we suppose the entire force upon the piston to act upon a single wheel, we have:

$$PR = (17)^2 \times 0.7854 \times 147 \times 10 = 33{,}366 \times 10$$

The bore of the wheel is 7.72″ hence $r = 3.86''$ while $l = 7.87''$. This gives

$$\frac{PR}{r} = \frac{333{,}660}{3.86} = 86{,}440 \text{ lbs.}$$

The moment 333,660 is that which the friction of the wheel upon the axle should be able to resist without slipping. Hence it follows that Q must necessarily be greater than 86,440. If we take a value of $Q = 154{,}000$ lbs., thus giving ample margin against slipping, and also use a wrought iron hub, making $S_2 = 7120$ lbs., taking $f = 0.2$ as before:

$$\frac{\delta}{r} = \sqrt{\frac{2\pi \times 3.86 \times 7.87 \times 0.2 \times 7120 + 154{,}000}{2\pi \times 3.86 \times 7.87 \times 0.2 \times 7120 - 154{,}000}} - 1$$

$$\sqrt{\frac{425{,}656}{117{,}656}} - 1 = 0.92$$

hence $\delta = 3.86'' \times 0.92 = 3.55''$

The actual thickness of the hub was 3.54″.

The ring form is not the only form of construction which may be used for joining members by forcing, since other forms may also be used. An example may be found in Erhardt's flange joint, Fig. 178*. In this case clamps of hardened steel

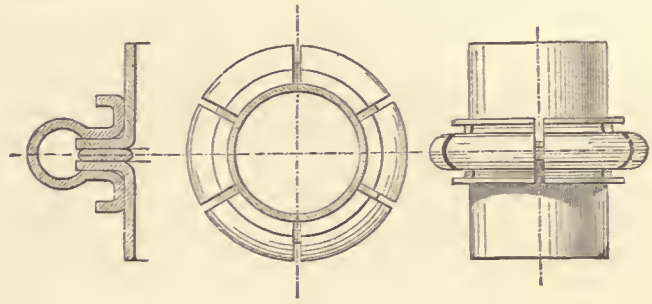

Fig. 178.

are used to create the pressure. These clamps serve to press the light flanges together, and they may be forced on by use of a screw clamp or other suitable press. Tests of such joints under steam, pneumatic and hydraulic pressure have shown the joint to be tight and serviceable.

The system of forced connections has grown into extensive use, and appears to be applicable to many forms of construction, and it is to be hoped that the forcing press, for which the firm of Schaeffer & Budenberg have made suitable pressure gauges, may be found an indispensable tool in all large workshops.

CHAPTER III.

KEYING.

§ 66.

Keyed Connections.

The simplest form of keyed connection consists of three parts, viz.: the two parts to be connected, and the key itself. The key is made with a slight amount of bevel on both sides, or a greater angle on one side, according to the manner in

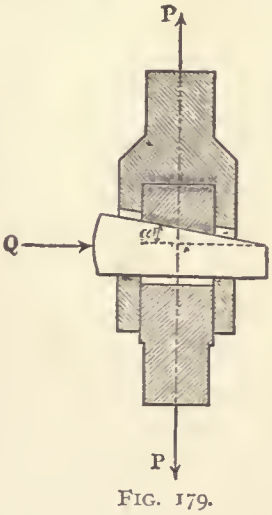

Fig. 179.

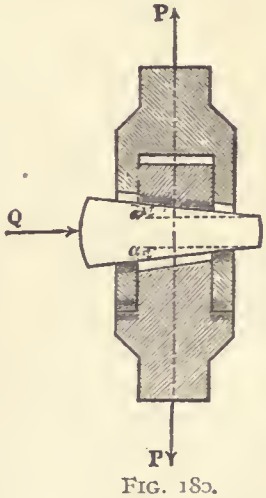

Fig. 180.

which the connection is made. The trigonometrical tangent of this angle is called the draft of the key. In Figs. 179 and 180 are shown both forms of keys. For the latter form we will assume that both sides have the same angle.

Let:

a = the angle of draft,
P = the force to be transmitted,
Q = the driving force upon the key, normal to P,
Q' = the opposing force, tending to drive out the key,
$f = tg\,\phi$, the co-efficient of friction between the surfaces of the three parts.

For keys with draft upon one side, we have:

$$\left.\begin{aligned} Q &= P\,2\,tg\,(a + 2\phi) \\ Q' &= P\,2\,tg\,(2\phi - a) \end{aligned}\right\} \quad \ldots\ldots\ldots (67)$$

In order that Q' should not be negative and the key come out of itself, we must have $a < 2\phi$. For $f = 0.1$, this gives $tg\,a < \frac{1}{5}$.

* Royal Prussian Patent, May 23, 1876. No. 159. Illustrated by drawing, model and description.

For keys with draft on both sides we have approximately:

$$\left.\begin{aligned} Q &= P\,2\,tg\,(\alpha+\phi) \\ Q' &= P\,2\,tg\,(\phi-o) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (68).$$

In this case it is necessary to keep below the full value of f, for each edge of the key in order that the connection may not force itself apart. The total draft will be found to have approximately the same minimum value as in the previous case.

In practice it has been found that keys which have shown endurance and resistance under load, have been made with a total draft of $\frac{1}{30}$, $\frac{1}{50}$, and even $\frac{1}{100}$ or less, while others made with $\frac{1}{24}$, $\frac{1}{12}$ or sometimes $\frac{1}{6}$ are less secure.

The load upon a key may act in three different manners each of which may again be positive or negative. In the first, the

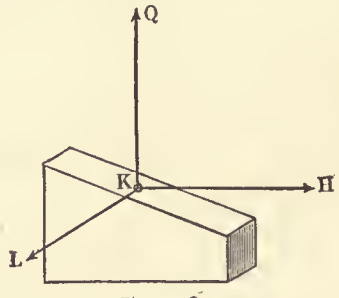

FIG. 181.

load acts normal to the base of the wedge, as at QK, Fig. 181, or as P, in Fig, 179 and 180; and for this form, the term Cross Key may be used. The second case occurs when the load acts normal to the plane KHQ, as KL, in Fig. 181, which may be called a Longitudinal Key. The third case is that in which the force acts normal to the plane QKL, as KH, Fig. 181, which may be called a wedge key.

§ 67.

CROSS KEYED CONNECTIONS.

In Fig. 182 we have an example of a cross keyed connection. The rod and the key are both of wrought iron, the boss is cast iron. The stresses for a given force P upon the rod are: the bending stress upon the key, as in Case VIII. § (6) (Stress S_1); the shearing stress between the key and the inner edge of the

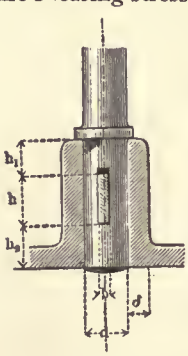

FIG. 182.

FIG. 183.

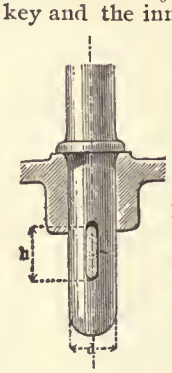

FIG. 184.

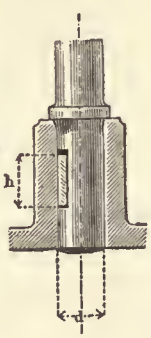

FIG. 185.

boss (Stress S_2), and the tension upon the segment shaped sections of the rod on both sides of the mortise for the key (Stress S_3). If, according to § 2, we make $S_2 = 0.8\,S_1$, and $S_1 = S_3$, we have:

$$\left.\begin{aligned} h &= \frac{6}{5}\,d \\ b &= 0.267\,d, \text{ or say } \frac{d}{4} \end{aligned}\right\} \quad \ldots\ldots (69).$$

If we make $h_1 = 0.8\,d$, $h_2 = d$, $\delta = 0.5\,d$, we shall have good practical proportions. In Fig. 183 we have two wrought iron rods coupled by wrought iron keys. In this case a wrought iron sleeve is used, whose thickness $\delta = 0.25\,d$. Fig. 184 shows a form similar to Fig. 182, except that the key passes below the boss, instead of going through it, while in Fig. 185 the key is let into the side of the rod.

The pressure upon the base surface of the key in the case of Fig. 182 may be taken as:

$$p = \frac{P}{b\,d} = \frac{(0.7854\,d^2 - b\,d)\,S_3}{bd}$$

which gives $p = 2.14\,S_3$ . . . . . (70)

quite a high enough value, especially if we take, in Fig. 183, $\delta = 0.25\,d$. The pressure becomes yet higher for the method shown in Fig. 185, for which case the value of S_3 should not be taken too great. If the connection is intended to be taken apart frequently, the value of p should not be allowed to be too great. This may be accomplished either by reducing the value of S_3, or by providing an increased cross section of metal about the mortise for the key, or by extending the surface by means of cotters or gibs, as shown in Fig. 186. The key may then be made smaller than already given above. The forms of keyed connection shown are used for example in the rod connections of water wheels, and in similar cases.

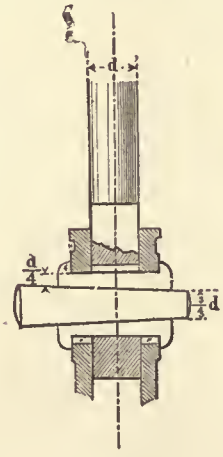

FIG. 186.

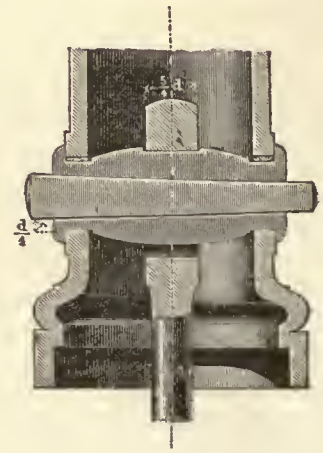

FIG. 187.

In Fig. 187 is shown a method of keying a foundation bolt. The gibs or cotters are used to increase the strength. Following the calculations of § 12, the depth of the three pieces may be made alike in the middle. The anchor plate in the foundation masonry should be arranged so as to give access to a nut on the lower end of the bolt, and this can be tightened by hand until the bearing is thrown upon the key, and the driving in of the latter binds all the parts firmly together.

§ 68.

LONGITUDINAL KEYS.

Keys of this class are principally used to secure the hubs of wheels to their shafts or axles. For this purpose they may be considered as divided into three classes, as follows:

Concave, or hollowed keys, Fig. 188, 1;
Flat Surface keys, Fig. 188, 2, 4, 5,
and Recessed keys, Fig. 188, 3.

The Concave key is only suitable for constructions involving small resistance, and acts merely by the friction due to the pressure which it causes. The flat surface key is capable of

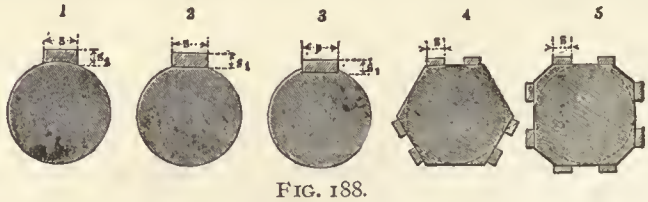

FIG. 188.

resisting much greater force and vibration, and when used in the multiple manner shown in 4 and 5, it makes a secure and efficient fastening. The recessed key, shown in 3, affords a very secure method of fastening hubs to shafts to which they have been closely fitted, and is simply and readily made. Keys of this kind are also used as an additional precautionary fastening for hubs which have been forced on.

In determining the dimensions of keys it will be found most convenient to use empirical methods, except in cases of great vibration; the following formulæ will be found to cover the usual range of work. The material for the key is taken as steel, and distinction is also made between cases in which the hub is subjected merely to endlong pressure, and those where torsional stresses exist. The former may be called draft-keys, the latter torsion keys.

If we call the diameter of the shaft D, the breadth of the key S, and the middle depth of the key S_1 we have:

$$\left.\begin{aligned} &\text{for Draft keys, } S = 0.24'' + \frac{D}{7};\ S_1 = 0.16'' + \frac{D}{12} \\ &\text{for Torsion keys, } S = 0.16'' + \frac{D}{5};\ S_1 = 0.16'' + \frac{D}{10} \end{aligned}\right\} \quad (71)$$

The taper of such keys is made about $\frac{1}{100}$

For the more commonly occurring diameters we have the following proportions :

$D =$ 1″ 2″ 3″ 4″ 5″ 6″ 7″ 8″ 9″ 10″

For Draft Keys :

$S =$ $\frac{3}{8}$″ $\frac{1}{2}$″ $\frac{5}{8}$″ $1\frac{3}{16}$″ 1″ $1\frac{1}{8}$″ $1\frac{1}{4}$″ $1\frac{3}{8}$″ $1\frac{1}{2}$″ $1\frac{5}{8}$″

$S_1 =$ $\frac{1}{4}$″ $\frac{5}{16}$″ $\frac{7}{16}$″ $\frac{1}{2}$″ $\frac{9}{16}$″ $\frac{5}{8}$″ $\frac{3}{4}$″ $\frac{13}{16}$″ $\frac{7}{8}$″ 1″

For Torsion Keys.

$S =$ $\frac{3}{8}$″ $\frac{9}{16}$″ $\frac{3}{4}$″ 1″ $1\frac{3}{16}$″ $1\frac{3}{8}$″ $1\frac{9}{16}$″ $1\frac{3}{4}$″ 2″ $2\frac{1}{8}$″

$S_1 =$ $\frac{1}{4}$″ $\frac{3}{8}$″ $\frac{1}{2}$″ $\frac{9}{16}$″ $\frac{11}{16}$ $\frac{3}{4}$″ $\frac{7}{8}$″ 1″ $1\frac{1}{16}$ $1\frac{3}{16}$″

For shafts of less diameter than 1″, we may make $S = \frac{D}{3}$, $S_1 = \frac{D}{5}$. If several keys are to be used, they may be made the same dimensions as single keys. For hubs which have been forced on, and hence would be secure without any key, the dimensions for draft keys may be used.

§ 69.

EDGE KEYS.

When the pressure upon a key acts at right angles to the plane of its height, the difference between the positive and negative direction of the forces is readily distinguishable.

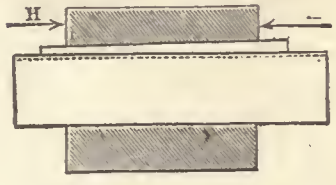

FIG. 189.

FIG. 190

When the pressure acts as in Fig. 189, the combination is insecure, since the only binding action of the key is that due to the pressure, and consequent friction between the parts. If the base of the key is rough, and the inclined face smooth, the action of a force in the direction H', tends to tighten the parts together. An application of this action is shown in the curved key of Kernaul, shown in Fig. 190. When the hub is rotated

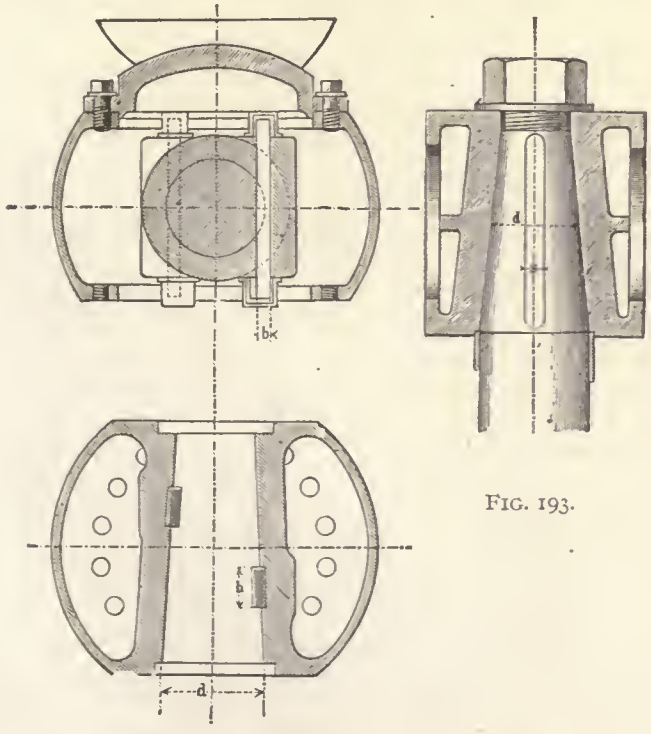

FIG. 192.

FIG. 193.

in the direction of the arrow, the action is the same as that of the force H', in the preceding case, and the shaft is firmly grasped. A countersunk screw at a, is used to tighten the key, and a similar one at b, to loosen it. This principle will be discussed later, under the subject of couplings.

§ 70.

METHODS OF KEYING SCREW PROPELLERS.

In securing the propellers of steamships the greatest care must be observed in the methods employed, and in their application. In Fig. 191 is shown Rennie's method of securing one of the blades of a Griffith's double bladed propeller. In this case a rectangular key is used, passing through a cylindrical pin which is cast in one piece with the blade and which is in turn held firmly by the four smaller keys shown. These latter keys are held in their places by caps secured firmly by jam nuts. (See §71.) The blade and hub are both of bronze.

Example. In a propeller by Penn & Son, $d = 15''$, $h = 7\frac{1}{2}''$, $b = 2\frac{1}{2}''$.

Fig. 192. This shows a method used by Maudslay, Sons and Field, Ravenhill & Hodgson, and others. Two rectangular

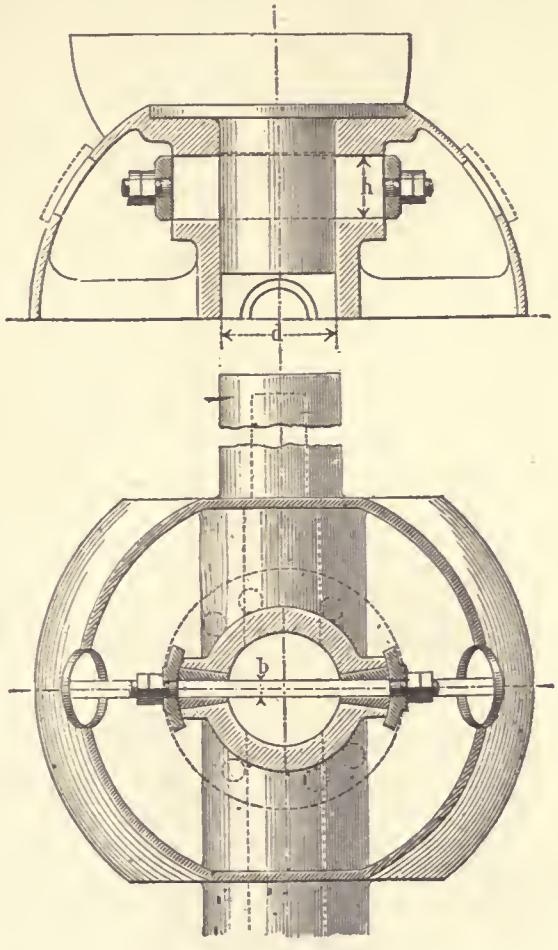

FIG. 191.

keys, passing through the hub of the propeller boss, and recessed into the metal of the shaft, act at the same time to receive the thrust of the screw and to prevent rotation upon the shaft. In this case the hub is made of bronze.

Example. In the "Lord Warden," the middle diameter of $d = 19''$, $l = 52''$, $h = 8\frac{1}{2}''$, $b = 3\frac{1}{8}''$; in the "Lord Clyde," $d = 20\frac{1}{8}''$, $l = 54''$, $h = 10''$, $b = 3''$.

Fig. 193. This shows a method of using two longitudinal keys. The hub is bored with a quick taper, and a heavy bronze nut holds the hub upon the cone, while the longitudinal keys resist the action of torsion.

Example. In the "Minotaur," engined by Penn & Son, the mean diameter $d = 18\frac{1}{2}''$, $l = 48''$, $S = 3'$.

The ordinary rectangular keys are also used to secure screw propellers, as well as special forms of fastenings.*

§ 71.

UNLOADED KEYS.

The force P, which under ordinary conditions bears upon a key, may by various methods be supported by other means; the key in such a case may be said to be unloaded. Such constructions offer a much greater security, and permit much lighter keys, than the methods previously described. A few examples will serve to illustrate.

* See N. P. Burgh, Modern Screw Propulsion, London, 1869.

Fig. 194. This shows a form of connection used for mine pump rods; the interlocked notches receive the load of tension upon the rod, and the hollow key only serves to bind the parts together without itself supporting any of the weight. Fig. 195 shows Wiedenbruck's rod connection.* The hub is made

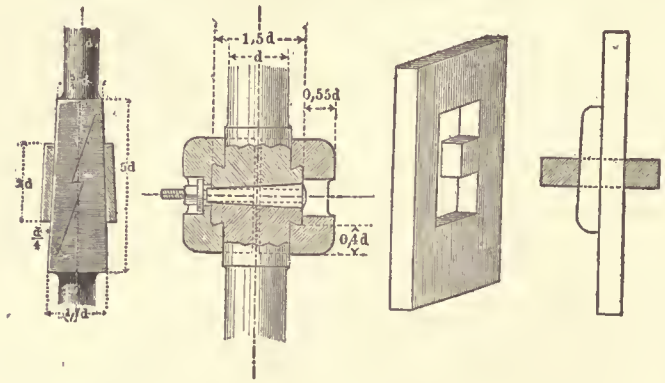

Fig. 194. Fig. 195. Fig. 196.

in halves and the reversed conical seats receive the load. In Fig. 196 is shown a connection for two intersecting plates; by Bayliss.†

The method of keying shown in Fig. 192, *H*, may be made quite secure by relieving the key from the load, and examples of this form are often found.

§ 72.

Methods of Securing Keys.

In order that a key may not back out under its load, the angle of taper should be less than $\frac{2}{10}$, or if it is symmetrical in form, each side should be less than $\frac{1}{10}$, providing the co-efficient of friction is equal to $\frac{1}{10}$. Even when the taper is made

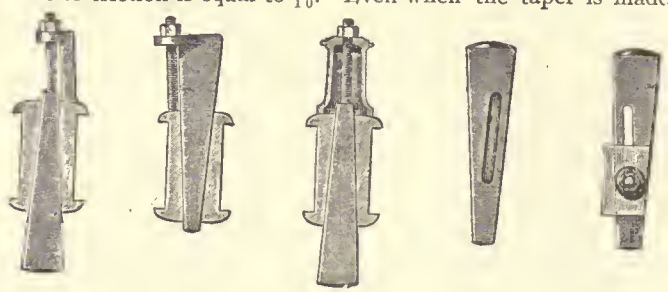

Fig. 197. Fig. 198. Fig. 199. Fig. 200. Fig. 201.

less than this, however, keys are very apt to become loose if they are subjected to much vibration, and to sudden and irregular changes of load. In order to provide against such emergencies, and also in order to permit the use of greater taper, various methods of securing keys are employed.

The simplest method consists in splitting and spreading the small end of the key, and for some purposes this is sufficient. In order to prevent a key from flying back, or jumping out, the projecting end may be drilled and fitted with a split pin. For the keys used in connecting rod ends various methods are used, examples of which will be found in the following figures. Figs. 197, 198, and 199 use screws under tension. When these are used in locomotives or marine engines, the screw is again secured by the use of a jam-nut. Fig. 200 is used with a set-screw, the point of which bears in a shallow channel in the side of the key, so that if the pressure of the set-screw is unable to hold the key, it will at least keep it from flying out. The channel is also of service in preventing the point of the set-screw from marring the finished surface of the key. Fig. 201 shows a form of screw-clamp. This clamps the key by drawing the two blocks tightly against its sides; the screw passes through a slot in the key.

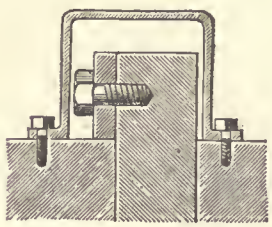

Fig. 202.

Fig. 202 shows the method employed in securing the key used in the form of fastening for screw propellers, used by Maudslay, and shown in Fig. 192. A small block is bolted fast to the projecting end of the key, and a brouze cap is screwed down over all.

*German Patent, No. 1507. See also patent No. 510 of H. Rademacher for improved rod connections.

†See Pract. Mech. Journal, Vol. III, 3d Series. P. 342.

CHAPTER IV.

BOLTS AND SCREWS.

§ 73.

Geometrical Construction of the Screw Thread.

Screws are used in machine construction to produce three kinds of effects, viz.: for clamping or joining parts together, for producing pressure, and for the transmission of motion. We shall now only consider the first two classes. Screws may be classified with regard to the shape of the cross-section of their threads into:

Triangular or V,
Square or Rectangular,
and Trapezoidal.

All these forms belong practically to the so-called axial screw thread surfaces.* By this is meant the surface which is described by a right line *A B C*, Fig. 203, when one of its points remains upon a directrix, in this case the axis *O D*, of the screw, while the generating line itself maintains a constant angle α, with the axis, proportional to the advance which the directing point makes upon the axis. The angle α is called the angle of advance, and its complement β, is the base angle of the screw thread. These are either V, or square, according as the angle α

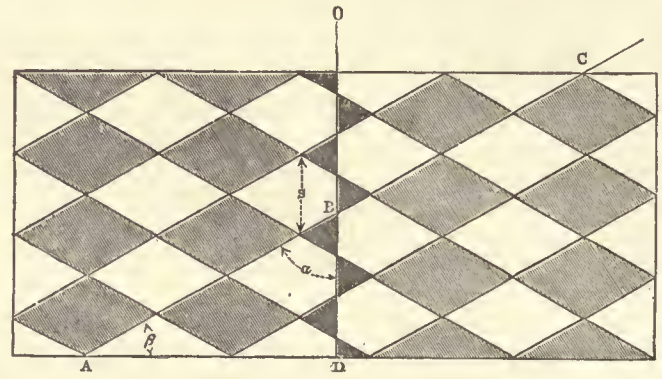

Fig. 203.

is an acute or right angle. The normal cylinder upon the axis *O D*, upon which the screw thread is described, is called the pitch cylinder. This cylinder is supposed to pass through the threads of the screw at such a point that two adjoining sections bear the relation of screw and nut.

The space passed over by the directing point during one rotation around the axis is called the pitch of the screw, and will hereafter be designated by the letter s; and the angle which a line tangent to the screw-thread at any point makes with the base of the pitch cylinder, is called the pitch angle, called δ. From this it will be seen that threads described upon concentric cylinders may have the same pitch, but different pitch angles.

The area of a V screw thread may be taken as equal to the sum of the surfaces the two halves of the thread, opened out to an angle of 180°. For rectangular threads the area is simply that of the corresponding simple surfaces. For trapezoidal threads the area is equal to the sum of the inclined and parallel surfaces (see § 86).

§ 74.

Dimensions of V Screw Threads.

For any given force P, acting parallel to the axis of a screw, the resistance of the metal of the body of the screw may be determined according to Case I., page ——, but the stress may instead be taken as a simple case of tension if the value of S, be not made too great. If we take for wrought iron, $S = 3600$, and let d_1 be the diameter at the base of the thread, we have:

$$\left.\begin{aligned} d_1 &= 0.02\sqrt{P} \\ P &= 2750\, d_1^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (72)$$

The nut is generally hexagonal, but is sometimes made square; we will here limit ourselves to the former shape. The thickness of the nut is usually made equal to the outside diameter d, of the screw. This makes it much stronger than the threads of the screw;† but the depth is desirable, as it distributes the pressure over a greater area of screw thread. For the superficial pres-

*See the article: Concerning some Properties of Screw Threads. *Berliner Verhandlung*, 1878, p. 16.

†Recent investigations made at the Stevens Institute at Hoboken, show that the resistance of the thread is reached when the thickness of the nut is made 0.45 to 0.4 d. See *Railroad Gazette*, 1877, November, p. 483.

sure p, we have for a depth of thread t and n threads in the nut, both for V, and square threads:

$$p = \frac{S}{4} \cdot \frac{1}{n} \frac{d}{t} \left[1 - 3\frac{t}{d} + \left(\frac{t}{d}\right)^2 \right] \quad \ldots\ldots\ldots (73)$$

Introducing the pitch S, and making $n\,s = d$, we have:

$$p = \frac{S}{4} \cdot \frac{s}{t} \left[1 - 3\frac{t}{s}\frac{s}{d} + \left(\frac{t}{s}\right)^2 \left(\frac{s}{d}\right)^2 \right] \quad \ldots\ldots (74)$$

In both equations the third member may be neglected.*

The value of p should not be permitted to exceed 1440 lbs.

If $n = 8$, and $\frac{d}{t} = 12$, we have, taking S, as above,

$p = 3600 \times \frac{3}{8}\,(1 - \frac{1}{4} + \frac{1}{144})$ or about $= 1000$ lbs.

In the consideration of this subject, the friction of a screw should not be neglected.

Let:

Q = the force acting at the mean diameter of a screw, normal to the plane of the axis;

δ' = the pitch angle of the thread at the mean diameter,

$f = tg\,\phi$, the coefficient of friction;

we then have, in order to overcome the thread friction, due to the force P, for square thread:

$$\left.\begin{aligned} Q &= P\frac{f + tg\,\delta'}{1 - f\,tg\,\delta'} = P\,tg\,(\phi + \delta') \\ \text{or for the resistance:} \\ Q' &= P\frac{f - tg\,\delta'}{1 + f\,tg\,\delta'} = P\,tg\,(\phi - \delta') \end{aligned}\right\} \quad \ldots\ldots (75)$$

while for V threads, we have:

$$Q = P\frac{f' \pm tg\,\delta'}{1 \mp f'\,tg\,\delta'} = P\,tg\,(\phi' \pm \delta') \quad \ldots\ldots\ldots (76)$$

in which $f' = \frac{f}{\cos\beta}$,

In order to overcome the friction on the base of the nut also, the value of Q must be more than twice as great. For $tg\,\delta'$, we may take then $tg\,\delta$. This value is usually so small that the friction often cannot resist the load, and the value of Q' becomes negative.

§ 75.

The Whitworth Screw System.

By a system of screw threads is meant a collection of rules or formulæ by which the profile of thread, pitch, diameter, and other details of screws and nuts may be determined. Such a system was first formulated by Whitworth in 1841, and since that time the subject has been more and more studied, until it is now considered one of the greatest importance,† especially in regard to the metric system.

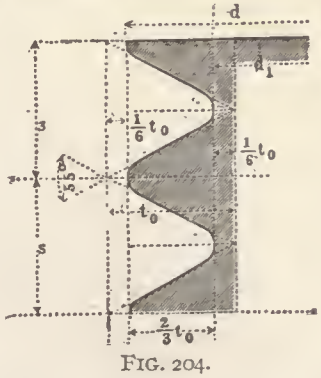

FIG. 204.

A united opinion on the subject has not yet been reached. Many weighty reasons have been advanced for the introduction of the Whitworth system into Germany, while others, equally strong, have been advanced for the metric system.

The Whitworth system takes for the form of a section of a screw thread an isosceles triangle whose base is equal to the pitch s, and whose angle at the apex $= 55°$, from which the height $t_0 = 0.96\,s$. The thread is rounded at top and bottom to an amount equal to $\frac{1}{6}\,t_0$, so that the working depth $t = \frac{2}{3}\,t_0 = 0.64\,s$. The values of the pitch s were given by Whitworth in a table which extended to 1″.* The use of this system developed some deficiencies, among others the difficulty of originating the cross-section of thread, and the gradation of diameters. The original table of diameters was not altogether satisfactory to Whitworth himself, and in 1857 he extended the old table by a new one, which, since that time, has been known in England as the standard system for bolts and nuts.† In Germany, however, the whole subject is yet under active discussion.

The following table gives the old and new scales, the values of d and s being in English inches. The values for $\frac{5}{16}$″ and $\frac{7}{16}$″ are only given approximately.

WHITWORTH'S SCREW THREAD SCALES.

New Scale. d.	Old Scale d.	$\frac{1}{S}$.	New Scale. d.	Old Scale. d.	$\frac{1}{S}$.
0.100		48	1.125	1⅛	7
0.125	⅛	40	1.250	1¼	7
0.150		32	1.375	1⅜	6
0.175		24	1.500	1½	6
0.200		24	1.625	1⅝	5
0.225		24	1.750	1¾	5
0.250	¼	20	1.875	1⅞	4½
0.275		20	2.000	2	4½
0.300	$\frac{5}{16}$	18	2.125	2⅛	4½
0.325		18	2.250	2¼	4
0.350		18	2.375	2⅜	4
0.375	⅜	16	2.500	2½	4
0.400		16	2 675	2⅝	4
0.425		14	2.750	2¾	3½
0 450	$\frac{7}{16}$	14	2.875	2⅞	3½
0.475		14	3.000	3	3½
0.500	½	12	3 250	3¼	3¼
0.525		12	3.500	3½	3¼
0.550		12	3.750	3¾	3
0.575		12	4.000	4	3
0.600		12	4.250	4¼	2⅞
0.625	⅝	11	4.500	4½	2⅞
0.650		11	4.750	4¾	2¾
0.675		11	5.000	5	2¾
0.700		11	5.250	5¼	2⅝
0.750	¾	10	5.500	5½	2⅝
0.800		10	5.750	5¾	2½
0.875	⅞	9	6.000	6	2½
0.900		9			
1.000	1	8			

WHITWORTH'S PIPE THREAD SCALE.

$d =$	⅛	¼	⅜	½	¾	1	1¼	1½	1¾	2
$n =$	28	19	19	14	14	11	11	11	11	11

The regularity of the progression might be improved upon. This may be more clearly illustrated in the following diagrams. The greatest irregularity lies between the sizes from ¾″ to 2½″; and the gradation of diameters is also uneven. The cause for this lies in the system of measurement used. Whitworth evidently perceived the desirability of introducing a decimal notation, but also wished to retain the fractional divisions in halves, quarters, eighths, &c.; this has partly been secured, neglecting sixteenths, by having the gradation based upon fortieths, and their combinations as shown in Fig. 206.

For the pressure p, we have from (74), taking $t = 0.64\,s$:

$$\frac{P}{S} = \frac{1}{4 \times 0.64}\left[1 - 1.9\frac{s}{d} + 0.4\left(\frac{s}{d}\right)^2 \right]$$

If we make $S = 3555$ lbs. we have for $d = 0.1''$, $3''$, and $6''$, the values of $p = 938$ lbs., 1152 lbs. and 1209 lbs. For $t\,g\,\delta$, we have, when $d = 0.1''$, $3''$, and $6''$, the values 0.0663, 0.0303, and 0.0212.

---

* $P = p\,\pi\,t\,(d - t)\,n$, hence $p = S\frac{\pi}{4}d_1^2 : \pi\,n\frac{t}{d}\left(1 - \frac{t}{a}\right)d^2 = \frac{S}{4}\frac{1}{n}\cdot\frac{d}{t}\left(\frac{d - 2t}{d}\right)^2 : 1 - \frac{t}{d}$, which, by neglecting $\left(\frac{t}{d}\right)^3 + \left(\frac{t}{d}\right)^4 +$, etc., gives the above result. In this case, p is the pressure upon the projected area of the screw thread.

† See The Metrical Screw System, etc., by a Committee of the Society of German Engineers. Berlin, Gärtner, 1876.

* Briggs stated the relation of pitch and diameter of the Whitworth system to be approximately:—

$$s = 0.1075\,d - 0.0075\,d^2 + 0.024.$$

† See *Eng. and Arch. Journal*, 1857, p. 262; 1858, p. 48; also Shelly, Workshop Appliances. London, 1876, p. 102.

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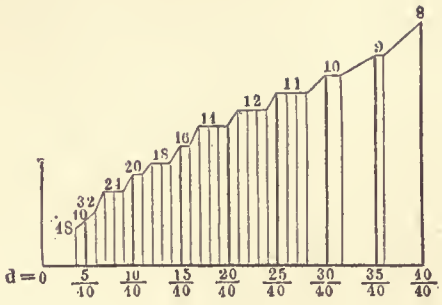


FIG. 206.

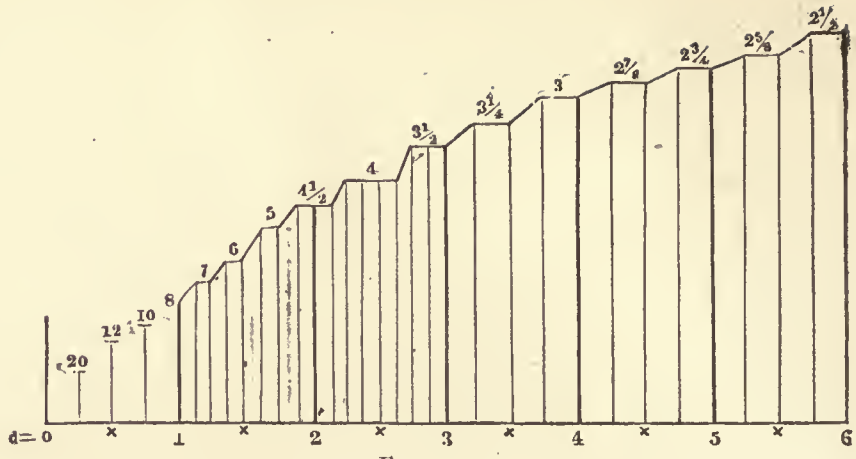
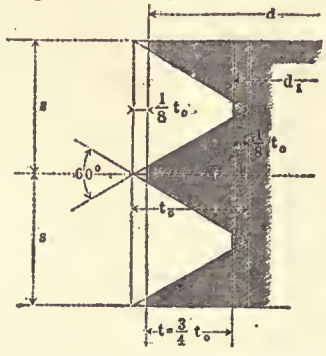


FIG. 205.

§ 76.

SELLERS' SCREW THREAD SYSTEM.

The confusion in the use of screw threads having become very troublesome in the United States, Mr. William Sellers brought before the Franklin Institute, in 1864, a system which he proposed for general use.* A committee of the Institute reported upon the system in December of the same year, and recommended its general adoption.† This system is now



FIG, 207.

generally known as the Sellers System. The profile of this thread is shown in Fig. 207. The thread angle $2\beta = 60°$; the depth $t = 0.75\ t_0 = 0.65\ s$. The pitch is determined by the formula $S = 0.24\sqrt{d} + 0.625 - 0.175$, the result, as with Whitworth's system, being so modified that the number of threads per iuch shall be a whole number.

The following table gives the adopted number of threads per inch for various diameters:

$d =$	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
$\frac{1}{s} =$	20	18	16	14	13	12	11	10	9	8
$d =$	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2		
$\frac{1}{s} =$	7	7	6	6	5 1/2	5	5	4 1/2		
$d =$	2 1/4	2 1/2	2 3/4	3	3 1/4	3 1/2	3 3/4	4		
$\frac{1}{s} =$	4 1/2	4	4	3 1/2	3 1/2	3 1/4	3	3		
$d =$	5	5 1/4	5 1/2	5 3/4	6					
$\frac{1}{s} =$	2 1/2	2 1/2	2 3/8	2 3/8	2 1/4					

The Sellers System compares very favorably with the Whitworth System, and notwithstanding the difference in profile, it gives almost the same depth of thread. The angle is very convenient, and the simplicity of the profile is such that a suitable tool may easily be made and used in the shop. These facts explain the rapid introduction of the system in America. The progression of the pitch is also more uniform than in Whitworth's System; and the uncertainty about the thread of the 1/2″ screw, which was always a stumbling block in the original Whitworth Scale, is avoided. The values for 5/16″ and 7/16″ are retained as in the Whitworth Scale of 1857, and 9/16″ is also provided for, so that the requirements of the English system of measurements are fully met, up to 2″.

* Journal of the Franklin Institute, 1864, Vol. 47, p. 344.
† Journal of the Franklin Institute, 1865, Vol. 49, p. 53.

§ 77.

METRICAL SCREW SYSTEMS.

Recognizing the advantages which have followed from the introduction of the Whitworth System, various attempts have been made to devise a system of screw-threads which shall be adapted to the metric system of measurements. The following fourteen systems have been suggested:

Armengaud, Redtenbacher, Paris-Lyons-Mediterranean R.R., Northern Railway of France, J. F. Cail, the French Navy, Bodmer, two systems proposed by Ducommun, of Mulhouse, Alsace; the Engineering Society of Mulhouse, Reishauer & Bluntschli, of Zurich; the Pfalz-Saarbrück Society of German Engineers, and two systems of Delisle.

The formulæ and tables given in the previous editions of this work have also been spoken of as systems, but they are not entitled to any such position, as they were merely adaptations of the Whitworth system. The number of proposed systems may be taken as an indication of the difficulty of the task. Indeed, it is only by very carefully weighing the respective merits of the various plans, that it is possible to say which is the best. The following requirements should be kept in mind as essentials in considering any system:

1. *The profile of the thread should be such as may be readily made with requisite accuracy.* In this respect Whitworth's system is deficient, and the profile of the Sellers thread is to be preferred.
2. *The pitch should be given, so far as possible, directly by the formula, without requiring any modification of its results.* Both Sellers and Whitworth are deficient in this point, since they both modify the results of their formula.*
3. *The gradation of bolt diameters should be so disposed that fractions of millimeters should not occur in diameters, and that their gradation should conflict as little as possible with the decimal system.*

All three of these requirements should be attained within the limits of generally used sizes, and should at least extend to bolts of 80 mm. in diameter. The last three systems, viz.: the Pfalz-Saarbrück system and the two of Delisle, are the only ones which appear to have considered these points, and these we shall examine somewhat in detail.

§ 78.

METRICAL SCREW THREAD SYSTEMS.

DELISLE *I*, PFALZ-SAARBRUCK AND DELISLE *II*.

The following three diagrams show the gradation of pitch and diameter for the three systems, the ordinates representing the pitch being shown on five times the scale of the bolt diameters, and the values being also given for d and s in the annexed tables. In the first two cases the profile of thread is exactly the same as in Sellers' system, while in the third, the base angle is made 26° 34′. This has been chosen for the purpose of making the theoretical height of the triangle of the thread equal to s. The thread is flattened as in the Sellers system.

All three of these systems are marked by simplicity and intelligibility. These features have been attained by abandoning the idea of representing the relation of s to d by a single equation (such as that of a parabola), and using two or more equations of straight lines. A noticeable irregularity exists in the Pfalz-Saarbrück system between the diameters of 26 and 28 mm., indicating that a somewhat finer pitch is used in proportion to the diameter below 26 mm.

The second system of Delisle is rather simpler than the first; there is also an important difference in the angle of thread, as will be seen subsequently.

* In the old Whitworth scale all 33 values were modified; in the Sellers system this is done with 31 out of 34 sizes.

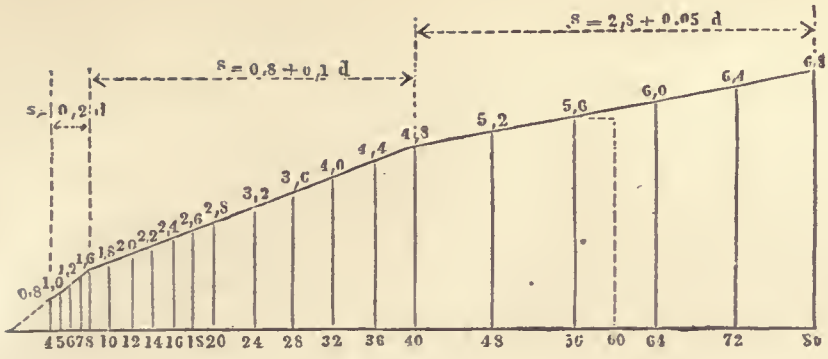


FIG. 208.

DELISLE *I.* FIG. 208.

$d =$	4	5	6	7	8	10	12	14	16	18	20
$s =$	0.8	1.0	1.2	1.4	1 6	1.8	2.0	2.2	2.4	2.6	2.8

$d =$	24	28	32	36	40	48	56	64	72	80
$s =$	3.2	3 6	4.0	4.4	4.8	5.2	5.6	6.0	6.4	6.8

Diameter and pitch both in millimeters.

For any interpolated diameter the next lesser ordinate is to be taken, as for example $d = 60$.

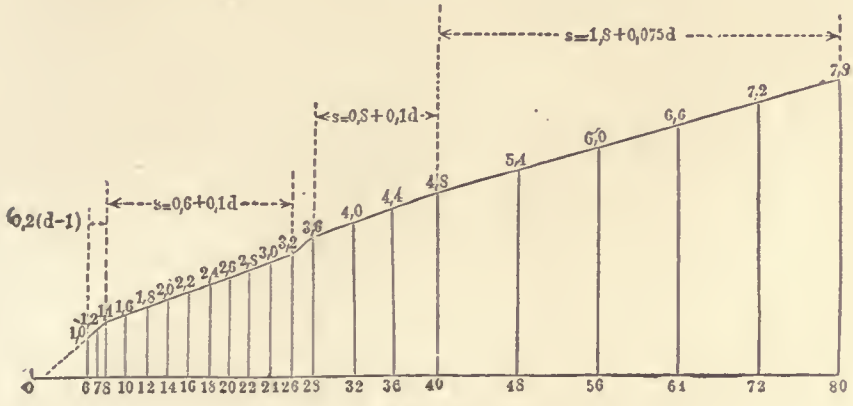


FIG. 209.

PFALZ-SAARBRUCK SYSTEM. FIG. 209.

$d =$	6	7	8	10	12	14	16	18	20	22	24
$s =$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0

$d =$	26	28	32	36	40	48	56	64	72	80
$s =$	3.2	3 6	4.0	4.4	4.8	5.4	6.0	6.6	7.2	7.8

No interpolation to be made.

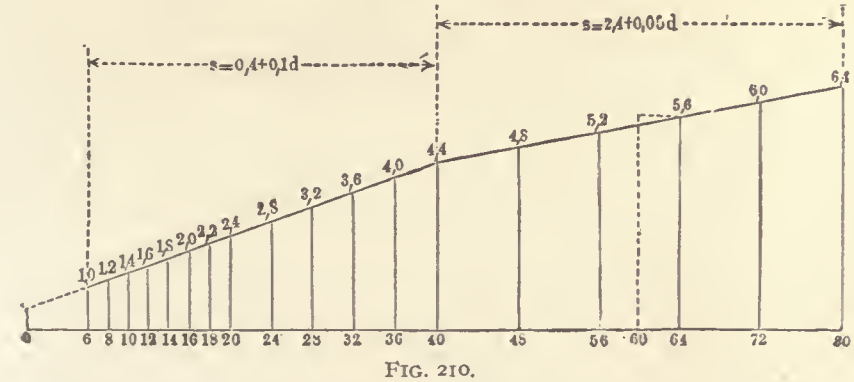


FIG. 210.

DELISLE *II.* FIG. 210.

$d =$	6	8	10	12	14	16	18	20	24	
$s =$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.8	

$d =$	28	32	36	40	48	56	64	72	80
$s =$	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0	6.4

For any interpolated meter the next greater ordinate is to be taken, as for example $d = 60$.*

In all three systems the superficial pressure is quite satisfactory. According to formula (74), taking $S = 3600$ we obtain for values of p—

Delisle I . . . . . 8600 to 11,500 lbs.
Pfalz-Saarbrück . . 8600 to 11,000 "
Delisle II . . . . . 7600 to 10,000 "

§ 79.

NEW SYSTEMS.

A thorough investigation of the proposed systems of the German Society of Engineers failed to produce any definite results, and the whole subject of a metrical screw thread system is still unsettled. For this reason it has been thought advisable to offer a further discussion of the problem.*

It might seem a shorter plan to adopt some one of the three preceding systems, yet they all seem capable of improvement. The subject will bear further investigation in two main points one being the gradation of diameters and the other the profile of thread. The actual diameters and their gradation are of more practical importance than the gradation of threads. This is shown by the fact that the Whitworth profile has long been in use with the bolt diameters taken in Prussian inches, and more recently with dimensions in millimetres with Whitworth profile. One of the first requisites of such a series is that the diameters should follow the decimal divisions (see the third condition of § 77). This point is not met by the preceding systems, since they lack the natural divisions 30, 50, 60 and 70. The removal of this objection introduces a new difficulty, but not an iusuperable one.

The critical feature of the screw thread system is really the relation which the diameter bears to the profile. A thread should not be said to be coarse or fine, implying the ratio $s : d$, but rather should the depth of thread be cousidered, or the ratio $t : d$.

This can best be illustrated by an example :

If we select two equal sizes from the systems Delisle I and I I, we shall find that for the same pitch the threads are not alike.

If $d = 60$ mm. we shall have (see the dotted lines in Fig. 208 and 210) in both cases $s = 5.6$, hence the angle of thread is the same.

The working depth t, however, is :

in. I : $t = \frac{3}{4} t_0 = 0.65\, s = 3.64$ mm.
in II : $t = \frac{3}{4} t_0 = 0.75\, s = 4.20$ mm.

* This is especially necessary for use in technical instruction, which will afford the surest method of introducing a metric screw thread system into practical use. The advocates of the Whitworth system urge the desirability of an international standard, in view of the widespread use of the American system, which is indeed already in use to some exteut in Germany. In this case the conflict between the two systems of measurement has been met by proposing to take for any dimension in English units the next higher dimension in millimetres. Such a system would be impracticable for educational purposes and would lead to many errors in actual practice. It also seems only to be practicable for the old Whitworth scale, and for the new scale, with its close divisions, its application would be impossible. A comparison between the preceding diagrams will show that a close adherence to the Whitworth system would result in a complication of dimensions which would be most undesirable.

*In both his systems Delisle has provided for the interpolation of intermediate diameters, but these have been omitted from the diagrams and tables to avoid obscurity.

This gives for the diameter of the bolt at the bottom of the thread

in I : $d_1 = 57.70$—cross section 2182 sq. mm.
in II : $d_1 = 51.60$— " " 2091 " "

which shows a difference in resistance of about 5 per cent. between the two bolts, the second having the coarser thread. We see here that a choice of the relation of s to d affects the profile of thread, and it is this which led Delisle to suggest two systems.

Whether the angle of 53° 8′ is preferable to the Sellers angle of 60° is uncertain. Among the preceding systems may be noted two for the latter, five for the former, and three for

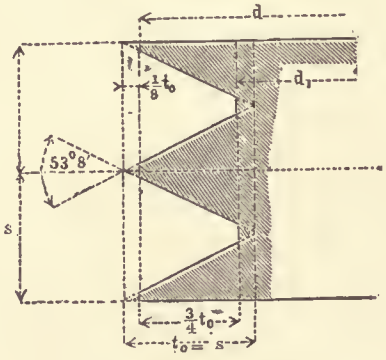


FIG. 211.

still smaller angles; and if the choice be given, it seems rather better to go below the Whitworth angle of 55° than above it.

We prefer the angle as shown in Fig. 211:

and hence $2\beta = 53° 8'$ or $t_0 = s$; $t = \frac{3}{4} t_0 = \frac{3}{4} s$ . . . . . (77)

For sizes of d from 4 to 40 mm. the pitch may be

$$s = 0.4 + 0.1\, d \quad . . . . . (78)$$

and for sizes of d, from 40 to 80 mm. and over*

$$s = 2 + 0.06\, d \quad . . . . . (79)$$

with the following series of diameters:

		4	5	6	7	8	9	10		
12	14	16	18	20	22	24	26	28	30	32
	36	40	.	45	.	50	60	70	80	

Formula (78) is the same as in Delisle II, from 6 to 40 mm. Interpolation for intermediate diameters seems unnecessary;

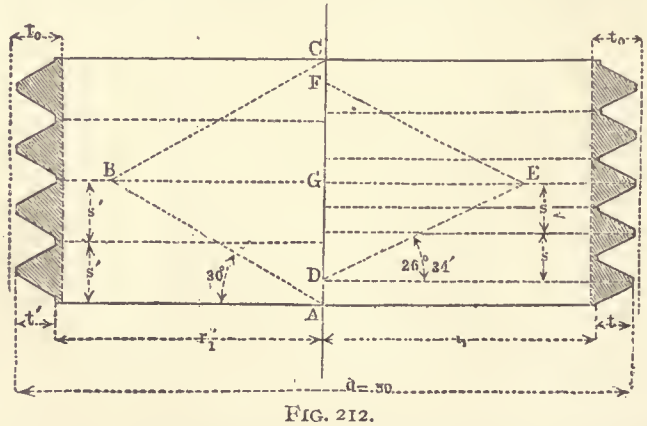


FIG. 212.

should it be done, however, the formula should not be departed from, since the values in the second and third groups above will give round numbers, and offer no difficulty for their production on the screw cutting lathe. If it is still desired to use the angle of 60°, and yet retain the other proportions, we may take

for $d = 4$ to 8, $s' = 0.2\, d$ (as in Delisle I)
for $d = 8$ to 40, $s' = 0.8 + 0.1\, d$ (as in Delisle I
for $d = 40$ to 80, $s' = 1.6 + 0.08\, d$ . . (80)

in which arrangements the sizes 30, 45, 50, 60, 70 remain in the series, which may also be extended above 80 mm. The two plans may be compared to Fig. 212, in which the formulæ are respectively applied to a diameter of 80 mm. The radii to the bottom of thread r'_1 and r_1, are almost identical, as are also working depths, although the profiles differ, as shown by the triangles A B C and D E F. Instead of numbering the sizes arbitrarily, it seems preferable to use the bolt diameter for the number. Screw No. 20 would then stand for $d = 20$ mm., No. 4 for $d = 4$ mm. Any establishment could omit numbers not desired without impairing the system, while for fine work smaller numbers could readily be added.

*For sizes over 80 mm. we have not yet established relations. If we take $d = 150$ mm. which is about as high as Whitworth or Sellers have gone, $s = 11$ mm., which seems a good proportion. See § 87.

§ 80.

NUTS, WASHERS AND BOLT HEADS.

The thickness of metal in a nut bears a close relation both to the depth of thread t, and to the pitch s. It is desirable that the formula to be used should give the dimensions readily in order to avoid the necessity of approximating.

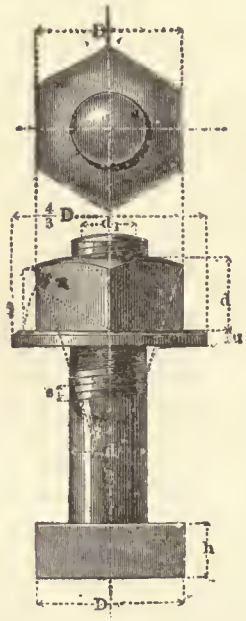


FIG. 213.

For the diameter D, of the inscribed circle of the hexagon we may take for finished nuts:

$$D = .04 + d + 0.5\, s \quad . . . . . (81)$$

The maximum pressure upon the base of the nut in this case (for $d = 3''$) = about 2400 lbs. per square inch. Unfinished nuts are made somewhat heavier, and for them we have

$$D_1 = 0.14'' + d + 5\, s \quad . . . . . (82)$$

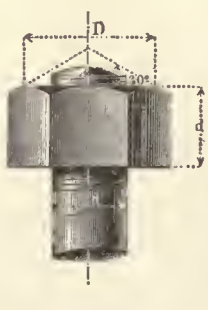


FIG. 214.

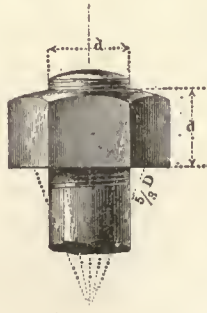


FIG. 215.

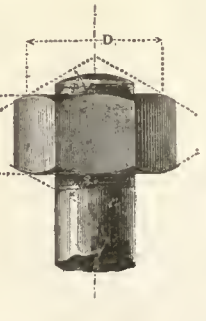


FIG. 216.

The use of the washer insures a better bearing for the nut in case the surface is not true. Its dimensions may be taken as

diameter $= J = d + 10\, s$
thickness $= a = \frac{5}{4} s$ . . . . . (83)

Bolt heads are often made square, but are preferable hexagonal, and for them we may take D and D_1, the same as for nuts, and the height $h = 0.7\, d$. Fig. 213.

For finished nuts the upper surface may be finished with a bevel of a frustum of a cone whose base $= D$, and a base angle of 30°, Fig. 214, or as a portion of a sphere with a radius of $\frac{5}{8} D$, Fig. 215, while unfinished nuts have the corners beveled off above and below, as shown in Fig. 216.

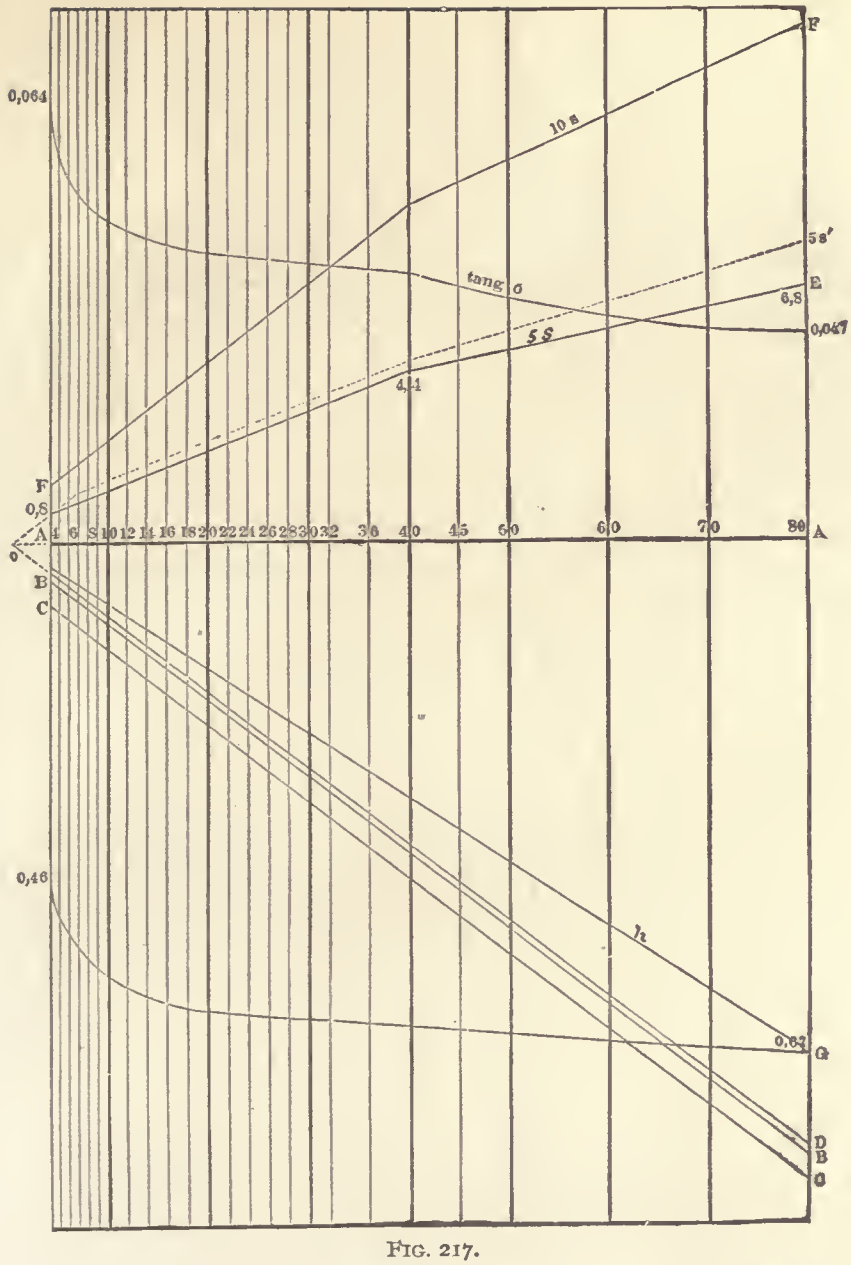


FIG. 217.

§ 81.

TABLE AND PROPORTIONAL SCALE FOR METRICAL BOLTS AND NUTS.*

The preceding table contains a summary of the preceding discussion, and Fig. 217 is a diagram in which the relations of the parts are shown graphically. The value for s is shown on a five-fold scale. The dotted line gives the value for s' of formula (80).

The diagram Fig. 217 shows the pitch of thread and the pressure upon a unit of area, for the dimensions of nuts and bolt heads for the preceding metric screw thread system for diameters from 4 to 80 mm.

The pitch is shown five times full scale (line E) and ten times full scale (line F); the bolt diameter in its actual size (line D), all measured from the base line A. The line B is 1 mm. from D, and C is 4 mm. from D while the distance between A and G is 0.7 d.

The various details may be summed up as follows:

Between A and E = the fivefold pitch,

" E and B = dia. of finished nut,

" E and C = dia. of rough nut,

" F and D = dia. of washer,

" A and G = height of bolt head.

The tangent of the pitch angle ranges from 0.064 to 0.047, and the pressure per sq. mm. on the thread, from 0.46 to 0.67 kilogrammes.

§ 82.

WEIGHT OF ROUND IRON.

The weights in the following table are given by the Formula

$$G = 2.617\ d^2,$$

the bars being one foot long and d = diameter in inches. For cast iron, multiply the values in the table by 0 93 and for bronze by 1.092. A hexagonal rod whose inscribed diameter = d is 1.103 time the weight of a bar of wrought iron of the same diameter.

BOLTS AND NUTS. (Metric system).

Bolt Dia. d mm.	Pitch. s	Depth of Thread. t	Bottom Dia. of Bolt. d_1	Nut. D	Nut. D_1	Washer. U	Washer. u	Bolt Head n	Load. P kilos.
4	0.8	0.60	2.80	9	—	12	1	3	16
5	0.9	0.68	3.65	10.5	—	14	1	3 5	27
6	1.0	0 75	4.50	12	—	16	1	4	41
7	1.1	0 83	5.35	13.5	—	18	1 5	5	57
8	1.2	0.90	6.20	15	—	20	1.5	6	77
9	1.3	0.98	7.05	16.5	—	22	1.5	6	99
10	1.4	1.05	7.90	18	21	24	1.5	7	125
12	1.6	1.20	9.60	21	24	28	2	8	184
14	1.8	1.35	11.30	24	27	32	2	10	255
16	2.0	1.50	13.00	27	30	36	2	11	338
18	2.2	1.65	14.70	30	33	40	3	13	432
20	2.4	1.80	16.40	33	36	44	3	14	538
22	2.6	1.95	18.10	36	39	48	3	15	655
24	2.8	2.10	19.80	39	42	52	3	17	784
26	3.0	2.25	21.50	42	45	56	4	18	841
28	3.2	2.40	23.20	45	48	60	4	20	1076
30	3.4	2.55	24.90	48	51	64	4	21	1240
32	3.6	2.70	26.60	51	54	68	4	22	1415
36	4.0	3.00	30.00	57	60	76	5	25	1800
40	4.4	3.30	33.40	63	66	84	5	28	2231
45	4.7	3.53	37.95	70	73	92	6	32	2880
50	5.0	3.75	42.50	76	79	100	6	35	3613
60	5.6	4.20	51.60	89	92	116	7	42	5325
70	6.2	4.65	60.70	102	105	132	7	49	7369
80	6.8	5.10	69.80	115	118	148	8	56	9744

WEIGHT OF WROUGHT IRON RODS. One Foot Long.

d	G	d	G	d	G
1/4	.163	1 3/8	3.68	2 1/8	11.82
5/16	.255	1 1/4	4.09	2 3/16	12.50
3/8	.368	1 5/16	4.50	2 1/4	13.25
7/16	.500	1 3/8	4.94	2 5/16	13.95
1/2	.654	1 7/16	5 36	2 3/8	14.76
9/16	.826	1 1/2	5.89	2 7/16	15.54
5/8	1.02	1 9/16	6.39	2 1/2	16.36
11/16	1.23	1 5/8	6.91	2 7/16	17.14
3/4	1.47	1 11/16	7.43	2 5/8	18.03
13/16	1.72	1 3/4	8.01	2 11/16	19.11
7/8	2.0	1 13/16	8.57	2 3/4	19.79
15/16	2.29	1 7/8	9.20	2 13/16	20 61
1	2.61	1 15/16	9.79	2 7/8	21.63
1 1/16	2.94	2	10.47.	2 15/16	22.52
1 1/8	3.31	2 1/16	11.02	3	23.56

§ 83.

SPECIAL FORMS OF BOLTS.

Instead of being made with square or hexagon heads, bolts are sometimes fitted with special heads, instances of which are shown in Figs. 218 to 222; the last being countersunk. These are all furnished with means to prevent the bolt from turning when the nut is operated.

*This table has been kept in the metric system for obvious reasons. Trans.

In Fig. 223 is shown an anchor bolt with cast iron plate for brickwork, the bolt being inserted from above and locked by being turned 90°. The area of the anchor plate should in no case be less than 100 d_1^2

In Fig. 224 is shown a form of anchor bolt for masonry with a cast iron washer, secured by a key. The washer should be not less than 25 d_1^2 in area. Such plates are often made of wrought iron.

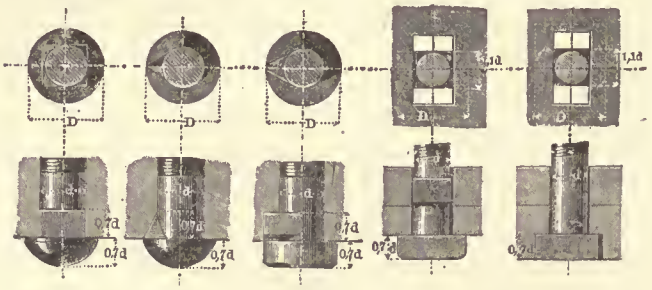


FIG. 218. FIG. 219. FIG. 220. FIG. 221. FIG. 222.

In Figs. 225 and 226 are shown bolts secured by cross keys and side keys. In these two figures the nuts are shown in different positions, the latter being the more convenient to use the proportions shown in Figs. 214 to 216. Figs. 227 and 228 are forms

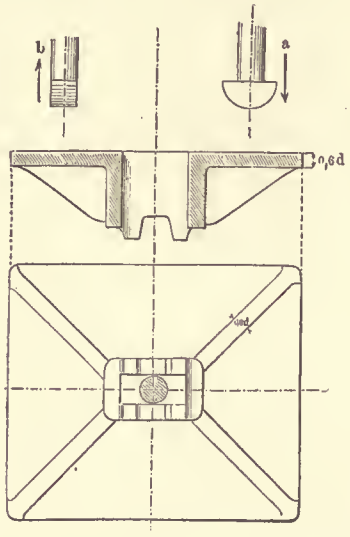


FIG. 223.

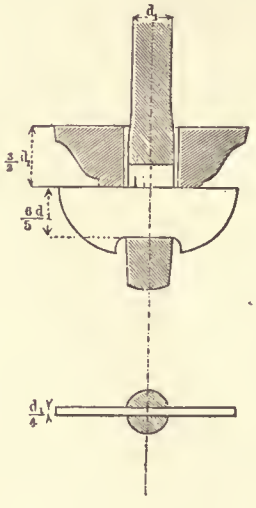


FIG. 224.

of stud bolts. Fig. 229 is a cap screw. For small work these cap screws are often made with cylindrical heads with slots for use with a screw-driver.

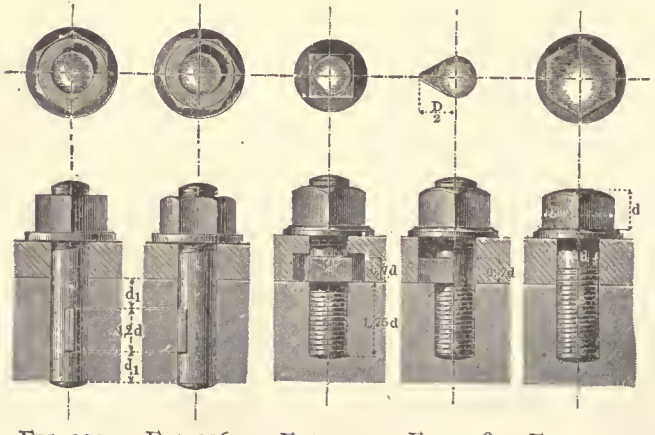


FIG. 225. FIG. 226. FIG. 227. FIG. 228. FIG. 229.

§ 84.

WRENCHES.

A wrench is a lever adapted to tighten and loosen nuts and bolt heads. The simple wrench, shown in Fig. 230, in two forms, consists of a flat or round handle fitted to the shape of the nut, the dimensions being based upon the unit D, which is the diameter of the nut as given in formula (81). The double wrench, Fig. 231, is arranged to receive nuts of different sizes at the opposite ends of the handle. If the ends are inclined so as

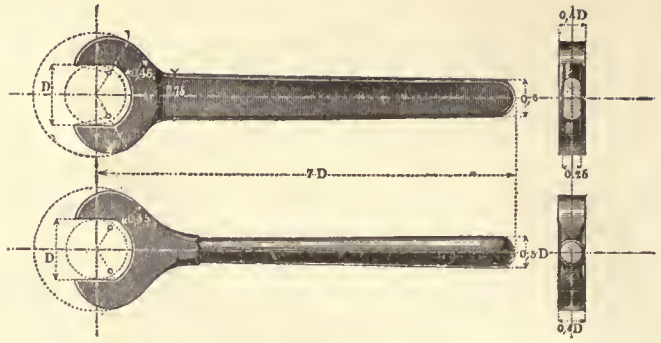


FIG. 230.

to bring the corners of the hexagon at 15° and 45° with the axis of the handle the wrench will be able to operate in contracted spaces by $\frac{1}{12}$ revolution of the nut.*

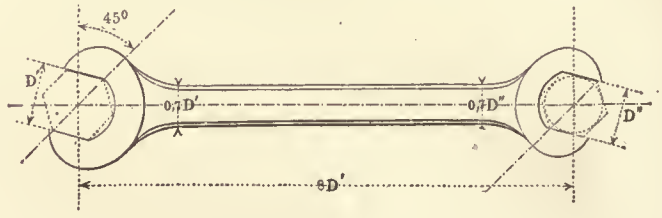


FIG. 231.

§ 85.

NUT LOCKS.

For **bolts** made according to the preceding proportions, the angles of pitch are not steep enough to allow the pressure in the direction of the axis of the bolt to overcome the resistance of friction and cause backward rotation. If, however, there is much vibration, lost motion may appear and gradually cause the connection to work loose. This is true of foundation bolts as well as of those in moving parts of machinery and in locomotive and marine engines. For these and similar cases it is necessary to have some method of securing the bolt or nut from coming loose, and a variety of such nut locks are here shown.

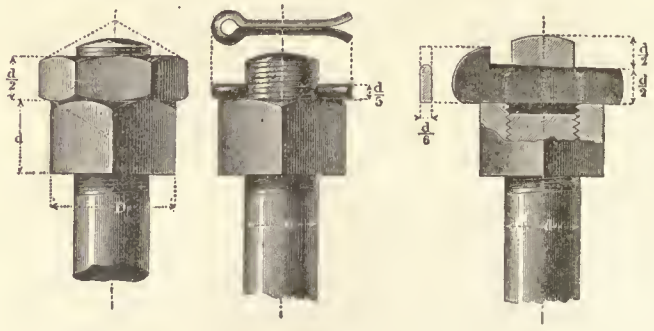


FIG. 232. FIG. 233. FIG. 234.

One of the oldest and most useful forms is the jam nut, Fig. 232. Both nuts should be truly faced so that they will bear fairly upon each other. The thin nut is frequently placed under the thicker one, but this is immaterial since a nut of a thickness of 0.45 to 0.4d is as strong as the bolt thread. The security obtained by the use of the jam nut is not very great, and the form with right and left hand thread, as shown in Fig. 244, is to be preferred when greater security is essential. In Fig. 233 is shown a split pin, often used in connection with a jam nut. Fig. 234 shows an arrangement with a key upon the nut, making a very convenient and secure combination. In the three preceding cases the action is such as to tighten the nut upon the thread. The three following methods are made to hold by fastening the nut or bolt, or both, to the parts which they are intended to hold together. Fig. 235 is used in the spring hangers of Borsig's locomotives, Fig. 236 on an oil cup lid, and Fig. 237 on a set screw for a connecting rod end, arranged to lock at any 1-12 part of a turn.

In the following methods the nut is held from turning by being locked to one of the stationary pieces, the bolt itself being secured in a similar manner. The form shown in Fig. 238 is used for bearing cap bolts, the support at the middle of the

* This idea is due to Proell.

split pin keeping it from bending. The method shown in Fig. 239 is used for the bolts in a steam piston, while that in Fig. 240 is for a bearing cap. The latter form is arranged by means of the sever notches, to lock at every $\frac{1}{7}$ of a turn, while the other two require $\frac{1}{6}$ of a turn between successive positions.

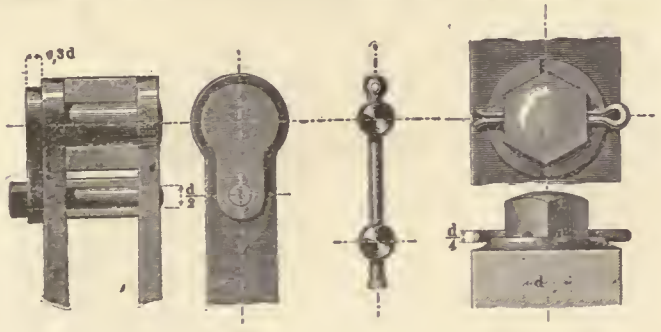


FIG. 235. FIG. 236. FIG. 237.

Fig. 241 shows a device for securing the nuts of stuffing box bolts as applied to locomotive engines. The ratchet wheels are attached to the nuts, and similar notched nuts may be used to advantage in many places.

FIG. 238. FIG. 239. FIG. 240.

A method of securing the bolts for locomotive springs, used by Borsig, is shown in Fig. 242. The teusiou of the spring keeps the bolt from turning, and the cap which secures the nut is fitted to the end of the bolt as shown; this locks for every $\frac{1}{6}$ of a turn. Fig. 243, shows a nut arranged to be locked by a set screw. This method, used by Penn, is a very nseful form

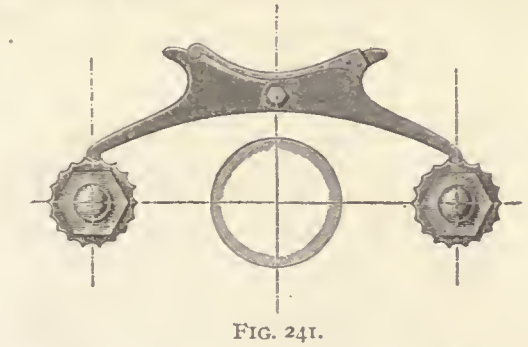


FIG. 241.

for bearings, spring hangers, and other situations, since it permits any fraction of a turn to be made. The uut, in this case, should be a little thicker than usual in order that the lower cylindrical portion may not be too weak. The diameter D_1, is in this case taken from formula (82). The small set screw should be made of steel and hardened. This form of nut lock is especially useful on marine engines.

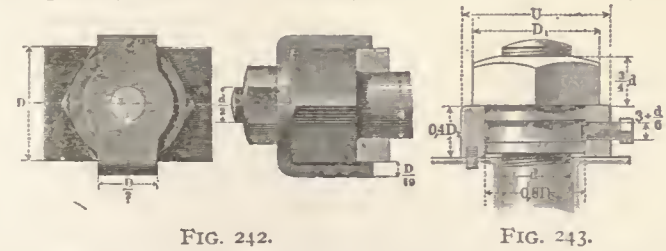


FIG. 242. FIG. 243.

A different class of nut locks depends for its action upon the introduction of an elastic resistance betweeu the bolt and the nut.* The elastic washer of Pagel and similar devices have found many applications. Parsons' bolts belong to this class.†

* See Ludewig Nut Locking Devices. Bavarian Industrial and Technical Journal, 1870, pp. 17, 144, 283; also Journal of the Society of German Engineers.

† Engineer, July, 1867, p. 16; Nov., p. 391; Engineering, 1867, Nov., p. 411; Railroad Journal, 1868, pp. 77, 117.

In this form the body of the bolt is fluted, so that the cross section is reduced to about the same area as that of the bolt at the base of the thread. This increases the elasticity of the bolt and enables the nut to be tightened so that it is much less likely to come loose. Fig. 244 shows a modification of this form used by Gerber for bridge connections. The security is still further increased by the use of a left hand jam nut. Instead of being fluted, the body of the bolt may be flattened on four sides, or the reduction of area may be obtained by drilling a hole into the bolt from the head to the beginning of the thread.

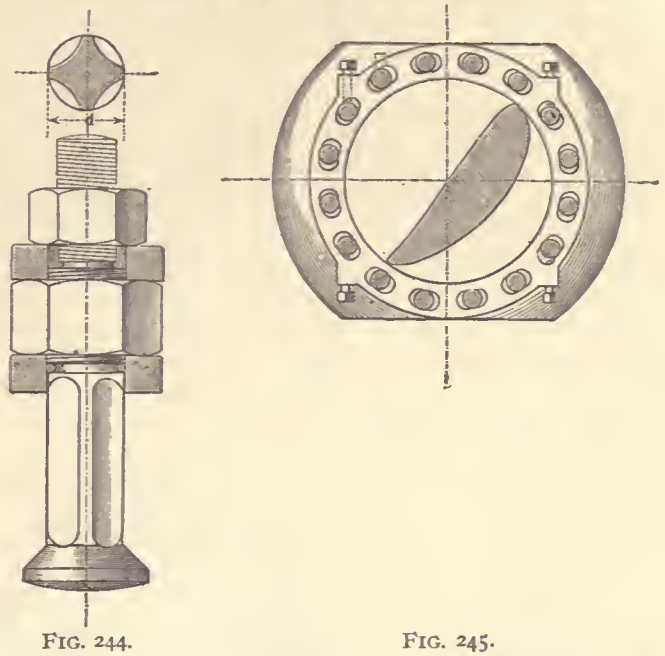


FIG. 244. FIG. 245.

One of the most important instances of screw fastenings may be found in the construction of built-up screw propellers, in which the blades of the screw are bolted fast to the hub, a connection requiring the greatest strength and security. Fig. 245 shows the base of such a propeller blade, from the same example as shown in Fig. 192. The flange of the blade is secur-

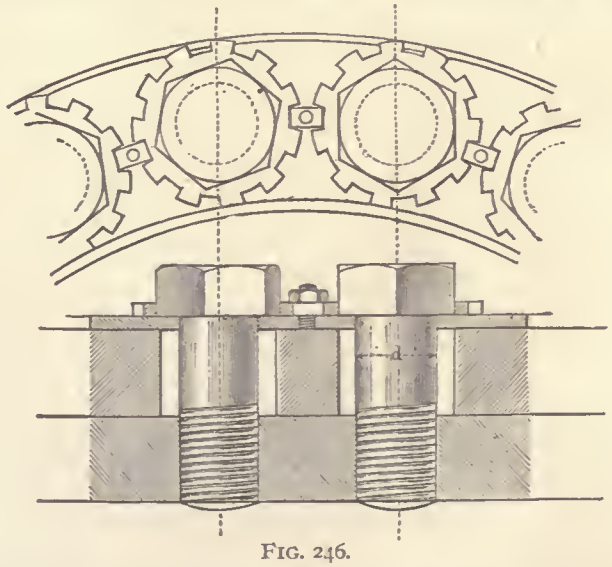


FIG. 246.

ed to the hub by sixteen cap-screws. Four set screws serve to provide a small adjustment of the blade within the range of motion of the oval bolt holes.

All of the cap screws are secured. Fig. 246 shows the method adopted by Penn. The bolts, which in the case of the Minotaur are $3\frac{1}{4}''$ diameter, have a common ring washer under the heads. When the bolts have been screwed up as tightly as possible, a ratchet washer with hexagonal hole is slipped over each bolt head. These ratchet washers are prevented from turning by the introduction of small locking pieces which are bolted fast to the large ring washer, being held down by the thin nuts shown. The ratchet washers have 11 teeth, and hence each bolt may be locked at $\frac{1}{66}$ part of a turn. Fig. 247 shows a method by Maudslay. Here each pair of bolts is held by a flat key which permits locking at $\frac{1}{18}$ part of a revolntion.

A continuous washer ring is not used with this method, but one washer is put under each pair of bolt heads, to which the locking key is bolted. Another method by Maudslay is shown in Fig. 248. A double washer is placed under two adjacent bolt

FIG. 247

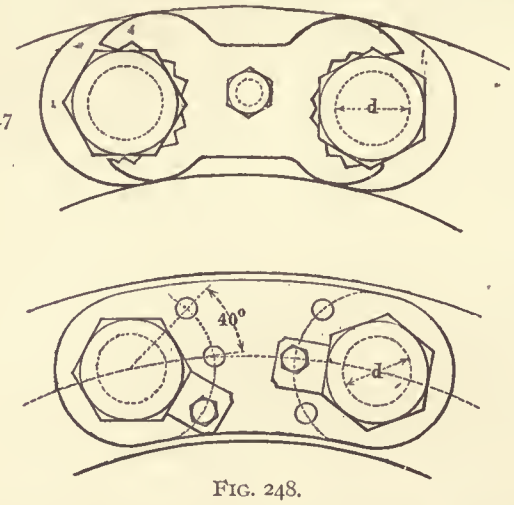


FIG. 248.

heads, and each bolt is locked by a small block held against one of the faces of the bolt head by a small bolt. Three bolt holes situated 40° apart are tapped in the washer for each block, thus giving an adjustment of $\frac{1}{18}$ of a turn. The method by Penn gives the best opportunity for adjustment.

§ 86.

SPECIAL FORMS OF SCREW THREADS.

Screw threads of square or trapezoidal section may be used for bolts, but in their use it is desirable to use a deeper nut in order to secure a sufficient number of threads in the nut to keep the pressure per square inch on the thread within the prescribed limits. Trapezoidal threads are well suited for bolts, since the relation between s and d permits the use of the same proportions as those given for V threads in Fig. 211. In fact the thread in Fig. 250 may be given the same proportions as that in Fig. 211, for depth t, and pitch s, making the angles respectively equal to 0° on one side and 45° on the other. These forms of screw-threads are principally used for screw-presses and for similar uses.

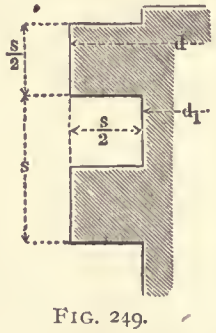


FIG. 249.

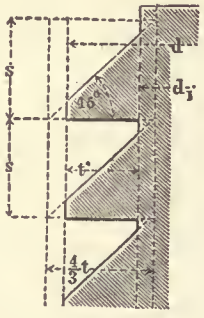


FIG. 250.

For such screws the diameter d_1, at the bottom of thread, is generally determined from formula (72). If, however, it is desired to make the diameter d_1 a minimum, we must consider the pressure to act only on one side of the thread in the nut and then take the pressure per square inch at double the previous allowance, or $1 = 7110$ lbs. We then have,

$$\left.\begin{aligned} d_1 &= 0.0134\sqrt{P} \\ P &= 5568\, d_1^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (84)$$

The depth of thread, both for square and trapezoidal threads, is,

$$t = \frac{d}{10} = \frac{d_1}{8}$$

and for square threads—

$$s = \frac{d}{5} = \frac{d_1}{4} \quad \ldots\ldots\ldots\ldots \quad (85)$$

and for trapezoidal threads—

$$s = \frac{2}{15}d = \frac{d_1}{6}$$

Formula (84) is applicable to screws of locomotive springs, since in this case the conditions are well complied with.

In order that the nut may not wear or grind out, the working pressure on the threads should not exceed say 700 lbs. per square inch. These conditions will obtain, according to (73), when the number of threads n, in the cast iron or bronze nut is not less than

$$\left.\begin{aligned} n &= 0.00035\, S \frac{d}{t}\left(1 - 3\frac{t}{d}\right) \\ \text{or,} \quad n &= 0.0014 \frac{P}{\pi d_1^2}\frac{d}{t}\left(1 - 3\frac{t}{d}\right) \end{aligned}\right\} \quad \ldots\ldots \quad (86)$$

If $t = \frac{1}{10}d$, we have

$$n = 0.00245\, S = 0.00312 \frac{P}{d_1^2} \quad \ldots\ldots\ldots \quad (87)$$

Example. For a pressure of 55,000 lbs., we have, under the preceding formulæ, from (84) the diameter at the bottom of the thread

$$d_1 = 0.0134\sqrt{P} = 0.0134 \times 234.5 = 3.14''$$

The depth of thread, from (85) $= \frac{3.14}{8} = 0.392''$, which gives $d = 3.92''$, or about $3\frac{15}{16}''$.

From (87) we have, making $s = 7710$ lbs., the minimum number of threads in the nut $n = .00245\, S = 17.4$ which gives for the height of the nut for square thread $h = ns = 17.4 \times .785 = 13.65''$, while for trapezoidal thread $h = 17.4 \times .523 = 9.1''$.

In many cases the diameter of such screws is made greater than the normal diameter indicated in the preceding discussion

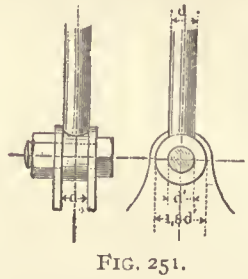


FIG. 251.

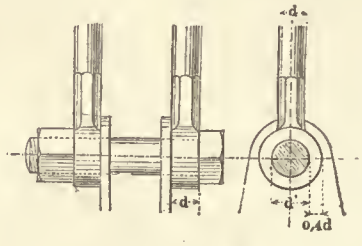


FIG. 252.

for the given load. Such screws may be called enlarged screws, as compared with the normal dimensions as previously determined. For such screws the same cross section of thread and the same height of nut may be given as for the normal screw of the same load, in which case the wear will practically be the same for both examples. Enlarged screws are frequently used for presses, where the diameter must be made greater than indicated by formula (84) in order to resist bending stresses.

§ 87.

SCREW CONNECTIONS, FLANGE JOINTS.

In screwed connections a distinction may be made as to whether the force acts parallel to the direction of the axis, or at right angles to it. The latter condition, which produces shearing stresses, is shown in the examples given in Figs. 251, 252 and 253. If we take d, as the diameter of the rod through which the force acts, we may call d', the bolt diameter, and

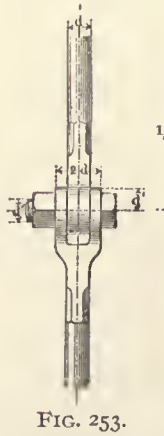


FIG. 253.

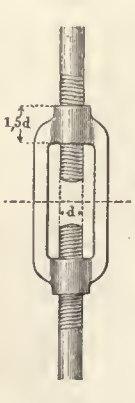


FIG. 254.

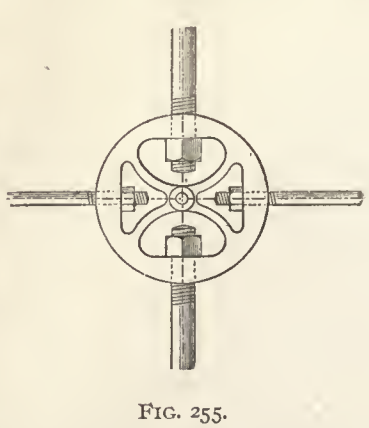


FIG. 255.

then determine their relation for various cases. In Fig. 251, $d' = d$; in Fig. 252, $d' = 1.4\, d$; in Fig. 253, $d' = d$; the increased diameter for Fig. 252, being given because it is possible in that case for the load to act so unequally that the greater portion may pass through one of the rods. Fig 254 shows a turnbuckle

with right and left hand thread. In this it is desirable to make the nut somewhat deeper than d, as shown. A form of junction piece for a point where four members meet is shown in Fig. 255. Such examples as the preceding are of frequent occurrence in bridge and roof construction.*

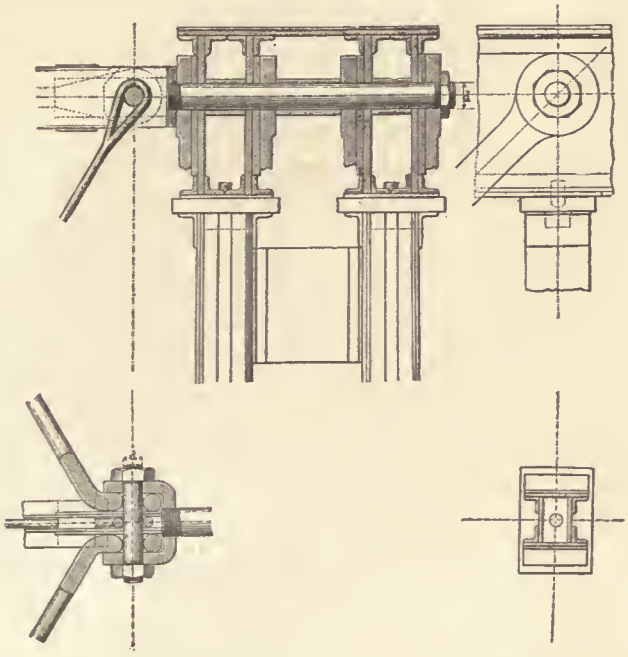


FIG. 256.

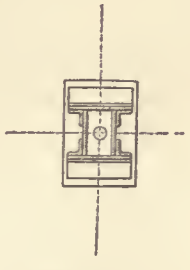


FIG. 257.

Bolt connections which bring shearing stresses upon the bolts are of frequent occurrence in bridges built with pin-connections, the general method in use in America. These designs exhibit very fully the substitution of bolt or pin-connections for riveting, and the method has been carried to great perfection. Some examples are here given. Figs. 256 aud 257 show an intersection of several members of the bridge over the Ohio, at Cincinnati. The top chord and the posts are double, and are

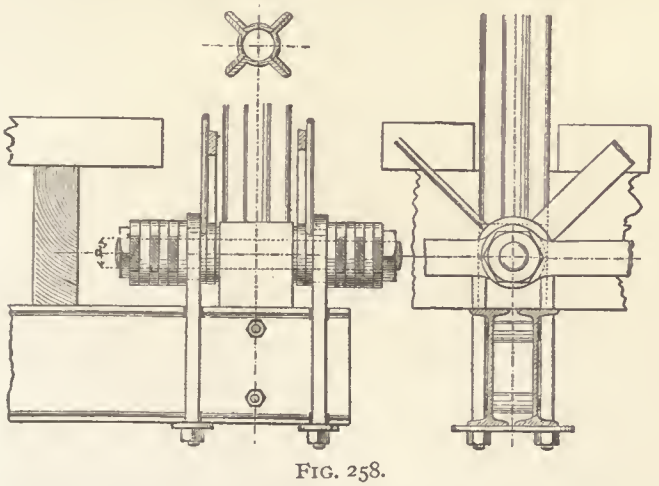


FIG. 258.

made of plate T and angle iron. The diagoual rods and braces to resist the action of the wind are connected to the upper chord by means of a bolt passing entirely through the beams and threaded at both ends. The nut ou the left end is in the form of a fork to receive the ends of the braces, while the right hand end is fitted with a thin octagonal nut. The ends of the braces are held by a bolt passing through the fork, with a nut at each end. The pins are carefully turned and closely fitted ;† after years of service they show no signs of looseness.‡ The proportions are such that stress on the bolts does uot exceed 15,000. lbs, in most cases not more than 10,000 to 12,000 lbs. The e connection of the posts to the chords (iu the illustration the rivets are omitted) is both simple and strong. The posts are provided with cast iron ends, which are fitted with square projections entering into the tops of the posts ; in these capitals are wrought iron dowel pins which pass through the lower angle iron and lower plates of the top chord. The diameter d, of the main bolts varies from 4 to 5½ or 6 inches or even heavier, according to the load. Their dimensions are based upon as bearing stress of 8000 lbs., while the diagonal braces and the lower chord are proportioned upou a tensile stress of 10,000 lbs. (a ratio of 0.8, see § 5). The compressive stress in the top chord is about 8,500 lbs., and in the posts, owing to tbe bending action, only about 5000 lbs.

Fig. 258 shows an intersection on the lower chord of the Niagara railway bridge (9 spans over a total width of stream of about 1900 feet). In this case the posts and top chord are made of the ingenious Phœnix columu of quadraut iron. The illustration especially shows the method hy which the cross beams are conuected to the longitudinal members. In this case the stress in the body of the screw bolts is about 8000 lbs., rather more than given for press screws iu § 86. A cast iron base, through which the large pin bolt passes receives the thrust of the post, and to it the cross beams of I shape are bolted. On these cross beams are wooden stringers to which the roadway is secured.

It will be noticed that these examples of bolt work far exceed the limit of size set by the Society of German Engineers for bolt dimensions, viz., 80 mm. or $3\frac{3}{16}''$. Should such sizes be necessary the formulæ in § 79 should be reconsidered.

FIG. 259. FIG 260. FIG. 261.

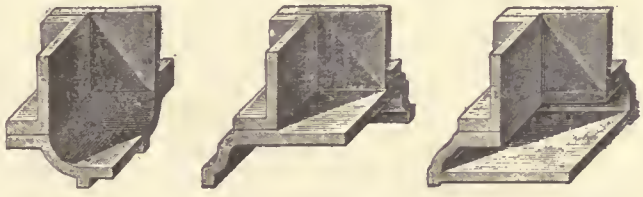


FIG. 262. FIG. 263. FIG. 264.

In uniting the various parts of iron constructions, flange joints are very frequently used. These are made iu a great variety of forms for various conditions. The following figures show some examples of corner junctions with flanges. Fig. 259 shows three external flanges, with a dished base. Fig. 260, also three external flanges, with an external pliuth on the base. Fig. 261 shows one external flange, and two which are half external and half internal. Fig. 262 has three half external flanges and a base as in Fig. 260. Fig. 263 has also three half external flanges, and Fig. 264 two external and one half-external flauge. The last three examples produce a more pleasing external appearance thau the preceding forms. If the form shown in Fig. 262 is used, with the flanges all turned inward, the bolts cannot be unscrewed from without.

Proportions for flange joiuts are shown in Fig. 265, the bolt diameter d, being obtained from the thickuess of metal δ. The distance between bolts is usually 2½ to 3 D, D being the width of the nut across the flat dimeusion. The width of flange is given in the illustration for metric sizes = 10 mm. + $2.8\,\delta$ = $\frac{3}{8}'' + 2.8\,\delta$.

If the flanges are finished on the planing machine, a ledge is left for finishing, as shown on the left of Fig. 265, in order that a fair bearing may he secured. Flange joints which are to be bolted together without finishing are made as shown in Fig. 266, with a gasket of some form of elastic packing. Such flanges are sometimes made for vessels with very thin walls, and on the left of Fig. 266 is shown the method of assembling a cylindrical vessel, such as a water tauk. The base has internal flanges for the bottom pieces, with an external flange for the connection to tbe body. By turning the flanges of the bottom inward a flat exterior base is obtained, well adapted to sustain the load of the water. The walls are very light, δ = only about $\frac{1}{4}''$, the bolts are $\frac{3}{8}''$ diameter, and their distance from centre to centre, in the base, 13.5 d, and in the vertical joints of the walls 15 d, and in tbe circumferential joints 20 d.

* Other good examples of similar work in roof construction may be found in E. Brandt's "Iron Constructions," Berlin, Ernst and Korn, 1871, 2d Edition.

† It is well known that variations in temperature during the boring of the holes for the pins in the eye bars may make sufficient difference to materially affect the fit. This has been overcome by the use of a double boring machine which the author saw at work in the notable bridge works at Phœnixville, whereby both ends are bored simultaneously, the distance being gauged by a wrought iron jig bar, which varied in length to the same extent as the eye-bars themselves.

‡ See H. Fontaine, "l'Industrie des Etats Unis," Paris, Baudry, 1878. Roller, Highway Bridge's New York, Wiley, 1878.

§ 88.

UNLOADED BOLT CONNECTIONS.

Various methods have been adopted to relieve bolts, in a connection, from the direct stresses due to the load, much in the same manner as has been described in § 71, for keyed connections. In Figs 267 and 268 are shown methods of notching two plates together. The bolts are relieved from the action of tensile or compressive stresses which act normal to the direction of the tongue and groove.

Fig. 269 shows a method of constructing a prismatic intersection so as to relieve the bolts from transverse stresses; while

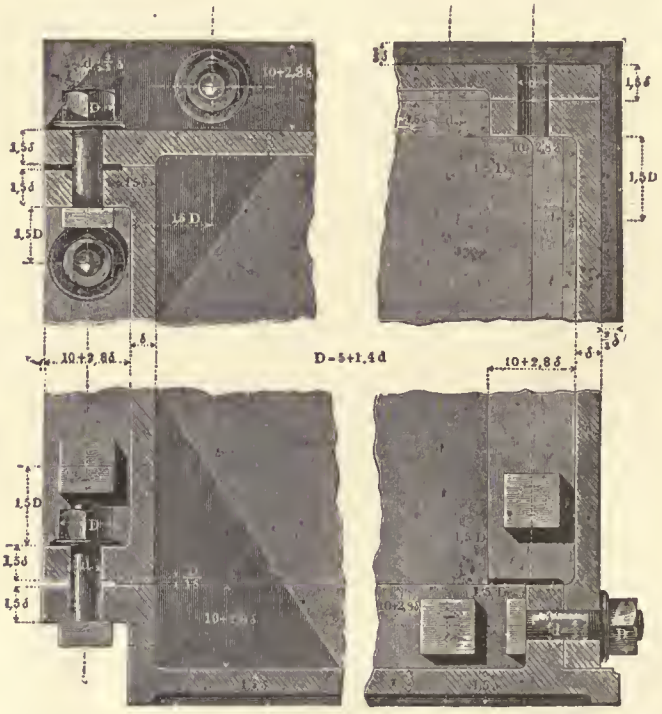


FIG. 265.

Fig. 270 shows a very convenient and useful form in which the projections on each piece lip over the other, greatly increasing the security of the connection. The bar may be made of wrought iron and the fitting should be made to conform carefully to the position of the bolt holes.

If the parts are large they are often both made of cast iron, and in some cases a turned dowel is let into both parts. The

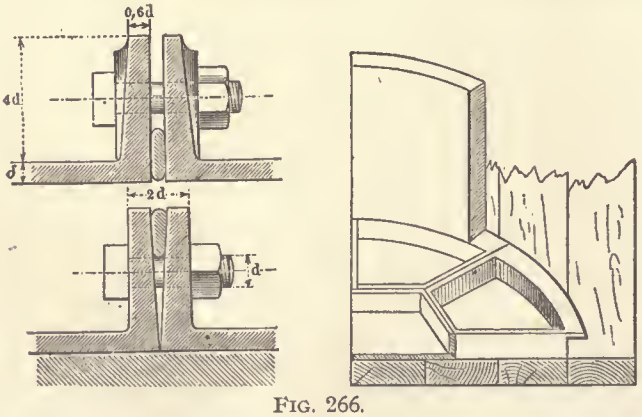


FIG. 266.

constructions shown in Figs. 269, 270, are used in the framework of large water-wheels, in which case the lower piece is made flat, thickened wherever it may be found necessary.

In many cases the lateral stresses are not great, while at the same time it is not desired that the bolts shall be made to fit closely. In such positions dowel pins are frequently used, being made of steel and fitted to reamed holes.

An example of bolt connection of large proportions, in which the lateral stresses are relieved, is shown in Fig. 271.

This is taken from the bridge over the Mississippi at St. Louis, and shows the bearing of the end of the lower member of one of the arches, which are composed of steel tubes. There are four such bearings at the end of each arch, or 24 bearings in all. The shoe to which the end of the tube is fitted is made of wrought iron, and the sole plate, of cast iron. Three bolts pass through both plates, the diameter d at the thread being $6\frac{1}{4}''$, and in the body $5\frac{7}{8}''$. The shoe is tongued into the sole plate

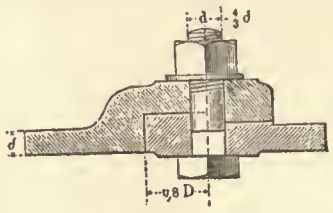


FIG. 267.

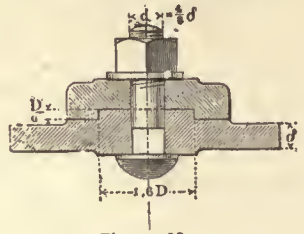


FIG. 268.

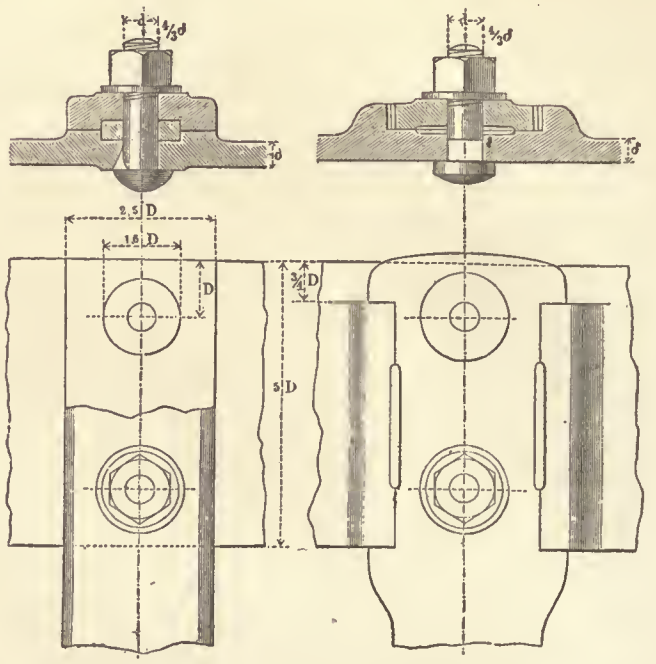


FIG. 269.

FIG. 270.

and the latter is supported by the masonry of the pier. The hole across the shoe is for the reception of the bolt by which the adjoining bearings are braced together.

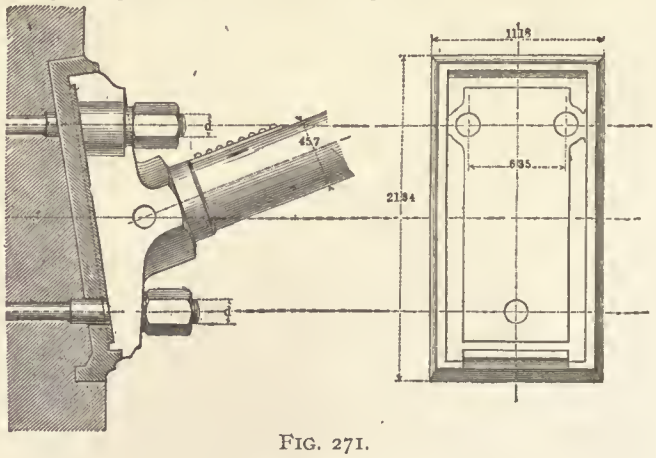


FIG. 271.

CHAPTER V.

JOURNALS.

VARIOUS KINDS OF JOURNALS.

§ 89.

Journals are made for the purpose of permitting parts of machinery to rotate about a geometrical axis and hence they are necessarily round, and their use involves some form of bearing or box for a support.

A journal may be subjected to pressure upon its side, or rather, normal to its axis; or the pressure may act lengthwise, in the direction of the axis. This gives us the two divisions:

1, Lateral journals.

2, End, or thrust journals.

In the calculations relating to these, both the questions of strength and of friction must be considered. In machine construction many forms of journals are employed, the most important of which will be here considered.

A. LATERAL JOURNALS.

OVERHUNG JOURNALS.

§ 90

A lateral journal which is connected on one side only to the member to which it belongs is said to be overhung. Such journals are usually made cylindrical, as in Fig 272, with a collar at the outer end, the height of the shoulder e above the diameter d being—

$$e = \tfrac{1}{8}'' + 0.07\,d \quad \ldots\ldots\ldots\ldots (88)$$

If the lateral pressure $= P$, the length of the journal $= l$,

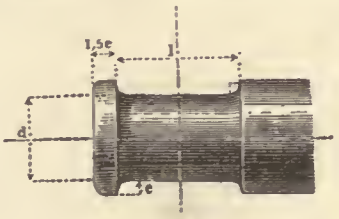


FIG. 272.

and the permissible stress at the root of the journal $= S$, we have for the diameter to resist the pressure

$$d = \sqrt{\frac{16}{\pi S}\left(\frac{l}{d}\right)}\sqrt{P} \quad \ldots\ldots\ldots\ldots (89)$$

The ratio of $l : d$, determines the superficial pressure between journal and bearing. In ordinary circumstances the pressure per unit of area on the lower half of the bearing is $p = \frac{P}{ld}$. When the journal is revolving, this pressure is not the same at all points, but has at the base line a value $= p^o = \frac{4}{\pi}\,p$, and at any angle β, from the base line, a value $p' = \frac{4}{\pi}\,p\cos\beta$. Since the relation between p^o and p is constant, we may use the latter value for all purposes of calculation.

For any given value of p, we have from the preceding:

$$\frac{l}{d} = \sqrt{\frac{\pi}{16}\frac{S}{p}} = \sqrt{\frac{S}{p^o}} \quad \ldots\ldots\ldots\ldots (90)$$

In order that the wear may not be too great at high rotative velocities, it is advisable to take p, somewhat less than the maximum value given above, and it may be made proportional to n, the number of revolutions per minute, or:

$$\frac{l}{d} = \sqrt{\frac{\pi}{16}\frac{S}{a}n} \quad \ldots\ldots\ldots\ldots (91)$$

in which a is a constant, dependent upon material and lubrication.* By combining (91) with (89) we get:

$$d = \sqrt[4]{\frac{16}{\pi S}\frac{n}{a}}\sqrt{P} \quad \ldots\ldots\ldots\ldots (92)$$

These four equations should be applied and the greater values of d, and $\frac{l}{d}$ used. The maximum value of $p = \frac{n}{a}$.

For the value of the constant, the following considerations obtain. If the pressure on the journal acts constantly in the same direction it produces a higher superficial pressure on the lubricant than when, for example, the pressure is reversed frequently, as in a steam-engine crank pin. In the latter case there exists a kind of pumping action between the journal and bearing, which constantly draws the oil into the bearing surfaces, keeping them lubricated so that a higher value of p may be taken than when the pressure is acting continuously in one direction. Such bearings, however, are frequently subjected to violent thrusts and shocks, so that a lower value of S should be taken than with journals in which the directions of pressure is constant. For journals which only make a partial revolution, much higher pressure may be permitted, than for revolving journals. The former may be classed as journals at rest, as distinguished from running journals. The constant a in equation (91) must be determined from practical considerations. It will be found that in practice, wide variations in the value of a occur, but while fair results are obtained with the smaller values, the increased value of a secures greater durability. Good lubrication is of the highest importance, and especially a good distribution of the lubricant over the bearing surfaces.

For bronze bearings under favorable conditions when the pressure is constantly in one direction, a may be taken $= 75$, while if the direction of pressure is periodically reversed, a may be taken $= 150$.

The following table will give the general proportions for lateral journals:

PROPORTIONS OF JOURNALS.

Constant Pressure.

		Wrought Iron.	Cast Iron.	Steel.
Slow Moving Journals.	p_o =	8500	4260	14,220
	S =	8500	4260	14,220
	$\frac{l}{d}$ =	0.5	0.5	0.5
	d =	$0.017\sqrt{P}$	$0.0248\sqrt{P}$	$0.0135\sqrt{P}$

Intermittent Pressure.

		Wrought Iron.	Cast Iron.	Steel.
Slow Moving Journals.	p_o =	8500	4260	14,220
	S =	8500	4260	14,220
	$\frac{l}{d}$ =	0.5	0.5	0.5
	d =	$0.017\sqrt{P}$	$0.0245\sqrt{P}$	$0.0135\sqrt{P}$

Constant Pressure

		Wrought Iron.	Cast Iron.	Steel.
Running Journals. $n \vee 150$	p =	700	360	700
	S =	8500	4260	14,220
	$\frac{l}{d}$ =	1.5	1.5	1.94
	d =	$0.03\sqrt{P}$	$0.043\sqrt{P}$	$0.027\sqrt{P}$

Intermittent Pressure

		Wrought Iron.	Cast Iron.	Steel.
Running Journals. $n \vee 150$	p =	1422	700	1422
	S =	7000	3500	11,840
	$\frac{l}{d}$ =	1.0	1.0	1.3
	d =	$0.027\sqrt{P}$	$0.037\sqrt{P}$	$0.024\sqrt{P}$

Constant Pressure.

		Wrought Iron.	Steel.
High Speed Journals. $n \wedge 150$	a =	75	75
	S =	8500	14,220
	$\frac{l}{d}$ =	$0.13\sqrt{n}$	$0.17\sqrt{n}$
	d =	$0.0244\sqrt{\frac{l}{d}\sqrt{P}}$	$0.019\sqrt{\frac{l}{d}\sqrt{P}}$

Intermittent Pressure.

		Wrought Iron.	Steel.
High Speed Journals. $n \wedge 150$	a =	150	150
	S =	7000	11,840
	$\frac{l}{d}$ =	$0.08\sqrt{n}$	$0.10\sqrt{n}$
	d =	$0.0273\sqrt{\frac{l}{d}\sqrt{P}}$	$0.02\sqrt{\frac{l}{d}\sqrt{P}}$

If $n > 150$, the ratio of $l : d$, is first approximated and the value substituted in the last formulas of the table.

For hollow journals the following proportions may be adopted. Let d_0 = the external and d_1 the internal diameter of the equivalent solid journal, $\psi = \frac{d_1}{d}$, we have:

$$\frac{d_0}{d} = \frac{1}{\sqrt[3]{1 - \psi^4}} \quad \ldots\ldots\ldots (94$$

the length of both solid and hollow journals being the same.

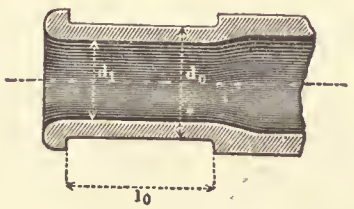


FIG. 273.

If, however, the ratio of diameter to length is to be the same then

$$\frac{d_0}{d} = \frac{1}{\sqrt{1 - \psi^4}} \quad \ldots\ldots\ldots (95)$$

from which the following series is obtained.

$d_1 : d_0 = \psi =$	0.4	0.5	0.6	0.7	0.75	0.8
$1 : \sqrt[3]{1 - \psi^4} =$	1.01	1.02	1.05	1.10	1.14	1.19
$1 : \sqrt{1 - \psi^4} =$	1.01	1.03	1.06	1.15	1.21	1.30

In both cases there exists a smaller superficial pressure for the hollow journal than for the solid one. A common ratio of internal to external diameter is 0.6, and such journals were frequently used in cast iron work and are again being used in connection with hollow steel shafting and axles.

Bronze boxes or their substitutes, such as white metal or other combinations, belong more especially to the subject of bearings (§ 96), and their use permits a higher superficial pressure without creating an excessive increase in the coefficient of friction. For moderate speeds, boxes of cast iron give results which are as satisfactory as can be obtained with bronze. This is especially the case with machines which are actuated by hand. For heavier or continuous service cast iron boxes are only suitable when the pressure is not great, and examples of such bearings will be given in a later chapter. Bearings of wood may be operated satisfactorily at a pressure double that which is used with bronze, if the journal runs in water, or is kept wet. For heavy mill shafting making from 60 to 80 revolutions per minute, wooden bearings lubricated with grease are often used. For mill spindles, boxes with bearings of willow wood are sometimes used with good results. In this case the speed sometimes exceeds 100 revolutions per minute, but the pressures are light.

§ 91.

Examples and Tables of Journals.

In the following tables are collected the results of the formulæ (93) in which the number of revolutions of the journal is assumed to be not greater than 150.

1. Example. A water wheel weighing 66,000 pounds carries a load of 212 cubic feet of water. The axis of the wheel is of cast iron, and the load is equally distributed between the two journals, giving a load upon each journal of 33,000 + 6605 = 39,605 lbs. The nearest value to this in the table is 40,058 lbs., which would give a diameter of 8½ inches, and a length of 12¾ inches.

2. Example. A wrought iron shaft for a similar load, but subjected to alternating action, should have, according to the table, a diameter of about 5⅜", and the same length.

If in cast steel, with alternate action, diameter should be about 4¾ inches, and length of 4.75 × 1.3 = 6.175".

3. Example.* The centres of the walking beam of the water engine at Bleyberg in Belgium each bear a load of 309,210 lbs. The journals are hollow, with a ratio of external to internal diameter of 0.5. We have from (93) and (94)

$$d_0 = 1.02 \times 0.043 \sqrt{309{,}210} = 24.38''$$

and a length $l_0 = 24.38 \times 1.5 = 36.57''$

which gives a pressure of about 350 pounds per square inch of projected area.

The actual dimensions of these journals are $d_0 = 19\frac{3}{4}''$, $l_0 = 18''$, which gives a stress at the base of the journal of a little over 4000 lbs., but the actual bearing is only $15\frac{3}{4}''$ long, which gives a pressure of nearly 1000 lbs. per square inch, which appears to be too great; and in actual practice these journals are obliged to be kept cool with water.

In actual practice there is very little uniformity in the proportions of journals. Sometimes the distinction between constant and alternate action of load is considered but often it is neglected. In the case of locomotive crank pins, for example, p is frequently as high as 1500 to 3000 pounds per square inch, and on the cross head pin, as high as 4500 pounds. On the other hand quite low values of p are sometimes found on the crank pins of marine engines. † In all cases careful lubrication is of the utmost importance. When the number of revolutions is very great the length of the journal should be made greater than is given above.

Table of Journals.

Value of P.

d	e	Direction of Load Constant. Wrt. Iron $\frac{l}{d} = 1.5$	Cast Iron $\frac{l}{d} = 1.5$	Steel $\frac{l}{d} = 1.94$	Direction of Load Varying. Wrt. Iron $\frac{l}{d} = 1$	C't Iron $\frac{l}{d} = 1$	Steel $\frac{l}{d} = 1.3$
1	0.20	1121	554	1419	1419	724	1833
1.25	0 20	1752	866	2217	2217	1113	2188
1.50	0.25	2523	1247	3193	3193	1629	4124
1.75	0 25	3434	1698	4346	4346	2218	5163
2.00	0.28	4485	2218	5677	5677	2896	7331
2 25	0.28	5677	2807	6870	6870	3666	9278
2.50	0.32	7009	3465	8871	8871	4526	11455
2.75	0.32	8481	4193	10734	10734	5476	13861
3.00	0 32	10093	4989	12774	12774	6517	16495
3.25	0.36	11845	5856	14992	14992	7649	19359
3.50	0.40	13738	6792	17387	17387	8870	22452
4.00	0.40	17943	8870	22709	22709		29325
4.25	0.40	20256	10014	25637	25637		33106
4.50	0 44	22709	11227	28742	28742		37115
4.75	0.46	25303	12509	32025	32025		41353
5.00	0 48	28036	13860	35484	35484		45821
5.50	0.50	33924	16771	42935	42935		55443
6.00	0.52	40373	19959	51096	51096		65982
6.50	0.60	4738	23424	59967	59967		79260
7.00	0.62	5495	27167	69548	69548		89809
7.50	0.64	63082	31187	79838	79838		103097
8.00	0.68	71773	34483	90868	90868		117301
8.50	0.72	81025	40058	102520	102520		132422
9.00	0.74	90838	44909	114915	114915		148460
9.50	0.76	101212	50037	128097	128097		165413
10.00	0.80	112141	55443	141935	141935		183284
10.50	0.85	123641	61126	156483	156483		202070
11.00	0.90	135696	67087	171741	171741		221773
11.50	0.92	148313	73324	187709	187709		242394
12.00	0.95	161489	79838	204386	204386		263934

4. Example. An axle on a railway carriage makes from 200 to 300 revolutions per minute; n may taken = 270, and from (93) we have $\frac{l}{d} = 0.13\sqrt{270} = 2.14$. In practice the ratio is made from 1.8 to 2.0. The journals of fan blowers are often operated at more than 1200 revolutions; hence we get, in such cases $\frac{l}{d} = 0.13\sqrt{1200} = 4.5$, or for steel $\frac{l}{d} = 0.17\sqrt{1200} = 5.9$. The blowers made by Sturtevant, of Boston, have steel shafts, with the journals 5 to 6 diameters in length.

§ 92.

Neck Journals.

When a journal is placed between two loaded parts of a shaft, as shown in Fig. 274, it is called a Neck Journal.

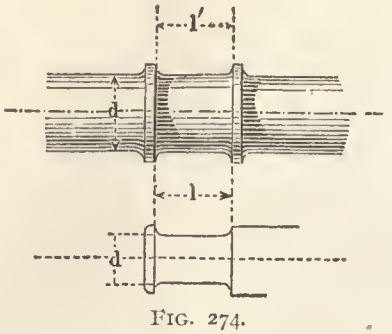


FIG. 274.

In such cases the diameter d' is dependent upon other conditions than those of mere pressure. In order that the wear

* Portfeuille de John Cockerill, I. p. 189.

† See Marks, "Crank Pins and Journals," Philadelphia, Kildare, 1878, where the following values of p are given: Swatara, 400; Saco, 412; Wampanoag, 725; Wabash, 470. The third of these engines had a cylinder 100" diameter, and crank pin 16" dia., 27" long, and the stress in the preceding cases was respectively, 4039, 3071, 10,537, and 2745 lbs.

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may not be greater than in the case of overhung journals, the conditions of speed, lubrication, bearing metal, being the same, the length should not be made less than the corresponding eud journal. If it is practicable to make the length greater, it may be done to advantage, aud the weai thereby greatly reduced.* In many cases, however, the lack of space limits the length, as for example, in the case of crank axles for inside connected locomotives. Such journals are properly considered merely as enlarged end journals.

For hollow journals of this type formula (94) may be used.

1. Example. Borsig's Express Locomotive at the Vienna Exposition.† The journal of the rear drawing axle of steel was loaded with 13,200 lbs. $d' = 7\frac{1}{8}''$, $l' = 7\frac{5}{16}''$ According to the table the corresponding journal is given as $d = 3\frac{1}{8}''$, $l = 3.125 \times 1.94 = 6.1''$, and $p = \frac{13,200}{3.125 \times 6.1} = 692.4$ lbs.

In this case l' is much greater than l, and for the given values of l', and d' we have $p = \frac{13,200}{7.125 \times 7.3125} = 253.3$ lbs.

while if $l' = l$, the pressure $p = \frac{13,200}{7.125 \times 6.1} = 303$ lbs.

2. Example. In the same locomotive the forward axle carried the crank pin journal upon whicb the entire force of the piston was exerted. The total pressure on the piston was 32,120 lbs., and the dimensions of tbe pin were $d' = 4\frac{1}{8}''$, $l' = 4\frac{1}{2}''$.

The corresponding values from the table of the preceding section give $d = 4\frac{1}{4}''$, $l = 4{,}25 \times 1.3 = 5\frac{1}{2}''$ $p =$ about 1400 lbs. The actual value of p, for the sizes used is $\frac{32,120}{4.125 \times 4.5} = 1730$ lbs. In this case l' is less than l, on account of lack of room, which accounts for the increase in superficial pressure.

§ 93.

Fork Journals.

A Neck Journal which is held at both ends in a yoke or fork, as shown in Fig. 275, may be called a Fork Journal. Such journals may safely be made of lesser diameter than those which are overhung. If we let $P =$ the load, $l =$ length, and $d =$ diameter, and s, the maximum permissible stress, we have from case VIII. § 6,

$$d = \sqrt{\frac{4}{\pi S}} \sqrt{\frac{l}{d}} \sqrt{P} \quad \ldots\ldots\ldots (96)$$

and if, as in the beginning of § 90, we put $p = \frac{4}{\pi} p_o$

$$\frac{l}{d} \sqrt{\frac{\pi S}{4 p}} = \sqrt{\frac{S}{p_o}} \quad \ldots\ldots\ldots (97)$$

Proceeding as in § 90 we obtain the following collection of proportions.

Formulæ for Fork Journals.

Constant Pressure.

			Wrought Iron.	Cast Iron.	Steel.
Slow Moving Journals.	p_o	=	8500	4250	14,220
	S	=	8500	4250	14,220
	$\frac{l}{d}$	=	1	1	1
	d	=	$0.0121\sqrt{P}$	$0.0171\sqrt{P}$	$0.0095\sqrt{P}$

Intermittent Pressure.

			Wrought Iron.	Cast Iron.	Steel.
Slow Moving Journals.	p^o	=	8500	4250	14,220
	S	=	8500	4250	14,220
	$\frac{l}{d}$	=	1	1	1
	d	=	$0.0121\sqrt{P}$	$0.0171\sqrt{P}$	$0.0095\sqrt{P}$

Constant Pressure.

			Wrought Iron.	Cast Iron	Steel.
n = or V 150.	p	=	711	355	711
	S	=	8500	4250	14,220
	$\frac{l}{d}$	=	3	3	4
	d	=	$0.0212\sqrt{P}$	$0.029\sqrt{P}$	$0.0185\sqrt{P}$

Intermittent Pressure.

			Wrought Iron.	Cast Iron.	Steel.
n = or V 150.	p	=	1422	711	1422
	S	=	7110	3550	11,845
	$\frac{l}{d}$	=	2	2	3.5
	d	=	$0.0185\sqrt{P}$	$0.026\sqrt{P}$	$0.0158\sqrt{P}$

High Speed journals of this sort are seldom used, and need not be considered here. It will be noticed that these Fork

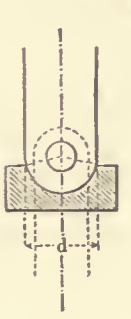

Fig. 277.

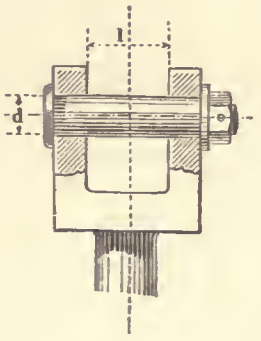

Fig. 275.

Journals are comparatively small in diameter and of greater length ratio than the preceding forms.

Example. A Fork Journal of wrought iron bears a load $P = 4400$ lbs., acting constantly in one direction and revolves at a moderate speed. We have tben $d = 0.0212\sqrt{4400} = 1.4''$, $l = 1.4'' \times 3 = 4.2''$. For an overhung journal under similar conditions we have, from the table of § 91, $d = 2''$, $l = 3''$. The product of the length and diameter is approximately the same in both cases. If the length 4.2" is found inconveniently long, it may be diminished, providing d be proportionally increased. The strength will then be unnecessarily increased and the resistance of friction somewbat greater. Tbese are only examples of the many variations which are to be met among the many conditions of practice.

§ 94.

Multiple Journals.

In some cases the resistance of friction becomes so great that a modification of the fork journal is resorted to in order to reduce it within practical limits. Such an arrangement is shown in Fig. 276, which may be called a multiple journal. If we assume the load to be equally distributed among the plates, this

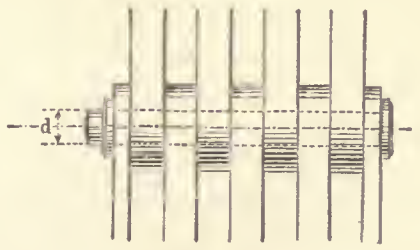

Fig. 276.

arrangement may be considered as a series of fork journals. If the number of members on each side be taken $= K$, each pair will support a Kth portion of the load P, and d will be $\sqrt{\frac{1}{K}}$ times as large as would be required for a simple fork journal.

If $K =$	2	3	4	5	6	7	8
We have $\sqrt{\frac{1}{K}} =$	0.7	0.57	0.5	0.45	0.41	0.38	0.35

Journals of this kind are generally of the slow-moving class, with a length ratio = 1. The total length of journal is the $= 2Kd$. Journals of this sort will be found is some varieties of chain links, of which examples will be given later. *

* See § 109.

† See Berliner Verbandlung, 1874, p. 389.

* Joints of this kind may sometimes be subjected satisfactorily to a greater pressure than the calculation would indicate. Engineer Vollhering bas used sucb a joint in a system of levers to operate a heavy drawbridge. In tbis case the load was about 95,000 lbs. $K = 10$, the thickness of eacb plate $\frac{3}{4}''$, $d = 1\frac{5}{16}$, botb plates and journal being of steel.

§ 95.

HALF JOURNAL.

In those cases in which the reduction of the moment of friction is of great importauce, the length of a journal may be somewhat increased, if the bearing surface is limited to oue-half the circumference, as shown in Fig. 277, which shows such a bearing, the load acting constantly in one directiou and the movement extending only through a small angle. In such cases it is desirable to have a small supplementary journal as showu iu the figure, in order to meet uuexpected lateral pressures. In such half journals, provided the uuused side of the material is proportionally increased, d is independent of P, and p only is to be considered. We have for

		Wrought Iron.	Cast Iron.	Steel.
Po	=	8500	4250	14,220
p	=	6700	3340	11,160

Example: For a pressure P = 220,000 lbs., acting in a constant direction upon a slow moving journal of wrought iron, we have from (93) $d = 0.017\sqrt{220{,}000} = 7.97''$, say 8", and $l = 4''$; for a fork journal, according to (98) $d = 0.0121\sqrt{220{,}000} = 5.67''$, and l is the same; for a multiple bearing with eight parts on a side $d = 0.35 \times 5.67 = 1.98''$, say 2", and a total length $7 = 2 \times 16 = 32''$. If now we take for a half journal the same conditions and make $d = 2''$, we get $l = 2 \times 8 = 16''$. We may, however, make $d = 1.5''$, in which case $l = 1\frac{3}{8}'' \times 16 = 21\,28''$. The journal friction will in this case be $\frac{1}{2}$ that of the overhung journal, $\frac{5}{28}$ that of the fork journal, $\frac{1}{4}$ that of the multiple bearing journal, which latter is nearly 60 per cent. longer.

An application of this form of journal will be seen in Fossey's Coupling. Woolf has also used it on the cast iron crosshead of a large pumping engine.*

The principle of the half journal may be seen carried to its extreme limit in the knife edge bearings of weighing machine in which the friction is reduced to a minimum. The superficial pressure upon these very small surfaces is correspondingly high, ranging from 15,000 to 150,000 lbs. per square iuch. The hardened steel edges aud bearings seem to be able to stand these pressures without injury.†

§ 96.

FRICTION OF JOURNALS.

New journals show greater frictional resistance than those which have worn to a good bearing.

At first the journal only comes in coutact with the metal of the bearing in a limitcd number of spots until after a moderate amount of wear the superficial pressure is distributed over the projected area of the bearing, giving the value of p, as indicated in § 90.‡

For a diameter d, and load P, for a cylindrical journal, whose cœfficient of friction $= f$, we have for the initial force F, which the resistance of friction holds in equilibrium,

for new, unworn journals

$$F = \frac{\pi}{2} f P,$$

and for smoothly worn journals

$$F = \frac{4}{\pi} f P$$

The reduction in frictional resistance is equal to $\frac{8}{\pi^2}$; or about 0.81 times less in a smoothly worn bearing than in a new one. The actual value of F is, however, greatly dependent ou f. This, however, is not only dependent on the lubrication and condition of surfaces, as according to the theories of Morin and Coulomb, but also upon the superficial pressure p, and speed of rubbing surfaces v. §

Additional researches upon this subject are yet greatly to be desired. ‖

Renuie's experiments with cast iron journals in bronze bearings, with copious lubrications:

When p =	3.2	175	315	492	668	739
f =	0.157	0.225	0.215	0.222	0.234	0.234

no account being taken of v, in these experiments.

Hirn experimented with cast iron on bronze with full lubrication, the value of v being equal to 335 feet per minute:

When p =	3	5.26	7.54	9.71	12
f =	0.0376	0.0211	0.0226	0.0199	0.0183

and these experiments showed that for small values of p, f diminishes as p increases.

Hirn also found that if p remained constant, aud equal to 12 lbs., that when

v =	92	164	184	275	327	335	367
f =	0.0086	0.0121	0.0128	0.0165	0.0181	0.0183	0.0191

thus being at all times quite small, but still constantly increasing with the increase of velocity.

Morin's researches gave with pressures of 14 to 20 pounds per square inch, values of f, from 0.05 to 0.11 for journals lubricated with oil, and from 0.08 to 0.16 when lubricated with grease.

The following results were obtained at the Royal Technical Academy from experiments after Morin, upon Clair's apparatus. The journal was of wrought iron in brass bearings, freely lubricated with oil.

	First Test.	*Second Test.*
Bearing Surface	12.800 sq. mm.	128 sq. mm
Total pressure P	16.5 kilo	16.5 kilo
Pressure pr. sq. mm.	0.00129 kilo	0.129 "
Observed frictiou	1.25 kilo	2.65 "
Coefficient f	0.076	0.160

The author's experiments with an apparatus resembling a Prowny brake with surfaces of wrought iron on bronze with good lubricatiou and velocities of 30 to 35 feet per minute, gave the following results:

P =	50	122	192	335	484	624	711
f =	0.090	0.087	0.095	0.118	0.171	0.184	0.180

Here the value of f was doubled, while p increased 15 times. If p remained coustant aud equal to 470 lbs. we have

for v =	0 79	14.17	34.64	55.1
f =	0.222	0.210	0 191	0.167

In this case the coefficient of friction diminishes for an increase in the value of v, contrary to the results in Hirn's observations, the value of p being above 40 times greater thau Hiru used.

These latter results appear to be more in accordauce with Morin's, in that the friction of rest is greater than the friction of motion, and hence for small velocities the friction should be greater than with higher velocities. This law appears to hold good only between certain limits for v, either side of which f increases for iucreasing velocity. Hirn's researches lay beyond these limits. Those of the author are only preliminary to a fuller series of observations.

The following table give some results of the wear on boxes of various kinds in railway service:

Kind of Alloy.¶	Distance, Km. for a wear of 1 kilogramme from 4 boxes.	Wear on 4 boxes in grammes for 1000 Kilometres.
	Kilometres.	Grammes.
1. Gun Metal 83 Cu. 17 Sn. . .	90,390	11.06
2. " " 82 Cu. 18 Sn. . .	99,900	10.01
3. White Metal 3 Cu. 90 Sn. 7 Sb.	72,280	13.83
4. " " 5 Cu. 85 Sn. 10 Sb	88,145	11.34
5. Lead Composit'n 84 Pb. 16 Sb	81,280	12.30
6. Phosphorbronze	429,200	2.33
7. Parsons' White Brass . . .	385,275	2.60
8. Dewrance's Babbit Metal .	637,679	1.57

mixture of 7 parts copper and 1 part tin. They all worked under the same car and all had the same lubrication. In running 28000 miles the losses were as follows:

	Journals.	Boxes.
1.	$d = 3\frac{1}{4}$, $l = 5\frac{1}{2}$, loss $= \frac{1}{32}''$;	loss = 5 lbs.
2.	" $= 3\frac{1}{2}$, " $= 6\frac{1}{2}$, " $= \frac{1}{64}$;	" = 3 "
3.	" $= 3\frac{3}{4}$, " $= 7''$, " $= \frac{1}{100}$;	" $= 2\frac{1}{2}$ "

Taking the journal load as 11,000 lbs., the value of p in the three cases is 612, 554 and 427 lbs.

¶ Nos. 1 to 6 are from the work of Dr. Kunzel on Bronze bearings, Dresden, 1875. The others are from *The Engineer*, Vol. 41, 1876, pp. 4 and 31, all being given in metric quantities as readily comparable.

* See Tredgold, "Cornish Pumping Engines."

† In large track scales, pressures as high as 425,000 lbs. per square inch are found upon bearings less than $\frac{1}{32}''$ wide. The knife edges on the large Werder Testing Machine at the Royal Technical Academy are 360 mm. long, and sustain a maximum pressure of 100,000 kilograms, or 277.8 kg. per mm, or at $\frac{1}{2}$ mm., in width is equal to 556.6 kilograms per square millimetre, or 810,000 lbs. per square inch, and this pressure has been sustained without apparent injury.

‡ See Reye, Theorie der Zapfenreibung, Civ. Ing. VI., 1860, p. 235, also Grove, Trag-und Stuzzapfen, Mitth. d. Gen. Vereins für Hannover, 1876.

§ See Hirn, Études sur les frottements médints, Bulletin von Mülhausen, 1854. p. 188, also the researches of Reunie, Sella, Bochet, and others.

‖ *Engineer*, Nov., 1873, p. 312, contains a brief, but valuable discussion upon the action of railway axles in their actual conditions of operation. The following abstract gives the results:

The brasses were all poured from the same crucible and consisted of a

B. THRUST BEARINGS.

§ 97.

PROPORTIONS OF PIVOTS.

A thrust bearing which is formed on the end of a shaft and bears the pressure upon its sectional area, is termed a pivot. For ordinary cases these are made in the form shown in Fig. 178. The pressure p is uniformly distributed over the area of the end of the shaft, and the velocity is proportional to the distance ρ of any given element from the centre. A small oil chamber of a radius r_1 is formed in the middle of the bearing.

If the outer radius is r_0, we have

$$p^1 = \frac{0.5\, p\, (r_1 + r_0)}{\rho}$$

and for the elements on the outside radius

$$p_0 = \frac{0.5\, p\, (r_1 + r_0)}{r_0}$$

In the formulæ for a uniformly distributed pressure p, we have taken $r_1 = \frac{1}{3}\, r_0$ and the two diametral oil channels are made of a width $= \frac{1}{12}\, d$. We then have for a given load P:

$$P = 816\, p\, d^2 \qquad \ldots\ldots (101)$$

In order that there may not be too much wear for fast running bearings (see § 90) we may take $p = \frac{a}{n}$, and have for high speed pivots :

$$P = 816\, d^2\, \frac{a}{n} \qquad \ldots\ldots (102)$$

Alternating pressures do not occur in these bearings and need not be considered. The value of a may be taken for wrought iron on brouze as $= 75$.

Bearings of lignum vitæ running in water may bear loads of 1500 pounds per square inch even at high speeds.*

The following formulæ and tables will serve for the proportions for end pivots :

FORMULÆ FOR PIVOTS. . . . . (103)

	Wro't Iron or Steel on Bronze.	Cast Iron on Bronze.	Iron or Steel on Lignum Vitæ.
Slow moving Pivots	$p = 1422$ $d = 0.035\sqrt{P}$	700 $0.05\sqrt{P}$	— —
$n =$ or < 150	$p = 700$ $d = 0.05\sqrt{P}$	350 $0.07\sqrt{P}$	1422 $0.035\sqrt{P}$
$n > 150$	$a = 75$ $d = 0.004\sqrt{Pn}$	— —	$p = 1422$ $d = 0.035\sqrt{P}$

FLAT PIVOTS.

$d =$	$0.035\sqrt{P}$	$0.05\sqrt{P}$	$0.07\sqrt{P}$
1	816	398	204
1.25	1275	622	319
1.50	1836	895	459
1.75	2500	1219	625
2.00	3265	1592	816
2.25	4132	2016	1033
2.50	5102	2483	1275
2.75	6173	3011	1543
3.00	7347	3494	1836
3.25	8622	4205	2155
3.50	10000	4877	2500
3.75	11479	5599	2869
4.00	13061	6370	3265
4.25	14745	7192	3686
4.50	16530	8063	4132
4.75	18418	8983	4604
5.00	20498	9954	5102
5.25	22140	10974	5535
5.50	24694	12044	6673
5.75	26990	13164	6747
6.00	29388	14334	7344
6.25	31890	15630	7972
6.50	34490	16900	8623
6.75	37190	18220	9298
7.00	41690	19600	10000

* Penn has used lignum vitæ bearings with pressures of $p = 7000$ to 8000 pounds. (See Burgh.)

Example 1. A crane in the barbor of Cherbourg carries a load of 33,000 lbs. on an end pivot $6\frac{3}{4}$" diameter. Adding its own weight of 6600 lbs. gives a value $P = 39{,}600$ lbs. This is a slow moving pivot and we have from the table for this load a diameter between $6\frac{3}{4}$" and 7.00". A similar crane of 4000 pounds weight and 20,000 pounds load has $d = 6\frac{5}{16}$", while the table would give about $5\frac{1}{2}$".

Example 2. A driving shaft making 100 revolutions per minute, with a load of 2200 lbs., should have, by the table, a diameter of about $2\frac{3}{8}$".

Example 3. A turbine, making 200 revolutions per minute and 3080 lbs. load, should have a step, according to (103), of $0.004\sqrt{Pn} = 0.004\sqrt{3080 \times 200} = 3\frac{1}{8}$".

The length of journals in the case of such pivots is usually made from 1 to 1.5 d, its value being sufficiently great to provide for the lateral pressure.

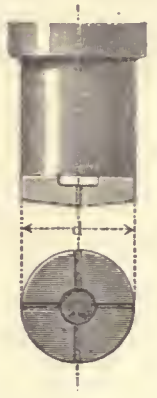

FIG. 278.

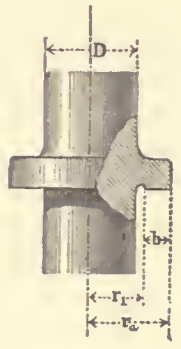

FIG. 280.

There is a general tendency in machine practice to use smaller diameters for pivot bearings,† in order to reduce the resistance of friction.

In order to reduce the effect of higher speeds upon pivots bearing heavy pressure a series of disks is often used. If, in Fig. 279, the number of plates between the end of the spindle and the step is 1, 2, 3, 4, . . . i, we have for the proportion of turns between each pair of surfaces $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{1+i}$ times n.

This device has been used for steps of turbines, mill spindles, etc., by Escher, Wyss & Co., Reiter and others. But few examples now remain of this firm for the thrust bearings of screw propeller shafts; the disks bound together and were

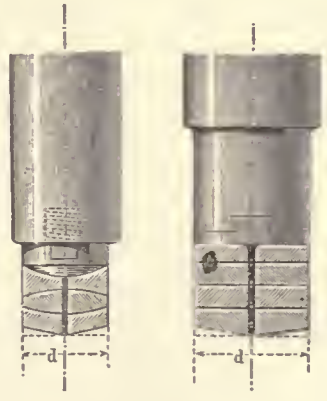

FIG. 279.

overheated and injured. So far as experience indicates, such thrust bearings are capable of standing pressures of 1400 pounds per square inch or even more. The important point to be considered is, therefore, the reductiou of the superficial pressure p.

The use of other materials than iron, wood or bronze, and their substitutes, such as white metal, Babbitt metal, etc., has often been attempted. The subject of wooden bearings will be considered hereafter. Besides the use of hardened steel, which is of small value for great pressures, such bearings have also been made of stone, glass,‡ or hard burned clay,§ but none of

† At the establishment of Gruson, in Madgeburg, a boring mill is made with cast iron spindle in cast iron bearings, with a superficial pressure of more than 20,000 pounds, without ill results.

‡ Bearings of glass bave been used for more than twelve years at the works of E. Acker & Co., at Graggenan, near Rastatt. These bearings are very durable and cheap and require but little lubrication.

§ Shown at the Exposition of 1867, by Leoni, of London, with good results.

these materials have come into general use. Girard used a pump to keep a film of water between the friction surfaces, and after deducting the power to operate the pump showed a very light resistance.* A similar device was shown by Girard at the Exposition of 1867, in which the water jet was operated by a blast of air. This apparatus was rather of the nature of a scientific apparatus, than as a practical application. There were also exhibited journals which ran in bearings in which water was inclosed.† The experience of general practice, however, shows that the ordinary forms are sufficient, without requiring the use of any of these complicated devices.

§ 98.

FRICTION OF FLAT PIVOT BEARINGS.

If a flat pivot bearing with annular bearing surface, as in Fig 278, has an inner radius r_1, and an outer radius r_0 with a load P, we have for the tangential frictional resistance

$$F = \frac{2}{3} f P \frac{1 - \left(\frac{r_1}{r_0}\right)^3}{1 - \left(\frac{r_1}{r_0}\right)^2} \quad \ldots\ldots\ldots\ldots \quad (103)$$

in which f is the coefficient of friction. For rapidly running pivots we have

$$F = \frac{f}{2} P \left(1 + \frac{r_1}{r_0}\right) \quad \ldots\ldots\ldots\ldots \quad (104)$$

The second value is rather less than the first, since, from the previous proportions $r_1 = \frac{1}{3} r_0$, which gives for running pivots $F = \frac{2}{3} f P$, and the ratio of the two values is as 7 to 6, while if $r_1 = 0$, it is as 4 to 3. For values of f see § 96.

Example. In the crane of example 1, § 97, $P = 39{,}600$ lbs. $r_0 = 3\frac{1}{8}''$. $\frac{r_1}{r_0} = \frac{1}{3}$, $f = 0.15$. This gives in (104)

$$F = 0.075 \frac{4}{3} \times 39{,}600 = 3960 \text{ lbs.}$$

The force required to overcome this resistance, if acting at a lever arm 40 inches from the axis would be—

$$\frac{3960 \times 3.25}{40} = 322 \text{ lbs.}$$

§ 99.

COLLAR THRUST BEARINGS.

The use of collars to receive thrusts on horizontal bearings is similar to such use on vertical shafts, and a form is shown in Fig. 280. In this case the inner diameter $2r_1$ cannot be less than the diameter D of the shaft. It is best to make it sufficiently greater to permit a small oil channel to be used as shown in the figure, and oil ways should also be cut in the bearing surface.

FIG. 281.

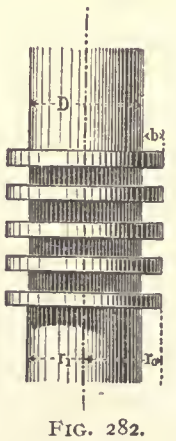

FIG. 282.

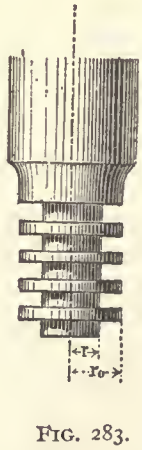

FIG. 283.

If $r_0 - r_1$ is made the same as before, good proportions will be obtained, although the rubbing surfaces will move at a somewhat higher velocity. For this reason such bearings are not to be recommended when high values of P must be carried. The resistance of friction may be calculated by the formulæ of the preceding section.

§ 100.

MULTIPLE COLLAR THRUST BEARINGS

Thrust bearings are frequently made with a row of collars on the shaft as shown in Figs. 281–283. If the collars are similar the pressure may be taken as distributed uniformly among them. If f be a constant value we have for m collars but $\frac{1}{m}$ part of the value is given by (104) for each collar, although the total frictional resistance will be the same, being the sum of the resistances of all the collars. Nevertheless the results of experience, especially with screw propeller shafts, shows the necessity of making m large in order to keep the pressure p as small as possible. This is due to the fact that heavily loaded shafts give, according to (104), so great frictional resistance as to case excessive heating and consequent injury, and experimental researches have shown the value in reducing p, and consequently f. The best values of p lie between 40 to 80 pounds per square inch. When bearing of this kind is placed at the end of a shaft it may be reduced in diameter as shown in Fig. 283, and in such cases p may be made somewhat greater, even as high as 350 pounds, but in such a case there is a great tendency to heat.

Example 1. Screw Propeller Engine, by Indret. Thrust on shaft 39,600 lbs., $n = 55$ lbs. $2r_1 = D = 15''$. Breadth of collars $= b = r_0 - r_1 = 2''$. Number of collars $M = 9$.

$$\text{Here } p = \frac{39600}{9\pi \times 17 \times 2} = 40 \text{ lbs.}$$

The velocity v, at a radius $r_0 = 275$ ft. This gives in (104) taking $f = 01$.

$$F = \frac{0.1}{2} 39600 \left(1 + \frac{7.5}{9.5}\right) = 3465 \text{ lbs.}$$

and the friction horse power

$$\text{H P} = \frac{3465 \times 275}{33000} = 29 \text{ H. P.}$$

Example 2. Turbines on the Rhine at Schaffhausen. $P = 30800$ lbs. $n = 48$. $2r_1 = D = 9''$. Collar width $b = r_0 - r_1 = 1\frac{5}{8}''$, $m = 9$.

$$\text{This gives } p = \frac{30800}{9\pi \times 10{,}625 \times 1.625} = 63 \text{ lbs.}$$

$v = 133$ feet.

$$F = \frac{0.1}{2} 30800 \left(1 + \frac{9}{12.25}\right) = 2664.$$

$$\text{H. P.} = \frac{2664 \times 133}{33000} = 10.7 \text{ H. P.}$$

Example 3. Girard Turbine at Geneva‡

$P = 33{,}000$ lbs. $n = 16$, $2r_1 = D = 9.8''$

$b = r_0 - r_1 = 1 = \frac{3}{8}''$, $m = 12$

$$\text{This gives } p = \frac{33{,}000}{12\pi \times 11.175 \times 1.375} = 57 \text{ lbs.}$$

$v = 46.7$ ft. From (104) we get $F = 2970$ and the friction horse-power is

$$H.\ P. = \frac{2970 \times 46.7}{33{,}000} = 4\ H.\ P.$$

Example 4. Langdon lays down the rule that for collar thrust bearings of screw propeller engines there should be ¾ square inch of surface for every indicated horse-power.§ If N = the horse power and c the velocity of the ship

$$N = \frac{Pc}{33{,}000}$$

$$\text{This gives } p = \frac{33{,}000 \times P}{0.75\ Pc} = \frac{44{,}000}{c}$$

add if $c = 1000$ ft. ser minute, $p = 44$ lbs.

Example 5. A large centrifugal machine by Langen & Sons, in Cologne, has a collar step of the following proportions:

$$P = 4400 \text{ lbs. } n = 800,\ 2r_1 = 1'',\ 2r_0 = 1.57''\ m = 11$$

$$p = \frac{4400}{11\pi (0.78^2 - 0.5^2)} = 366 \text{ lbs.}$$

which is an excessive pressure, liable to cause heating, and demanding most careful lubrication. In this case $v = 275$ and taking $f = 0.1$ we get as before

$$F = 260 \text{ and } H.\ P. = \frac{360 \times 275}{33{,}000} = 3\ H.\ P.$$

---

*See Armengand, "Vignole des Mecauiciens," p. 139.

†Exhibited by Jouffray. See Armengand "Progrès de l'industrie à l'exp. universelle," Vol. I, Pl. 8.

‡Oppermann, Portefeuille écon des machines, Vol. 17. Also Engineering, 1872, Vol. 14, p. 238.

§See Burgh.

In all these examples the co-efficient of friction f has been taken = 0.1, and for the moderate pressure of the first three examples a lower value might have been taken. The examples will suffice to show the importance of the selection of a suitable value for p, and other cases will be examined in § 122.

§ 101.

THE COMPOUND LINK AS A THRUST BEARING.

In the previously examined cases it has been the object of the various plans to reduce the journal friction to a minimum, but there are sometimes occasions in which it is desired to give a journal a definite amount of frictional resistance, without danger of its sticking fast, so that it may be rotated with a moderate force, and may also be readily clamped in any desired position. This may be accomplished, for example, by a thrust journal made in the form of a truncated cone. If the radii of the large and small ends are respectively r_0 and r_1 and the half angle a, we have for the force F, instead of (104),

$$F = \frac{f}{2} \frac{P}{\sin a}\left(1 + \frac{r_1}{r_0}\right) \ldots\ldots (105)$$

and by varying the angle a, may give any desired value to F.*

Very acute pivots sometimes bind in an injurious manner, and hence the increase of F cannot be carried to an extreme in this way. Clamping of this sort may better be accomplished

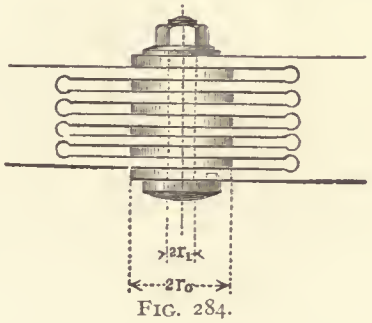

FIG. 284.

by the use of compound bearing surfaces, so arranged as to press on each other, as shown in Fig. 284. Each plate then transmits the axial pressure to the next. If m is the number of contact surfaces, the friction at the radius r_0 of the bearing is found by an analogous equation to (104),

$$F = m\frac{f}{2}P\left(1 + \frac{r_1}{r_0}\right) \ldots\ldots\ldots (106)$$

Example. Let $F = P$, and let $f = 0.1$

$$m = \frac{20}{1 + \frac{r_1}{r_0}}$$ whence, if $r_1 = \frac{1}{2} r_0$, $m = 13$

This arrangement has been used by the writer with success in many parts of machines where a clamp was desired. Formerly the joints of dividers were made with four plates at the pivot.

§ 102.

ATTACHMENT OF JOURNALS.

When a journal cannot be made in one piece with the rest of the shaft, various methods of attachment may be used; such devices are mainly necessary in fitting iron journals to wooden shafts, as for water-wheels.

In Fig. 285 is shown a form of attachment in which a cross anchor piece is forged on the shank of the journal, and a slot mortised in the end of the shaft to receive it. After the journal

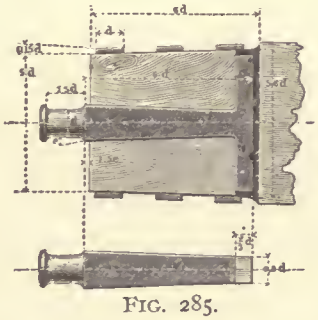

FIG. 285.

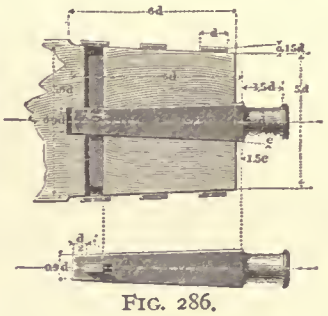

FIG. 286.

is in place it is clamped by driving on the previously heated metal bands (see § 62). The angle of taper is $\frac{1}{20}$. Fig. 286 is a very good form in which the shank of the journal is keyed in

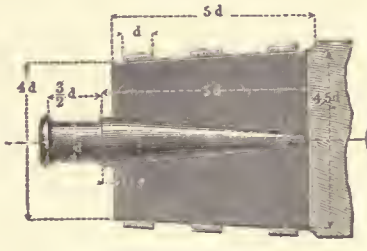

FIG. 287.

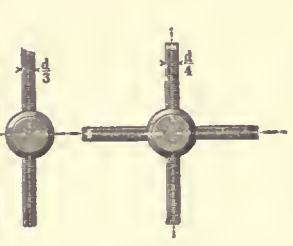

FIG. 288.

place. In Fig. 287 is shown a cast iron journal with two wings, arranged to be driven in, and Fig. 288 shows the proportions of the same when four wings are used. If three wings are desired their thickness may be made equal to $\frac{3}{10} d$.

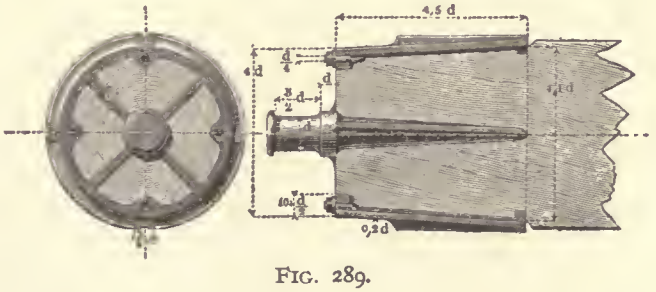

FIG. 289.

Fig. 289 shows a form in which the four wings are surrounded by a conical shell, which is held in place by bolts and anchor plates. The shell is sometimes made with keyways cast in it to act as a centre for the hub of a gear wheel.

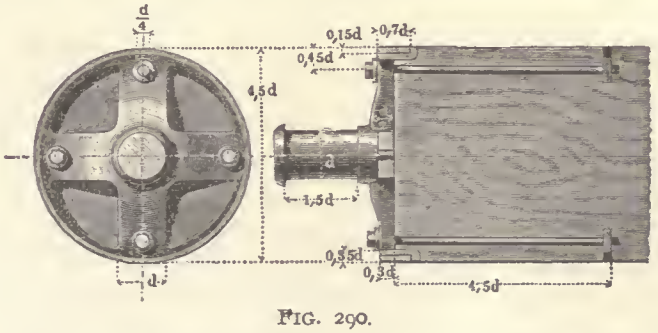

FIG. 290.

Fig. 290 shows a very practical form. The journal is cast on a plate strengthened by heavy cross arms, and a wrought iron ring is shrunk on, while the whole is fastened to the shaft by the four bolts, whose nuts are let into the wood, as shown.

* Applications of this principle may be seen in the spindles of astronomical and surveying instruments. Formula (105) may be used to determine the friction of stop-cocks.

CHAPTER VI.

BEARINGS.

§ 103.

DESIGN AND PROPORTION.

The mechanical devices by which the journals of shafts and axles are carried are called bearings. A complete bearing may be divided into three portions: 1, the boxes; 2, the body or frame; 3, the connecting parts.

The various forms may be divided according to their uses into the two main classes:

A. Bearings for Lateral journals or Lateral Bearings.
B. Bearings for end-long pressure or Thrust Bearings.

Under these classes the principal distinction is to be made as to the side on which the bearing is to be supported. If we

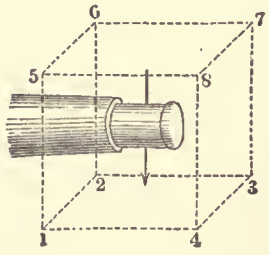

FIG. 291.

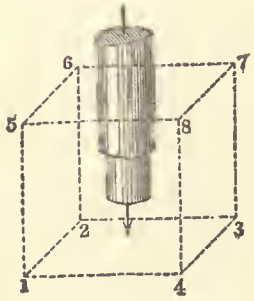

FIG. 292.

suppose the journal to be inclosed in a cube 1.2 . . . 8, Figs. 291, 292, we have for lateral bearings

A Pillow Block, when the base lies in 1, 2, 3,
A Wall Bearing, " " " " 1–8 or 2–8,
A Front Bearing, " " " " 1–6 or 4–7,
A Hanger, " " " " 5–7.

For Thrust Bearings we may have Foot Step Bearings, Wall Step Bearings, or Hanging Step Bearings.

Especial care is to be taken for the equalization of wear and for efficient lubrication, and these points affect mainly the boxes.

The examples which follow have only been selected from the vast number of forms to show typical cases.

The dimensions are based upon a proportioual scale. As the unit for the thickness of the brasses we have $e = 0.07\,d + \frac{1}{8}''$, d being the bore of the boxes, and volues of e are given in the second column of the table in § 91. The modulus for the body of the bearings is:

$$d_1 = 1.15\,d + 0.4'' \quad \ldots\ldots\ldots\ldots\ldots (107)$$

A.—LATERAL BEARINGS

§ 104.

PILLOW BLOCK

In Fig. 293 is shown a form of pillow block suitable for journals from $1\frac{1}{4}''$ to $8''$. The proportions of the body and cap are based on the modulus d_1 (see 107), with the exception of the oil cup on the cap, which would then be rather too large for small bearings, iu which it is made in length equal to the width of the cap, and iu width equal to $0.7\,d_1$.

The length of the boxes is dependent upon the length of the journal, which, as discussed in § 90, may be $1.5\,d$, $2\,d$, etc. For the form shown a good proportion is $l = 2\,d$, the projecting portion of the boxes beiug governed by the proportion of length to diameter adopted.

The bolts for the base plate are made somewhat heavier than those for the cap, as they are screwed up much tighter, and they are often made with special heads to fit a separate sole plate as shown in Fig. 294. The ends of the base are given a bevel in order to permit the use of side keys. The coring out of the sole plate reduces its weigh and also simplifies the machine work. The spaces betweeu the cap and the body of the bearing are filled with slips of wood so that the cap bolts may be tightened without binding the shaft. In cases where the load is great, the pressure alternating, the joiut is closely fitted without spaces, and if wear in the journal is to be taken up the surfaces are filed down.

§ 105.

PROPORTIONAL SCALE FOR PILLOW BLOCKS.

The proportional scale may be used to great advantage in the construction of bearings, and the following example will show its use: *

The poles O, O_1, O_2, Fig. 294, are used for the journal diameter d; the poles P, P_1 and P_2, for those dimensions which depend on the modulus $d_1 = 1.15\,d + 0.4''$. This gives $d_1 = O$,

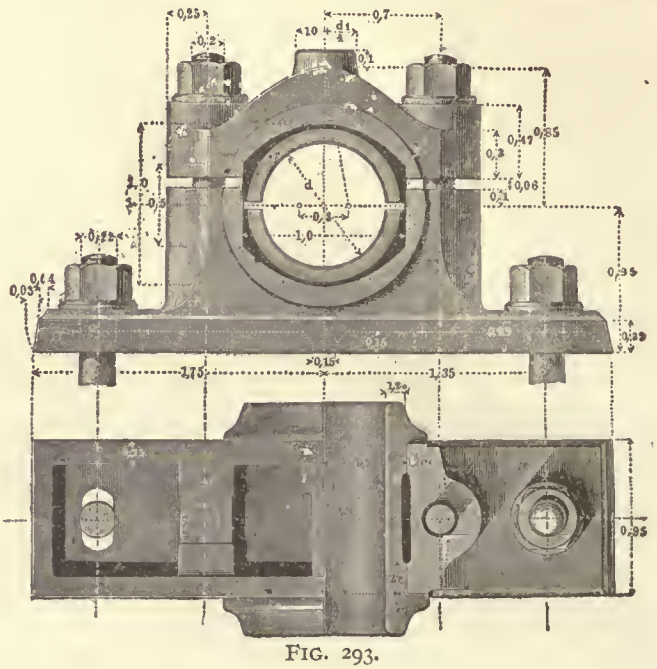

FIG. 293.

when $d = \frac{(-0.4'')}{1.15} = 0.34''$, hence P must be placed when the vertical space between the rays Oa aud Ob is equal to $-0.34''$. The intersection of the rays from O and from P, by the ordinates I, II, etc., give the dimensions of the correspondiug sizes. The dimensions of the boxes must be obtained from another pole, as they depend upon another modulus. This modulus is $e = .07\,d + \frac{1}{8}''$ and becomes $= O$, when $d = \frac{0.125}{-.07} = -1.78''$. The poles E and E_1, therefore, are placed on the

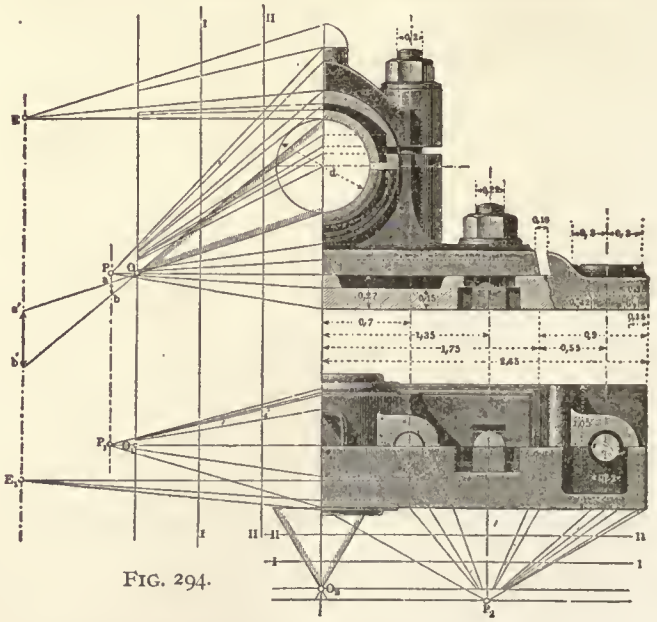

FIG. 294.

vertical line on which the distance $a'\,b'$ equals $1.78''$. For the oil cup in the cap the width is:

$$0.25\,d_1 + 0.4'' = 0.4'' + 0.25\,(0.4'' + 1.15\,d) = 0.29\,d + 0.5'' = 4.16\,(.07\,d + \tfrac{1}{8}'') = 4.16\,e$$

Hence E is also the pole for the oil cup.

* The firm of Escher, Wyss & Co, in Zurich, have used the proportional scale very well for designing bearings, both in determining the geometrical proportions throughout and also by the excellent method of a single pole.

§ 106.

VARIOUS FORMS OF JOURNAL BOXES.

It is often found convenient to give the boxes of a pillow-block other forms than those of the preceding illustrations, as for example octagonal, as in Fig. 295, or cylindrical, as in Figs. 296 and 297. The last two forms are suitable for bearings in lathe headstocks, and in such cases the boxes are kept from

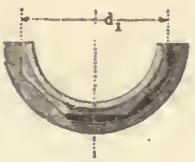

FIG. 295.

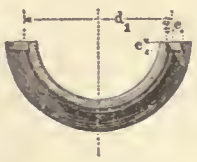

FIG. 296.

FIG. 297.

slipping out of place by the flanges whose width is $2e$, as shown in Fig. 296, or by projecting pins, Fig. 297, fitting into recesses in the base and cap. Each of these forms has its advantages and objections, and it is hardly possible to decide which form is the most desirable, special conditions being generally present. The modifications in the base and cap to admit the forms shown in Figs. 296 and 297 are readily made without requiring detailed instructions.

Boxes in which white metal or similar compositions are used require special construction, since these materials are not strong enough to resist the stresses with the same security as solid bronze boxes; for such bearings a cast-iron or bronze shell is made, in which a lining of the softer metal can be poured, as in Fig. 298. In such cases the shell should be cleaned with acid and tinned before pouring the lining metal.

Boxes of lignum vitæ (see §§ 97–117) must be made of simple shape. A convenient shape is shown in Fig. 299, which the general form of the bearing may be made.

In America examples are often found of bearings in which

FIG. 298.

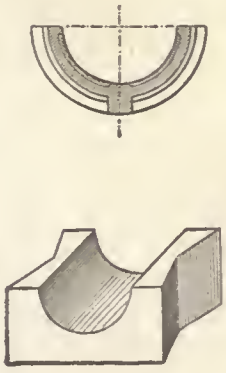

FIG. 299.

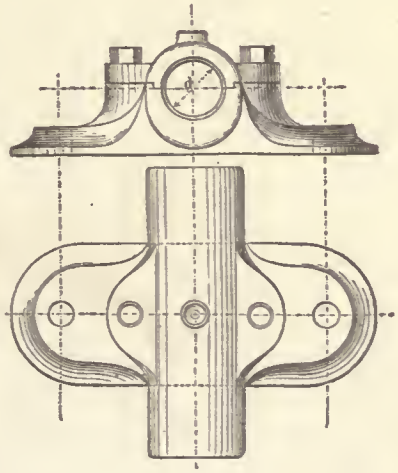

FIG. 300.

the soft metal is run directly into recesses in the base and cap. Fig. 300 shows such a bearing as made for the journals of fan-blowers and shafting, by Sturtevant, of Boston. The base is hollowed out to serve as an oil chamber, and the oil is fed to the journal by a wick. The details are shown in Fig. 301. These journals are made very long ($l = 4d$), and hence the superficial pressure is small.

§ 107.

NARROW BASE BEARINGS. LARGE PILLOW BLOCKS.

It is often desirable, when space is limited, to make bearings with narrow bases, and this may be done by making the cap-bolts with collars as shown in Fig. 321, and also Fig. 312. This permits the holding down bolts to be dispensed with, and space saved. Such collared bolts are also used for pillow blocks, which are subjected to both upward and downward stresses, since the boxes are firmly bound together (see § 88). Fig. 302 shows a form of pillow block for journals of 8 to 12 inches in diameter. It is made with four cap bolts and four base bolts, by which it is secured to the base plate. The base bolts are of the form shown in Fig. 294, § 105, so that the base may be removed from the base plate when necessary without disturbing

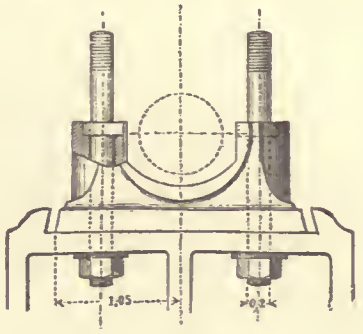

FIG. 301.

the solidity of the latter. The body of the pillow block is cored out to a greater extent than in the previous form, and when

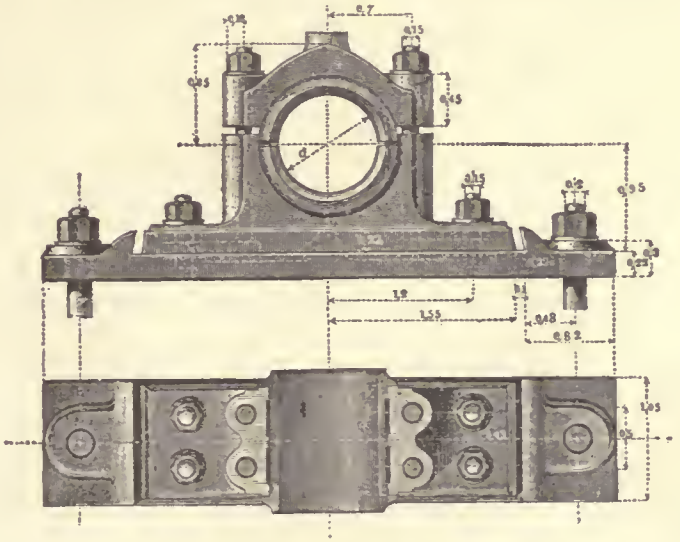

FIG. 302.

the journal is used for a crank shaft, or is subjected to jarring strains, the cap bolts should be provided with jam nuts, or some of the other forms of security, such as is shown in § 85.

§ 108.

PILLOW BLOCK WITH ADJUSTABLE BEARING.

In many cases it is only necessary to adjust the height of pillow blocks from time to time by inserting liners beneath the

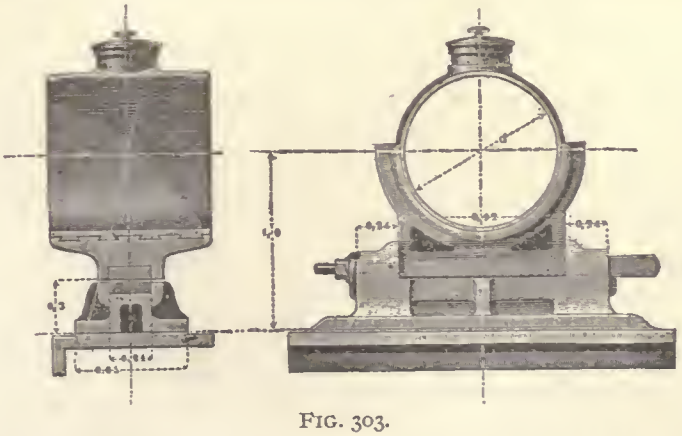

FIG. 303.

base, but in some situations it is desirable to provide a special means of obtaining such an adjustment. In Fig. 303 is shown such an adjustable bearing for use in screw propeller shafts.

The body of the bearing is not bolted down, but rests solely by its weight upon the wedge system, by means of which it can be raised or lowered as may be found necessary. The upper box is provided with flanges through which the cap bolts (omitted in the illustration) pass. The lower box is lined with white metal, which is poured into the recessed bearing.

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§ 109.

ADJUSTABLE PILLOW BLOCKS.

Many attempts have been made to arrange the boxes in a pillow block so that they may be self-adjustiug and so adapt themselves to various positions, which the journal may assume and secure for it at all times a full beariug and support.* For this purpose, among other methods, the plan has been adopted of making the boxes with central spherical portion fitting into corresponding recesses in the body of the pillow block. This form of bearing has been widely introduced in America by Messrs. Wm. Sellers & Co., and adapted to a great variety of positions.

Sellers has always urged the desirability of the principle of keeping the pressure between journal and bearing at a minimum.† This practice permits the use of cast iron boxes, for which a pressure of not more than 15 pounds per square inch is used.‡

The use of moderate superficial pressures is most practicable in the case of bearings for line shafting in which the proportions may be made such as to give but light pressure. This advantage will be seen on reference to § 92.§

Fig. 304 shows Sellers' form of pillow block. The cast iron boxes are made with a spherical enlargement in the middle, which is held between corresponding recesses in the cap aud base. The boxes are prevented from revolving by the hollows

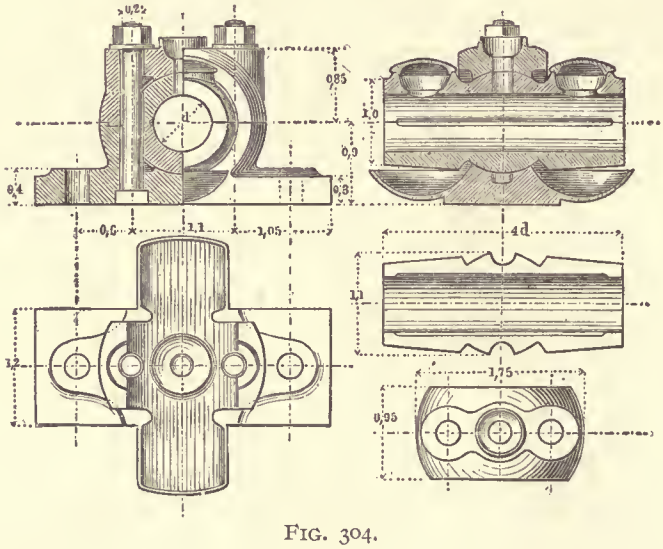

FIG. 304.

in the sides which receive the bodies of the cap bolts. Three openings are made for oil or grease and two drip cups, which are cast on the base plate, serve to receive the superfluous oil.‖

The modulus upon which the proportions of this bearing are based, is not that given iu (107), but the following :¶

$$D_1 = 1.4\,d + 0.' \qquad \ldots\ldots (108)$$

The length of the boxes $= 4\,d$. The shape adopted by Sellers shows the care in modelling which is characteristic of the American designs of engineers. The Sellers' bearings have been used to a considerable extent in Germany.

Another form of adjustable pillow block is shown in Fig. 305. This is used by Sturtevant in some of his fan blowers. In this case the ratio of l to d is very great (see example 4, § 91). The adjustability is obtained by pivoting the bearing A upon a

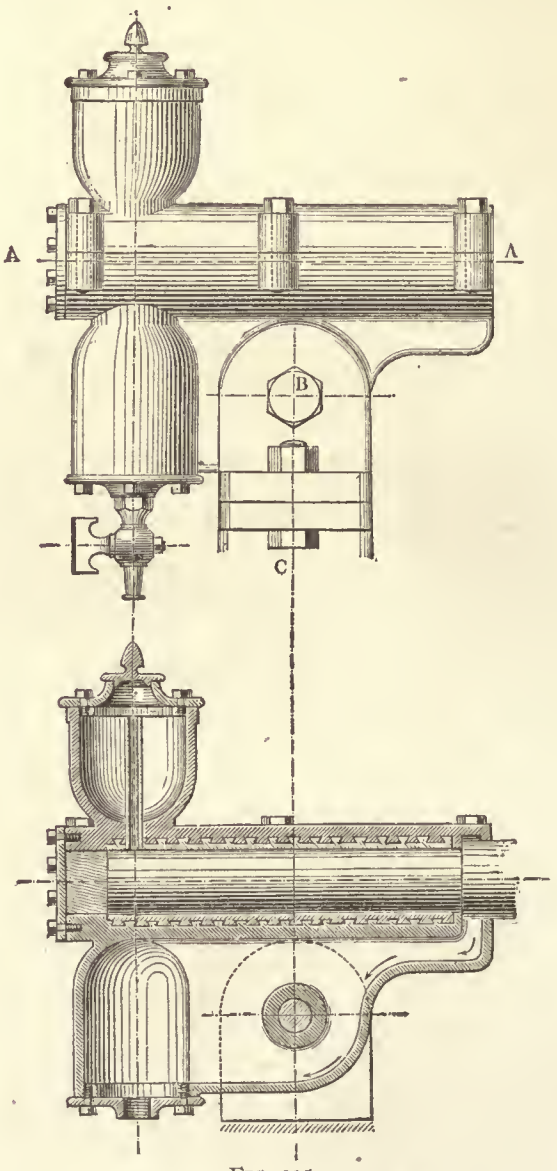

FIG. 305.

cross bolt B, which passes through the cheeks of the pedestal also ; the latter beiug adjustable about the axis BC. The bearing is lined with white metal, aud the eud thrust is taken up by a block of lignum vitæ. If an adjustment in the directiou AA is required, the bolt C may be looseued and the required movement made. The provisiou for lubrication is especially noteworthy both iu the uanuer of supply aud in the collectiou of the overflow.

§ 110.

BEARINGS WITH THREE-PART BOXES.

In horizontal steam engines and iu similar service, the pressure upon the journal is thrown first on one side and then on the other, while at the same time there is a constant vertical pressure, such for instance as is due to the weight of a fly wheel. Attempts to remedy the tendency to overwear by making the boxes inclined, have proved but a partial remedy, and the best method of construction in such cases is to make the box iu three parts, one of which receives the constaut vertical pressure, while the other two provide for the backward and forward thrust. Such a bearing is shown in Fig. 306. The modulus $d_1 = 1.15\,d + 0.4''$. The bottom box rests on two wedges which are tapped with screw threads and can be adjusted and locked at any desired point by the bolts shown. The side boxes are each held up by two steel set screws ; a wrought iron plate being interposed between the screws and the boxes. If it be-

* Various designs have been made by Bodmer at Manchester, Schönherr at Chemnitz, Stehelin at Thann, and Zimmerman at Karlsruhe.

† Treatise on Machine tools, etc., as made by W. Sellers & Co., Philadelphia, Lippincott, 1873, p. 161.

‡ As an example of the performance of these cast iron bearings Sellers cites a bearing which had been in service for sixteen years and in which the lower box was not yet worn to a polish over its entire surface. The shaft made 50 revolutions per minute and was 4½ in. diameter, and carried near the bearing a 72 in. pulley, of 20 in. face, transmitting 52 horse-power. In other examples it is shown that after a year's use the tool marks were still visible. The small superficial pressure does not force the oil out, and hence the journal is carried on a film of lubricant The consumption of oil is very small, and Messrs. Sellers state that a shaft making 120 revolutions per minute consumed but 2½ ounces of oil in six months.

§ As an example of the impracticable results which would follow from an attempt to obtain such light pressures to overhung journals, we may take Sellers' value of 15 pounds per square inch and apply it to an example. If $P = 17{,}600$ lbs., we have for a wrought iron shaft with a constant direction to the pressure, from the table in § 91, $d = 4''$, $l = 6''$. If $p = 15$ lbs. we have from formula (90) $\frac{l}{d} = 11.9$ and from (89) we have $d = 11.2''$; hence $2 = 11.2 \times 11.9 = 133''$! !

‖ Sellers recommends a mixture of tallow and oil, which becomes more liquid should the bearing grow warm.

¶ See Berliner Verhandlungen, 1876, p. 89.

comes necessary to remove the side boxes the cap is first taken off, and the iron plates taken out, when the boxes can be separated far enough from the shaft to permit their removal without

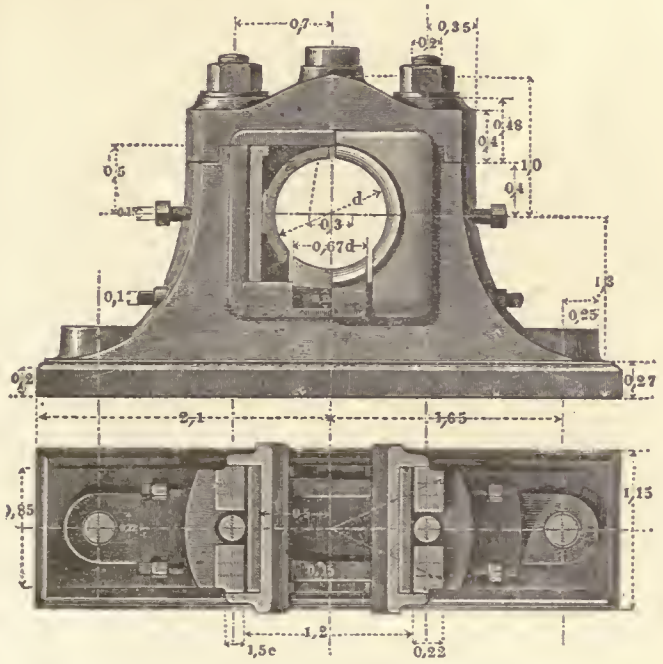

FIG. 306.

interference with the shaft. The body of the bearing is increased in width in order to provide for the increased lateral pressure.

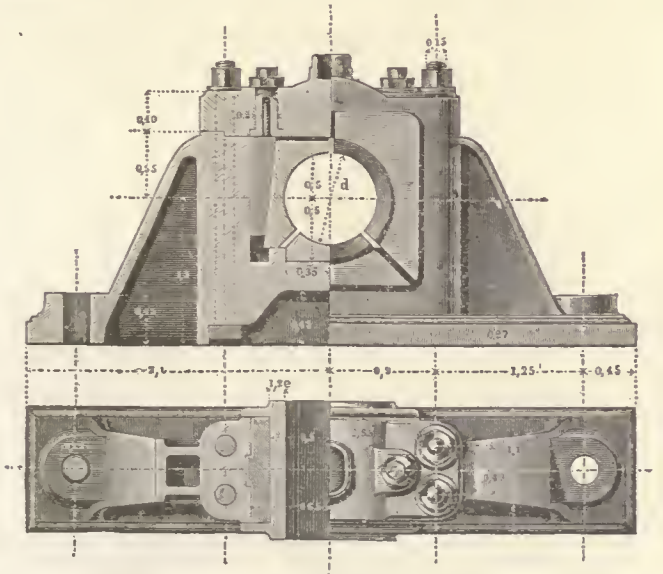

FIG. 307.

Another three-part bearing* is shown in Fig. 307. In this case there is no vertical adjustment to the lower box—and if necessary it must be raised by packing underneath. The side boxes are set up by wedges which are adjusted by set screws through the cap. Each wedge carries a screw on its upper end, and the nuts for these screws are fitted so as to revolve in the cap, being turned by a wrench on the hexagonal head, and then clamped in position by the thin jam nut shown. The heavy inclined ribs stiffen the body of the bearing to resist the stock and thrust of the piston. It is often convenient (as in the case of the original of the figure) to cast the body of the bearing in one piece with the bed plate of the engine.

A third, and simple form of three-part bearing (by Schultz Brothers in Mayence) is shown in Fig. 308. It is suitable for

* From a steam engine by the Soc. Fives-Lille in Paris.

those cases in which an alternating up and down pressure is combined with a constant lateral pressure. The latter would not be provided for in an ordinary pillow block, but here it

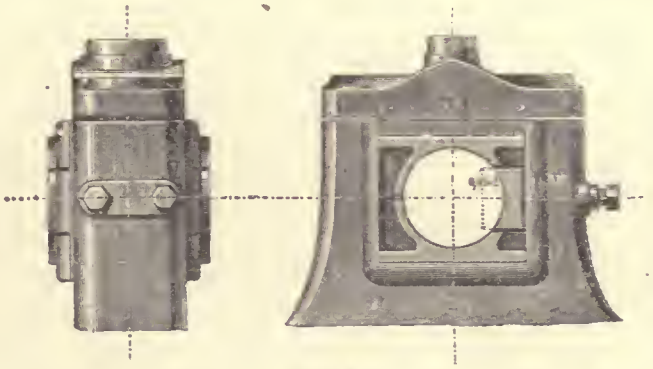

FIG. 308.

is taken up by the small side box. This form is suited for small vertical engines in which the pull of the belt is toward one side.

§ 111.

PEDESTAL BEARINGS.

Bearings which are not placed directly upon a base plate, but are raised upon feet or pedestal are called pedestal bearings

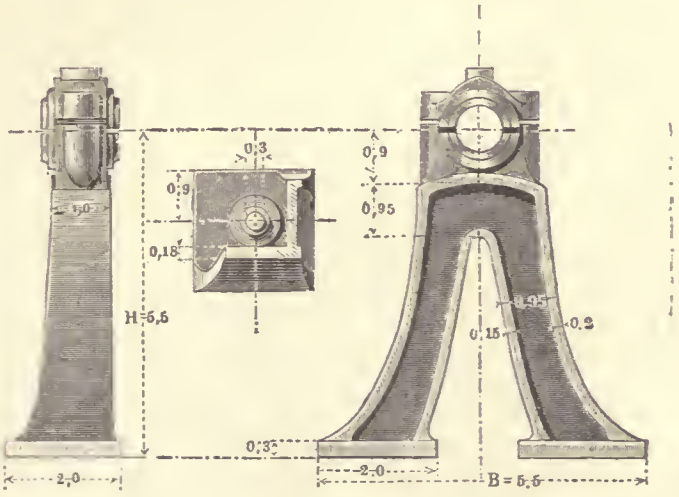

FIG. 309.

That shown in Fig. 309 is similar to the one in Fig. 293, placed upon a pedestal. Such pedestals vary greatly both in form and height. The width of the foot is made equal to the height of the journal in the form shown, which gives the base and the legs a sufficiently slender appearance.

§ 112.

WALL BEARINGS.

The wall bearing shown in Fig. 310 is the same as shown in Fig. 293, with the addition of the bracket. The base here is placed at right angles to the joint in the boxes and parallel to the axis of the bearing, the whole being made in the bracket form shown.

The cap and the boxes are of the same form and proportions as for a pillow block for the same size journal. The bolts may either be tapped into the body of the bearing, or made as stud bolts, using the forms shown in Figs. 225 and 226 § 83, with key.

For larger sizes the opening in the plate should be surrounded with a rib of a thickness $0.1d_1$ and width $= 0.4d_1$, the latter being measured in the direction of the axis of the journal.

Fig. 311 shows an adjustable wall bearing by Sellers. In this case the cast iron boxes are somewhat lighter than for pillow blocks and are made with a cylindrical cross piece in the middle, in which the spherical seats are placed. The especial feature is the method by which the vertical adjustment is made. The two plugs which support the boxes have cast upon them a very shallow screw thread, and the nuts in the sockets have also their threads cast in them. The thread only extends along

a portion of the length of the plugs as shown, in order to permit securing them in position. This is done by the two self screws which clamp them firmly in their places.

The opening through the upper plug gives access for the tube of a lubricator.

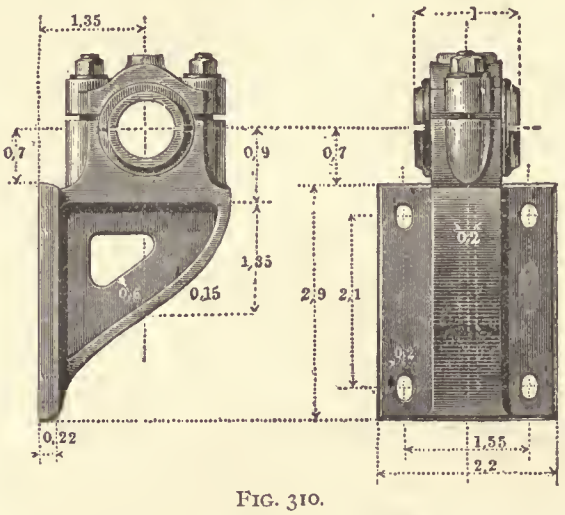

FIG. 310.

The projection from the wall *a* is made constant for bearings for journals 2″ to 4″ in diameter and equals 6″. The elegance of the form is noticeable in the principal elevation and also in the horizontal section.

§ 113.

YOKE BEARINGS.

The bearings used on vertical shafts may be considered as a variety of wall bearings. In situations where the space is limited the forms shown are not always convenient, the first, because it is not symmetrically disposed about the parting of the

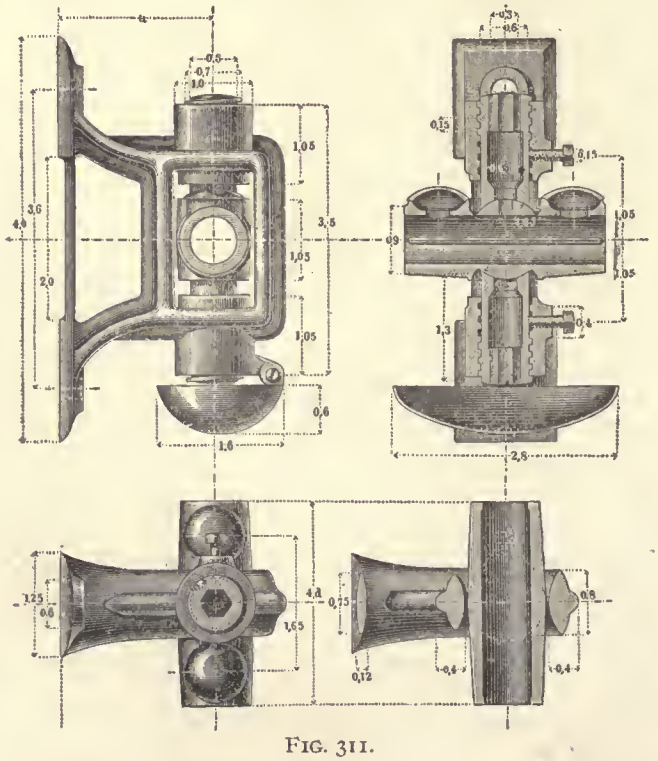

FIG. 311.

boxes, and the second, because of the space it requires. For this service a compact, symmetrical bearing, whose base is at right angles to the parting of the boxes, is often very desirable. Such a construction is shown in Fig. 312, and may be called a Yoke Bearing. In this case the cap and body together form a rectangular yoke, in which the bronze boxes are placed in a transverse direction. In the illustration the wear can only be taken up in one direction, but if it is desired in both directions the cast iron block on the right may be replaced by a wedge as shown on the left.

By removing the cap, the wedge and the block can be easily removed and the shaft moved sideways to a sufficient extent to permit the removal of the boxes. The cap bolts are provided with collars forged upon them and serve also to fasten the bearing in place. The modulus for the dimensions is the same as (107), $d_1 = 1.15\,d + 0.4''$.*

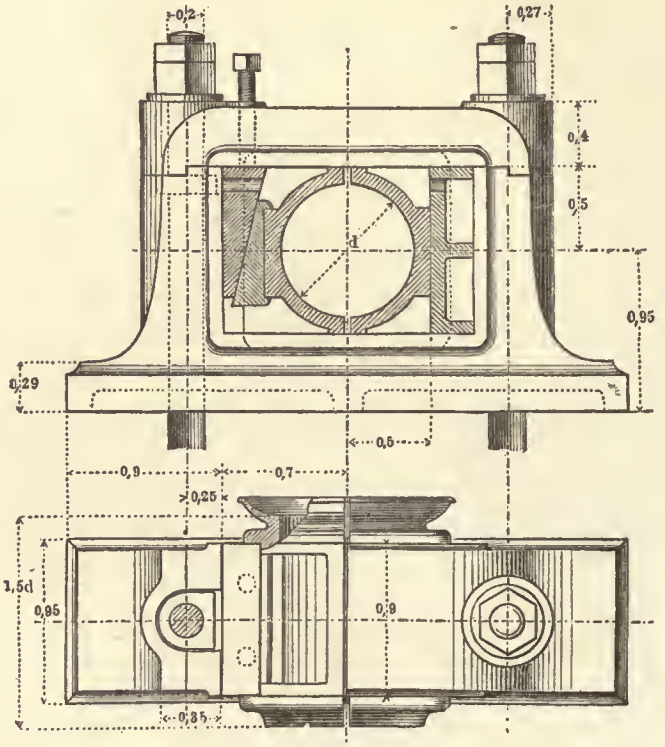

FIG. 312.

§ 114.

WALL BRACKETS.

In Fig. 313 is shown a form of bearing similar to Fig. 293, which may be called a wall bracket bearing. The cap bolts are inserted from below, which permits their ready removal and replacement. If only two bolts are used in the wall plate, it is

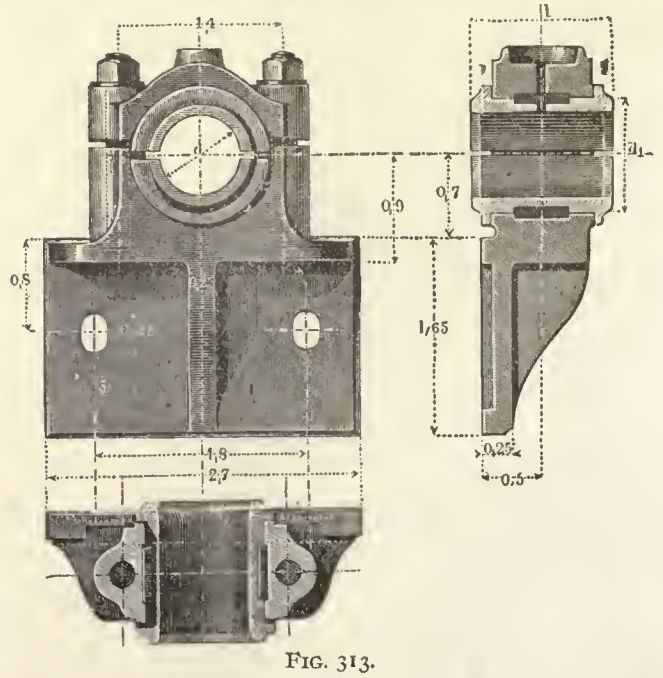

FIG. 313.

desirable that it should be held from lateral motion between wedges, and should also be firmly secured against vertical mo-

* For such a Yoke Bearing, see Engineers' and Machinists' Assistant, London, 1854, Pl. I.

tions by some of the methods given in the following chapter. Where it is not practicable to secure it in this manner, four bolts should be used.

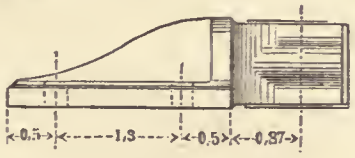

FIG. 314.

Another form of wall bracket is shown in Fig. 314. It is similar to the Yoke Bearing, and can often be of service, as for example in Fig. 350, § 126, although it is not of as general application as the preceding form. The bolts for the cap are made with heads, of the ordinary cap screw form.

Various other wall and bracket bearings may be made by combination of a wall plate and pillow block in different positions, and these may be grouped in the general class of Arm Bearings, each form being governed by the conditions of the special case under consideration.

115.

HANGERS.

According to the definition in § 103 a pillow block by inversion becomes a hanger, the pressure of the journal falling upon the cap box. If the journal is one of wrought iron proportioned to bear the loads given in § 91, the bolts for the cap and base plate will not be strong enough if determined from the same

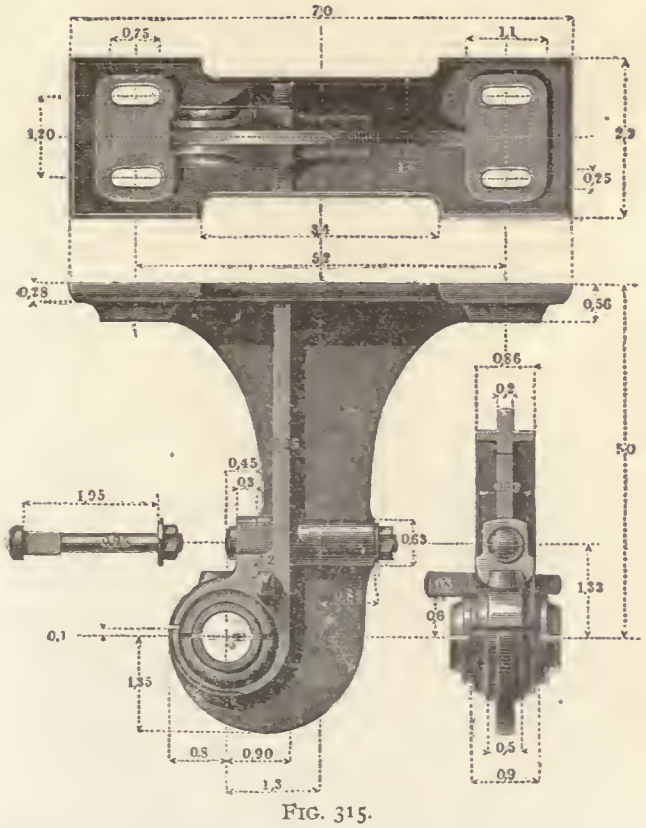

FIG. 315.

unit of proportion as already given for such bearings. This is also true for the cap, and feet of the base.

For this service, good dimensions may be obtained by using for the boxes the previous modulus $d_1 = 1.15\,d + 0.4''$, and also E as before, and for all other portions the special modulus,

$$D_1'' = 1.75\,d + 0.4'' \quad \ldots\ldots\ldots (109)$$

If a pillow block is to be used as a hanger for a neck journal, the cap bolts should be increased to such size as would be given by the use of formula (109), in which d is the diameter of the neck journal corresponding to an equivalent end journal.

Example: A load of 17,600 lbs. would give, according to the table in §91 for a wrought iron journal a diameter of about 4 inches. If this load is carried on the cap of the bearing we use the modulus,

$$D_1'' = 1.75\,d + 0.4'' = 1.75 \times 4'' + 0.4'' = 7.4''$$

This gives for the diameter of the cap bolts $7.4 \times 0.2 = 1.48''$, say 1 ½″. A neck journal of 6¾″ diameter to bear the same load would have for its normal unit $d = 1.15 \times 6.75 + 0.4 = 8.15''$, which is greater than the preceding value and hence may be used safely, even should the full load be carried by the cap.

Sellers makes a short hanger which resembles in form and dimensions the corresponding size pillow block, with the boxes turned 180° and the drip cups cast on the cap instead of the base. In most cases, however, a greater distance is required between the shaft and the base plate for hangers than is given in pillow blocks, for which reason they are best considered as a separate form of construction.

The hanger shown in Fig. 315 is called, from its form, a Ribbed Hanger. The boxes are carried in the hook-shaped portion below, their form being the same as we have already shown. The cap is secured with a key and clamped in the desired position by the bolt shown.

For journals of less than 2 inches diameter, but one bolt need be used in each foot, and in such case their diameter is $= 0.3\,d_1$, the bosses on the plate be altered to correspond.

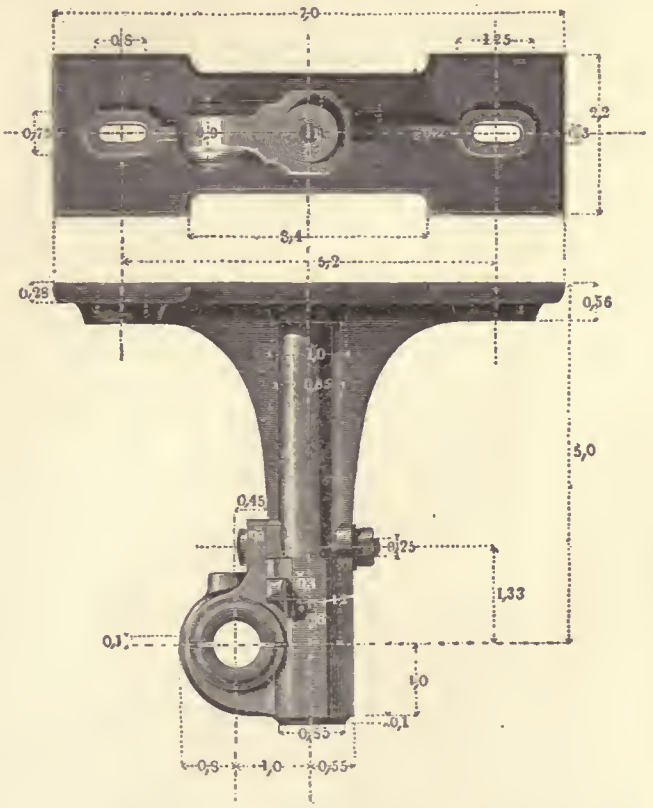

FIG. 316.

In the Post Hanger, Fig. 316, the general arrangement is the same as in the preceding form, the principal difference being in the frame. The column is made hollow and its internal diameter $= 0.55\,d_1$. For the larger sizes four bolt holes are made in the base plate, as shown in Fig. 315.

Hangers are not generally bolted directly to the ceiling beams, but to strong pieces, or intermediate timbers, and by

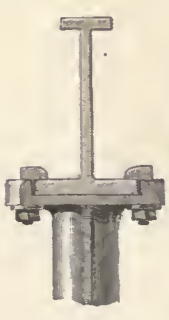

FIG. 317.

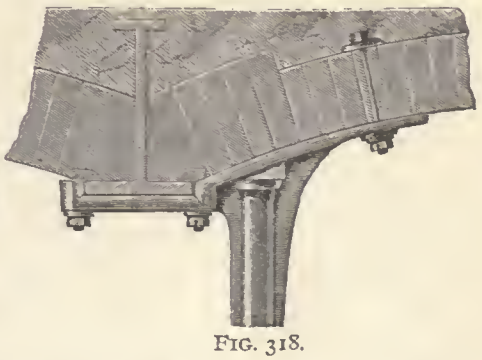

FIG. 318.

varying the thickness of these pieces any desired amount of drop may be obtained. If the variation is too great to be secured in this manner a different depth hanger must be used.

If the building is of so-called fire-proof construction, with

ceilings of iron beams and brick arches, the form of the base of the hanger must be correspondingly modified. A practical method is shown in Fig. 317, in which hook bolts are used. The bolts, which are four in number, pass through sockets cast in the base of the hanger, and their method of attachment avoids weakening the beam. The base of the hanger is made with ledges which fit over the edge of the beam and permit the use of wedges on each side.

The form shown in Fig. 318, which is due to Fairbairn is intended to bring the shaft parallel to the beam, while the previous form carries the shaft at right angles to the beams. The attachment of the hanger both to the beam and the arch makes a very secure fastening, but the inaccessibility of the bolt head is an objection. In this case also the beam is not weakened by drilling, hook bolts and keys being used, as in the previous case.

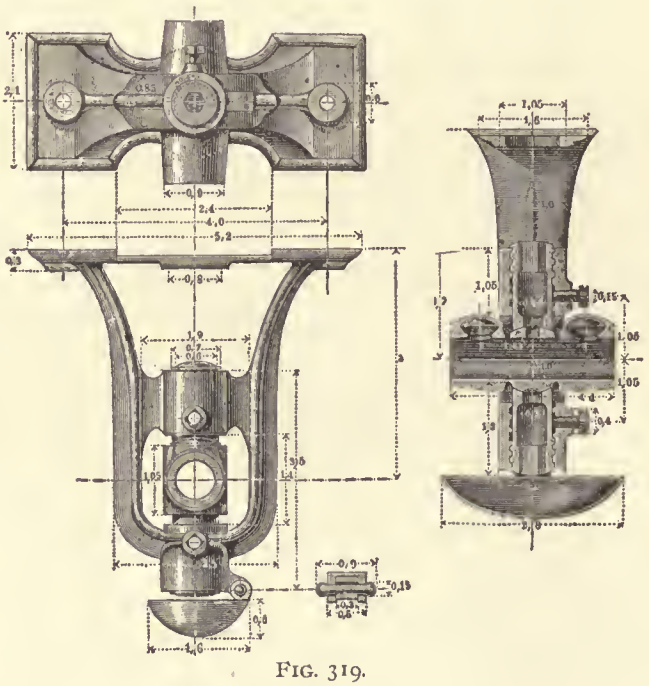

FIG. 319.

§ 116.

ADJUSTABLE HANGERS.

The most generally used of the Sellers' adjustable bearings is the hanger shown in Fig. 319. The method of holding and

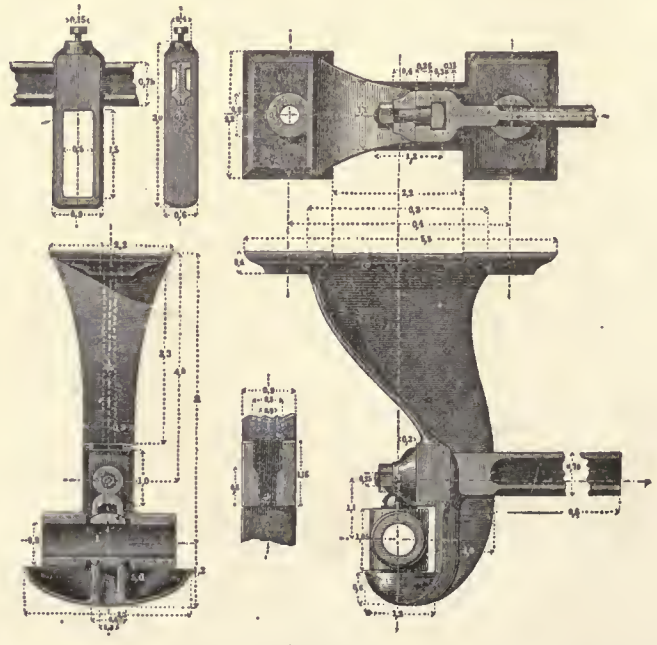

FIG. 320.

adjusting the boxes by means of screw plugs is the same as shown in the wall bearings, Fig. 311. Especially to be noted is the attachment of the drip cup, which may be easily removed by withdrawing the small pin with enlarged ends.

The drop, or distance from base to centre of shaft, $= a = 3.5\ d_1'$ in the illustration, but in some cases it must be made greater. These hangers, like all of Sellers' bearings, show very careful modeling and proportioning, which the small size of the illustrations can only imperfectly show.

In Fig. 320 is shown Sellers' countershaft hanger. In this form the shaft is put in place from the side, and the amount of wear in the boxes is so slight that they are made solid, instead of in halves. The cap—which is secured by a bolt, holds the box in place, and the drip cup is cast in one piece with the body of the hanger and provision is made for a drip cock to remove the waste oil.

The illustration shows also the arm for carrying the belt shifter.

Sturtevant uses ball and socket hangers also for the countershafts of his fan blowers. These are somewhat different from the preceding. Fig. 321 shows the boxes in perspective and in cross section. The section shows the white metal lining and also the arrangement of double oil chambers, which, by means

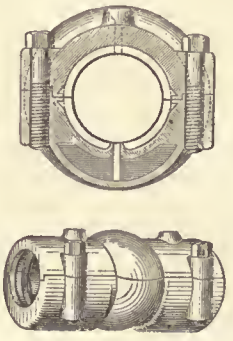

FIG. 321.

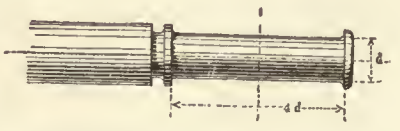

FIG. 322.

of wicking, keep the journal lubricated. The outer ends of the box casting are formed into drip channels, and also receive the shoulders on the shaft. These shoulders, as shown in Fig. 322, run freely in the boxes without contact. The journal as shown is on the end of the shaft, and the pressure is so small that the wear is inappreciable.

§ 117.

SPECIAL FORMS OF BEARINGS.

In propeller shafts where the screw is arranged to be lifted it is necessary to design bearings which are to be entirely immersed in water. Penn's practice is to line such bearings with wood, which has proved especially satisfactory. In Fig. 323, is given an illustration of such a bearing as constructed by Ravenhill & Hodgson, the diameter of the shaft being about 19 inches. The body of the bearing is of bronze, the boxes are of cylindrical section fitted with strips of lignum vitæ set in a

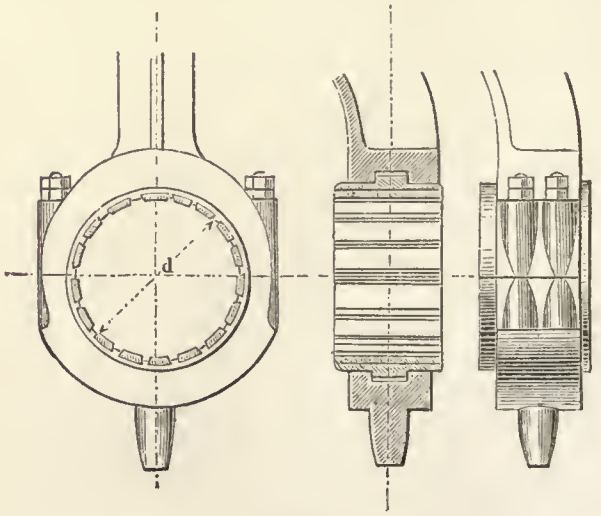

FIG. 323.

special lining metal. The pin, projecting from the bottom, enters into a corresponding recess in the stern frame, when the screw is lowered into place.

On the Prussian State Railway there have recently been adopted two standard forms of bearings for use under cars—one form being for bronze, the other for white metal boxes. In

Fig. 324 is shown details in partial section of the latter form, with a few dimensions. The bearing is made in two principal parts, the body and the lower portion, both being provided with oil chambers having openings and covers to keep out the dust. The joint between the two parts is in the horizontal plane passing through the axis of the journal, the parts being

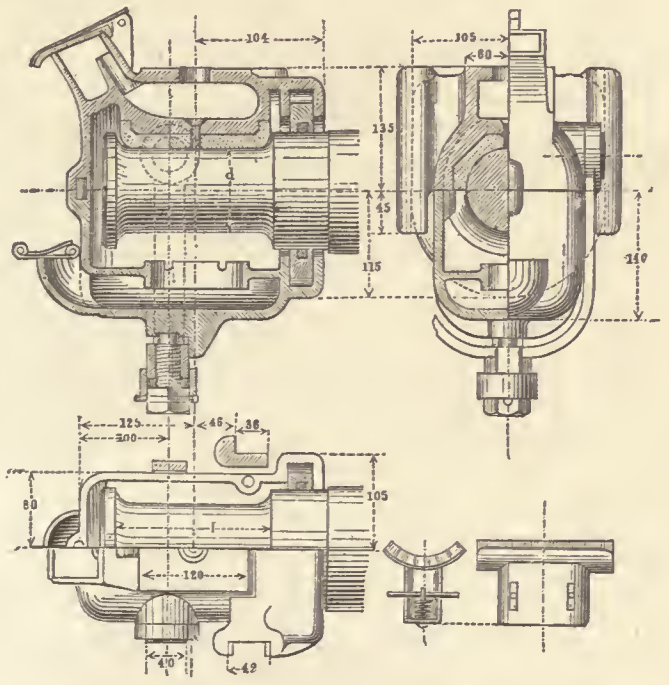

FIG. 324.

kept in position by three dowel pins. A wrought iron yoke holds the lower portion up to the body of the bearing by means of the bolt shown, the head being secured by the internal hexagonal socket shown.

The white metal lining is cast in the body of the box by being poured upon the journal. The inuer end of the journal is provided with a wooden dust guard packed with a ring of felt.

As will be seen, lubrication is provided both above and below. The upper chamber contains wicking and affords a means of prompt and copious lubricatiou in case the journal grows hot. The principal source of lubrication, however, is from below, the oil being wiped upon the journal by a brush, which is fed with oil by a wick reaching into the chamber below. The oil brush is shown, with its spring holders in the lower right hand corner of the illustration.

In order to permit the boxes to adjust themselves to the journal when the axle assumes an inclined position with regard to the bearing a certain amount of play is given, as is shown in the plan view, where the ledges cast upon the bearing are made parallel for a short distance and then diverge from below upward from a width of 34 mm. to 42 mm.

All the dimensions in Fig. 324 are in millimeters, as this is a standard Prussian railway journal box.

This construction is undoubtedly well adapted to meet the requirements, but it is a question whether the results might not be attained by simpler means.*

The second form of standard bearing of the Prussian Railways differs from the first mainly in the boxes. These are cast of bronze with semi-cylindrical projections on the track, which enter into corresponding recesses in the bearing, and permit the boxes to adjust themselves to the journal.

The guides for the bearings are given an amount of play similar to the previous form, and there is no change in the details of the lower portion.

Fig. 325 shows a form of American axle bearing. This is similar to the older pattern designed by Lightner † It is only arranged for lubrication from below and is designed so as to permit a box to be removed and replaced in the shortest possible time. The body is of very simple form and is cast in one piece and a large opening and lid renders it readily accessible from without. The box is made of bronze, and between it and the body of the bearing is a filling block, somewhat similar to that used in the bearing shown in Fig. 312, arranged so that its removal facilitates the changing of boxes.

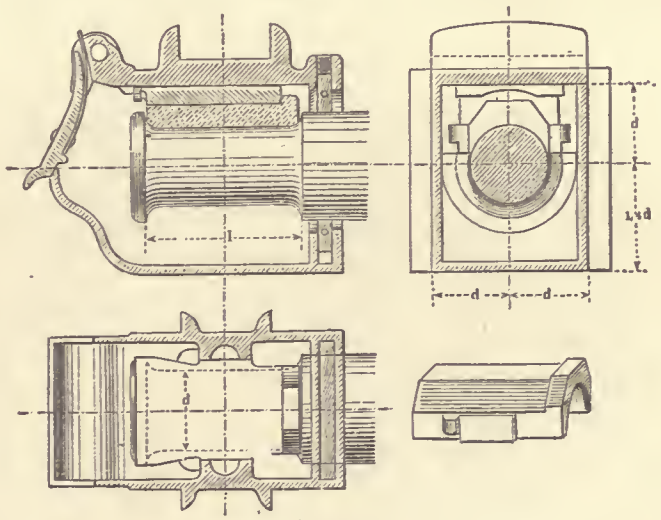

FIG. 325.

This filling block, which is sometimes rounded on top to provide adjustmeut, is held between two small projections, but can easily be removed when the pressure is removed by use of a lifting jack. The change of boxes can be effected in a few minutes.

A brush or pad for distributing the oil is not used, but instead the vacant space in the bearing is packed with waste, which feeds the oil to the journal. This form of journal box has proved very efficient in service.*

B. THRUST BEARINGS.

§ 118.

STEP BEARINGS.

In Fig. 326 is shown a form of step bearing for vertical shaft. The bearing piece or step proper is made with very obtuse

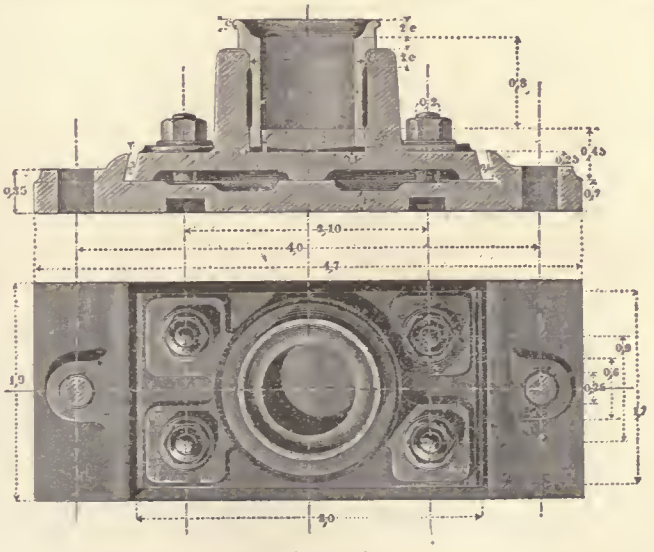

FIG. 326.

point on the under side in order that it may be able to adjust itself to the shaft. In order to provide for adjustment in the position of the bearing the bolt holes in the base plate are elongated in a cross-wise direction, while those in the bearing are elongated length-wise, thus permitting adjustment in any direction.

§ 119.

WALL STEP BEARINGS.

The following is a modified form of step bearings, and is intended to be used with the wall plate supported on a key beneath its lower edge; this key may be made = 0.8 d_1 deep, so

* This question of railway journal boxes is an instructive example of the importance of constructive simplicity as applied to machine elements. Since in the year 1877 in Prussia there were in use 315,000 axles or over 630,000 boxes. The cost of these represents an investment upon which every penny economized in construction foots up an important total.

† See Heusinger, Schmiervorrichtungen (*Lubrication*), Wiesbaden, 1864, p. 88.

* A standard axle and journal box were adopted in the United States in 1873, and at that time there were over 1,200,000 axles in service.

that by its removal the bearing may be taken from under the journal, without removing the shaft from its place.

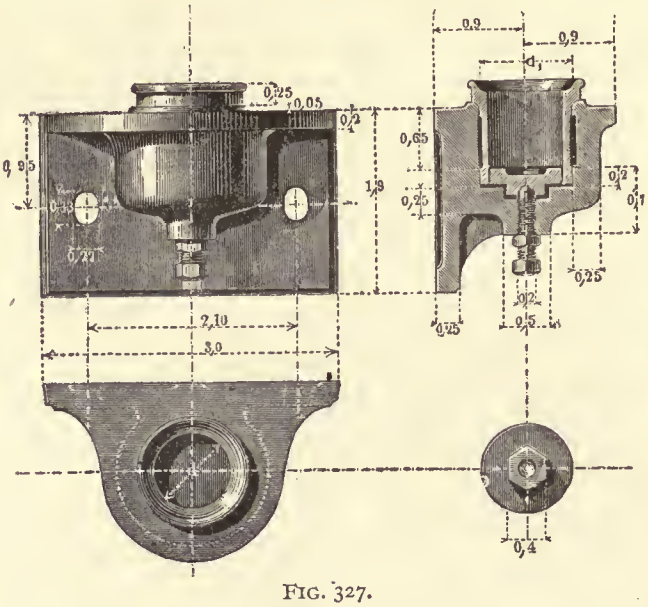

FIG. 327.

The recess in the step plate serves an oil chamber; endlong wear may be taken up very conveniently by the adjustment provided by the set screw.

§ 120.

INDEPENDENT STEP BEARINGS.

In many cases, as in examples by Belgian designers, the lower bearing of a vertical shaft is divided into two independent parts, a pure lateral bearing and a pure thrust bearing. For the lateral bearing may be used a pillow block or yoke bearing of one of the forms already described, while the vertical thrust is taken by a simple step quite close to the preceding bearing. This makes the step bearing readily accessible and also readily adjustable in the direction of wear.

The following example is selected from among a number of such bearings.

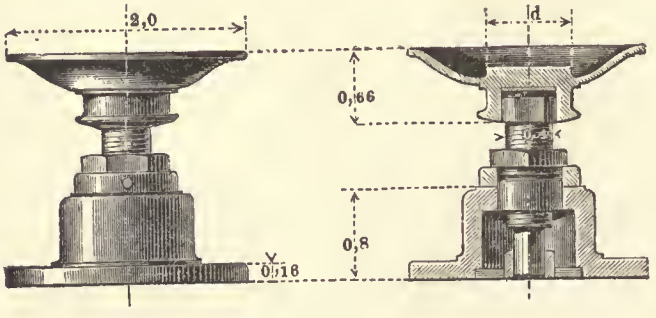

FIG. 328.

The step itself is made of brouze. This is carried on the bluntly coned head of the stout set screw, a steel plate being interposed, while the prismatic form of the screw head prevents rotation of the step. The screw itself is kept from moving by Penn's method within the bearing, and the whole is bolted down to a base plate. The modulus for the dimensions is the same as before. An application of this form is shown in § 126.

§ 121.

THRUST BEARINGS WITH WOODEN SURFACES.

For bearings which are operated wet, the use of Lignum Vitæ has been found to give the best results. The wood is inserted in a similar manner to that shown in § 117, the pieces being made in the form of plugs. In Fig. 329 is shown the step of a screw propeller shaft of this type. The plugs are inserted in a bronze plate, and the end of the shaft faced with bronze.

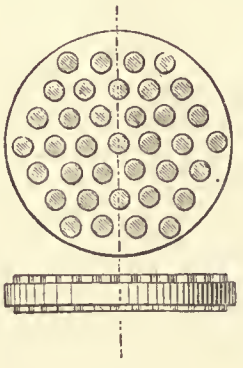

FIG. 329.

A bearing of this form on the "Orontes" had 37 plugs each 1¾" diameter, and on 50 H. P. nominal English gunboats the thrust plates have 7 plugs each 2" diameter. Both these examples are by James Watt & Co.

Collar bearings with surfaces of wood are often made; these should be always worked under water. Penn, to whom the introduction of such wooden bearing surfaces is mainly due, has especially used them in various bearings in the length of a screw propeller shaft, the lower half of the shaft running in a water trough. The usual construction of the thrust ring between the hub of the screw propeller and the stern post is shown in Fig. 330. *A* is the shaft with a bronze sleeve fitting into the wooden lining of the hole through the stern tube; *B* is the hub of the screw propeller; *C*, the thrust ring with its wooden plugs; *D* is the nozzle on the end of the stern tube showing the stiffening ribs which assist in receiving the thrust. The parts *B*, *C*, *D* and *E* are of bronze.

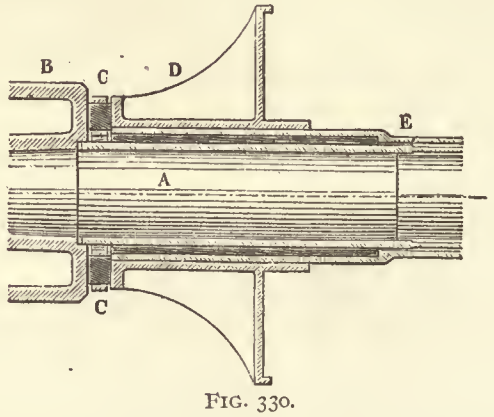

FIG. 330.

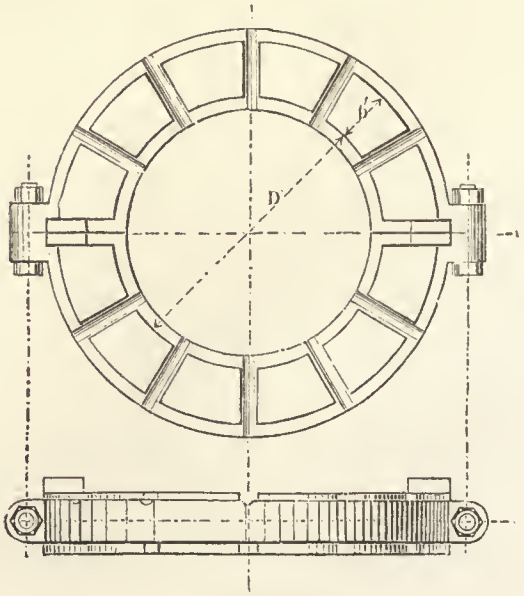

FIG. 331.

Fig. 331 shows a form of thrust ring used on the imperial steamships "Kaiser," "Friedrich Karl," "Preussen," "Vineta," "Freya," "Ariadne," "Nautilus" and "Cyklop." The ring is made in halves, and can readily be removed and replaced.

The two axial projections enter into recesses in the flange on the end of the tube, and prevent the thrust ring from revolving. The dimensions of the wooden bearing surfaces on the various ships above named are approximately as given in the following table :

	Kaiser.	Friedr. Karl.	Preussen.	Vineta.	Freya.	Ariadne.	Nautilus.	Cyklop
D'	28″	24¾″	26¾″	16½″	19¼″	19¼″	10¼″	8½
b'	6½″	1⅝″	7⅛″	3¼″	4¼″	4¼″	$2\frac{9}{16}$″	1⅝
Surface sq. ft.	4.078	0.740	2.629	1.188	1.840	1.840	0.476	0.238

In the "Wasp" the thrust ring is made with 6 sectors of 3 186 sq. ft. surface, in the "Leipzig" there are 80 small sectors with a total surface of 2.422 sq. ft. The use of such thrust rings filled with blocks of lignum vitæ has been most successful in vessels of the German navy, and the wear on the wood has been so slight that renewal is rarely necessary.

§ 122.

Multiple Collar Bearings.

For thrust bearings which are subjected to heavy service, the multiple collar bearing is most valuable. These are very generally used to receive the thrust of screw propellers, but are also used in other situations, as, for example, large turbines, also centrifugal machines of great size and weight, such as are used in sugar refineries. The forms which may be given to these bearings are quite varied; but in every case the most important consideration is the pressure to which the various surfaces are subjected.

For pillow blocks in which the shaft is made with several collars, the boxes may be cast in bronze with internal collars

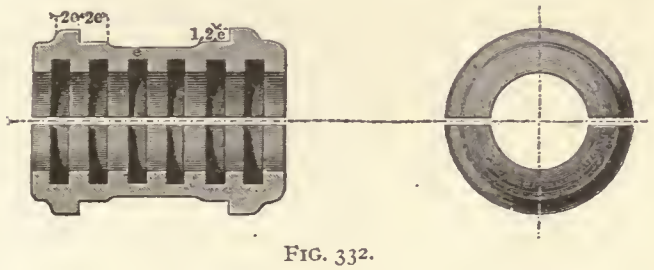

Fig. 332.

as shown in Fig. 332.* For larger dimensions, the boxes may be strengthened by ring shaped ribs, let into recesses in the cap and body of the bearing.

Example:—The thrust bearing on the "City of Richmond," built by Todd & MacGregor, of Glasgow, from the designs of Jaffrey,† has 12 rings; inside diameter, 19″; outside diameter, 23″; total length of the bearing, 43¾″. The boxes are strengthened by three ribs of ½″ depth by 4″ wide. The engines indicate 3340 H. P., and the speed of the vessel is about 1342 feet per minute

James Watt & Co. make the boxes free in the bearing, and support them by set screws at the ends, as shown in Fig. 333.

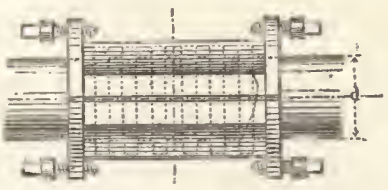

Fig. 333.

On the "Medusa" and "Triton" four set screws are used in each flange, the shaft being 7″ diameter, with six rings. In the "Jason," by the same firm, there are six set screws in each flange, the shaft being 12″ diameter, with eight rings.‡

Boxes of cast iron lined with white metal are sometimes used by various makers, as, for example, in the "Mooltan" by Day & Co., in which the shaft is 13¼″ diameter, and has twelve rings. The design shown in Fig. 334, which is a French pattern, uses an adjustable bearing lined with white metal.

In Fig. 335 is shown a form of thrust bearing in which the rings are made of bronze separately, and fitted to the body and cap. This form is the design of Ravenhill & Hodgson. Especially to be noted is the arrangement of bolts. These are in two sets, the first securing the body of the bearing to the sole plate, and the second being the cap bolts. The ledge or tongue which is let into the sole plate is arranged with a space as shown on the left, in which a key is fitted to provide for the take-up of the wear upon the rings. The cross section in upper right hand portion of the illustration shows the construction and application of the bronze rings. The arrangement provides for a constant distribution of grease, thus preventing the rusting of the journal by the application of water for cooling.

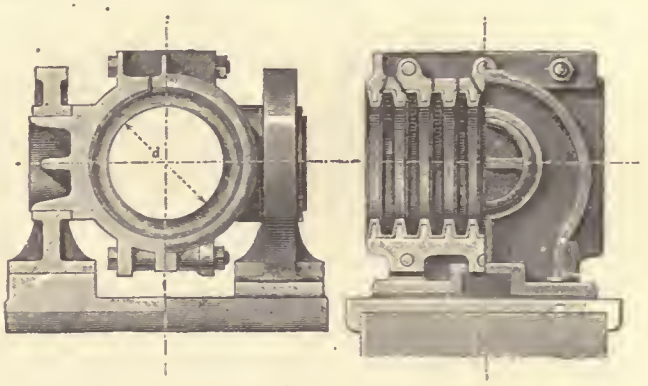

Fig. 334.

In Figs. 336 and 337 is shown a thrust bearing by Penn, as used on the "Kaiser." Here the bearing surfaces are made in separate rings of still simpler form than the preceding. These

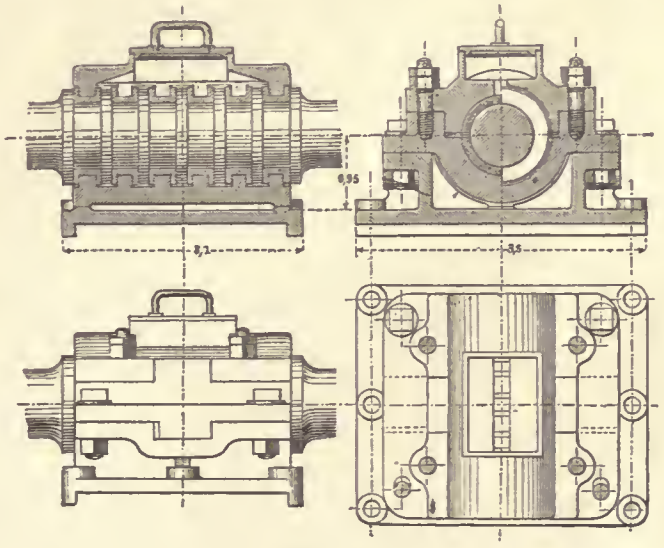

Fig. 335.

rings, which are made of bronze, are in halves for convenience of construction. In the "Kaiser" d is equal to 18½″, and there are eight rings on the shaft and in the bearing. The six bolts

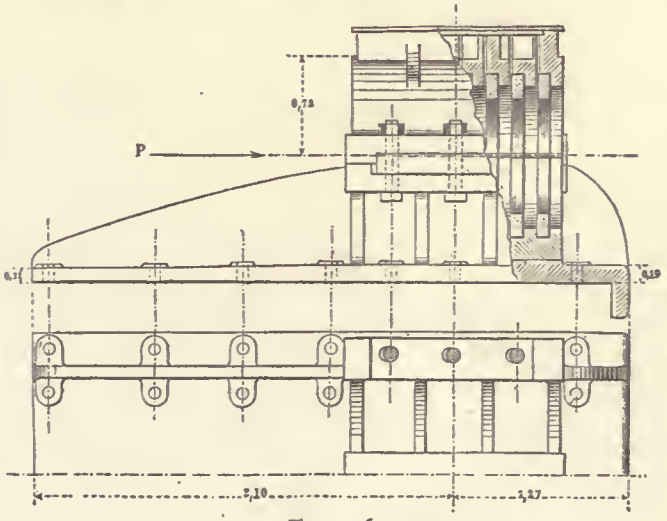

Fig. 336.

are arranged so as to act both as cap bolts and fastenings for the bearing. The adjustment for wear is similar to the preceding case. The dimensions are based on the same modulus as already given, viz.: $d_1 = 1.15\,d + 0.4''$.

A most noticeable form of thrust bearing is that of Maudslay,

* See Armengaud, Vignole des Mécaniciens. Pl. 13, Fig. 32.
† Engineering, May, 1875, p. 493. ‡ See Burgh.

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shown in Figs. 338 to 340, as used on the "Elizabeth." For each collar on the shaft there is provided a separate ring and support, with means for ample lubrication. The bearing rings are made of horse shoe form, and are of cast iron lined with white metal. The collars on the shaft dip into an oil trough. They are also provided with oil cups above, so that as in the case of the car axle box previously described, lubrication is supplied both above and below. Each ring may be adjusted by its own set screws, or all can be adjusted together. The proportions are all based upon the previous modulus, $d_1 = 1.15\,d + 0.4''$, and the shape and dimensions give an excellent appearance. In the "Elizabeth" the shaft is 12½" diameter.

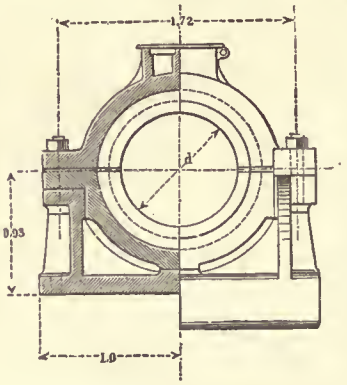

FIG. 337.

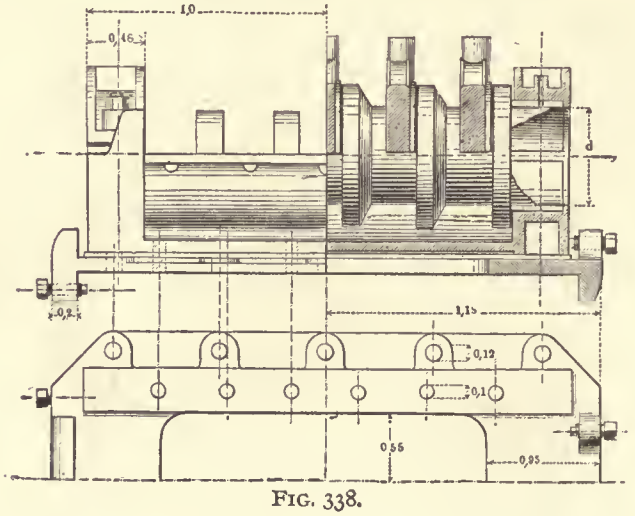

FIG. 338.

§ 123.

EXAMPLES OF THRUST BEARINGS.

The following examples are taken from twelve of the most important vessels of the German navy, the data being furnished to the author with the approval and authority of the Chief of Admiralty. The power and speed of the engines and the velocity of the vessel are all most important data, and are obtained from official tests. From these may be obtained, as

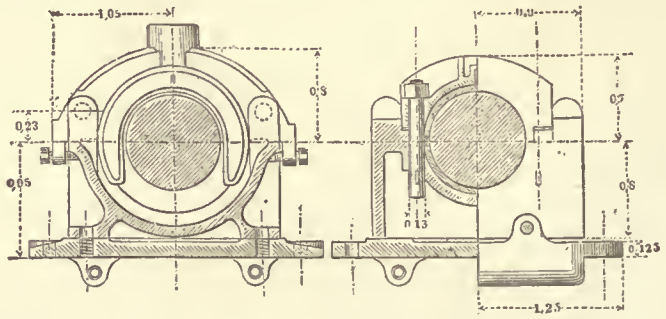

FIG. 339.

in § 100, the maximum pressure upon the thrust bearing surfaces. It is important to observe that in only two cases out of the twelve was a thrust ring used between the stern post and

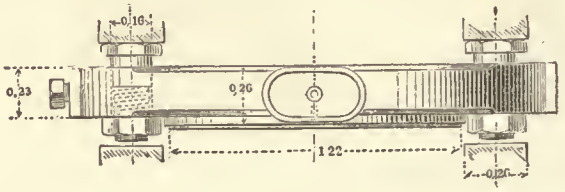

FIG. 340.

propeller hub. The elasticity of the hull of the ship may sometimes cause the entire force to be thrown on the thrust bearing, and at other times much may be taken by the thrust ring. The data given in the table will also be found valuable for other purposes.

EXAMPLES OF THRUST BEARINGS.

No.	Name of Vessel	Builder of Engines.	Indicated Horse Power.	Speed of Vessel, Feet per Minute.	Dia. of Propeller Shaft.	Revolutions per Minute.	No. of Collars and Bearings.	Material of Bearings.	Bearing Surface in Square Feet.	External Dia. of Thrust Collars.	Bottom Diameter between Collars.	Lubricant.	Results in Operation.	Remarks.
1	Armored Frigate König Wilhelm.	Mandslay Sons & Field, London.	8325	1491	18"	63.86	6	Anti-mony.	8.467	24¼"	18"	Oil Bearing cooled with Water.	Worked well.	No thrust ring in stern post.
2	Armored Frigate Kaiser.	John Penn & Sons, Greenwich.	7803.3	1457	18"	77.00	8	Brouze.	7.104	23"	18 9/16"	Ditto.	Ran warm before the thrust ring was applied.	Thrust ring in stern post, § 121.
3	Armored Frigate Friedrich Karl.	Societé des Forges et Chantiers de la Mediterranée, Marseilles.	3503	1328	15"	61.82	11=18⅞" 1=20½"	White Metal.	8.004	18⅞"	15½"	Ditto.	Made without thrust ring and ran warm. Since its application, works well.	Ditto.
4	Armored Frigate Preussen.	Stettiner Maschinenbau Aktiengesellschaft Vulkan in Bredow bei Stettin.	4386.7	1408	16¾"	64.5	8	Bronze.	5.371	20⅛"	16¼"	Ditto.	Worked well	Ditto.
5	Decked Corvette Leipzig.	Ditto	3519 3	1437	16"	72.4	8	Bronze.	4.816	19⅛"	16"	Ditto.	Ditto.	Ditto.
6	Decked Corvette Vineta.	John Penn & Sons, Greenwich.	1359.3	1120	10½"	67.9	6	Bronze.	1.489	12⅜"	10⅜"	Ditto.	Ditto.	Ditto.
7	Decked Corvette Freya.	Märkisch-Schlesische Maschinenbau, und Hütten Aktiengesellschaft.	2598.8	1557	12¼"	82.52	8	Bronze.	2.528	15"	12¼"	Ditto.	Ditto.	Ditto.
8	Decked Corvette Ariadne.	Ditto.	1726.9	1282	11⅝"	80.24	7	Brouze.	3.391	14⅞"	11⅝"	Ditto.	Ran warmed first, afterwards worked well.	Ditto.
9	Decked Corvette Augusta.	Mazeline & Co., Havre.	1127	1245	11"	62.09	11	Anti-mony.	5.177	14 9/16"	11¼"	Ditto.	Worked well.	No thrust ring.
10	Gunboat Nautilus.	Möller & Hollberg in Grabow.	504.2	1047	7¼"	109.30	6	Anti-mony.	1.159	9¾"	7¾"	Ditto.	Ditto.	Fitted with thrust ring.
11	Gunboat Cyklop.	Stettiner Maschinenbau Aktiengesellschaft Vulkan in Bredow bei Stettin.	245.4	894	5⅜"	143.89	4	Lignum Vitæ.	0.496	7¼"	5⅜"	Ditto.	Ditto.	Ditto.
12	ArmoredGunboat Wespe.	Aktiengesellschaft, Weser in Bremen.	799.7	1054	6¾"	138.85	1=10⅞" 8=9⅜"	Bronze.	1.728	9⅜"	7½"	Ditto.	Ditto.	Ditto.

CHAPTER VII.

SUPPORTS FOR BEARINGS.

§ 124.

GENERAL CONSIDERATIONS.

The function of a support for one or more bearings is to hold them in a firm and definite position with regard to the frame or other parts of a machine. Such supports are nearly always made of cast iron, and in the following treatment of the subject this material is the only one considered.

Simple supports are those which are intended to hold but one bearing, in distinction from those supports which are arranged to receive several. In both cases the following considerations should be observed as closely as may be, when, as is

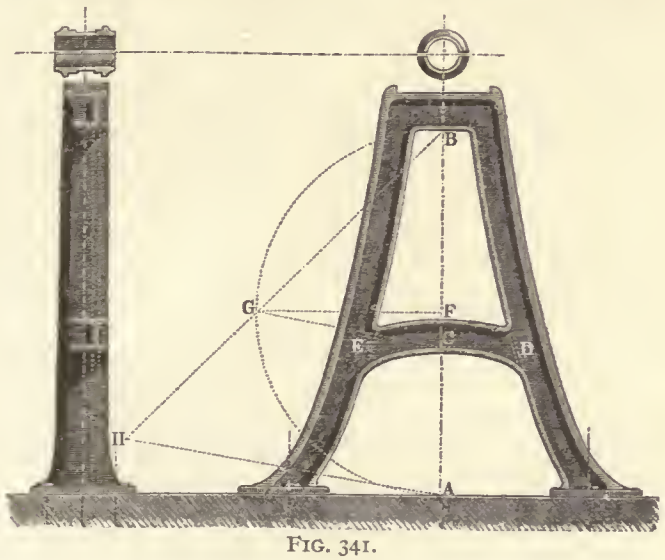

FIG. 341.

usually the case, the shafts which the bearings carry are fitted with gear wheels which should be near the bearings.

1. The bearings should be as near to the hubs of the gear wheels as practicable.

2. The pressure upon the journal should, in no case, act in the direction of the joint between the boxes.

3. The support for the boxes should be so arranged as to allow the easy removal of shafts and gear wheels.

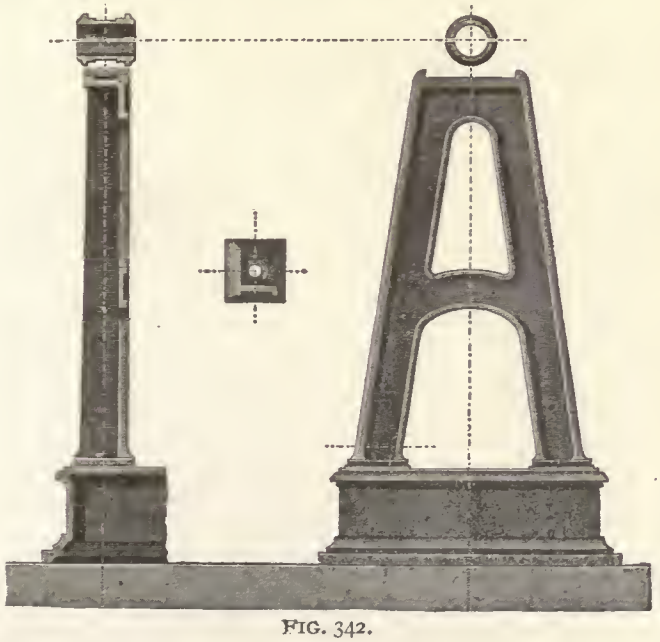

FIG. 342.

4. The number of bearing surfaces should be made as few as possible, and all finished surfaces should be capable of being finished at one setting on the planing machine.

5. Whenever possible, and especially in situations of difficult access, the bearings should be so disposed that the boxes may be removed and renewed without involving the removal of the shafts from their position.

§ 125.

SIMPLE SUPPORTS.

A simple support for a single pillow block is shown in Fig. 341. It is intended for a bearing such as is shown in § 107; hence the upper portion is made correspondingly narrow. The two legs which form the main portions are reinforced by a cross girth, DE. The position of the points D and E may always be well placed by observing the following method: Taking the total height AB as a diameter, draw from the centre F a semi-circle AGB; take the middle point of the arc AGB at G; join BG, and prolong it, making $GH = AF$; then join H to A, and draw GC parallel to HA, and AC is the height from the base to the cross girth. The dimensions of the various parts are dependent upon the pressure on the bearing, and must usually be governed by the dimensions of the pillow block and by the judgment of the designer. In order to meet the requirements of Rule 5 of the preceding section, there should be under the pillow block a removable plate, which may be given a thickness of $0.3\,d_1$.

Fig. 342 is a similar form of support suitable for heavier dimensions.

Fig. 343 is a support for a wall bearing. This is arranged to be built into the wall, and forms an opening through which the shaft can pass, and resembling what a builder calls a bull's eye window. The pressure of the journal is received by the

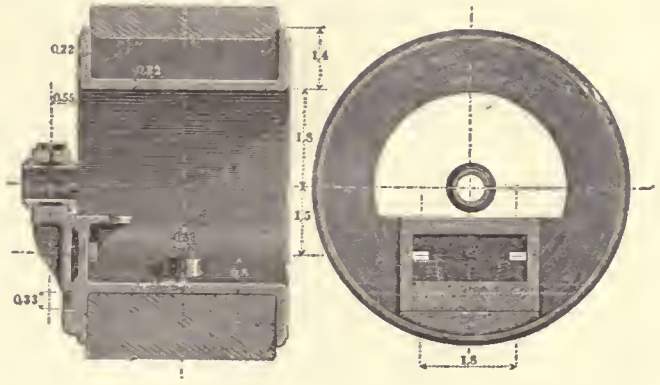

FIG. 343.

bracket bearing, which is supported on the key beneath, and can be removed without disturbing the shaft. One point which should not be overlooked is the bearing plate in the wall, shown in tangential dotted lines below the cylinder. The dimensions in the illustration are based on the modulus d_1 of the bearing.

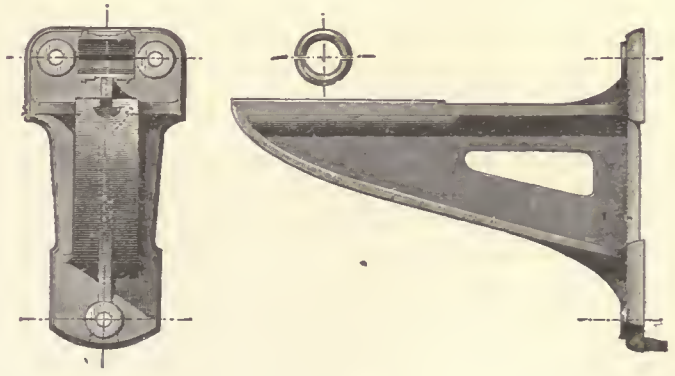

FIG. 344.

A wall bracket support is shown in Fig. 344. This is intended to carry a pillow block, and the T slot for the bolt heads enables the distance of the bearing from the wall to be adjusted. This form may be used for bearings of various sizes. A simpler and lighter form of bracket is shown in Fig. 345. This is merely an arm attached to a wall and adapted for a horizontal shaft.

Frequently the joint between the base of a bearing support and its foundation is made with cement. When this is done, the base is adjusted to its position, resting upon wedges, and the joint being closed with clay, the liquid cement is run in; this

will hardeu iu a few days so that the wedges may be driven out and the bolts fully tightened.

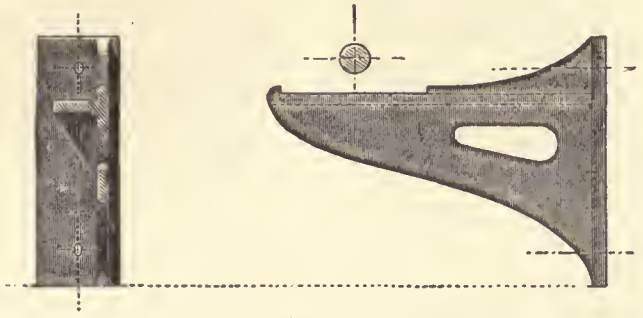

FIG. 345.

§ 126.

MULTIPLE SUPPORTS FOR BEARINGS.

Fig. 346 represents a bridge support. The vertical shaft *A B* comes from below, as for example, from a turbiue, aud transmits its motion to the horizontal shaft *C D*. The journal pressure acts at *E*, at right angles to the plane of the two shafts,

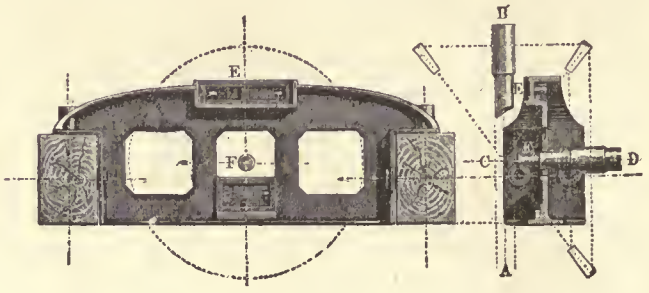

FIG. 346.

and at *F* it acts in an inclined direction downward, both from the pressure of the gear teeth, and also because of the weight of the wheels and shafts. These pressures are best received at *E*, by a yoke bearing as shown in § 113, and at *F*, by a bracket bearing, § 114, supported on an adjusting key.

Fig. 347 shows a support for a step-bearing. Here the horizontal shaft *A B* runs in a bracket bearing at *C*, and transmits motion to a vertical shaft which is supported at *D*, by a step-

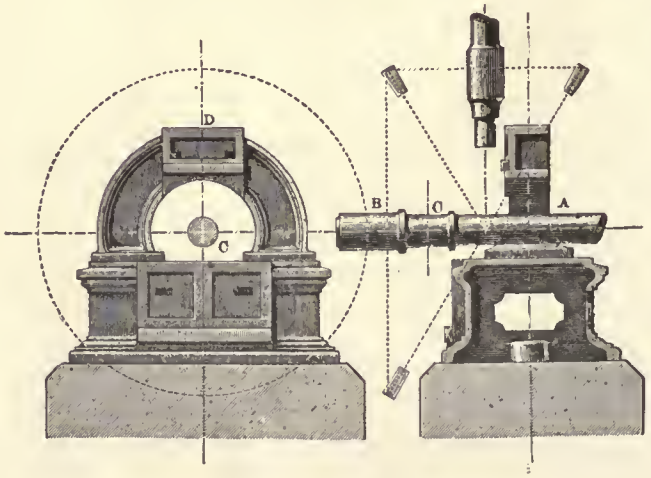

FIG. 347.

bearing, § 119. The latter, as the illustration partially shows, is carried on an adjusting key in such a manner that it can readily be removed from below. The bridge which carries the step-bearing is bolted to the box-shaped base and the nuts for the foundation bolts are placed inside the base.

Another form for similar service is shown in Fig. 348. The shaft *A C*, for the large gear-wheel terminates in the support and is provided with a small bracket bearing at *C*. On account of the position of the wheel, this is not very accessible. The bearings for the vertical shaft *D E F*, are intended to be of the form described in § 120, a yoke bearing being fitted into a space cast in the upper part of the frame at *E*, while an independent

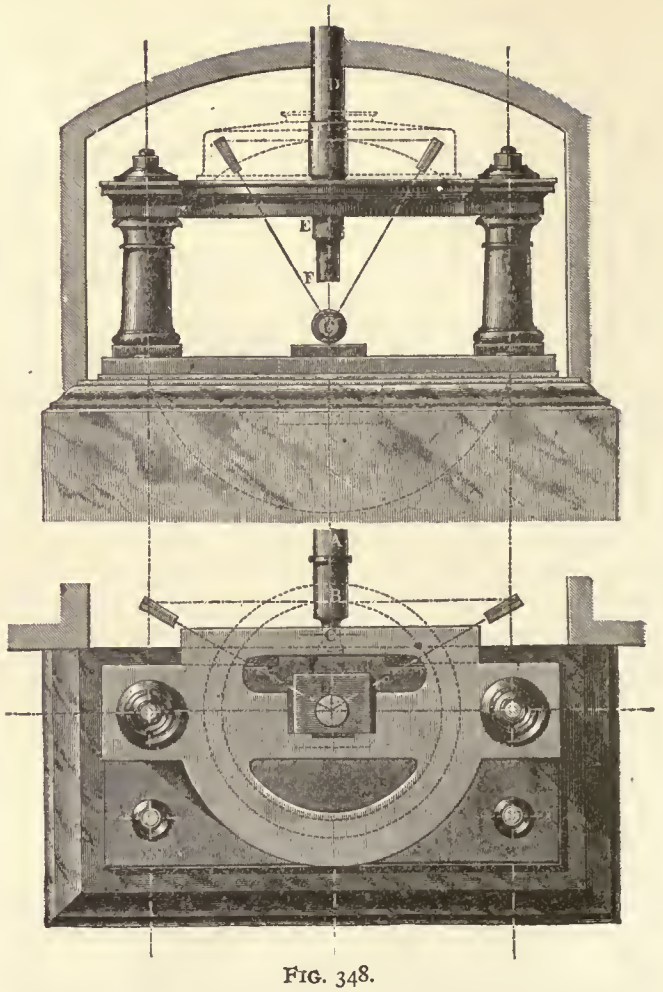

FIG. 348.

step at *F* is used similar to that shown in Fig. 328. The upper part of the frame is made circular in shape, so that a cast-iron cover may be placed over the pinion, as shown in the dotted lines. The base plate is held dowu to the stoue foundation by four bolts; two of the bolts pass through the columns, as shown in the illustrations, and so bind the two plates firmly together. The plan view shows how the columns are keyed iuto the entablature. The base of the columns are let into the base plate as shown in Fig. 349, and an iron cemeut is used.

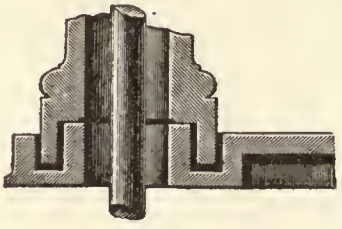

FIG. 349.

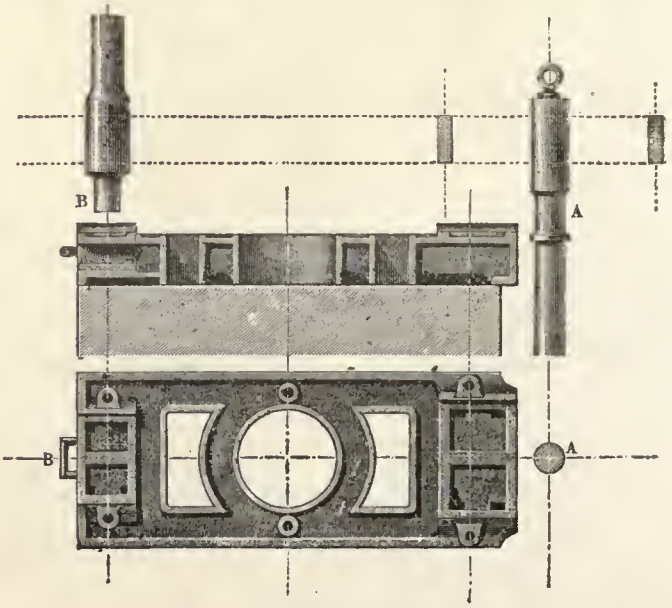

FIG. 350.

In Fig. 350 is shown a support for two vertical shafts, *A* and *B*, the motion being transmitted from one to the other by means of spur gears. The shaft *A*, for instance, may be that of a turbine wheel, and *B*, the main driving shaft of the mill.* At *A* there is a bracket bearing such as shown in Fig. 314, and at *B* a step bearing, with a removable block beneath it, so that the bearing may be removed or examined without removing the wheel or shaft.

Fig. 351 shows a frame for a vertical shaft *A B*, which transmits its motion to a horizontal shaft *D E*. At *C* is a yoke bearing and at *E* a bracket-bearing. The horizontal bevel gear is

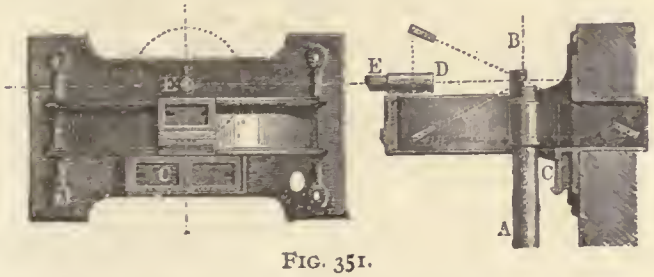

FIG. 351.

inclosed in the semi-circular frame, so that a cover may easily be adapted, as in the previous case. The removal of the vertical shaft is not quite so convenient in this form as in some others, but presents no serious difficulty. In some cases the lower part of the frame is entirely closed and the shaft inclosed in a sort of pilaster, to avoid accidents.

For a shaft running parallel to a wall, as at *A B*. Fig. 352, and transmitting its motion to one *D E*, at right angles, the frame shown in the illustration is suitable. The bearing for the

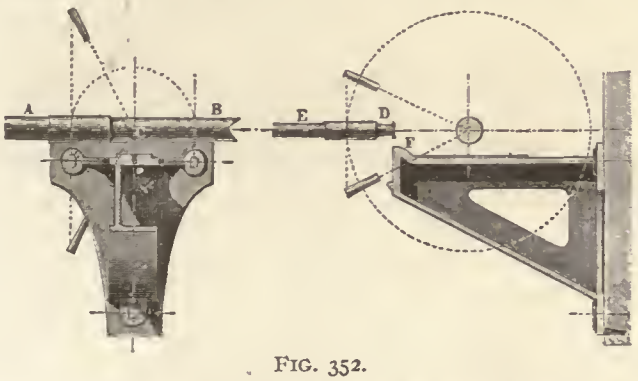

FIG. 352.

main shaft at *C* may be a pillow-block, while a bracket bearing is suitable at *F*. The distance of the pillow-block from the wall is adjustable (as in Fig. 344). If the gears are equal in size the form may be as shown in plan in Fig. 353. In this case the journal at *C* runs in a bracket bearing. If the construction is

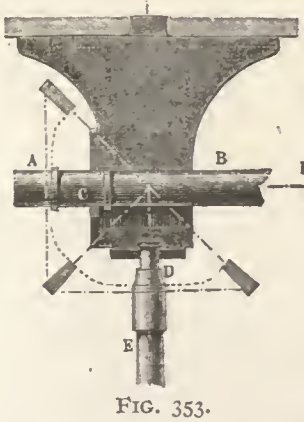

FIG. 353.

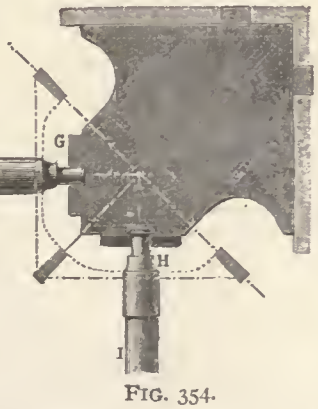

FIG. 354.

intended to fit in the corner of a building, the frame is modified as shown in Fig. 354; the bearings at *G* and *H* are then the same. Both these forms are shown in Fig. 355 and 356 in pseudo-perspective.

Very often a main overhead driving shaft is required to transmit motion both to horizontal and vertical shafts from one point, and the combination of Fig. 357 is an example. Here the frame-work is made a portion of one of the columns of the building and is really simple in construction; at *A* should be

*Such a frame is used in a spinning-mill at Chur, the frame and one-half of the large gear-wheel being in an archway in the large end wall of the building.

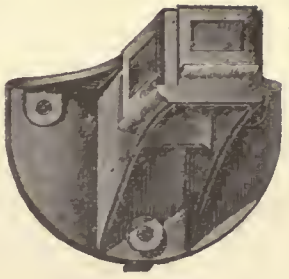

FIG. 355.

FIG. 356.

used a bracket like Fig. 313; at *E* and *F*, wall brackets like Fig. 310, and at *C*, a step bearing like Fig. 327.

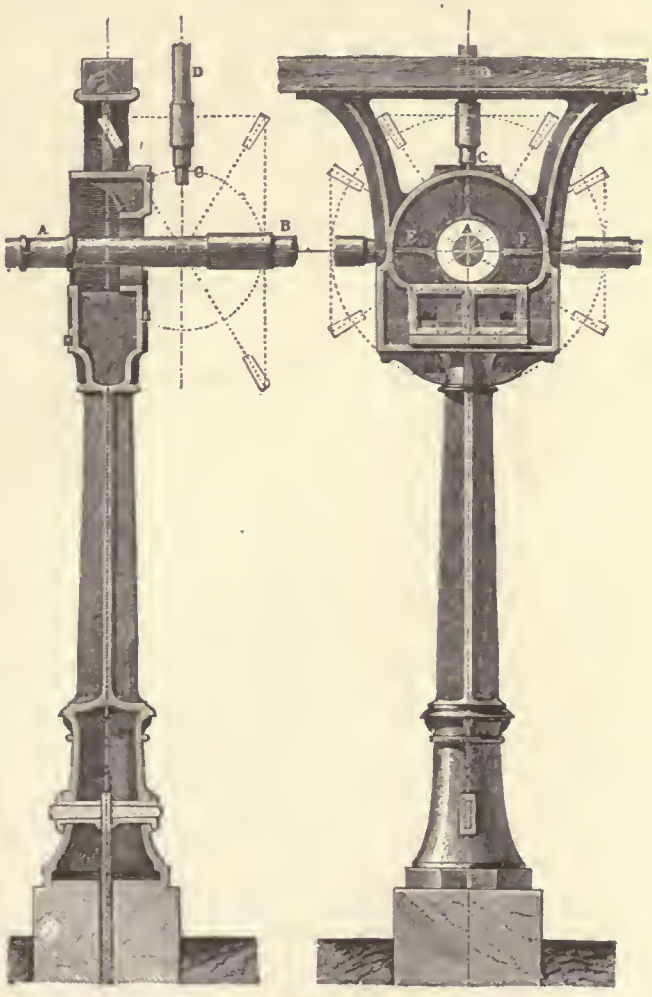

FIG. 357.

Fig. 358 shows a wall frame for four bearings. A horizontal shaft *A B*, is to transmit motion to the vertical shaft *C D*, and two horizontal shafts *E* and *F*, by means of bevel gears. At *B*

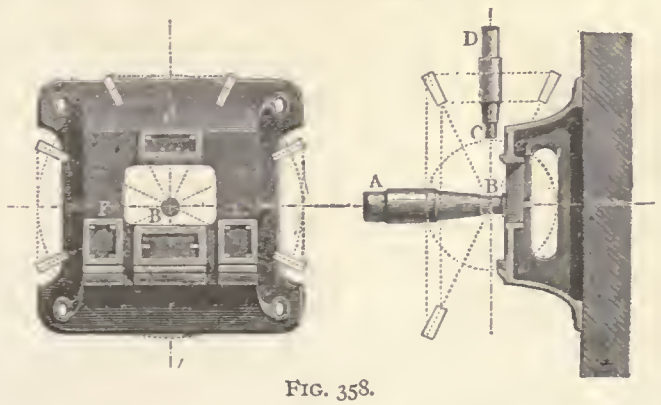

FIG. 358.

is a bracket, and at *C* a step bracket, as in Fig. 327, while the bearings at *E* and *F* are wall-brackets, like Fig. 310.

By a proper choice of journal diameters and clearances the seats for the four bearings may be brought into one plane, and the other conditions of § 124 readily complied with.

An examination of the fundamental principles of construction of supports for bearings will show that all forms may be represented by a rigid piece adapted to hold in fixed relation two or more revolving bodies, in such manner as to permit the application of the various details of construction such as boxes, caps, bolts, etc. It is often desirable to sketch out in the first place a general scheme of the construction in order that the direction and manner of resistances and arrangement of parts may be examined more readily. The frame shown in Fig. 350 is similar to the elementary shape of Fig. 359, which resembles a simple connecting rod; which indeed the base plate really is, the

FIG. 359.

variations being due to the especial conditions and not to any fundamental difference. The bridge frame, Fig. 346, is in elementary form Fig. 360. The step supports of Fig. 347 and 348 may be shown in principle either in Figs. 360 or 361, since in these elementary schemes a bearing may be shown either by

FIG. 360.

FIG. 361.

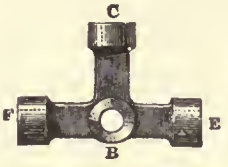

FIG. 362.

the journals or the reverse. The four-fold bearing support just described may be sketched in Fig. 362.

To show how these elementary sketches may serve, the following application to one of Lémielle's ventilators will indicate.

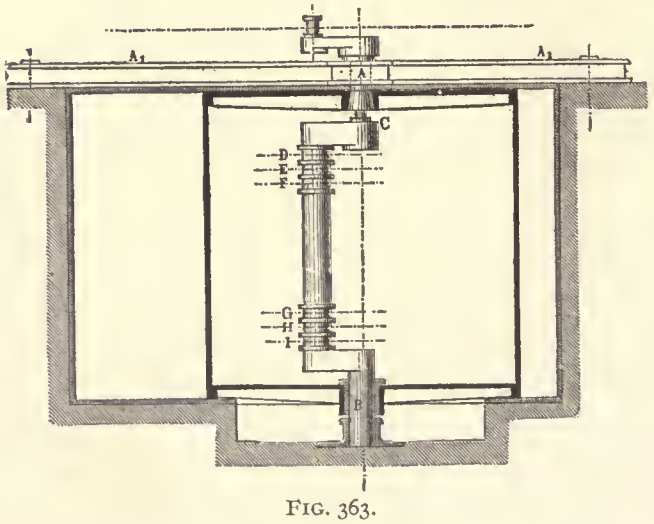

FIG. 363.

Here, Fig. 363, nine bearings are to be supported. Three of these are for the drum, which is fast to the driving crank; it is carried by the two neck bearings at *A* and *B*, and the thrust bearing at *C*. The six bearings at *D*, *E*, *F*, and *G*, *H*, *I*, are for the rods of the buckets; the supports for all of these are then the beams A_1 A_1, the masonry, and the cranked rod *B*, *I*, *D*, *C*.

§ 127.

CALCULATIONS FOR IRON COLUMNS.

The calculation of the proportions of iron columns often becomes necessary in machine construction, for besides serving merely as portions of building construction they are often combined with machine details, and also enter into the design of framework as supports and similar relations. Their consideration in this place is therefore appropriate.

Iron columns are generally considered as being subjected to stresses of compression, and, also within certain limits, to bending stresses; it is therefore important to allow sufficient latitude in the calculations to provide for variations in the manner of application of the load.

The various methods of application may be treated as indicated in the following illustrations, Fig. 364, which show the three Cases II, III, and IV, of § 16. The first shows a column

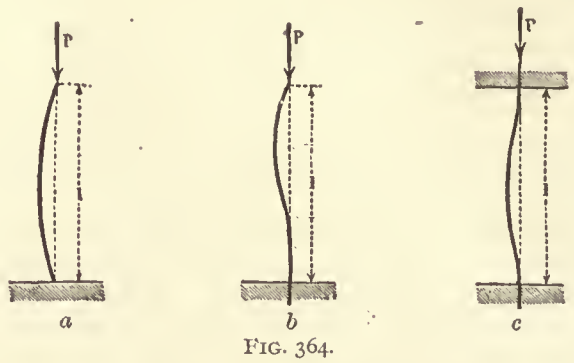

FIG. 364.

hinged at both ends, the second is hinged at one end, while the third is rigidly held at both ends. The breaking loads of the respective forms are:

$$\overset{a}{\pi^2 \frac{JE}{l^2}} \qquad \overset{b}{2\pi^2 \frac{JE}{l^2}} \qquad \overset{c}{4\pi^2 \frac{JE}{l^2}}$$

the columns being of prismatic form and of a height l; J being the moment of inertia of the cross section and E the modulus of elasticity of the material; l being taken in inches. As already stated in § 16, experiment has shown that columns whose ends are faced off square and true fall under Case *c*, even though not held at the ends. If, therefore, a load smaller than that indicated for Case *a*, be chosen for all cases, security will be assured, even should both ends of the column be jointed.*

We may therefore take for the greatest permissible load in the direction of the axis:

$$P = 0.4\,\pi^2 \frac{JE}{l^2} = 3.94 \frac{JE}{l^2} \quad \ldots\ldots \quad (109)$$

If d is the diameter for a solid circular cross section, we have for cast iron, in which $E = 14{,}200{,}000$.

$$P = 2{,}750{,}000 \frac{d^4}{l^2}, \quad d = 0.0245 \sqrt{l} \sqrt[4]{P} \quad \ldots \quad (110)$$

For Wrought Iron, $E = 28{,}400{,}000$. This gives

$$P = 5{,}500{,}000 \frac{d^4}{l^2}, \quad d = 0.0206 \sqrt{l} \sqrt[4]{P} \quad \ldots \quad (111)$$

Example 1. For a load $P = 33{,}000$ lbs., a solid cast iron column 157.5 in. high, the diameter $d = 0.0245 \sqrt{l} \sqrt[4]{P} = 4.15''$, or about $4\frac{5}{32}''$. Under the same conditions a wrought iron column would be $3\frac{1}{2}''$ diameter.

An inspection of the formula shows that the shorter l becomes, the smaller is the value of d. The cross section must, however, never be allowed to become so small that the limit of permissible stress shall be passed.

In order that the stress upon the cross section shall not exceed 8500 lbs. for either cast or wrought iron (their modulus for compression in either case being 21,300 lbs.), d should in no case be taken as less than

$$\left.\begin{array}{c} d = 0.0122 \sqrt{P} \\ \text{or the load should not be greater than} \\ P = 6397\, d^2 \end{array}\right\} \quad \ldots \quad (112)$$

The following table for round solid cast iron posts is calculated from formulas (110) and (112), and gives the loads which may safely be put upon columns of the respective heights and diameters given.

The quantities marked with an asterisk are calculated from formula (112) and are a marked reduction upon the loads otherwise obtained.

---

* Drewitz has tested cast iron columns with a load equal to $\pi^2 \frac{JE}{l^2}$ without observing perceptible alteration. Erbkam's Banzeibung, V., p. 534.

STRENGTH OF SOLID CAST IRON COLUMNS.

d	l=8 ft.	10 ft.	12 ft.	14 ft.	16 ft.	18 ft.
1 in.	297	191	132	97	75	59
1½	1,504	994	671	493	377	298
2	4,753	3,055	2,122	1,559	1,193	942
2½	11,600	7,460	5,180	3,806	2,914	2,302
3	24,060	15,830	10,740	7,892	6,043	4,774
3½	44,470	28,660	19,900	14,620	11,200	8,845
4	76,040	48,890	33,950	24,950	19,100	15,090
4½	121,800	78,310	54,380	39,950	30,590	24,170
5	154,400*	119,300	82,890	61,460	47,060	37,180
5½	186,900*	174,700	121,400	89,160	68,260	53,930
6	222,400*	222,400*	171,900	126,250	96,680	76,400

Hollow Columns.—Cast iron columns are generally made hollow. The dimensions in this case may readily be determined from the formulæ for solid columns.

If the external diameter is d_0, the internal diameter d_1, and the diameter of a solid column of equal strength, d, we have

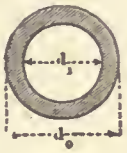

FIG. 365.

$$\frac{d_0}{d} = \frac{1}{\sqrt[4]{1-\left(\frac{d_1}{d_0}\right)^4}} \quad \ldots\ldots\ldots\ldots (113)$$

The ratio of iuternal to external diameter $\frac{d_1}{d_0} = \psi$ is conveniently made 0.7 to 0.8. We have for:

ψ =	0.5	0.6	0.7	0.75	0.8	0.85	0.9	0.95
$\frac{d_0}{d}$ =	1.016	1.035	1.07	1.10	1.14	1.20	1.31	1.52

The limits of stress fall within the formula for compression and the above results are close approximations. It is to be observed that d_0 should in no case be taken less than:

$$d_0 = \frac{0.0122\sqrt{P}}{\sqrt{1-\psi^2}} \quad \ldots\ldots (114)$$

or the load greater than

$$P = 6397\, d_0^2\,(1-\psi^2)$$

We have for:

$\psi = \frac{d_1}{d_0}$ =	0.5	0.6	0.7	0.75	0.8	0.85	0.9	0.95
$1-\psi^2$ =	0.75	0.64	0.51	0.44	0.36	0.28	0.19	0.10
$\frac{1}{\sqrt{1-\psi^2}}$ =	1.15	1.25	1.40	1.51	1.67	1.89	2.29	3.20

Example 2. The solid column of the preceding example to support a load of 33,000 pounds was found to be 4.15" diameter, and for a ratio of diameters of 0.8 for a hollow column for the same load we have $d_0 = 1.14 \times 4.15 = 4.73$", say 4¾", and the internal diameter $d_1 = 0.8 \times 4.73 = 3.78$" say 3¾", giving a thickness of metal of ½ inch. Substituting these values in (114) we have for the greatest safe load, $P = 6397 \times (4.73)^2 \times 0.36 = 51,530$ lbs. This shows the dimensions obtained above to be amply strong, *if the walls of the column are cast of uniform thickness.* If the ratio $\frac{d_1}{d_0}$ had been taken as 0.7 we should have obtained, from (114) $d_0 = 4.44$", and $d_1 = 3.10$, giving a thickness of metal of 0.67".

In practice it is often necessary to work to a given external diameter d_0, in which case, for cast iron, the internal diameter d_1, may be found from:

$$d_1 = d_0\sqrt[4]{1-0.000{,}000{,}36\,\frac{Pl^2}{d^4}} \quad \ldots\ldots (115)$$

and the load

$$P = 2{,}750{,}000\,\frac{d_0^4 - d_1^4}{l^2}$$

in which P is the difference in supporting capacity between two solid columns of the diameters d_0 and d_1 respectively.

It is necessary also in this case to observe that P should not be greater thau $P = 6397\,(d_0^2 - d_1^2)$ and d_1 not greater than

$$d_1 = d_0\sqrt{1-0.00015\,\frac{P}{d_0^2}} \quad \ldots\ldots (116)$$

in order that satisfactory castings may be produced.

Example 3. In a barracks in Berlin are hollow columns of 142 inches height, bearing loads of 37,180 lbs. These are made of diameter $d_0 = 6\frac{3}{16}$" According to (115) this should give the internal diameter:

$$d_1 = 6.1875\sqrt[4]{1-0.00000036\,\frac{37{,}180\times 142^2}{(6.1875)^4}} = 5.88"$$

According to (116) we have:

$$d_1 = 6.1875\sqrt{1-0.00015\,\frac{37{,}180}{(6.1875)^2}} = 5.71"$$

This would give a thickness of metal of about ¼". The empirical thickness for such a column is about ⅝", and the actual internal diameter was 4⅞".

Example 4. A cast iron column of 185 inches height and 9¼ inches outside diameter has to bear a load of 275,000 lbs., and was made with an internal diameter of 6¼ inches. According to (115), for direct resistance to thrust we get:

$$d_1 = 9.25\sqrt[4]{1-0.00000036\,\frac{275{,}000\times(185)^2}{(9.25)^4}} = 7.92"$$

but according to (116):

$$d_1 = 9.25\sqrt{1-0.00015\,\frac{275{,}000}{(9.25)^2}} = 6.65"$$

or very near the actual dimensions.

These examples show how important it is, to take all the conditions into account, in order to avoid errors, and a careful examination of the circumstances attending each case should always be considered.

Fluted Columns.—The cruciform section may serve as an example of such columns. The thickness and breadth, b and h, of the ribs may be determined by comparison with the diameter d, of an equivalent round solid column by making:

FIG. 366.

$$\frac{b}{h} = \frac{3\pi}{16}\left(\frac{d}{h}\right)^3 = 0.59\left(\frac{d}{h}\right)^3 \quad \ldots (117)$$

from which the approximate thickness b, for any breadth h, may be obtained. In order to keep within safe limits the cross section should not be less than:

$$bh = \frac{P}{17000} \quad \ldots\ldots (118)$$

or the load more than

$$P = 17000\,bh$$

Example 5. To suhstitute a cruciform column for the solid one of Example 1, we may take $h = 1.5d = 1.5 \times 4.15 = 6\ 225$".

We then have from (117).

$$b = 4.15 \times 0.59\left(\frac{4.15}{6.225}\right)^3 = 0.72"$$

The safe load according to (118) would he:

$$P = 17{,}000 \times 6.228 \times 0.72 = 76{,}200 \text{ lbs.}$$

For a direct calculation of b and h we may use the following:

$$b = \frac{30}{14{,}220{,}000\,\pi^2}\,\frac{Pl^2}{h^3} = 0{,}000{,}000{,}21\,\frac{Pl^2}{h^3} \quad \ldots (119)$$

and hence:

$$P = 4{,}762{,}000\,\frac{bh^3}{l^2}$$

Care should be taken that the load does not exceed the limit given by (118).

Example 6. In the new huilding of the sugar refinery of Waghäusel, built in 1859–60, are columns of cruciform section.

Those in the basement bear a load of 264,000 lbs., and are 78.74" high, the ribs being 2" × 14⅝". According to (119) these posts should sustain a load of

$$P = 4{,}762{,}000\,\frac{2\times(14.1875)^3}{(78.74)^2} = 4{,}386{,}000 \text{ lhs.}$$

According to (118)

$$P = 17{,}000 \times 2 \times 14.1875 = 482{,}300 \text{ lhs.}$$

which is much more than the actual load.

Columns of Angle and T Iron.—These are much used in bridge trusses, especially in America. (See § 87). The vertical posts may be considered as columns with jointed ends. Case I, Fig. 364, and the upper chord is in compression and may be considered as Case III, Fig. 364. The following figures show many of the forms, in section, which may be used for this purpose.

FIG 367.

The first is the column of the Phœnix Bridge Works at Phœnixville, Pennsylvania. This is shown made of four segments, but six or more are used. This form may be strengthened by rivet-

ing flat iron between the joints of the segments. The four following sections are from the Keystone Bridge Works.

The sectional distribution of material should be chosen so that the (equatorial) moment of inertia on both the principal axes are the same (see § 7). The fifth section shows a double T iron, in the middle in dotted lines. This is used in bridge chords, where two or more such shapes are sometimes introduced. The last form is a combination of four pieces of angle iron recently used for pump rods in mine shafts. The resistance to thrust is here dependent upon the distance between the guides of the rod.

Grouped Columns.—It is sometimes a question whether, in the support of very important loads, as well as for economy of material, it is not best to use two or three columns instead of one. If we let m be the number used, instead of one, we have, for the supposition that the columns are in compression, the relation for similar sections.

$$V^1 = \sqrt{m}\, V \quad \ldots\ldots\ldots \quad (120)$$

This shows that grouped columns use $\sqrt{m}$ times as much material as a single column. It is also economy of material to use a small number of heavily loaded columns to sustain a given load.

Example 7. This subject may also be treated by the aid of the preceding table. If we have a load of 2800 lbs. upon a column 18 feet high, the diameter for a solid round column would be 2¾", while for four columns of 2 inches diameter we have $4 \times 740 = 2960$, or about the same. The cross sections are to each other as $4 \times (2)^2 :: (2.75)^2$, or as $16 : 7.56$, or $\sqrt{4} : 1$.

Variations in the height of columns affect the economy of material, other things being equal, to a marked degree, since the resistance to compression varies directly as the height (l). It is sometimes desirable to make a column in several portions, when a proportional reduction in height can thereby be secured. The triple central core of the column shown in Fig. 368, is an example and is a form often used by architects in connection with columns of brickwork.* This is not as effective as a single column, since the volume ratio is $\frac{1}{2}\sqrt{m}$, *i.e.*, $\frac{1}{2}\sqrt{3} = 0.866$.

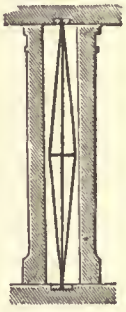

FIG. 368.

In conclusion it must be remarked that the columns which are used in machine construction are usually made much heavier than the preceding calculations indicate. This is due to the fact that such columns are often subjected to bending and tensional stresses, as well as to much vibration and the additional material is needed to meet these conditions. Columns of cast iron which are subjected to tension, as in the framing of vertical engines, should be made at least double the section given by (112), (114), (116), and (118). The security is also made greater in the case of buildings, as the result in Example 6 shows.

§ 128.

FORMS FOR IRON COLUMNS.

The columns which are used in machine construction must be held down to the iron base plates of the machines, or if used in connection with building construction are secured to foundations of masonry. Heavily loaded columns are often placed upon foundation stones with only a sheet of lead beneath, and no fastening, but otherwise some form of anchorage must be used.

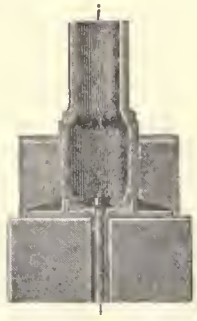

FIG. 369.

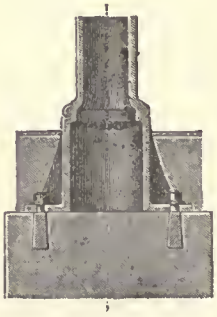

FIG. 370.

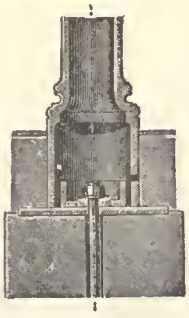

FIG. 371.

The illustrations show three forms of fastening. In each case the sole plate is placed beneath the pavement. In the first case a special form of sole plate is held down to the masonry by an anchor bolt; in the second the flange which is cast on the column is bolted to the keys shown; the third construction (by Borsig) is arranged with a short cylinder bolted to the faced sole plate and made so as to give a space in which melted lead may be poured after the column is set in its exact position. A hole is left in the side of the column to admit the melted metal. The portion of the base of the column which shows above the pavement is made to conform to the general style of the building. In Fig. 369 a simple moulding is used between the plinth and shaft; in Fig. 370 a bead is added; and in Fig. 371 a double moulding of more elaborate outline is used.

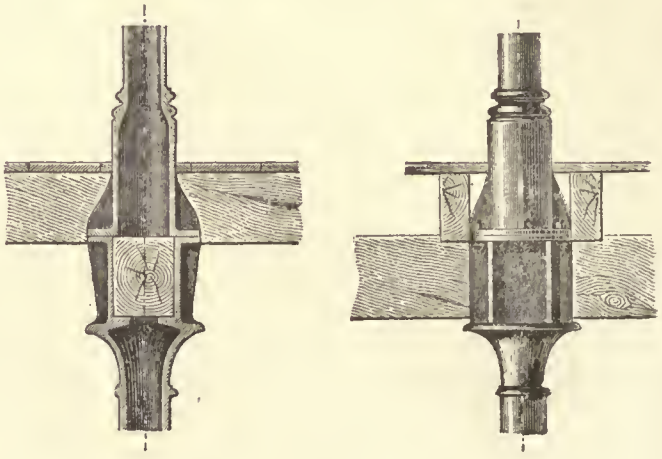

FIG. 372.

The capitals of such columns are made in many varied forms. Fig. 372 shows, in section and elevation, a capital arranged to carry a beam and also to support the base of the column of the floor above. A recess in the top of the column receives the main beam, and affords a good place for a joint. If iron beams are used, this recess is made proportionately narrower. The base of the upper column is securely bolted down as shown.*

FIG. 373.

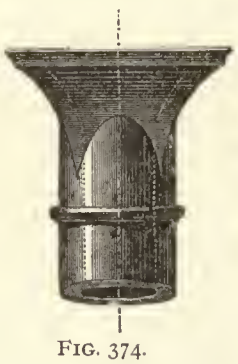

FIG. 374.

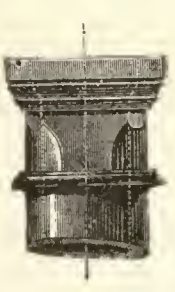

FIG. 375.

The capitals of iron columns afford much opportunity for effective decoration, which in many cases is neglected, although comparatively easy of execution. For the lower columns of heavy buildings the simple cubic capital so often found in Romanesque buildings is most suitable, and a good example is shown in Fig. 373.†

FIG. 376.

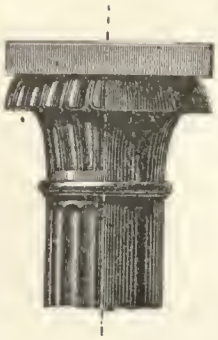

FIG. 377.

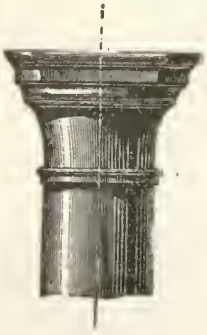

FIG. 378.

A somewhat lighter form is shown in Fig. 374, and for some situations the various Gothic capitals are suitable, Fig. 375. In

* For example, the columns in the vestibule of the theatre at Carlsruhe.

* Other forms will be found in Brandt's Eisenkonstruktion, Berlin, 1865.

† Shown among other places in the Österr. Lloyd, in Trieste, and in the Arsenal at Vienna.

all three examples the pattern making and moulding is not difficult. The form most used in machine construction is shown in Fig. 376, being something between the Roman Doric and the Tuscan orders, and having an echinus beneath the cap plate, and an astragal bead around the column a short distance below. By varying the distance of the latter from the former the effect can be modified for taller or shorter columns.

The heavier form of the Grecian Doric is unsuitable for machine construction and is seldom used. More appropriate is the modified Corinthian capital shown in Fig. 377. The top is a cornice of overhanging leaves, terminating in an astragal on the shaft. By omitting the ornament the same form may be retained, as shown in the right hand half of the illustration, and also in Fig. 348. The fluting of the column is by no meaus objectionable, at least in Germany. The fluted capital is readily cast by being made in a core box.

Fig. 378 shows a capital of Renaissance form with octagonal abacus, well suited for slender columns.

The support of beams, either iron or wooden, is best accomplished by the introduction of a plate between the column and the beam, and this may be treated simply, yet in harmony with the style of the rest of the work. Fig. 379 shows such a support on the cubic capital already shown, and is adapted for very

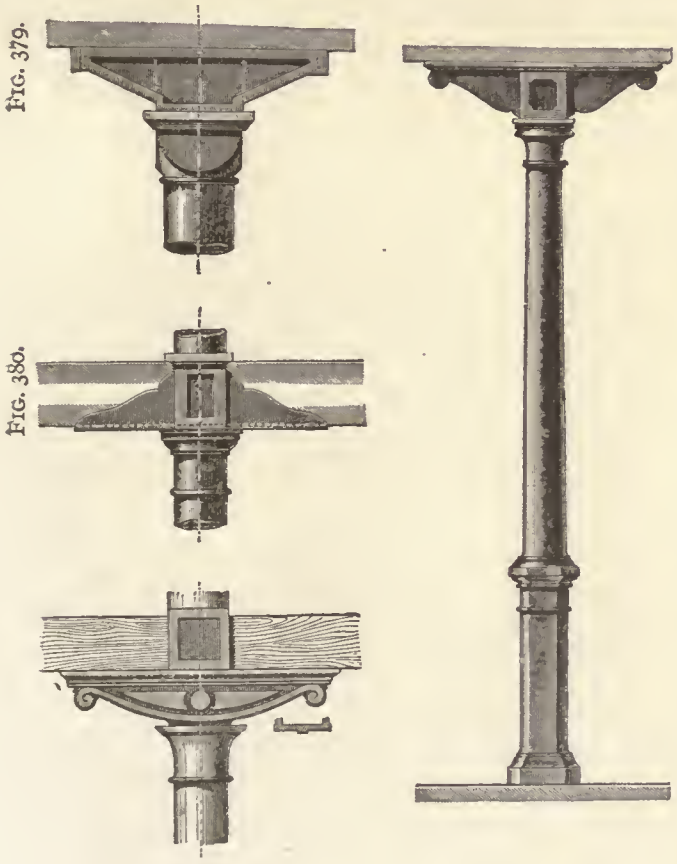

Fig. 379.

Fig. 380.

Fig. 382.

Fig. 381

heavy construction. Fig. 380 shows a lighter capital, in which the support for the beam is made of a box form; Fig. 381 is a still lighter design. This illustration also shows the effect of a high stylobate or base moulding, suitable for tall slender columns. As shown in this example, such bases are usually made octagonal in section, which approaches the Gothic style, but they are frequently made rouud. As in architecture, the columns are usually made tapering from below upward, the upper diameter being 0.8 to 0.7 that at the base. Fig. 382 shows a more elaborate form of capital and bearer.

Columns of cruciform section, already referred to, are often used in the construction of industrial establishments. They are sometimes to be preferred to hollow columns, since the latter are often cast of such unequal thickness as to be unreliable. Figs. 383 to 385 show such a column. Fig. 383 is from the Railway of St. Germain. Here the flutings extend from top to bottom and the column is swelled slightly in the middle. The form shown in Fig. 384, from the Tobacco Factory at Strasburg, is more elegant in its appearance. Here a rectangular base is used for the lower floor, but is omitted above. The method of connecting the base aud column, as well as the connection between capital, beam, and column above, is shown in Fig. 385; in which the solidity and substantial character of this form of construction is well shown.

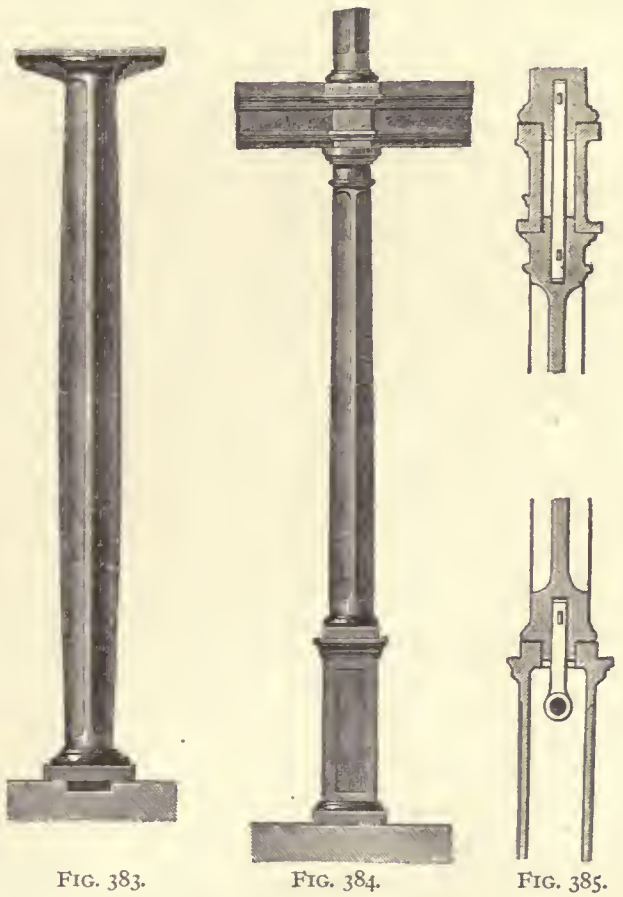

Fig. 383. Fig. 384. Fig. 385.

These examples will serve at least to show the variety of forms of columns which may be used, and the manner in which a little ornament may be introduced into machine construction.

CHAPTER VIII.

AXLES.

§ 129.

Various Kinds of Axles.

Axles may be considered as beams which carry revolving or oscillating loads, and hence are provided with journals at certain portions of their lengths; they may be subjected to deflection or to compression, as in the cases of journals already discussed, according as the load act normal or parallel to the axis. Axles which are subjected ouly to thrust, are not often found, the far greater portion being those bearing deflecting loads, although many are under combined stresses.

These may be divided into two classes: those which have the load applied at but one place, and those in which several loads are borne at various points. The first are called Simple-loaded Axles; the second, Multiple-loaded Axles.

The sections of axles of cast or wrought iron may be either circular or varied, and this gives rise to another subdivision in the calculations. The methods of graphostatics are especially applicable to the subject of axles, and in the following pages both numerical and graphical solutions will be discussed.

A. AXLES WITH CIRCULAR SECTION.

§ 130.

Simple Symmetrical Axles.

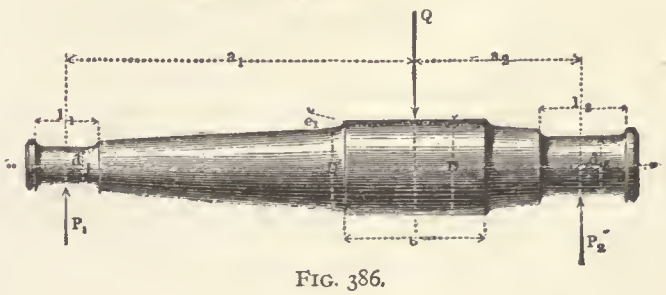

Fig. 386.

The load Q is in this case applied normal to the direction of the axis, midway between the two journals, upon a seat for a hub, as shown in Fig. 386. The portion between the hub-seat and journals is called the shank of the axle. The journals are proportioned according to the methods given in Chapter V, taking $P = \frac{1}{2} Q$, and the axle then proportioned so as to give approximately the same strength as the journals throughout.

Let:

d = diameter, l = length of journals,
e = height of shoulder or collar,
D = diameter of middle, or hub-seat, b = its breadth,
D' = diameter of shank at the junction with D,
$e' = \frac{1}{2}(D - D')$ the shoulder at the latter junction,
a = the length of shank,

then we have:

$$\frac{D'}{d} = \sqrt[3]{\frac{a - 0.5\,b}{0.5\,l}} \quad \ldots\ldots \quad (121)$$

This will give the axle the same security as the journal, so that the approximate stress will be $S = 8500$ lbs. for wrought iron, and 4260 lbs. for cast iron. If a higher or lower stress is desired, the journal should be proportioned for the desired stress, and the corresponding dimensions for the axle deduced.

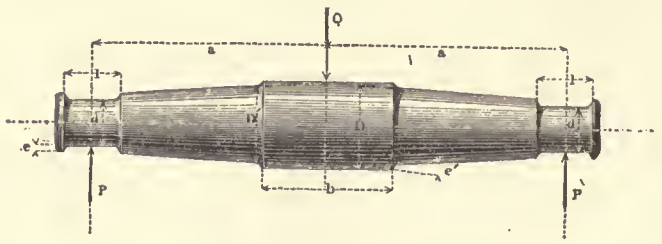

FIG. 387.

The strongest form for the shank of an axle is that of a cubic parabola (see § 10, No. VI), and the student will find this a valuable subject for investigation. In practice it is made a portion of a truncated cone whose larger diameter $= D'$, and the smaller diameter $= d = 2e$. The value of e' should only be made large enough to provide sufficient depth for a keyway.

§ 131.

NON-SYMMETRICAL SIMPLE AXLES.

If the two shank portions of a simple loaded axle are of unequal length, as in Fig. 387, the load on the two journals d_1 and d_2 will be unequally divided, and we have the proportion

$$\frac{P_1}{Q} = \frac{a_2}{a_1 + a_2}, \quad \frac{P_2}{Q} = \frac{a_1}{a_1 + a_2}, \quad \frac{P_1}{P_2} = \frac{a_2}{a_1} \quad \ldots\ldots \quad (122)$$

The hub-seat divides the axle into two parts, each of which may be considered as the half of a symmetrical axle, and so the

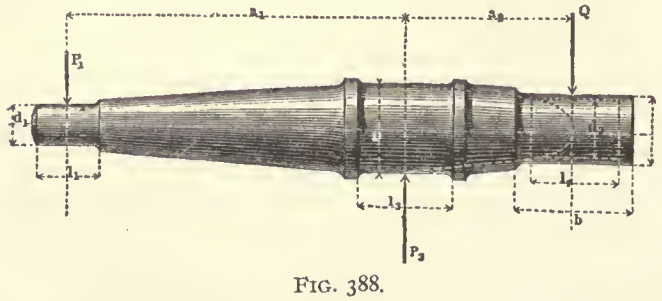

FIG. 388.

whole proportioned. The value of D' is determined for each shank, and the greater value taken for both sides. If $a_1 = a_2$ the axle becomes symmetrical.

If the seat for the load Q does not lie between the two journals, but projects, as in Fig. 389 (a_2 becoming negative), the load is said to overhang, and the journal D becomes a neck journal (see § 92).

We have for the relations of forces:

$$\frac{P_1}{Q} = \frac{a_2}{a_1}, \quad \frac{P_3}{Q} = \frac{a_1 + a_2}{a_1}, \quad \frac{P_1}{P_3} = \frac{a_2}{a_1 + a_2} \quad \ldots\ldots \quad (123)$$

The diameter of the journal d_1 is first determined, then a diameter for an assumed journal d_2 for the loaded point, and then a value D for a neck journal, taking for D the greater of the values for D' and D'', as given for the two ends by formula (121), the length of journal being made $l_2 = \sqrt{l_1^2 + l_2^2}$.

The cubic parabola is shown drawn in the shank a_2, being the shape for uniform resistance in this case, and we have for the diameter δ, at the root of the hub-seat:

$$\frac{\delta}{d_2} = \sqrt[3]{\frac{l}{l_2}} \quad \ldots\ldots \quad (124)$$

See (§ 10, Case VI, Remark).

Example.—Let the load Q, acting in one direction, be 14,520 pounds, $a_1 = 47.25''$, $a_2 = 23.625''$, $b = 13''$, material cast iron, number of revolutions, $n = 150$.

We have $P_1 = 0.5\,Q = 7{,}260$ lbs. $P_2 = 1.5\,Q = 21{,}780$ lbs. From the table of § 91 we have $d_1 =$ about $3\frac{3}{4}''$, $d_2 =$ about $6\frac{1}{4}''$, also $l_1 = 5\frac{5}{8}''$, $l_2 = 9\frac{3}{8}''$. We then have:

$$D = 6.25 \sqrt[3]{\frac{23.625}{4.6875}} = 10.72, \text{ say } 10\tfrac{3}{4}''.$$

$$l_3 = \sqrt{(5.625)^2 + (9.375)^2} = 10.93, \text{ say } 11''.$$

$$\delta = 6.25 \sqrt[3]{\frac{13}{9.375}} = 6.97, \text{ say } 7''.$$

§ 132.

GRAPHICAL CALCULATION OF SIMPLE-LOADED AXLES.

The determination of the forces acting upon the journals may be made according to the methods given in Cases I to V, of § 39. In a similar manner the cord polygon may be employed as in § 43 and § 44 to determine the statical moments of the parallel forces at various points, and the polygon so constructed may be called the surface of moments. The simple method about to be given will serve as a general graphical solution of the problem.

I. The Load Acts Normal to the Axis.

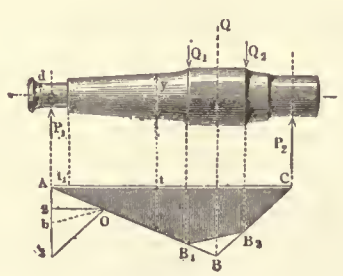

FIG. 389.

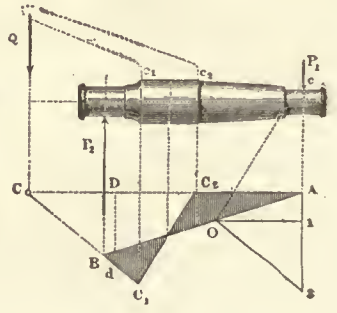

FIG. 390.

(a). *Hub and Load between the Journals.*—Draw the line $A\,C$, equal in length to the distance between centres of journals, and upon it construct any triangle $A\,B\,C$, whose apex lies on the line of the load Q. Draw $A\,3$ normal to $A\,C$, making $A\,3 = Q$; draw $3 \cdot O$ parallel to $B\,C$, and $2 \cdot O$ parallel to $A\,C$; then $A \cdot 2 = P_1$, $2 \cdot 3 = P_2$. By dropping the perpendiculars from the ends of the hub-seat we may divide Q into two forces Q_1 and Q_2, shown in the force polygon by $O\,b$, parallel to $B_1\,B_2$; giving $A\,b = Q_1$, $b \cdot 3 = Q_2$. The vertical ordinate t, at any point of the surface of moments is proportional to the statical moment M_y at its point of intersection with the axis, as for example the ordinate t_1, at the base of the journal for P_1. We have in any case:

$$y^3 = \frac{32}{\pi S} M_y. \quad d_1^3 = \frac{32}{\pi S} M_1$$

and hence:

$$\frac{y}{d_1} = \sqrt[3]{\frac{M_y}{M_1}}, \text{ or } = \sqrt[3]{\frac{t}{t_1}} \quad \ldots\ldots \quad (124)$$

from which y can readily be obtained.*

(b). *The Hub-Seat between the Journals and the Load Overhung*, Fig. 390.—Draw the line $A\,C$, parallel to the axis, construct a triangle with the points A, B and C on the lines of the directions of the forces, drop a perpendicular from the point D, where $D\,d = Q$, make $O \cdot 1$ parallel to $A\,C$, and equal to $C\,D$; make $A \cdot 1 \cdot 3$ normal to $A\,C$, also $O \cdot 3$ parallel to $C\,B$, and $1 \cdot 3$ will $= Q$, $A\,1 = P_1$, $3 \cdot A = P_2$. The force Q is decomposed into two forces at the ends of the hubs, and by dropping the perpendiculars, the points C_1 and C_2 are determined, and $O\,c$ drawn parallel to $C_1\,C_2$, giving the values $c \cdot 3$ and $1 \cdot C$ for the

* If it is desired to determine a series of values of t, beginning from t_1, it may readily be done by using a table of cube roots of numbers such as are given at the end of this volume; if the greatest value of y is the starting point, the table of cube roots of decimal numbers is useful, the space being divided into ten parts and the outline laid off correspondingly.

forces at C_1 and C_2 respectively. The diagram shows that at a point within the hub-seat the stresses are reversed and the bending moment is zero.

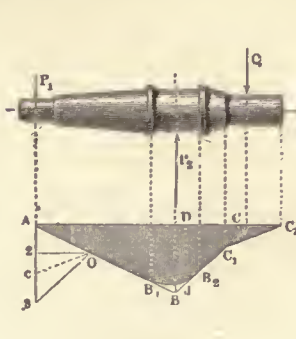

FIG. 391.

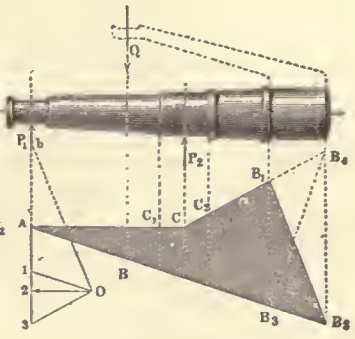

FIG. 392.

(c). *Overhung Axle with Load Outside the Journals*, Fig. 391—Construct the triangle $A\,B\,C$, as in the preceding case (b), and place D so that $D\,d = Q$, draw A . 3 normal to $A\,C$, make O . $2 = C\,D$, and parallel to $A\,C$, and draw O . 3 parallel to $C\,B$, and we have again A . $2 = P_1$, 3 . $A = P_2$. Divide Q into C_1 and C_2 and make $O\,c$ parallel to $C_1\,C_2$, giving c . 3 and 2 . c for the forces at C_1 and C_2. The journal at B being uniformly loaded, its moment surface is outlined by a parabolic curve (see § 42).

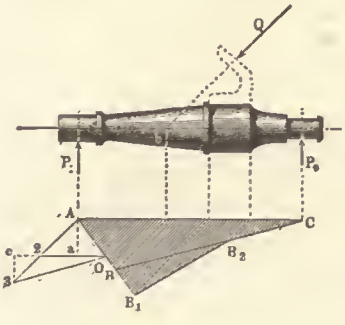

FIG. 393.

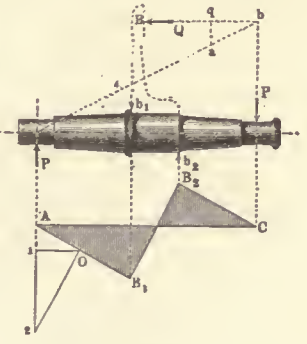

FIG. 394.

(d). *Overhung Axle, with Load between Journals*, Fig. 392.—Construct the triangle $A\,B\,C$ as in case (a), divide Q into B_1 and B_2, which gives the polygon $A\,C,\,B_1\,B_2$ (which is equivalent to the other one $A\,C,\,B_4\,B_3$). In the force polygon, 1 . 3 $= Q$, 2 . 1 $= P_1$, 3 . 2 $= P_2$, and by making $O\,l$ parallel to $B_2\,B_1$ we get b . 3 and 1 . b for the forces on $B_1\,B_3$ and $B_2\,B_4$.*

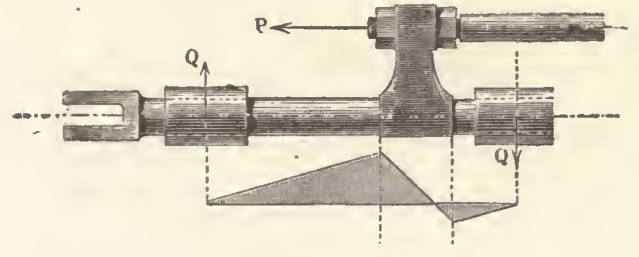

FIG. 395.

II. The Load Acts Inclined to the Axis, Fig. 393.

The construction is similar to L, except that the force and cord polygons are inclined according to the direction of Q. The vertical projections $a\,A$, and 3 . c give the journal pressures P_1 and P_2; the horizontal component of Q gives the axial thrust.

Another example is given the case of a rod worked from an overhung arm, as in some forms of locomotive feed pumps, Fig. 395. The directions are here periodically reversed, and the relations of the points continually changing.

III. The Load Acts Parallel to the Axis, Fig. 394.

We here have two couples: one consisting of the two equal journal pressures, and the other of the two pressures at the ends of the hub-seat (see § 38). Draw the lines $A\,B_1$ and $C\,B_2$ parallel to each other, and intersecting the perpendiculars dropped from the ends of the hub, join B_1 with B_2 and the surface of moments is $A\,B_1\,B_2\,C$. To find the forces, prolong Q from B until it intersects $C\,b$, join it to the middle of the other journal, make $q\,b = Q$, and drop the perpendicular $q\,a$, which is equal to

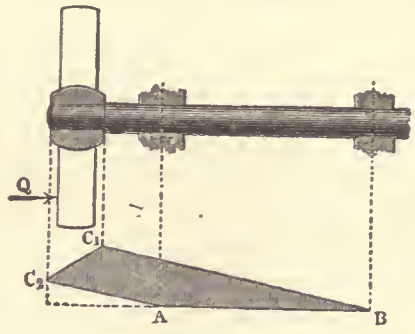

FIG. 396.

P. Make A 1 $= P$, draw 1 . O parallel to $A\,C$, and O . 2 parallel to $B_2\,B_1$, then 1 . 2 is the force at b_1 and 2 . 1 that at b_2. If the hub should overhang, as in the case of a screw propeller, Fig. 396, the diagram takes the form $A\,B\,C_1\,C_2$.

§ 133.

PROOF DIAGRAMS.

In order to calculate the resistance of a given axle to bending it is necessary to know the section modulus at various points. If all the sections are circular the moduli vary as the third power of the diameter. Hence the various diameters are to be cubed.

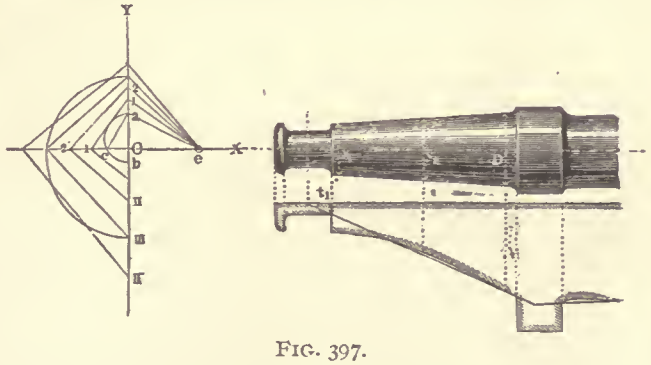

FIG. 397.

This may readily be done graphically by the method given in § 28. In order to compare such a diagram with one of the surface of moments as just discussed, it is necessary to construct them to the same scale. For this purpose take the origin O of the two axes X and Y, and make $O\,a$ equal to the diameter (or semi-diameter) of the shank of the axle, and lay off, below the corresponding value $O\,b$ of its ordinate t_1, draw on $a\,l$ a semicircle $a\,c\,b$, draw $a\,e$ normal to $a\,c$, and taking $O\,e$ as unity we have $O\,b = (O\,a)^3$. Make O . 1 $= y$, O . 2 $= y_2$, &c., and draw the moments to the axes of X and Y, as 1, 1′, I., 2, 2′, II., &c., and we have O I, O II, as the desired values of y_1^3, y_2^3 . . . which correspond to those of the principal diagram.

Such proof diagrams are very convenient to show what approximations may be made, and to detect possible errors in calculation, and shows at once any deficiency in security, since the relation of the actual stresses to the desired constant stress is that of the ordinates of the proof diagram to those of the theoretical surface of moments. This numerical series may be plotted in a curve, called the stress curve. By combining the theoretical diagram with the proof diagram on an exaggerated scale, as shown in the illustration, the unit can be chosen to a greater advantage.

§ 134.

AXLES LOADED AT TWO POINTS.

In an axle loaded at two points, as in Fig. 398, the end portions are called the shanks and the middle part the shaft. If Q_1 and Q_2 are the loads, s the length of shaft, we have for the journal pressures

$$\frac{P_1}{Q_1} = \frac{s + a_2\left(1 + \frac{Q_2}{Q_1}\right)}{a_1 + s + a_2}, \quad \frac{P_2}{Q_2} = \frac{s + a_1\left(1 + \frac{Q_1}{Q_2}\right)}{a_1 + s + a_2} \quad (125$$

If we take the diameters corresponding to these pressures as

*The conditions of this case, but with very light stresses, are found in the spindles of the American Ring Spinning Frame.

d_1 and d_2, and also have the shanks a_1 and a_2 given, we may determiue next the diameters at D_1 and D_2 at the points of application of the loads Q_1 and Q_2.

To fiud the diameters of the shaft at various points we have, taking y for the diameter at any point distant x from the load point Q_1:

$$\frac{y}{D_1} = \sqrt[3]{1 + \frac{x}{a_1}\left(1 - \frac{Q_1}{P_1}\right)} \quad \ldots\ldots\ldots (126)$$

an equation which gives the shaft the outline of a double cubic parabola, which iu practice may be replaced by two straight lines, giving the shape a truucated cone.

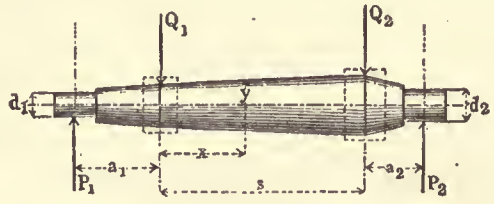

FIG. 398.

The two seats for hubs are formed so as to give shoulders for keyway, and have a determinate breadth b, governed by the piece to be carried. In many cases such axles are symmetrical and the two loads are equal to each other, hence $a_1 = a_2$, $Q_1 = Q_2$. We then have $P_1 = P_2 = Q_1 = Q_2$ and $y = D$, the shaft being cyliudrical. This is also the case when $P_1 a_1 = P_2 a_2$.

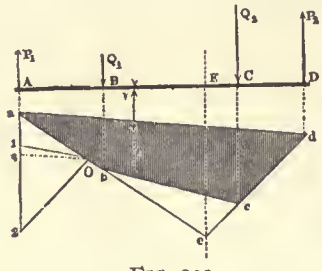

FIG. 399.

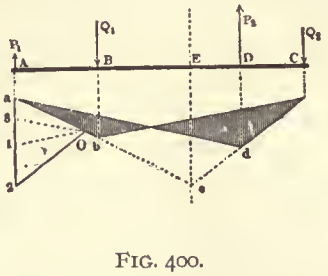

FIG. 400.

The graphical solution of the preceding problem is as readily made as in the case of single loads. If we draw normals to the axis $A\,D$, Fig. 399, correspouding to the given loads Q_1 and Q_2, also draw $A\,a$, make $a\,1 = Q_1$ and $1\,.\,2 = Q_2$, choose a pole O and draw the rays $O\,a$, $O\,1$, $O\,2$, prolong $a\,O$ to its intersection b with the line of the force Q_1, make $b\,c$ parallel to $1\,O$, $c\,d$ parallel to $2\,O$ and joiu d with a. Draw $O\,3$ parallel to $d\,a$ in the force polygon and we have $2\,.\,3 = P_2$, and $3\,a = P_1$ aud $a\,b\,c\,d$ the surface of moments whose vertical ordinates t may be used to determiue their correspouding diameters of the axle as in I, § 132.

The intersection e, of $a\,b$, and $d\,c$ prolonged determines the position $E\,e$ of the resultant of Q_1 and Q_2. If $E\,e$ is desired at

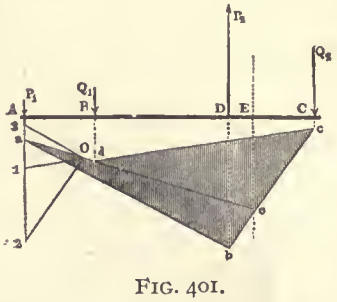

FIG. 401.

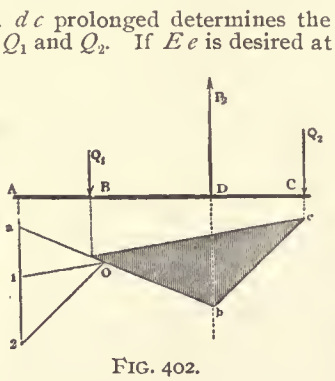

FIG. 402.

once, as in the method given in § 40, the previous case (§ 132, I) is applicable since the direction of the line $a\,d$ cau be chosen at will.

If one load acts beyond the bearings, Fig. 400, the reversal point in the elastic line will appear as before; this occurs when the resultant of Q_1 and Q_2 falls between A and D (see § 132, I). The above meutioned shearing stress is given by $1\,.\,3$.

If the resultaut of Q_1 and Q_2 falls outside both journals, Fig. 401, there will be no reversal, the force P_1 having the same direction as Q_1 and Q_2; in other respects the procedure is the same as before.

Finally the resultant may just equal the force at D, as in Fig. 402. In this case there will be no beuding stress in portiou $A\,B$, which in the previous case was quite small; the two lines of the surface of moments fall together. The shank $A\,B$ and the journal at A may therefore be made very light, unless other forces than those already cousidered act upon them.

The decomposition of the forces acting upou the hub-seat depend upon its breadth and the treatment is similar to § 132. Other variations may occur in the rclations of the loads and journals, but the prece diug examples will suffice.

§ 135.

INCLINED DOUBLE LOADED AXLES, RAILWAY AXLES, CRANE PILLARS.

The previous methods are almost as easily applied when the loads act iu an inclined direction. The incliued actiou is caused by various couditious, and as an example we will consider railway axles.

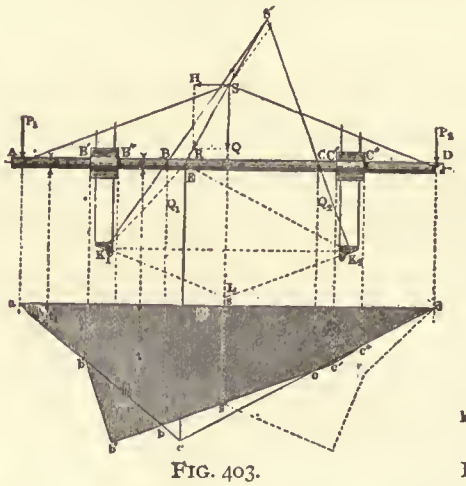

FIG. 403.

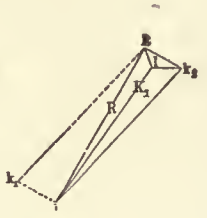

FIG. 404.

Besides the vertical load Q at the centre of gravity S, of the car, Fig. 403, there are forces due to centrifugal action and flexibility, which produce a horizontal force which Scheffler, according to Wöhler's researches, places at 0.4 Q,* so that there is an inclined resultant R, acting upon the axle. Since the value 0.4 Q was obtained by means of measurements on cars during long runs, it includes the action of the elevation of the outer rail iu passing curves. This force R is also acting on the wheel flanges at K_1 and K_2 as well as at the journals A and D. It must be noted that the wheel K_2 opposed to H can only resist forces acting normal to the coned face so that the angle $L\,K_2\,S'$ should be made $= 90°$.

The points of intersection B and C of the wheel forces on the axle give the positious for the verticals Q_1, Q_2, and the horizontal pressure may be neglected in determining the axle loads, these being P_1 and P_2. From these the journal diameters are found and the greater taken for both.

Then from the point of applicatiou E of the resultant R let fall a perpendicular $E\,e$ aud draw the triangle $a\,d\,e$, prolong the directious of Q_1 and Q_2 to b and c, and join b aud c by a straight liue. Then drop perpendiculars from $B'\,B''$, $C'\,C''$, to $b'\,b''$, $c'\,c''$ and join these latter, and $a\,b'\,b''\,c'\,c''\,d$ is the cord polygon for the given conditions. The ordinate t serves to determine the diameter y for any journal diameter d_1, and the ordinate t_2 gives the root of the journal

The direction $K_1\,B$ is readily determined as follows: Choose any point on the line of R, as for example, E, join it with K_1 and K_2 and decompose $R = E\,r$, Fig. 404, along the directions $E\,K_1$ and $E\,K_2$ into $E\,k_2$ and $k_2\,r = E\,k_1$, draw $k_2\,l$, horizontal and $E\,l$ parallel to the given direction $K_2\,S'$; then $l\,E$ is the force at K_2, and $r\,l$ that at K_1, whose direction is sought, while $E\,k_2$ and $k_2\,l$ are the inner forces at the corner K_2 of the cord polygon $E\,K_2\,K_1$, and in equilibrium with the force of the known direction $K_2\,S'$.

Since the horizontal force H acts either to the right or left, the larger side of the polygon $a\,s\,s'\,b''\,b'$ must be used for both halves of the axle, as shown in the dotted lines. The cord polygon for the direct vertical load should also be drawn, and if it gives a greater ordinate for the shaft than $s\,s'$, it should be used, the diameter of the shaft generally being smallest in the middle.

Axles of railway cars make from 250 to 300 revolutions per minute. For wrought iron the journals are generally made two diameters in length. In passing around curves these journals are subjected to considerable endlong pressure. The shoulder e is generally made $= \frac{1}{7}\,d$ to $\frac{1}{6}\,d$, and heavier than usual iu ordinary cases.

* Ad. Scheffler, Railway Axles. Braunschweig.

In many countries standard proportions for axles have been adopted. Those of the Prussian railways are as follows, the dimensions depending upon the value of Q, which is the total load on each axle.*

$Q =$	3800	kilogrammes.	$D =$	100	mm.	$d =$	65 mm.
" =	5500	"	"	115	"	"	75 "
" =	8000	"	"	130	"	"	85 "
" =	10000	"	"	140	"	"	95 "

The journal length l varies between $1\frac{3}{4}$ to $2\frac{1}{4}$ d, according to judgment. These proportions are for wrought iron; if steel is used Q may he increased by 20 per cent. For iron axles the

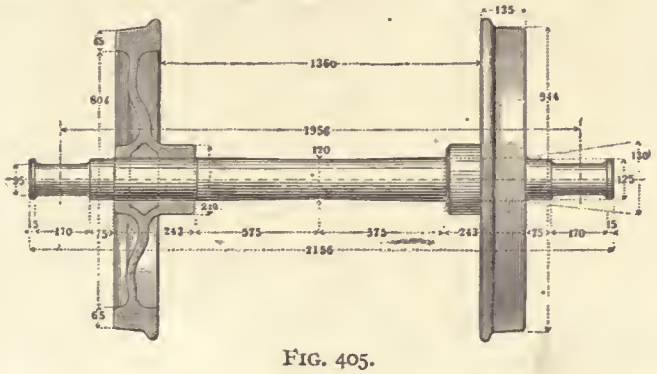

FIG. 405.

pressure upon any one journal should not exceed $\frac{2}{3}$ Q. These figures give a stress of 6.4 to 8.3 kilogrammes per square millimetre or 9000 to 12000 lbs., and the pressure p from 0.30 to 0.41 kilo. or 326 to 593 lbs.

In Fig. 405 is shown a steel axle for the Royal Eastern Railway, with its wheels, all dimensions heing in millimetres.

In England a standard axle has been adopted as shown in Fig. 406,† and the standard American axle is similar.‡ The

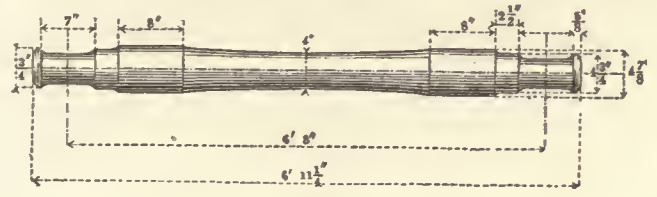

FIG. 406.

value of Q in this case is ahout 22000 lbs. In France there has been no general standard adopted, hut the various roads have adopted forms—for regular use. The Paris-Lyons-Mediterranean Railway has eight forms. The form No. 8 has $d = 85$ mm. ($8\frac{3}{8}$″), $l = 170$ mm. ($6\frac{3}{4}$″), length between centres of journals = 1925 mm. ($75\frac{3}{4}$″), diameter of hub-seat = 125 mm. ($4\frac{15}{16}$″), diameter of the axle in the middle = 105 mm. ($4\frac{1}{8}$″).

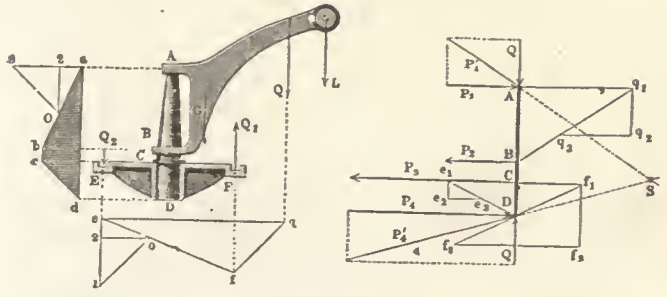

FIG. 407.

Crane pillars may he considered as axles subjected to inclined stresses, as the following example will show. The crane shown in Fig. 407 is suhject to the load L, and also its own weight G, and the resultant of these is at Q (see examples in § 34).

At A and B are hearings, and the pillar is held in a hase plate at C D, the plate heing secured at E F. In order to determine the forces at E and F, construct the cord polygon $e f g$, and force polygon e 2 1 O, in which 2 . 1 = Q, 1 . e = Q_1 the force at F, e . 2 the force Q_2 at E. All three external loads act parallel to the axis, so that we can use the method shown in Fig. 394. In the diagram to the right we make $q_1 q_2 = Q$, and $q_2 q_3$ parallel to $A q_1$ normal to $A B$. These lines then represent the horizontal forces P_1 and P_2 at A and B.

The hearing at A carries the entire vertical load, and hence we have at A the inclined resultant P_1' of Q and P_1. We now draw $C f_1$, normal to $A C$, $f_2 f_1 = Q$, draw $f_1 D$ and also $f_2 f_3$ parallel to $C f_1$, then $f_2 f_3$ will give the magnitude of a force acting right at D and left at C. In a similar manner, draw $e_1 e_2 = Q_2$, and draw $e_1 D$, and make $e_2 e_3$ parallel to $e_1 C$, and $e_2 e_3$ will be the magnitude of a force acting left at C and right at D.

We therefore have $P_3 = f_2 f_3 + e_3 e_2$ and $P_4 = e_2 e_3 + f_3 f_2$. The vertical pressure of the pillar itself is all taken at D, hence we get for its vertical componeut $Q = f_2 f_1 - e_1 e_2$, which combined with P_4 gives the resultant P_4'. This is proved hy the intersection of P_1' and P_4' at S must fall on the line of the resultant of P_2 and P_3.

If we neglect the compression in the direction of the axis, we may now draw the force polygon a 2 3 O of the forces P_1, P_2, P_3, P_4, as shown at the left of Fig. 407, and thus ohtain the surface of momeuts a b c d.

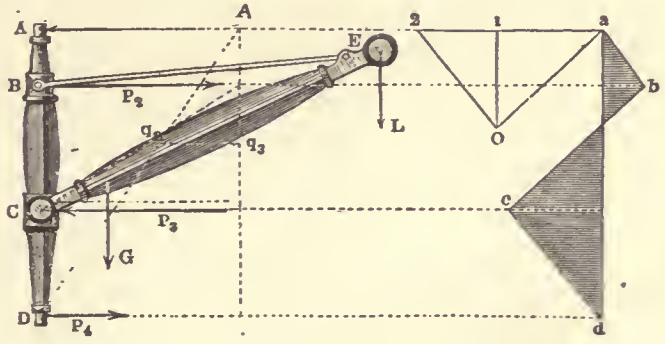

FIG. 408.

A crane with swivel column, to which the jib or boom is rigidly attached, may he examined as shown in Fig. 408. The position of $Q = L + G$ is taken as before, making $q_1 q_3$ represent Q, draw $A q_1$ normal to the axis, join $q_1 D$ and draw $q_2 q_3$ parallel to $A q_1$ till it intersects with $q_1 D$. We then have $q_3 q_2$ for the horizontal force P_1 at A, and $q_2 q_3$ the corresponding horizontal P_4 at D.

The step bearing at D will be subjected to an inclined thrust, the resultant of Q and P_4.

In a similar manner we obtain the horizontal forces P_2 and P_3 equal and opposite, and acting at B and C, and the resultant of the force at B with Q gives the inclined force due to the rod B E. The four horizontal forces have the same action as the load on the axle in Fig. 394. We may thus obtain the surface of moments a b c d, which shows a zero point for bending moments between B and D, and also indicates a forward bending above and a hackward helow. In the force polygon 2 a = P_2, a 2 = P_3, 2 1 = P_4 and 1 2 = P_1.

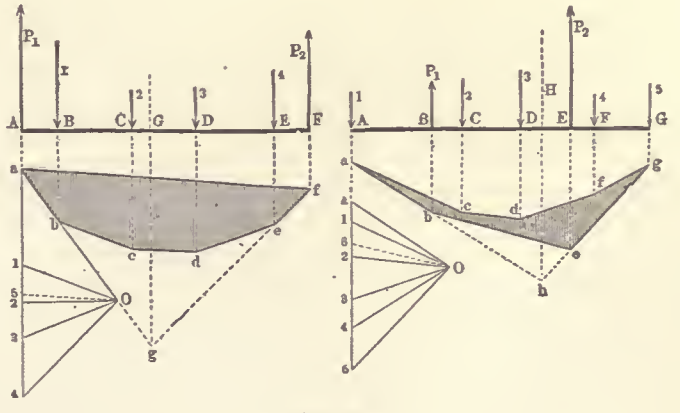

FIG. 409.

§ 136.

AXLES WITH THREE OR MORE BEARINGS.

The number of hearings for an axle is often as great as four. In such a case the forces and moments may he found as follows:

* These dimensions are given in the metric system as representing Continental practice.

† See Engineer, Nov., 1873.

‡ See Engineer, June, 1873. The M. C. B. standard varies slightly from the above (Trans.)

Starting at a, Fig. 409, with the given forces 1 to 5, we form the force polygon $a_5\ O$, and, according to § 40, the link polygon $a\ b\ c\ d\ e\ f\ g$, and join the closing line $g\ a$, parallel to $O\ 6$, in the force polygon; giving $5\ .\ 6 =$ the force P_3 at G, $6\ .\ a =$ the force P_1 at A. From P_1 and P_3 the journals d_1 and d_2 may be determined, and the ordinates of the cord polygon give the means of obtaining the axle diameter as before.

The intersection g, of $a\ b$ and $f\ e$, prolonged, is a point of the line of direction $G\ g$, of the resultant of the forces 1 to 4. If it is desired to find the successive resultants of the various forces as they are combined (see § 40), it will be found convenient to choose O, so that $a\ f$ will be parallel to $A\ F$. The inclined link polygon may also be transferred to a closing line parallel to $A\ F$.

If the shanks of the axle overhang the journals, as in Fig. 410, the procedure is similar to the preceding. Beginning at the point a, the force polygon $a\ 5\ O$ is constructed, and the first side of the cord polygon $b\ a$, drawn to the line of the first force, the second to the line $C\ c$, of the second force, and so on to the closing line $e\ b$. The first and nth line of the cord polygon intersect as before on the line $H\ h$ of the resultant. Variations on these examples may occur, as when the loads act in inclined directions, or opposed to each other, the methods being similar in all cases.

§ 137.

Axles with Inclined Loads.

The analytical investigation of axles becomes more difficult when, as in Fig. 411, the loads act in different planes, but the graphical method is readily applied. The force polygons $A\ O_1$

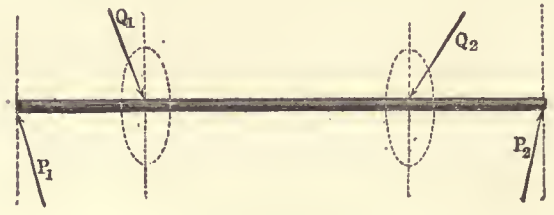

Fig. 411.

1, and $D\ O_2$ 2, are constructed for the forces Q_1 and Q_2, respectively, Fig. 412, the polar distances $G\ O_1$ and $H\ O_2$ being made equal to each other, so that the closing lines of the two cord polygons $A\ b'\ D$, and $A\ c''\ D$, coincide in $A\ D$. Then construct the second cord polygon with the inclined ordinates $B\ B'' = B\ b''$, $C\ C'' = C\ c''$, &c., making the angle u with the force plane of the ordinates of the first polygon, and inclined backwards as drawn. Then make $B\ b = B''\ b'$, $C\ c = C''\ c'$, $E\ e = E''\ e'$, &c., and draw the cord polygon $A\ b\ e\ f\ c\ D$, from which can be obtained (according to § 44) the bending moments

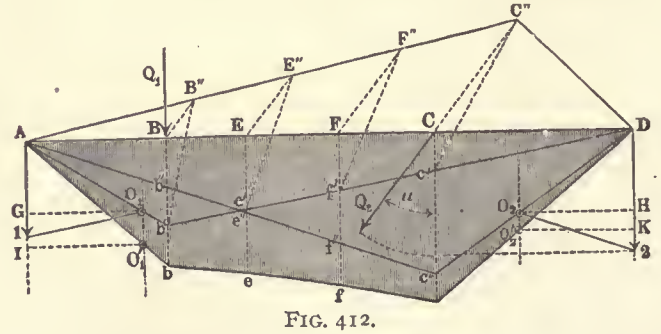

Fig. 412.

for the axle. The line $b\ e\ f\ c$ is a curve (hyperbola), $A\ b$ and $c\ D$ are straight lines. Draw $O_1\ O_1'$ parallel to A 1, $O_2\ O_2'$ parallel to D 2, and drop the perpendiculars $O_1'\ I$ and $O_2'\ K$, and $A\ I$ will be the force on the journal P_1, and $D\ K$ that at P_2, measured on the scale of the force polygon. Their directions are determined by combining $A\ G$ with H 2, and $D\ H$ with G 1 at the angle u.

B. AXLES WITH COMBINED SECTION.

§ 138.

Annular Section.

If it is desired to make an axle with annular section, or in other words, a tubular axle, the journals should first be calculated, according to the method given in § 90 for tubular journals, and then, retaining the same proportional thickness, determine the dimensions of the other parts in the same manner as for solid axles. The most commonly used ratio of internal to external diameters is 0 . 6. Instead of doing this, all the dimensions for a solid axle may be determined, and then having chosen a ratio for diameters, increase all the sizes according to formula (95). See also § 141.

§ 139.

Axles with Cruciform Section.

In cases where axles are made of cast iron the cruciform section, with circular centre and four ribs, is sometimes used. The shanks are then usually made of the ordinary conoidal form, Fig. 413, and in some cases the ribs gradually swell into a junction at the ends with the central core, Fig. 414.

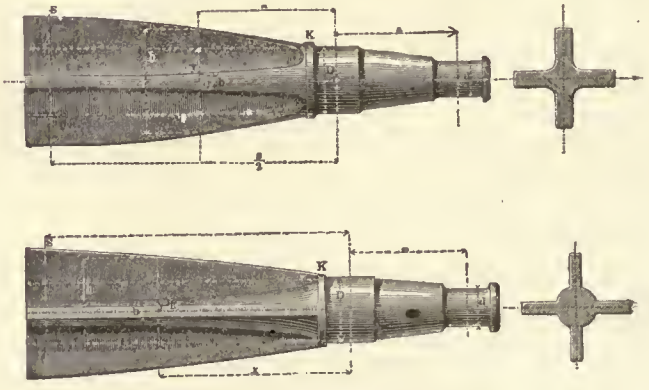

Fig. 413–414.

In designing such an axle, first proceed as if drawing a solid circular section as shown by the dotted lines, of the diameter corresponding to the portion K when the ribs join the head. Then for any point (x) of the shaft:

$y =$ the diameter of the assumed round axle, or equivalent conoid,
$h =$ height of ribs;
$b =$ thickness of ribs;
$k =$ diameter of core;

and the proportions are obtained from the following formula:

$$\frac{y}{h} = \sqrt[3]{\left(\frac{k}{h}\right)^4 + \frac{16}{3\pi}\left\{\left(\frac{b}{h}\right)^3\left(1 - \frac{k}{h}\right) + \frac{b}{h}\left(1 - \frac{k}{h}\right)^3\right\}} \qquad (127)$$

This formula serves for the pure cruciform section, without core by making $k = b$.

The results vary so slightly when $k = 0.2\ h$, that the following table may be used for both sections:

$\frac{b}{h}$	Values of $\frac{h}{y}$ when $\frac{k}{h}$												
	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20
0.05	1.30	1.40	1.50	1.61	1.72	1.84	1.94	2.04	2.15	2.18	2.22	2.26	2.27
0.06	1.30	1.39	1.48	1.58	1.68	1.79	1.87	1.95	2.02	2.07	2.11	2.13	2.14
0.07	1.29	1.38	1.46	1.56	1.65	1.74	1 82	1.89	1.94	1.98	2.00	2.02	2.02
0.08	1.28	1.36	1.45	1.53	1.62	1.70	1.76	1.83	1.87	1.91	1.93	1.93	1.93
0.09	1.27	1.35	1.43	1.51	1.59	1.66	1.72	1.77	1.81	1.84	1.86	1.87	1.87
0.10	1.27	1.34	1.4·	1.49	1.56	1.63	1.68	1.72	1.75	1.78	1.80	1.80	1.81
0.11	1.26	1.33	1.40	.47	1.54	1.60	1.64	1.68	1.71	1.73	1.74	1.75	1.75
0.12	1.25	1.32	1.39	1.45	1.51	1.57	1.61	1.64	1.67	1.68	1.69	1.70	1.70
0.13	1.25	1.31	1.38	1.43	1.49	1.54	1.58	1.61	1.63	1.64	1.65	1.65	1.65
0.14	1.24	1.30	1.36	1.42	1.47	1.51	1.55	1.57	1.59	1.60	1.61	1.61	1.61
0.15	1.23	1.29	1.35	1.40	1.45	1.48	1.52	1.54	1.56	1.57	1.58	1.58	1.58
0.16	1.23	1.2·	1.34	1.38	1.43	1.46	1.49	1.52	1.53	1.54	1.55	1.55	1.55
0.17	1.22	1.27	1.33	1.37	1.41	1.45	1.47	1.49	1.50	1.51	1.52	1.52	1.52

Example 1.—Simple Cruciform Section.—If the height of the ribs at any point is made double the diameter y, of the ideal conoid, we have in the third line of the table, first and last columns, the thickness of rib $b = 0.07\,h$.

Example 2.—Suppose a core to be used and at any given place $h = 1.5\,y$, and $k = 0.6\,h$, we have, according to line 8, columns 6 and 1, $b = 0.12$ of the height h at the same place.

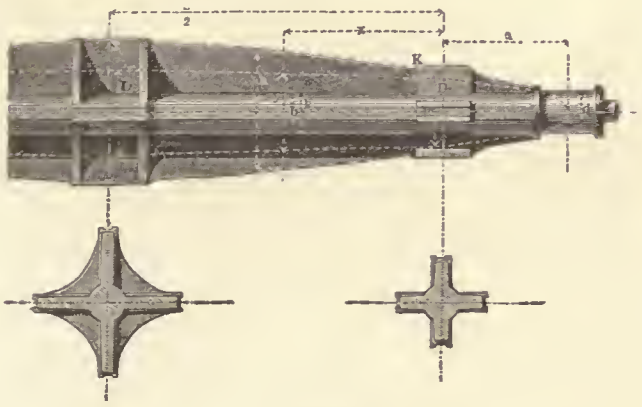

FIG. 415.

We may make b, constant and determine k, or let k be constant and b vary. The latter case is shown in Fig. 415. Here the shanks are also cruciform in section, and the hub-seats are made to receive keys, as shown in both sections, and the central one is strengthened by transverse ribs. A small auxiliary journal is shown at the end of the main journal, and is very useful in erectiou.

§ 140.

MODIFIED RIBBED AXLE.

For heavily loaded axles the form shown in Fig. 416 is suitable, the ribs being provided with flanges along the edge. Fairbairn has used such axles for water-wheels, and Rieter & Co., of Winterthur have made them for the same purpose. The proportions are determined by taking the diameter y, of an ideal shaft of circular section, and calculating h, as before. We may then make the flange thickness $c = b$, the thickness of the ribs, and then the flange breadth b_1 is obtained from the formula:

$$\frac{b_1}{b} = 1 + \frac{\frac{3\pi}{16}\left(\frac{y}{h}\right)^3 - \frac{b}{h} - \left(\frac{b}{h}\right)^3}{6\left(\frac{b}{h}\right)^2 - 12\left(\frac{b}{h}\right)^3} \quad \ldots\ldots (128)$$

from which the following table has been calculated:

$\frac{b}{h}$	VALUE OF $\frac{b_1}{b}$, WHEN $\frac{h}{y}$									
	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
0.05	——	——	——	——	——	7.94	6.17	4.81	3.64	2.75
0.06	——	——	——	——	6.99	5.38	4.13	3.17	2.34	1.07
0.07	——	——	——	6.70	5.12	3.91	3.45	2.24	1.61	1.01
0.08	——	——	6.82	5.16	3.91	3.45	2.24	1.61	1.17	——
0.09	——	——	5.45	4.11	3.10	2.33	1.73	1.01	——	——
0.10	——	6.00	4.48	3.37	2.53	1.89	1.39	——	——	——
0.11	——	5.05	3.77	2.82	2.11	1.57	1.15	——	——	——
0.12	6.56	4.34	3.23	2.42	1.80	1.34	——	——	——	——
0.13	5.73	3.78	2.81	2.10	1.56	1.15	——	——	——	——
0.14	5.06	3.34	2.48	1.85	1.38	1.01	——	——	——	——

The ratio between b_1 and b is never made greater than 6 to 7, and as it does not fall below unity the table is only given between these limits. The profile is determined for a few points and these are joined by a continuous line.

§ 141.

COMPOUND AXLES FOR WATER WHEELS.

In Fig. 417 is shown an axle for a water-wheel, made of cast and wrought iron. This was made to replace a broken axle of wrought iron, for a wheel 32.8 feet (10 m.) diameter, 19.68 feet (6 m.) in width.* The load is carried at four points, as shown,

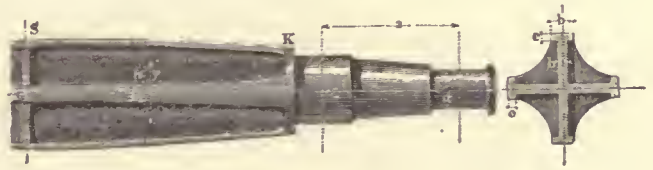

FIG. 416.

giving a total of 82,104 lbs.† The shaft consists of a drum of sheet iron ⅜″ thick and 44″ outside diameter, made in three sections riveted to the central spiders of the wheel. The two journals are fitted to the cast iron heads with a slight taper, the ends being prolonged into the middle of the drum, where they are drawn together by a right and left hand nut. The journals

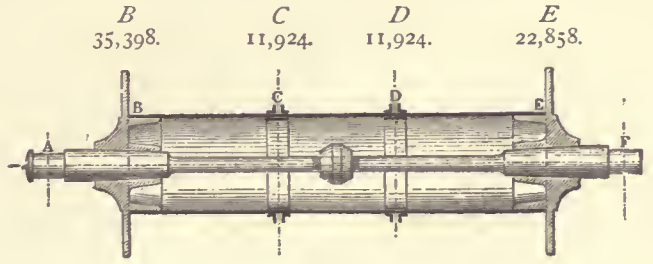

FIG. 417.

are 7⅞″ diameter and 11″ long. The circumferential joints in the drum are strengthened by pieces of angle iron as shown. The stress in the shell of the drum is only 3100 lbs., and on the riveting about 6400 lbs.

§ 142.

CONSTRUCTION OF RIB PROFILES.

In drawing the curved outline of ribs such as shown in the preceding designs, the following methods may be employed. In the various diagrams $A\,B$ is the geometric axis of the piece, S the highest point of the curve, and K the lowest point, these both being already determined.

1. *Circular Arc.*—This can only be used to advantage when on such a small scale that it can be drawn with compasses or trammel.

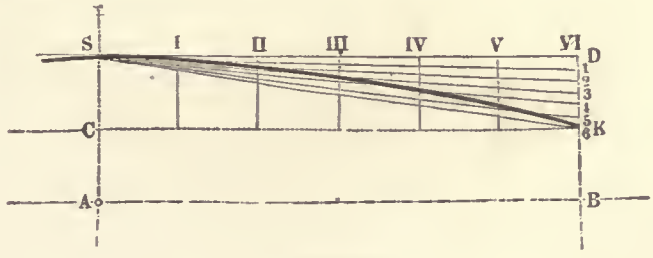

FIG. 418.

2. *Parabola.*—Draw $S\,D$ and $C\,K$ parallel to $A\,B$, divide $S\,D$ into any number of equal parts, as for example, six parts, and divide $D\,K$ into the same number. Drop perpendiculars from I, II, III, &c., join the lines S 1, S 2, S 3, &c., and the intersections of these with the perpendiculars I, II, III, &c., will be points in the parabola.

3. *Sinoide.*—Draw $S\,D$ and $C\,K$ parallel to $A\,B$; with a radius $A\,S$ draw a circle about A; divide the arc $S\,E$, cut off between $S\,D$ and $C\,K$ into six, or any number of parts; draw from the points of division, lines parallel to $A\,B$, and from I, II, III, &c., perpendiculars to $A\,B$, and the intersections will give points in the sinoide.

*This wheel belongs to the Société des Faux du Rhone, at Geneva. See Annales du Genie Civil, 1866 and 1872.

†See diagram in Fig. 409, where the loads are in this proportion.

4. *Elastic Line.*—By bending an elastic rod of uniform prismatic cross section, keeping it upon the points K_1, S, and K_2, the elastic curve may be drawn directly from the rod, using it as a ruler. For large sizes the rod may be ¾″ to 1¼″ thick, and kept under water: for smaller sizes, about ¼″ thick is sufficient.

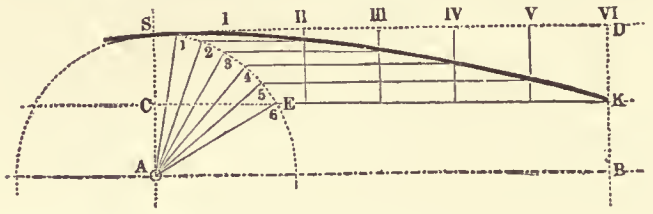

FIG. 419.

5. *Cardiode.*—The following method may be used for drawing the curve directly in the pattern loft. A wooden template $S'K$ $E\,C$ (Fig. 420) is made, in which $E\,C$ and $E\,S'$ are straight edges, and $C'\,S'' = C\,S$, and $C\,E = C\,K$. Guide points are placed at C and K, and the edge $C\,E$ kept against the point C, and the edge $S'\,E$ against the point K. The point S' of the

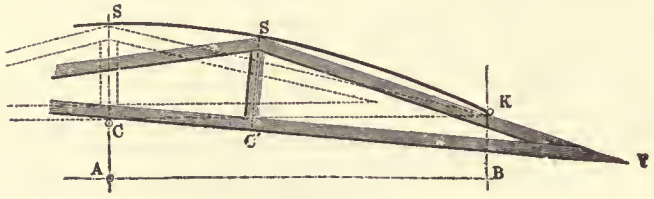

FIG. 420.

template will then describe a cardiode curve and by attaching a pencil point at S' it may be drawn directly for pattern makers' use.

The most convenient method in practice is to obtain a few points by (2) or (3), and then join them by a flexible spline or ruler.

§ 143.

WOODEN AXLES.

For some water-wheels axles of oak are still used, and these are made polygonal in section. They are made prismatic, the diameter being at all points equal to that necessary at the point of greatest stress, and the methods of attaching journals are shown in § 102. The diameter may be obtained by multiplying the diameter for cast iron by 1.55 (the cube root of the ratio of the modulus of cast iron and oak).

This must be the full actual diameter, as it is sometimes weakened by mortises cut for the arms of the wheel. Should this give a less diameter than required for the attachment of the journals, the diameter at the latter point must be taken for the whole axle. The choice between iron and wooden axles must be governed entirely by local reasons of cost and convenience.

Example.—A water-wheel axle with shanks 106.25″ long is loaded so that the journals of cast iron require to be 3½″ diameter and 5¼″ long. According to the formula in § 130 we have

$$D = 3.5\sqrt[3]{\frac{106.25}{2.625}} = 12''$$

For corresponding strength in wood, the axle should be at a minimum:

$$D' = 12 \times 1.55 = 18.6''.$$

CHAPTER IX.

SHAFTING.

§ 144.

CALCULATIONS FOR CYLINDRICAL SHAFTING.

In machine construction those axles which are used to transmit twisting moments are called shafting.

In order to fulfill this purpose two requirements must be met: first, the ultimate strength must be sufficiently great, and second, the torsional spring must be kept within proper limits. In actual practice, shafting is subjected not only to torsional stresses, but also to bending due to the weight and pressure of gears, pulleys, levers, etc., which are carried. These latter influences will not be considered at first, and the calculations made only for round, solid wrought- and cast-iron shafting.

Let:

P = the force acting to rotate the shaft;
R = the lever arm at which it acts;
N = the horse power transmitted;
n = the number of revolutions per minute;
d = the diameter of the shaft;
L = the length of shaft in feet;
ϑ = the angle of torsion in degrees;
S = the fibre stress at the circumference;
G = the modulus of torsion of the material = ⅔ of the modulus of elasticity.

We then have for strength:

$$d = \sqrt[3]{\frac{16}{\pi S}}\,PR \quad \ldots\ldots\ldots \quad (129)$$

and for stiffness:

$$d = \sqrt[4]{\frac{32}{\pi G}\,\frac{12.L}{\vartheta^\circ}\,\frac{360}{2\pi}}\,PR \quad \ldots\ldots \quad (130)$$

In order to have the same security for the shafting as already given to journals the value of S should be only ⅘ the fibre stress (see § 5), but in actual practice the stress is taken the same as for journals, viz.: for wrought iron $S = 8500$ lbs., and for cast iron $S = 4250$ lbs.

This gives the following results for strength:

For wrought iron shafts,

$$d = 0.091\sqrt[3]{PR} = 3.33\sqrt[3]{\frac{N}{n}} \quad \ldots\ldots \quad (131)$$

For cast iron shafts,

$$d = 0.114\sqrt[3]{PR} = 4.20\sqrt[3]{\frac{N}{n}} \quad \ldots\ldots \quad (132)$$

In taking the torsion of shafting into consideration the greatest allowable twist in degrees should not be over 0.075° per foot in length of shafting, that is $\vartheta^\circ = 0.075\,L$, which gives for stiffness against torsion:

$$d = 0.3\sqrt[4]{PR} = 4.7\sqrt[4]{\frac{N}{n}} \quad \ldots\ldots \quad (133)$$

and for cast iron shafts:

$$d = 0.357\sqrt[4]{PR} = 5.63\sqrt[4]{\frac{N}{n}} \quad \ldots\ldots \quad (134)$$

The quotient of effect $\frac{N}{n}$ is obtained from the relation to the statical moment $P\,R$ as follows:

$$PR = \frac{33000 \times 12}{2\pi} \cdot \frac{N}{n} = 63.020\,\frac{N}{n} \quad \ldots\ldots \quad (135)$$

From these formulæ the following table for round wrought iron shafts has been calculated. An inspection of the table will show that it is quite possible for a shaft to be strong enough to resist permanent deformation and yet be so light as to be liable to spring under its load. For example, a shaft 26 feet long, with a twisting force of 220 lbs. applied at one end, and acting with a lever arm of 20″, gives a turning moment $P\,R = 4400$ inch lbs., which would require a shaft only 1½ inches diameter (see column 2). This, however, would permit far too much torsion, and in order that the angular deflection should not exceed the limit of 0.075° per foot, a corresponding value of $P\,R$ in column 4, must be found, and against it in column 1 will be given the diameter, in this case about 2⅜″; which, by comparison with column 2, gives about five-fold strength.

For short shafts this examination of angular deflection is unnecessary, as for example, in the short lengths between two gear wheels, for here the value of ϑ will be small enough in any case. With longer shafts, and in all special constructions, it is important to consider the angular deflection and keep it within the given limit.

For shafting of cast iron the same table may be used by taking double the values for $P\,R$, or for $\frac{N}{n}$.

For steel shafts, whose modulus of resistance is ⅝ greater than wrought iron, the diameters in both cases may be taken as $\sqrt[3]{0.6}$, that is, 0.84 times that of correspondingly loaded wrought iron shafts.

Shafting which is subjected to sudden and violent shocks, as in rolling mills, etc., must be made much stronger than the preceding formulas require, and these must be classed with the special cases which occur in every branch of construction.

Example 1. A crane chain carrying a load of 5940 lbs. is operated by a drum of 7.3 inches radius, measured to the centre of the chain; required the

diameter of the shaft to resist the torsion thus brought upon it. Here $PR = 43{,}362$, which by reference to column 2 gives a diameter between 3 and 3¼ inches, say 3⅛". This would require to be somewhat increased for bending stress, for which see § 150.

Example 2. A turbine delivers 92 horse power to a wrought iron shaft, making 114 revolutions per minute, and of a length = 8½ feet. In this case $\frac{N}{n} = 0.807$, which, by column 2 in the table, would require about 3¼ inches diameter. If the deflection is not to exceed 0.075° per foot, we have, in column 4, a value of $\frac{N}{n} = 0.803$, which gives a diameter of 4½", and with this diameter the angle of torsion would be $8.5 \times 0.075 = 0.65°$. A similar case in practice has a shaft diameter of 5¼", which gives a still smaller angular deflection.

§ 145.

WROUGHT IRON SHAFTING.

d	FOR STRENGTH.		FOR STIFFNESS (Torsional)	
	PR	$\frac{N}{n}$	PR	$\frac{N}{n}$
1	1,327	0.021	123	0.0019
1¼	2,591	0.052	301	0.0048
1½	4,479	0.071	625	0.0099
1¾	7,112	0.114	1,157	0.0183
2	10,616	0.168	1,975	0.0313
2¼	15,115	0.239	3,164	0.0502
2½	20,730	0.329	4,822	0.0765
2¾	27,600	0.438	7,061	0.1120
3	35,830	0.568	10,000	0.1587
3½	56,890	0.902	18,520	0.2941
4	84,930	1.347	31,600	0.5015
4½	120,900	1.919	50,620	0.8032
5	165,800	2.632	77,160	1.2240
5½	220,800	3.503	11,000	1.7920
6	286,600	4.548	160,000	2.5390
6½	364,400	5.784	220,300	3.4960
7	455,200	7.222	296,400	4.7040
7½	559,800	8.883	390,600	6.2000
8	679,400	10.780	505,700	8.0240
8½	815,000	12.930	644,400	10.2300
9	967,400	15.350	810,000	12.8600
9½	1,138,000	18.050	982,700	15.6000
10	1,327,000	21.050	1,230,000	19.5900
10½	1,536,000	24.380	1,501,000	23.8100
11	1,766,000	28.020	1,808,000	28.6800
11½	2,018,000	32.020	2,159,000	34.2600
12	2,293,000	36.390	2,560,000	40.6200

§ 146.

LINE SHAFTING.

In the previous discussion we have assumed that the bending forces upon shafting might be neglected. As a matter of fact, this is rarely the case, only occurring when the turning moments are those due to a simple force couple. Nearly all the shafting used for power transmission is subjected to bending stresses due to belt pull, pressure of gear teeth, weight of gears and pulleys, and to take all of these into consideration would make a very complicated calculation.

In most cases ample strength will be given by taking the diameters according to the formulas (133) or (134). As already shown, these give ample strength, so that any ordinary bending stresses are provided for. These give reduced diameters for the higher speeds, shafting for high speed machinery running at 120, 140 or even 200 revolutions per minute.

First movers run a lower speed and are proportionally heavier, and the line shafting generally is gradually reduced in diameter in the successive ascending floors of a building. Such line shafting is only occasionally made of cast iron, when moderate power is to be transmitted.

The practice in the proportion of shaft diameters is not altogether consistent. In many cases very high stresses are permitted, as in the case of locomotives, in which stresses of 12,000 to 15,000 lbs. are borne by wrought iron cranked axles; shafts of screw propeller engines usually carry 7,000 to 8,500 lbs., while in many instances the stresses upon line shafting are very light, when the high rotative speed is taken into consideration. This is particularly the case in England, the shafting running at higher speeds with a proportional reduction in diameter. The greatest difficulty to be encountered lies in the fact that the forces are rarely given with sufficient accuracy, the so-called "nominal" horse power which a shaft is supposed to transmit bearing no definite relation to the actual power. In most cases, however, the use of the formulas above given for stiffness, with a slight increase for very long shafts, will give satisfactory results.

A few examples will serve to illustrate the manner in which the methods given may be applied, and the remarks which have been made should be borne in mind in connection with the application.

Example 1. The screw shaft of a large war ship is driven by two cylinders, each exerting a total pressure of 176,000 pounds, on cranks of 21.75" radius, situated at right angles to each other. The shaft is of wrought iron, and between the crank shaft and the propeller it is 72 feet long, by 15" diameter. Calculating this for strength by formula (131) we have:

$$PR = 2\sqrt{0.5 \times 176{,}000 \times 21.75} = 5{,}412{,}000$$
$$d = 0.091\sqrt[3]{5{,}412{,}000} = 15.98", \text{ say } 16".$$

If it is desired that the torsional deflection shall not exceed 0.075° per foot of length, formula (133) must be used, giving:

$$d = 0.3\sqrt[4]{5{,}412{,}000} = 14.47".$$

This is somewhat less than the previous dimensions, and consequently the deflection will be less than $72 \times .075 = 5.4°$.

Example 2. In the mills at Saltaire there is a cast iron driving shaft making 92 revolutions per minute, and transmitting 300 horse power, the diameter being 10 inches. According to formula (134) the diameter would be:

$$d = 5.63\sqrt[4]{\frac{300}{92}} = 7.56",$$

so that the actual shaft is ¼ stronger, and the other shafts in the mill are proportionally heavy.

Example 3. In the rolling mill at Rheinfall is a line of wrought iron shafting, 223 feet long, transmitting 120 horse power. The speed is 95 revolutions, giving the ratio $\frac{N}{n} = \frac{120}{95} = 1.263$. The diameter for strength, as given from column 3 of the table, would be about 3¾". The actual sizes are 3⅞" in the journals, and 4" in the body. The corresponding fibre stresses are 7,397 lbs. in the journals, and 6,541 lbs. in the body of the shaft. According to the formula of Fairbairn, who designed the mills at Saltaire, this shaft would have been made

$$d = 7.4\sqrt[3]{\frac{N}{n}} = 8.03",$$

or nearly eight times stronger than was actually used.

Example 4. In the spinning mill at Logelbach there is a cast iron shaft 8¼" diameter, making 27 revolutions per minute and transmitting 140 horse power by actual measurement. The ratio $\frac{N}{n} = \frac{140}{27} = 5.19$.

Taking the double value in the table, since the material is cast iron, we find in column 5, that $d = 8\frac{1}{2}$. The diameter, for strength only, would be found by column 3 to be about 7¾".

Example 5. In the same mill is a line of cast iron shafting, 84 feet long, transmitting 270 horse power, and making 50 revolutions, hence $\frac{N}{n} = 5.4$.

The journals are 6¾" diameter, and the body of the shaft is the section shown in Fig. 413, and its section approximates to that of a cylindrical shaft of 8½" diameter. For such conditions the table gives in column 3, taking double the value of $\frac{N}{n}$, we get $d = 8"$. The diameter 6⅞" in the journals gives a fibre stress of about 5,200 lbs. From the length of the shaft it is advisable to take the diameter for stiffness, which we get from the value corresponding to $2\frac{N}{n} = 10.8$ in column 5, which gives $d = 8\frac{5}{8}"$, which is quite close to the actual dimensions.

§ 147.

DETERMINATION OF THE ANGLE OF TORSION.

In a cylindrical shaft of a diameter d, which transmits a statical moment PR, throughout its length L, the modulus of torsion of the material being G, we have from No. I, § 14, the angle of torsion.

$$\vartheta = \frac{PRL}{JpG} = \frac{S}{G}\,\frac{L}{a}$$

$$\vartheta^\circ = \frac{32 \cdot 360\,PR \cdot 12\,L}{2\pi^2 \cdot d^4 \cdot G} = \frac{360\,S}{\pi G}\,\frac{12\,L}{d} \quad . \quad (136)$$

which for wrought iron, in which $G = 11{,}386{,}000$ gives

$$\vartheta^\circ = 0.00062\,\frac{PRL}{d^4} = 0.0001208\,S\,\frac{L}{d} \quad . \quad . \quad . \quad . \quad (137)$$

For cast iron these values are doubled, giving

$$\vartheta^\circ = 0.00124\,\frac{PRL}{d^4} = 0.0002436\,S\,\frac{L}{d} \quad . \quad . \quad . \quad . \quad (137)$$

Here L is taken in feet and S is the stress at the point of application on the shaft. It will be noticed that the angle ϑ can be determined very readily when S is known. It must be remembered that d and S are closely related, and that the value of d depends upon the value taken for S.

Various applications of twisting moments may be reduced to a single one for use in the formulas, by classification under some one of the following heads, taking the value for L as follow (see § 13, § 14):

(a). L = the whole length of the shaft, in feet, when the force is applied at one end and transmitted to the other.
(b). L = half the length of the shaft when the twisting forces are applied over the entire length uniformly.
(c). L = one-third the length of the shaft when the twisting forces diminish uniformly from one end to the other of the shaft, as in § 14, case III.
(d). In general, the distance of the point of application of a collected number of twisting forces distributed in any manner along the length of the shaft, may be found by multiplying the power applied at each point (in horse power), by its distance from the end of the shaft, adding the several products together and dividing by the total horse power transmitted.

The methods may be illustrated by the same examples which were given in the preceding section.

Example 1. The screw propeller shaft given in the previous Example 1, gives the following data:

$$S = 8{,}200 \text{ lbs.}, \; d = 15'', \; L = 72 \text{ feet.}$$

According to (137) we have

$$\vartheta = 0.0001218 \frac{8200 \times 72}{15} = 4.75°.$$

This will be reduced to $\frac{1}{10}$ this value, or $3\frac{1}{3}°$ when either of the cranks is on the dead centre.

Example 2. The line of shafting given in Example 3, of the preceding section, is made of two diameters in the bearings and in the body, and these must be combined. The bearings may be taken at 4 inches long each, and are 32 in number. We have then

$$\vartheta = 0.0001208 \left[\left(\frac{32 \cdot 0 \cdot 33 \cdot 7397}{3.875} \right) + \left(\frac{223 - 10) 6541}{4} \right) \right] =$$

$$= 0.0001208 \; (20138 + 348310) = 44\tfrac{1}{2}'' \; !!$$

a deflection which must be very marked, with variable loads, and entirely inadmissible with fine machinery.

Example 3. If the preceding shaft had been made 8 inches diameter, as by Fairbairn's formula, we have for $\frac{N}{n} = 1.263$.

$$\vartheta° = 0.00062 \frac{62500 \times 1.263 \times 223}{(8)^4} = 2.67°.$$

Example 4. In the twine factory at Schaffhausen there is a shaft made of Bessemer steel. The length is 489 feet, and it transmits 200 horse power from the Rhine up the bank and an angle of 23°. The diameter is $4\frac{13}{16}''$, $N = 200$, $n = 120$. This gives $S = 4756$, and if we take the modulus of elasticity of the steel the same as wrought iron, we have

$$\vartheta° = 0.0001218 \frac{4756 \times 489}{4.8125} = 58.34°.$$

Example 5. A shaft 164 feet long, and of a constant diameter, transmits 70 horse power at 100 revolutions. The power is taken off by a number of machines, ranged at uniform distances apart. According to the table for $\frac{N}{n} = 0.7$ the diameter should be about $4\frac{1}{4}''$. In determining the torsion the value of L is taken at one-half the length of the line (case b) giving:

$$\vartheta° = 0.00062 \frac{62500 \times 0.7 \times 82}{(4\frac{1}{4})^4} = \text{about } 6\tfrac{3}{4}°.$$

Since the formula is based on an angular deflection of 0.75° per foot, we might have obtained direct, $82 \times 0.75 = 6.35°$, or nearly the same value.

If in any case the calculated deflection appears too great, the diameter of the shaft must be increased, and since the denominator of the equation is the fourth power of the shaft diameter, a slight increase in its value effects a marked reduction in the deflection.

Example 6. If the angle in the preceding example is desired to be reduced to one-half its value, the diameter must be multiplied by $\sqrt[4]{2}$ or by 1.189, hence $d = 4.25 \times 1.189 = 5$ inches.

§ 148.

Journals for Shafting. Round Rolled Shafting.

The journals on shafting are either end journals, and treated as already shown, or, as in most cases, necked journals, and the length of bearing made as given in § 92. For line shafting the special determination of journals is unnecessary. Unless there is some apparent reason for a special determination of the journals (as in the case of locomotives), the journal length l is usually taken quite large, as $\frac{3}{2}d$, $2d$, $4d$ (see § 109 *et seq.*), care being taken to insure proper support of the journals in the hangers.

The Kirkstall Forge Company, of Leeds, have produced shafting which is rolled round and requires no turning. The round finish is given by the action of plane discs whose geometric axes are horizontal and parallel, about eight inches apart, and revolve rapidly. (See § 195.) The discs are placed so as to act upon the bar as it leaves the rolls, and are cooled with water, and their action produces a true cylindrical form to the shafting, and gives it a highly finished surface, so that it is at once ready for use without being turned in a lathe. By this process the modulus of resistance is also increased nearly 20 per cent. over that of shafting rolled in the ordinary manner, as shown by tests by Kirkaldy, and given in the catalogue of the Kirkstall Forge Company, for the Melbourne Exposition. This feature is not of as much importance as at first appears, although it is of some value.

The absence of turning is also of advantage, and the increasing use of this shafting is doubtless due to both causes. The principal objection to it lies in the fact that the hard outer skin cannot be disturbed without affecting the truth of its form. Keyways cut in it invariably cause springing.

Some of the modern methods of securing pulleys without cutting keyways may be used to avoid this difficulty. The journals and wheel seats on this kind of shafting do not require turning.

§ 149.

Combined Sections. Wooden Shafting.

The dimensions for shafting of various combined sections (tubular, cruciform, fluted) are determined by finding the size for round shafting of the same material, and then deducing the dimensions of the desired section in the same manner as given for axles in §§ 138 to 142.

Axles of wood (generally oak) are made of polygonal section described about a circle not less than 1.75 times the diameter of a cast iron shaft for the same work, this being the fourth root of the ratio of the moduli of elasticity of the two materials. Wooden shafting is now seldom used.

§ 150.

Shafting subjected to Deflection.

Shafting is often loaded in such a manner as to be subjected to bending stresses, and as already seen, this is the most general condition in which it is used. Under these circumstances the combined resistance must be taken into consideration. This is most conveniently done by assuming an ideal bending moment (see § 18).

Let M_d be the twisting moment for a given shaft section,
M_b be the bending moment for the same section;
then the ideal bending moment combining them both will be:

$$(M_b)_i = \tfrac{3}{8} M_b + \tfrac{5}{8} \sqrt{M_b^2 + M_d^2} \quad . \quad . \quad . \quad . \quad (139)$$

This formula may be simplified for numerical calculations by Poncelet's theorem, approximately:

When $M_b > M_d$ take $(M_b)_i = 0.975 \, M_b + 0.25 \, M_d$ . . (140)

and when $M_d > M_b$ take $(M_b)_i = 0.625 \, M_b + 0.6 \, M_d$ . . (141)

An examination will be made, first by the analytical, and then by the graphical method.

I. *Analytical Method.*—The axle or shaft $A \; B \; C$, shown in Fig. 421, carries a gear wheel R at C, which acts tangentially to

FIG. 421.

rotate the shaft with a moment $M_d = Q \, R$, and also acts to bend the shaft with a force whose reactions are parallel to Q, and are $P_1 = \frac{Q \, s}{a + s}$ at A, and $P_2 = \frac{Q \, a}{a + s}$ at B. The greatest stress is at C, for there both bending moments are at their maximum $M_b = \frac{P_1}{a} = \frac{P_2}{s}$, hence calculation should be made for this point.

Example.—Let $Q = 5500$ lbs. $R = 11\frac{3}{4}''$, $a = 19\frac{3}{4}''$, $s = 78\frac{3}{4}''$, then

$$P_1 = \frac{78.75}{98.50} Q = 0.8 \; Q = 4400 \text{ lbs.}$$

$$P_2 = \frac{19.75}{98.50} Q = 0.2 \; Q = 1100 \text{ lbs.}$$

Also $M_d = 5500 \times 11.75 = 64{,}625.$

$M_b = 4400 \times 19.75 = 86{,}900.$

Hence $M_b > M_d$ and formula (140) is used.

We have $(M_b)_i = 0.975 \times 86{,}900 + 0.25 \times 64{,}625 = 84{,}727 + 13{,}656 = 98{,}383''$ lbs.

From this the diameter at C can be calculated. If the shaft is of cast iron with cruciform section, we have for the diameter D,

$$(M_b)_i = S \frac{\pi}{32} D^3$$

and taking $S = 4250$ we have

$$D = \sqrt[3]{\frac{98{,}383 \times 32}{4250 \, \pi}} = 6\tfrac{1}{4}''.$$

The journal at A is found in the table of § 91, column 4, to be about $2\frac{7}{8}''$. For the neck journal at B, we have from the table of § 145, taking the double value for cast iron, $d_2 = 4\frac{1}{2}''$.

Graphical Method.—The same example may be solved graphically. In Fig. 422, with a horizontal closing line, construct the link polygon $a\,b\,c$, for the bending moments, and the force polygon a 10, giving the forces P_1 and P_2, and also $a\,c\,c'$, the surface of moments for the shank $A\,C$.

The moment Md is yet to be determined. In the force polygon with a distance R from the pole O, draw a vertical ordinate; this will be Md. Lay its value off at $c'\,c_1$, and $b\,b_1$, and ⅝ of these values give $c'\,c_0\,b_0\,b$ for the parallelogram of torsion for the shank $C\,B$.

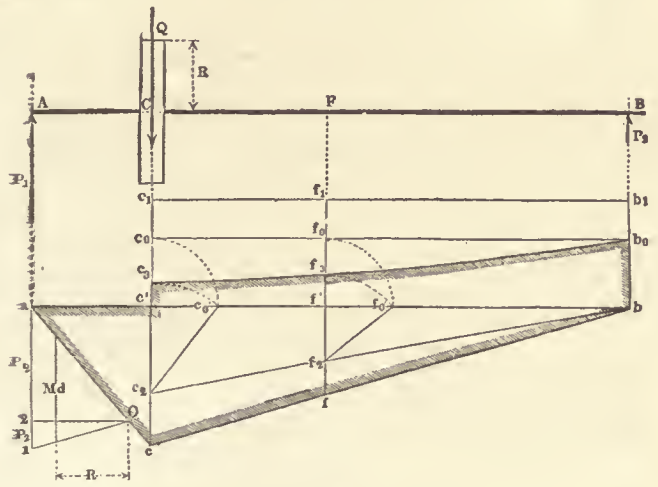

FIG. 422.

The combination of the bending and twisting moments may then be made by formula (139). Make $c\,c_2 = \frac{3}{8}\,c\,c'$ and join $c.\,b$, then at any point of the polygon, as for example at f, the distance $f\,f_2 = \frac{3}{8}\,f\,f'$. Now transfer $c'\,c_0$ to $a\,b$, at $c'\,c_0'$; then will the hypotenuse of the triangle $c_2\,c'\,c_0'$ divided by $c_2\,c_0' = \sqrt{(\frac{3}{8}\,c\,c')^2 + (\frac{5}{8}\,c_1\,c')^2}$, and the sum $c\,c_2 + c_2\,c_0' = c\,c_2 + c_2\,c_3$ the desired moment $(M_b)_i$ for the point C. In the same manner we obtain $f\,f_2 + f_2\,f_0' = f\,f_2 + f_2\,f_3$ the moment $(M_b)_i$ for the point F. The line $c_3\,f_3\,b_0$ is a curve (hyperbola) which may be taken approximately with sufficient accuracy as a straight liue $c_3\,b_{02}$. The various dimensions may be obtained from the polygon $a\,c\,b\,b_0\,c_3\,c'$ in a similar manner as shown in the discussion of axles.

Other discussions of this subject will be given when considering rock shafts and crank axles.

CHAPTER X.

COUPLINGS.

§ 151.

VARIOUS KINDS OF COUPLINGS.

The devices by means of which the different lengths of shafting are connected together so that the motion may be transmitted from oue piece to the next, are called couplings.

They may be classed as follows:

1. Rigid Couplings.
2. Flexible Couplings.
3. Releasing, or Clutch Couplings.

The first class includes the various forms of coupling for line shafting and the like, in which the coupling and the coupled portions have the same geometric axis. Flexible couplings are those which permit more or less change in the relative position of the coupled shafts; while clutch coupliugs are constructed so as to be thrown in and out of engagement, usually when the parts are in motion. These three classes are all shown in various forms in the following examples:

§ 152.

I. Rigid Couplings.

Rigid couplings may be made either in a single piece, or in several parts. Of the first sort is the so-called Muff Couplings, Fig. 423. The muff is fitted over both pieces of shafting, and a single key binds the parts all firmly together.

In giving the proportions of the various parts of the following couplings, we may take for a unit or modulus the thickness δ of the hub, making its value equal to:

$$\delta = \frac{d}{3} + \frac{3''}{16} \quad \ldots\ldots\ldots\ldots \quad (142)$$

d being the diameter of the shaft, whether of wrought or cast iron. The dimensions of the key may be taken as given in § 68, Formula (71) for torsion keys.

More recently, in exposed situations, the projecting end of the key is covered with a cap, in order to avoid accidents.

The form of coupling shown in Figs. 189 and 190, § 69, looks very practical, but the test of prolonged use will be necessary to demonstrate its merits.

FIG. 423.

The simplest two-part coupling is the well-known plate coupling, Fig. 424, and its form permits the nuts and bolt-heads to be kept below the projecting flanges, and thus out of the way.

The number of bolts in a plate coupling $i = 0.8\,d + 2$. The diameter d' of the bolts should be $0.125\,d + \frac{5}{16}''$, which gives a strength proportional to that of a shaft calculated by formula (133), or if d' is determined from formula (133) the bolt will be strong enough.*

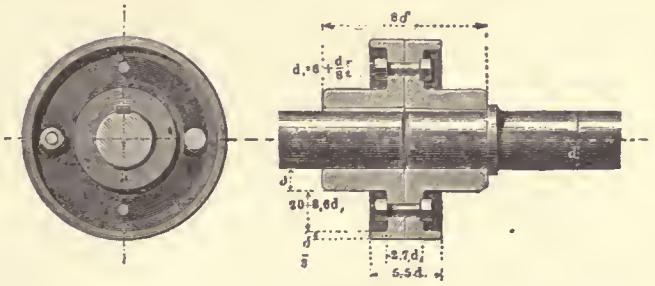

FIG. 424.

Plate couplings are extensively used in England and Germany, although they are being superseded by later forms. Their strength has caused them to be used for coupling the lengths of screw propeller shafts, and in this case the plates are forged

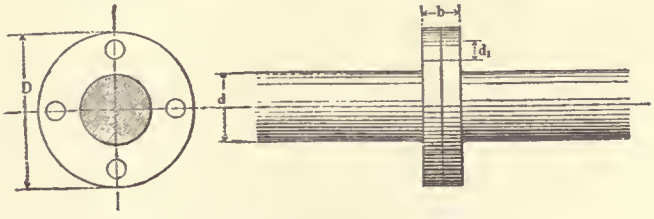

FIG. 425.

on the shafts, thus dispensing with the use of a key, Fig. 425. This form was introduced by Langdon in 1852, and is in general use, 4 to 6 bolts being used.

Examples: The following cases will serve to give the proportion of such plate couplings in executed designs.

Jason, James Watt & Co., $d = 12''$, $D = 24''$, $d_1 = 3''$, $b = 6''$, $i = 4$.
Warrior, John Penn & Son, $d = 17''$, $D = 37''$, $d_1 = 4''$, $b = 10$, $i = 6$.
Vessel by Ravenhill & Hodgson, $d = 12''$, $D = 25$, $d_1 = 3''$, $b = 6''$, $i = 4$.

Fig. 426 shows a clamp coupling divided into two parts longitudinally. This form is provided with two keys, and the manner in which it is bolted together. If it is desired to clamp the

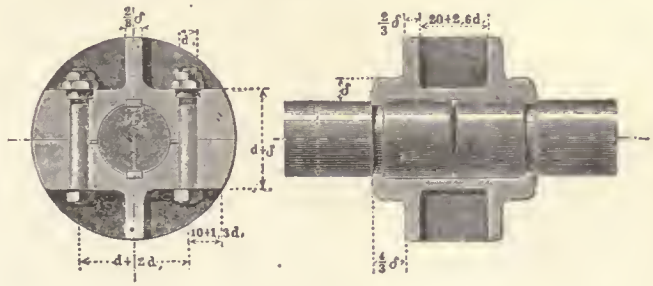

FIG. 426.

shafts together endwise, the small circumferential grooves and lips may be used as shown. Such grooves may be used in depth equal to $0.01\,d + \frac{1}{16}''$, but may be omitted where endlong clamping is unnecessary. If lock nuts are used on the bolts the main

* The dimensions in Fig. 424 are correct for English measurements, except the bolt diameter, which is as given above, and the distance from hub to inside of flange, which should be $2.6\,d + \frac{13}{16}''$.

nuts may be counter-sunk as shown in the illustration. The number of bolts is = 2, 4 or 6, rarely more, and of diameter as follows:

$$i = 2, \qquad 4, \qquad 6 \text{ or more.}$$
$$d_1 = \frac{d}{6} + \frac{3}{8}'', \quad \frac{d}{7} + \frac{11}{32}'', \quad \frac{d}{8} + \frac{5}{16}''.$$

Example: For a shaft $2\frac{3}{8}''$ diameter, with a coupling fitted with two bolts the diameter $d_1 = 0.77''$, say $\frac{13}{16}''$, for four bolts $d_1 = 0.72''$, say $\frac{3}{4}''$, for six bolts $d_1 = 0.67$, say $\frac{11}{16}''$.

This form of coupling has been made with bolts with differential thread passing through both parts and giving increased clamping force.*

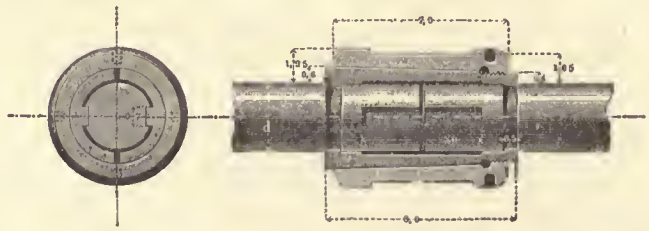

FIG. 427.

The cone coupling shown in Fig. 427 is the design of the author, and is a modification of the preceding form. The keys are cast in one with the halves of the inner cone, and are planed to fit the keyways in the shafts. The cone is made with a taper of $\frac{1}{40}$ on a side, which will hold the parts securely when driven on, without any other fastening. If there is much vibration, however, it is advisable to have a screw thread cut on the inner cones as shown, and the outer shell tightened by a spanner. In most ordinary cases the screw may be omitted, and a small steel countersunk set screw tapped into each side of the shell to clamp the inner cone. If endless motion need not be considered the circumferential grooves may be omitted.

With couplings for shafts larger than $2\frac{1}{2}''$, the bearing surfaces should be recessed to reduce the amount of finishing.

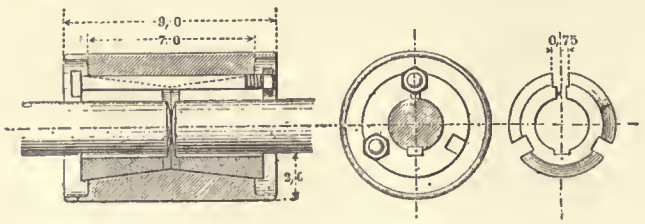

FIG. 428.

In America Sellers has introduced a clamp coupling in which two cones are opposed to each other and drawn together by three bolts, the whole being inclosed in a cylindrical shell bored out to fit the cones as shown in Fig. 428. The cones are cut through on one side so that they are compressed by the action

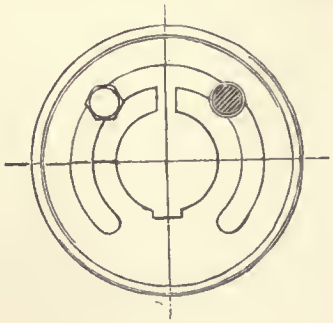

FIG. 429.

of the bolts. A key is let into the cones and shafts diametrically opposite the cut in the cones. An especial advantage which results from the double cone construction lies in the fact that it is not necessary for the two shafts to be exactly the same diameter.†

In England Butler's cone coupling has been used, and was designed for use with the cold rolled shafting described in § 148. It is similar in construction to Sellers', the three bolts being replaced by a single conceutric screw thread and nut at each end. The key which Sellers uses is omitted in Butler's coupling, the shafts being held only by the clamping action of the cones.

In the United States Cresson's coupling is also much used. Its construction is shown in Fig. 429. The clamping surfaces are cast in one with the outer shell, and forced upon the shafts by means of the tapering screws. This coupling possesses the same advantage as does Sellers', in being adapted to shafts of slightly unequal diameters.

II. Flexible Coupiings.

§ 153.

VARIOUS KINDS OF FLEXIBLE COUPLINGS.

Couplings which permit the shafts to change their positions may be required to meet three different conditions. The motion of the shafts may be:

(*a*) Lengthwise, or in the direction of the axis.
(*b*) Crosswise.
(*c*) Angular, the shafts being inclined to each other.

In some cases two, or even all three, of these conditions may be present. In the first place the axes of the two shafts coincide; in the second case they are parallel to each other; in the third case they intersect, while in the combination of (*b*) and (*c*) the axes pass each other. All these cases occur in actual practice and meet with useful applications.

§ 154.

COUPLINGS FOR LENGTHWISE AND PARALLEL MOTIONS.

Endlong motion of the coupled shafts may be provided for by giving various prismatic forms to the parts of the coupliugs. As an example, Sharp's Coupling, Fig. 430, may serve. This permits but slight movement lengthwise, and also a little angular displacement, and is therefore suitable for positions where

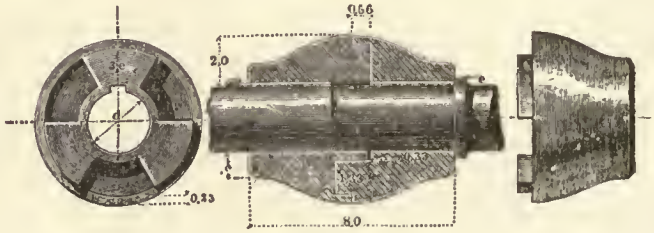

FIG. 430.

the bearings cannot be accurately placed. In some recent examples of this form of coupling, one half is first made and the second cast upon it, and in this case the outer recess is omitted.

In some French screw steamships in which the screw is arranged to be lifted, one of the shaft couplings is arranged so that sufficient endlong motion may be obtained to permit the end of the shaft to be withdrawn from the hub of the screw. ‡

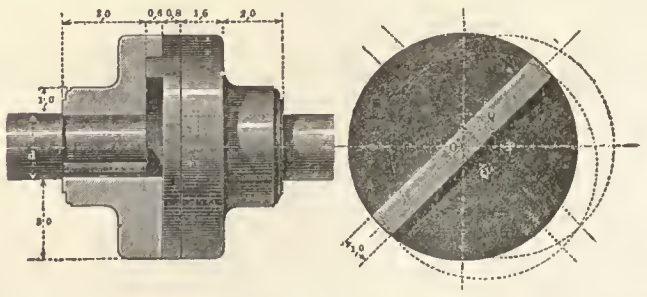

FIG. 431.

For shafts in which the displacement is crosswise, the axes of the shafts remaining parallel, Oldham's Coupling, Fig. 431, is most applicable. This consists of two end pieces and one intermediate piece. The latter has a prismatic tongue upon each side, the two being placed at 90° with each other, and fitting into corresponding grooves in the end plates, which latter are keyed on to the shafts. If the two axes of the shafts coincide

* Ruggles' Coupling, *Pract. Mech. Jour.*, 1866, p. 185. In Fig. 426 there are two dimensions which require transforming: for $20 + 2.6\,d$, use $\frac{3}{4}'' + 2.6\,d$, and for $10 + 1.3\,d$ use $\frac{3}{8}'' + 1.3\,d$.

† Among other tests the Sellers Coupling stood the following: Two shafts 10 feet long were fitted in three hangers, the middle hanger next the coupling being set $1\frac{3}{4}''$ out of line, and after several weeks' time, at 250 revolutions per minute, the coupling remained intact.

‡ Among various interesting examples of such couplings may be mentioned those described in Armengand's "Vignole des Mecaniciens," Paris, 1863, Plate 9; also Ledieu, App. a vapeur de navigation, Paris, 1862, and Ortolan, Mach. à vapeur marines, Paris, 1859.

at O, the tongues and grooves have no sliding action upon each other. If one of the axes is moved parallel to itself, say to P, the middle of the intermediate piece will describe a circle $O\,Q\,P\,Q'$ of the diameter $O\,P$ = the distance between the axes, making two revolutions for every revolution of the shafts, the other points of the disc describe cardiode paths. The velocity ratio remains constant.

Another form of coupling for the purpose consists of two cranks connected by a short drag-link to permit the necessary movement. This is frequently used in connecting engine shafts.

§ 155.

Jointed Couplings.

The best known of all the flexible couplings is the Universal Joint, known also as Hooke's Coupling, and also as Cardan's Coupling.* This form of coupling permits the connection of inclined axes within certain limits. It consists broadly of two end pieces, and a middle piece, the latter containing two pairs of journals placed at right angles with each other in the form of a cross, each pair fitting into journals on one and the other of the end pieces respectively.

The rate of transmission of motion is not uniform, and is dependent upon the angle of inclination a of the two shafts, the angular rotations ω and ω_1 of the driving and driven shafts being expressed in the following ratio:

$$\frac{\tan \omega_1}{\tan \omega} = \cos a \quad \ldots\ldots\ldots\ldots \quad (143)$$

which gives a periodical variation whose period is 180°. The following table gives the values of ω_1 for successive values of ω, for various angles a:

ω	$a = 10°$	20°	30°	40°
30°	29° 38′	28° 29′	26° 34′	23° 51′
45°	44° 34′	43° 12′	40° 54′	37° 27′
60°	59° 34′	58° 26′	56° 22′	53° 04′
90°	90°	90°	90°	90°
120°	120° 26′	121° 34′	123° 38′	126° 56′
135°	135° 26′	136° 48′	139° 06′	142° 33′
150°	150° 22	151° 31′	153° 26′	156° 01°
180°	180°	180°	180°	180°

For small values of a the variation is unimportant. For the angular velocities a, and ω_1, we have the relation:

$$\frac{\omega_1}{\omega} = \frac{\cos a}{1 - \sin^2 \omega \sin^2 a} \quad \ldots\ldots \quad (144)$$

which gives a maximum $\frac{1}{\cos a}$ and a minimum $\cos a$. These variations in velocity may be neglected when the moving masses are inconsiderable and the angle a is small †

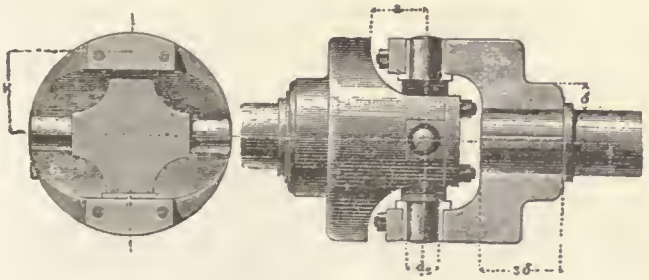

Fig. 432.

The detailed construction of the coupling admits of great variations. Fig. 432 shows a form with cast-iron end pieces and wrought iron middle piece. The relation $\frac{R}{d}$ is varied also. The journal diameter d_2 is determined by the methods already given, and from the moment of rotation $(P\,R)$ the journal pressure P_2 may be taken with sufficient accuracy as $= \frac{½\,(P\,R)}{R}$. The distance a should be made greater as the angle a is increased, a being made quite in the illustration. The joints in the boxes should be made in the plane of the shafts, not at right angles to them, in order to provide for the wear.

Universal joints are used to good advantage in screw propeller shafts in order to provide for the flexure due to the elasticity of the hull of the vessel. In such cases two universal joints are used on a shaft. A coupling for such service is shown in Fig. 433. Here all these pieces are forgings, one end piece being forged solid with the shaft. The middle piece is formed of a double ring, the bearings being held between the two parts,

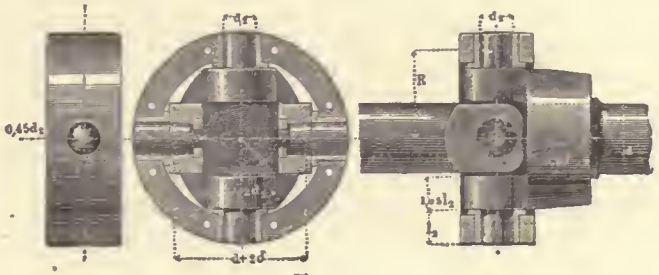

Fig. 433.

while the journals are secured to the end pieces. No provision is made for taking up the wear upon the journal boxes, as the angle a is so small that the wear is very slight. The length l_2 of the journals need not be great, about 1 to 1.25 d_2, and since R can be kept small, the dimensions of the entire coupling may be kept within reasonable limits.

Another form of universal joint is shown in Fig. 434. Here the cross journals are made in the form of through bolts, passing through both end and middle pieces. This requires a slight modification in the form, since the axes of the two bolts cannot intersect each other. A slight error in motion follows, causing

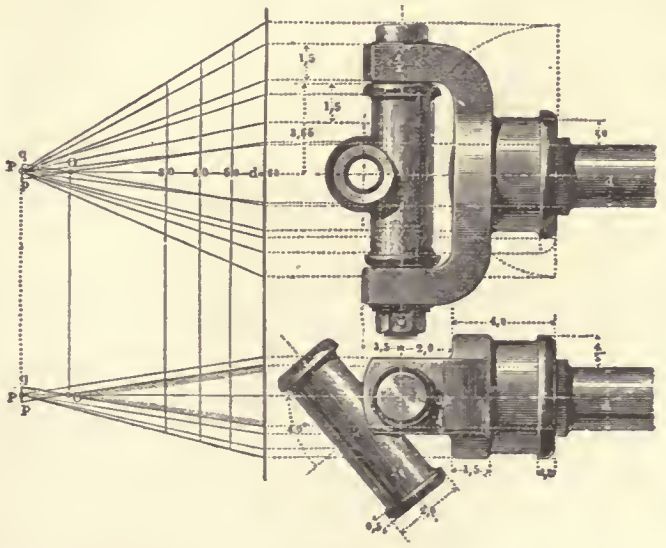

Fig. 434.

a very small endlong motion with each revolution in the coupled shafts, but this is generally insignificant. This form is suitable for agricultural machinery, horse powers, etc. In the illustration a proportional scale is given. The modulus is as before, $\delta = \frac{1}{8}\,d + \frac{3}{16}''$. The pole P is therefore taken, so that $\frac{1}{8}\,d + \frac{3}{16}'' = o$, or so that $d = -\frac{9}{16}''$.

The irregularity in motion shown in formula (144) is generally of little consequence, and need not be considered except in cases requiring geometrical exactness, as in the connections for large tower clocks, or in cases where large masses are drawn at high velocities, as in threshing machines. The variation can be obviated by the use of a double universal joint, which consists of two simple couplings. If, Fig. 435 a, the driving shaft A is coupled to B by means of a short intermediate shaft C, the connections being made by two similar couplings of the same angles, then the motion of A will be transmitted to B without variation. In this case the driven shaft may be given several positions with regard to A, being placed at B, making the angle $2\,a$, or at B', parallel to A, or at B'' on the surface of a cone of half the angle a, which is made with the intermediate axis C. The two universal joints are similarly placed when the cross axes belonging respectively to A and B lie in the same plane as the shaft C *at the same time*. In the positions B and B' all three shafts lie in the same plane, but not in position B''. In the last case A and B intersect.

* If not the original inventor of the Universal Joint, the Italian, Cardan, was the first to describe it (1501–1576), and the Englishman, Hooke (1635–1702), first applied it for the transmission of rotary motion.

† In the table the values of ω are so placed that when $\omega = O$, the cross journals of the coupling lie in the plane of the shafts.

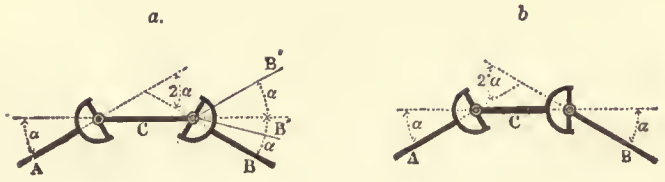

FIG. 435.

If the cross axes are not placed similarly, but, for example, at 90°, as in Fig. 435 *b*, the variations of motion are increased, and we have.

$\tan \omega_1 = \tan \omega \cos^2 a$, in which ω and ω_1 stand for A and B. If $a = 30°$, we have for $\omega = 45°$, $\tan \omega_1 = (\frac{1}{2}\sqrt{3})^2 = 0.75$, hence $\omega_1 = 36° 54'$ instead of $40° 54'$, as in the table above.

A concealed form of universal joint is that used in rolling mills, the cross being formed upon the end of the roller.

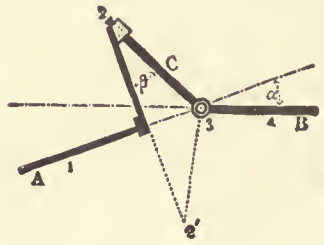

FIG. 436.

Fig. 436 shows a form of link coupling designed by the author. A is the driving, and B the driveu shaft, the arm C being jointed at 3 by journals at right angles to B, while at 2 it slides in a bearing rigidly attached to A. The axis B makes with A an angle a, and the piece $3\,C\,2$ makes with A the angle $90° - \beta$. For the relatiou between the angular rotation ω_1 and ω_4 of the shafts A and B, which are supposed to revolve in fixed bearings, we have:

$$\left.\begin{aligned} \tan \omega_1 &= \tan \omega_4 \left(\cos a - \frac{\sin a \tan \beta}{\cos \omega_1} \right) \\ \text{or} \\ \tan \omega_4 &= \frac{\sin \omega_1}{\cos a \cos \omega_1 - \sin a \tan \beta} \end{aligned}\right\} \quad . . (145)$$

In this case the transmissiou of motion is much more irregular than with the universal joint, since the angular velocity ratio between

$$\frac{1}{\cos a \mp \sin a \tan \beta} \text{ and } \frac{\cos a}{1 + \sin^2 a \tan \beta}$$

fluctuates. This coupliug is really only a modification of the universal joint, inclined to such an extent that the fork of the shaft A stands at right angles to the axis. (Compare formula (144) with (145)). By combining two such coupliugs symmetrically with each other, as in Fig. 437, the motion will be uniformly transmitted. The two sleeves at 3 and 4 are formed in

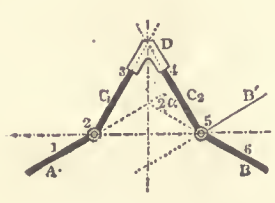

FIG. 437.

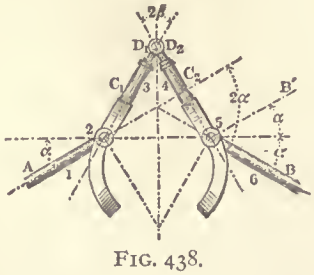

FIG. 438.

one piece, their axes making the angle 2β with each other. In practice it is better to make these parts in the form of journals and place the sleeves in the C_1 and C_2, Fig. 438. These parts are also prolonged beyond the shafts in order to counterbalance the weight. The pieces 3 and 4 can be bolted firmly together, since their relative position to each other is constant. It must be observed that a must not exceed $90 - \beta$, otherwise a dead point will occur.

The parts 3 and 4 uiay also be connected by ball joints $3'$ $4'$, Fig. 439, in which case the device becomes the coupling of Clemens.* Here the counter-weights are omitted, aud the parts

* Clemens' Angular Shaft Coupling, U. S. Patent, Nov. 10, 1869.

C_1 and C_2 connected on both sides. Clemens has used this form with the angle $2a = 90°$. The doubling of the parts has the

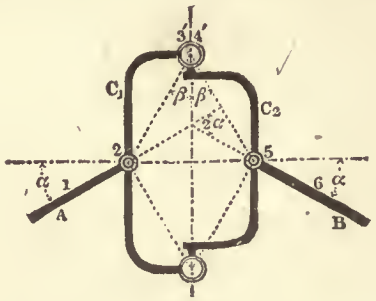

FIG. 439.

objection that it requires more accurate fitting of the parts than where only one side is connected. If the axis B is placed parallel to A, as at B' in Fig. 437, the rate of motion will be very irregular, for $a = \beta = 30°$ as in the illustration, the velocity ratio will vary between ½ and 2.

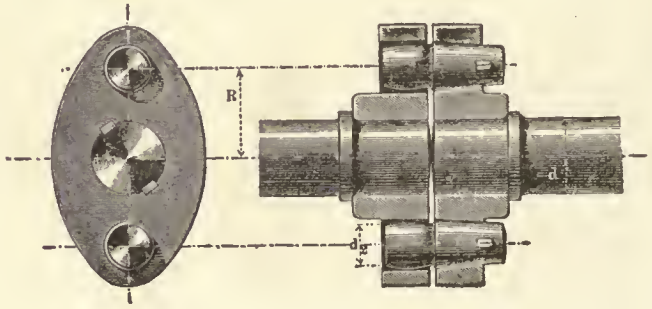

FIG. 440.

In many screw vessels a simple form of flexible coupling is used, suited for slight angular variatious. In Fig. 440 this is shown, and it will be seen to give slight flexibility similar to the universal joint, and sufficieut for many cases. A bearing should be placed back of the coupling on each shaft.

III. Clutch Couplings.

§ 156.

TOOTHED CLUTCH COUPLINGS.

Couplings of this form may be distinguished by their method of engagement, the clutch surfaces entering in and out of engagement axially, radially or iucliued.

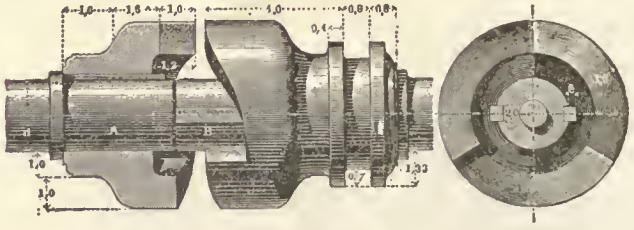

FIG. 441.

The oldest form of clutch coupliug, and one of the most widely used, is that shown iu Fig. 441. Here the eugagement is axial. The modulus for the proportions is the same as before, $\delta = \frac{1}{8} d + \frac{3}{16}''$; and an approximation to the number of teeth may be giveu by making $z = 1 + 0.6\,d$. The clutch is thrown in and out of gear by a lever which works in the groove in the portion of the clutch on B. Examples of suitable lever forks are shown iu Fig. 442.

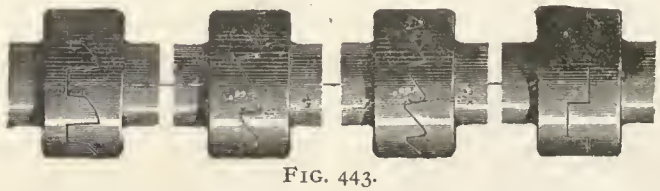

FIG. 443.

Various forms of clutch teeth are used. The forms in general use are given in Fig. 443. The first form is adapted for motion in either direction, but can only be operated when moving slowly. The second form is more readily throwu into action,

but is adapted to transmit motion only in one direction. The driving faces are inclined very slightly, from the normal to the direction of motion, the angle not being enough to cause any tendency to disengagement. In the third form the teeth are

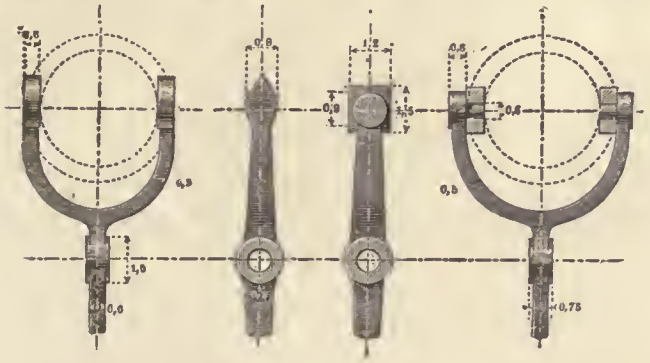

FIG. 442.

more blunt in shape at the point, which adds to their strength against breakage when subjected to shock. The last form is a combination of the preceding varieties, and like the first, may be driven backward. In spinning machinery, light couplings with many fine teeth are used and operated at high speeds. In some screw vessels in which there is no provision for raising the

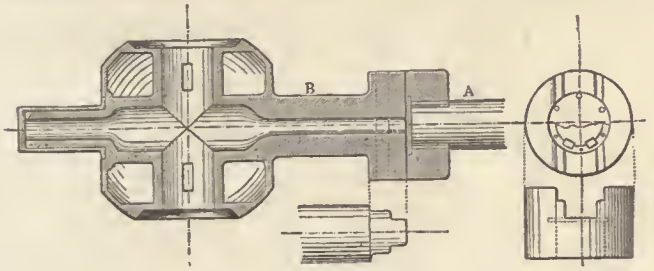

FIG. 444.

screw, it is desirable to disconnect it when proceeding under sail alone, and some form of clutch coupling is used. A very simple form is the so-called "cheese coupling," used in English vessels, Fig. 444. The hub of the propeller is provided with a bearing on each side, and formed with a T projection fitting into a corresponding recess in the heavy flange (or cheese) on the shaft *A*. The propeller blades are secured to the hub as already shown (Fig. 191).

§ 157.

FRICTION CLUTCHES.

Couplings in which one portion transmits motion to the other portion by means of friction, are often especially applicable, since by the mere removal of the frictional contact the parts are disconnected, and when they are thrown into contact the driven

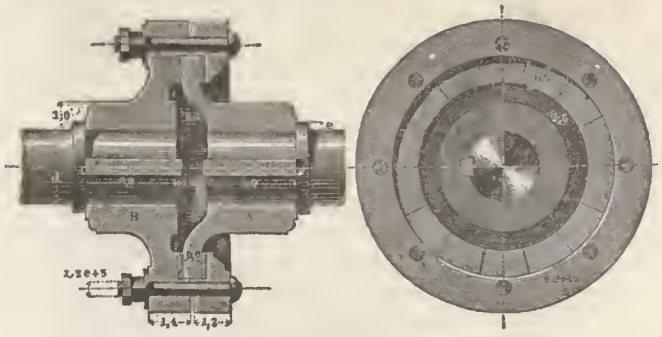

FIG. 445.

portion is put into motion gradually. By making friction couplings of large diameter, they may be used to transmit proportionally great rotative moments. In Fig. 445 is shown a friction coupling used by Ramsbottom in rolling mill work.* The part *A* is firmly clamped between the wood-lined surfaces of *B*; but the parts may be arranged so as to slip if undue resistance is encountered, thus making it a safety coupling. The modulus as before is $\delta = \frac{d}{3} + \frac{3''}{16}$.

*See Engineer, 1866, January, p. 44; also Genie Industrielle, Vol. 32, p. 101; and an older form of this coupling is shown in Salzenberg, Vortr. p. 173.

Cone couplings are used also, in many forms. In the example shown in Fig. 446 the driven portion *A* of the coupling carries a gear wheel shown in the dotted lines, to which motion is to be transmitted from the shaft. The two parts are forced into engagement by the screw and hand wheel *b*. If the parts are so arranged that the motion of the hand wheel *b* is in the same direction as the rotation of the part *B*, when the latter is thrown into engagement, it is only necessary to hold the wheel *b* stationary in order to throw the clutch out of gear. From the mean radius *R* of the cone surface, and the angle of taper α, we have for an axial pressure *Q*, for any circumferential force *P*:

$$Q = P\left(\frac{\sin\alpha}{f} + \cos\alpha\right) = \frac{(PR)}{R}\left(\frac{\sin\alpha}{f} + \cos\alpha\right) \quad . \quad (146)$$

in which *f* is the coefficient of friction between the cone surfaces, and (PR) is the statical moment tending to rotate the shaft.

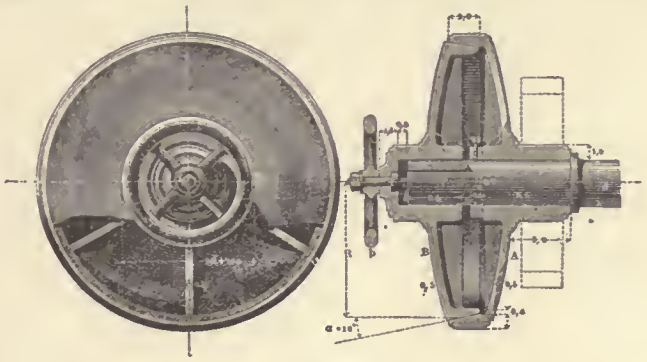

FIG. 446.

The angle α should not be taken at less than 10°, in order that the parts may not become wedged together; for iron on iron, *f* may be taken at 0.15. In order to keep both *P* and *Q* as small as possible, *R* should be made large, say between 3 and 6 *d*.

The relative motion of the screw and haud wheel is of course dependent upon the radius of the wheel *b*, and upon the pitch *s* of the thread.

Example: A wrought iron shaft of a diameter $d = 2$ inches, making 50 revolutions per minute, would transmit, according to the table of § 145. $0.0313 \times 50 = 1.5$ H. P., or have a statical moment $PR = 1975$ inch pounds. If the radius of the friction cone couple is equal to $5d$, or 10 inches, we have, according to (147) $Q = \frac{1975}{10}\left(\frac{\sin\alpha}{f} + \cos\alpha\right)$. If $\alpha = 10°$, and $f = 0.15$ we have

$$Q = 197.5\left(\frac{0.1736}{0.15} + 0.9848\right) = 423 \text{ lbs.}$$

Suppose the hand wheel to have a radius of 4 inches and the screw a pitch of $\frac{1}{4}''$, we then have for the force to be exerted on the rim of the haud wheel:

$$q = \frac{0.25 \times 423}{\pi \times 4} = 8.4 \text{ lbs.}$$

For the transmission of moderate force the cone coupling, or some of its various modifications, has very generally been used.*

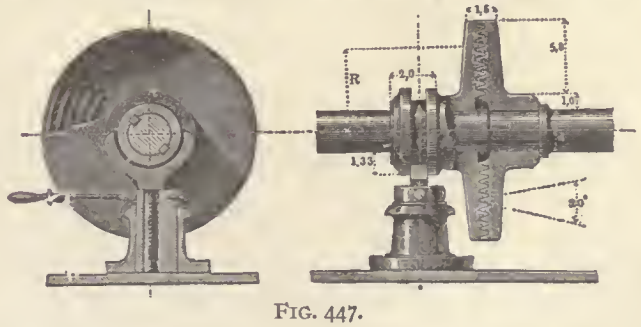

FIG. 447.

Instead of a single pair of external and internal cones, a number of small elements may be employed. This form is shown in Fig. 447. The general calculations are made as above, except that the lever arm *R* of the friction must be reduced, and may be taken with sufficient accuracy at a point distant from the outer circumference equal to one-third the width of the grooved frictional surface. The operating lever in this case need make but very little movement, and the arrangement of a fork mounted on an eccentric bearing, as shown in the illustration, may be conveniently adopted.†

When a cone coupling is intended to be used for the transmission of large forces, the apparatus for pressing the parts together may sometimes be so arranged that it is mounted on

* Many such applications will be found in the description of the Suez Canal; see Armengaud, Publ. Ind., Vol. 17, Pl. 9.

† See Armengaud, Vignole des Mecaniciens, Plate II.

the shaft, revolving with it, without creating so much pressure against the bearing. The fork and grooved collar shown in Fig. 447 is not suitable for heavy clutches on account of the excessive collar friction, hence the pressure is better applied by means of a screw mounted on oue of the shafts, and this may be conveniently arrauged so as to draw both shafts firmly together. Suppose tue shaft to be 4 inches in diameter, we have from the preceding, $R = 6d = 24''$, and an axial pressure $Q = \frac{31600}{24}\left(\frac{0.1736}{0.15} + 0.9848\right) = 2818$ lbs. This endlong pressure, instead of creating hurtful collar resistance, may be utilized by arranging the parts as shown in Fig. 448, which shows a friction clutch coupling of

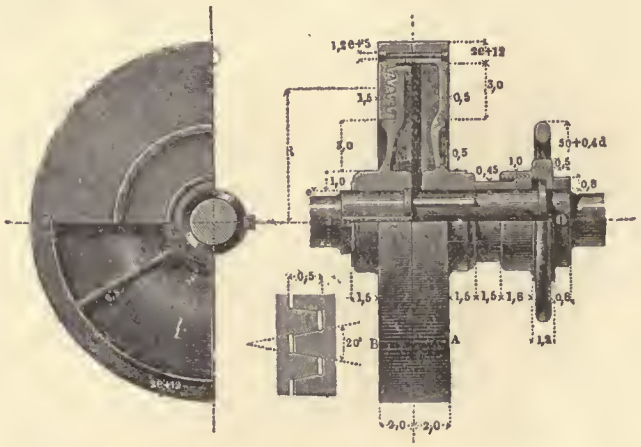

FIG. 448.

the author's design. As shown in the section, the part A extends over the part B, and both parts are drawn together by the action of the screw and hand wheel. The only modification in the screw gear is that the screw is made large enough to permit the shaft to be passed through it, the thread being thus cut upon the hub of the part A. This coupling runs very smoothly. The concentric channels should be arranged with clearance at the bottoms of the grooves, as shown in the sectiou, to provide for fittiug and wear. The modulus for the parts is the same as before, viz., $\delta = \frac{d}{3} + \frac{3''}{16}$.

In Fig. 449 is shown the cylinder friction clutch of Koechlin. In this case the clutch movement takes place radially. The part A is a hollow cylinder in which three iuternal clamp pieces are

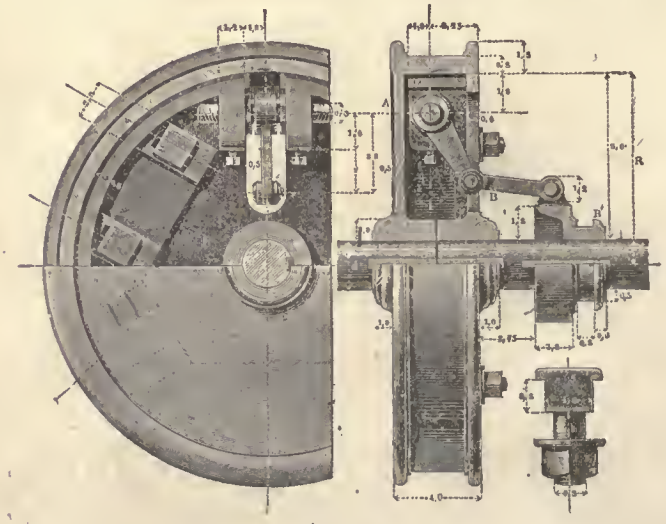

FIG. 449.

fitted, each being provided with a bronze shoe. These are thrown in and out of actiou by means of a sliding collar B', which operates right and left hand screws by means of the lever b. The clamps slide in radial grooves and the details are fully showu in the illustration. The nuts for the right and left hand screws cau be closely adjusted and clamped by set screws, so that a radial movement of less than $\frac{1}{16}''$ is sufficient. There is no danger of wedging the parts fast in this form of clutch, as may be the case in cone clutches, as the elastic reaction of the cyliuder assists in the direction to release the parts. At the same time the screws prevent the coupling from releasing itself and the axial pressure Q, upon the collar B', can be transmitted so that the screws need not have too quick a pitch.

If s is the pitch, b the length of lever arm, f the coefficient of friction of the clamping pieces, we have for the transmission of a given moment (PR), neglecting the friction of the screws,

$$\frac{2s}{2\pi b}\ \frac{P}{f} \text{ or } Q = \frac{s}{\pi b}\ \frac{PR}{fR} \quad \ldots\ldots\ldots (147)$$

which gives a very small value for Q.

If the parts are so arranged that B is the driven part, there will be no collar friction at B', when the coupling is not in action. When the shaft is vertical, a weight may be used instead of a collar and lever, and by gradually lowering it the apparatus may be started with very little shock. The first clutch made by Koechlin was designed for the transmission of 30 H. P.* The above value correspouds to a minimum value of R. The modulus is the same as before: $\delta = \frac{d}{3} + \frac{3''}{16}$.

A very excellent form of this coupling was designed by Bodmer, independeutly of Koechlin,† and a similar arrangemeut has been adapted to mill gearing with success.‡

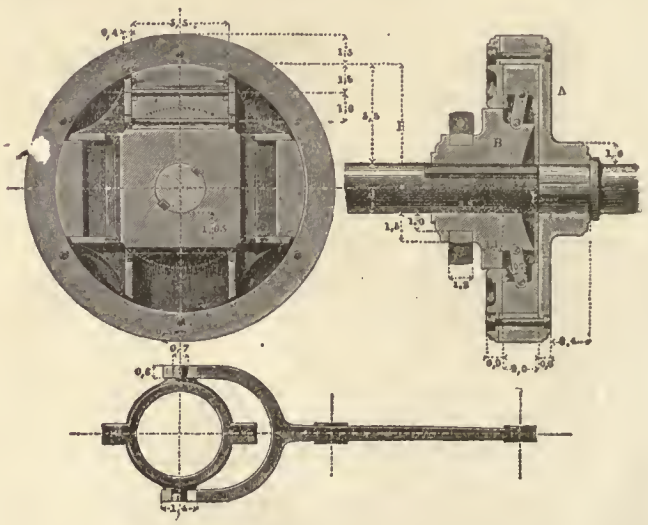

FIG. 450.

Cylinder couplings in which the clamping pieces are operated by toggle joiuts are also made. An example is shown in Fig. 450, which is a clutch by Fossey, as applied to mint machinery.§ This is a very compact design and is arranged with four clamps, which have no bronze shoes. The toggle links are as wide as the clamps and are fitted with half-journals to transmit the pressure outwards, while to draw the clamps back, light through bolts are used (see § 95). If the toggle links make with the axis an angle $90° + a$, we have for the axial collar-pressure:

$$Q = \frac{P \tan a}{f} = \frac{PR}{R}\ \frac{\tan a}{f} \quad \ldots\ldots\ldots (148)$$

The angle a may be taken very small, since there is no danger of clamping. The value may be as small as $a = 2°$, or even $1°$.

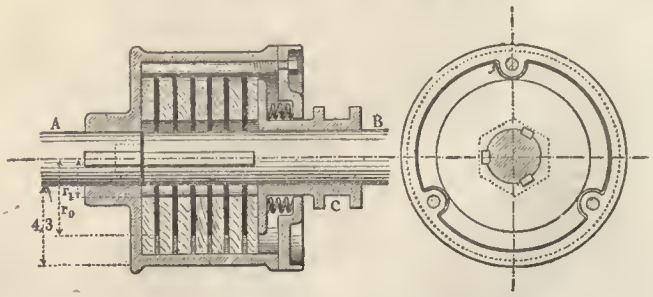

FIG. 451.

For $a = 1\frac{3}{4}°$ we have $\frac{Q}{P} = \frac{0.03}{0.15} = \frac{1}{5}$

Another form of cylinder coupling using toggle levers, has

* Bulletin von Mülhausen, 1854, p. 138.
† See Fairbairn, Mills and Millwork, Vol. II., p. 92.
‡ See Uhland's Prakt. Masch Konstrukteur, 1869, p. 97.
§ See Armengaud's Publ. Industrielle, Vol. XVII, Pl. 10.

been designed by Garand.* Jackson uses hydraulic pressure to force the clamps into contact.† Dohmen-Leblanc uses springs to throw the toggles out of action.‡ Schurmann uses, instead of the separate clamps, a ring, which is compressed externally;§ Napier also uses a ring, expanded from within.‖ Becker arranges the clamp blocks to be operated by centrifugal force.¶ These are only a few of various modifications of the cylinder coupling.

A form of axial friction coupling which acts with very slight pressure is the Weston clutch, made by Tangye.** This acts upon the principle of multiple plate friction (see § 101), as is shown in Fig. 451.

The wooden discs are engaged with the case, and the iron ones with the shaft. In the form shown the plates are pressed together by the springs, and released by drawing back the collar *B* and releasing the spring pressure. A larger example of Weston's clutch is shown in Fig. 452. *A* is a winding drum, *B* the shaft driven by the engine. The outer disc *C*, and the inner discs of the coupling are held apart by spiral springs, as shown at *a*. A light pull on the cord *c* holds the drum stationary; a strong pull engages the clutch for winding; if the cord is left slack the load on the drum runs backward.

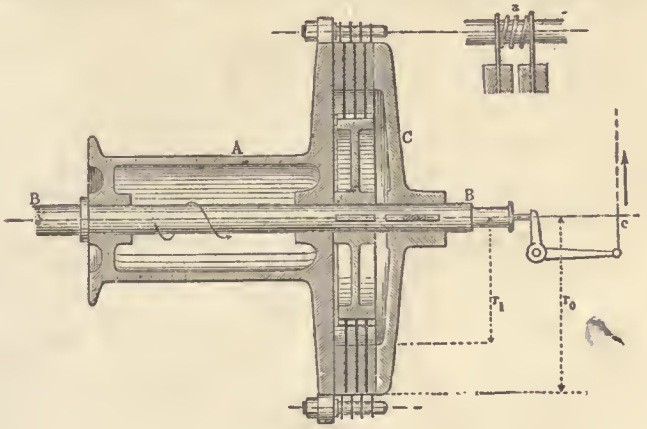

FIG. 452.

§ 158.

AUTOMATIC COUPLINGS.

When power is transmitted to a shaft from two different sources, as from two independent engines, it is desirable to have one or both of them connected by a coupling which will automatically release or engage with the shaft, according to the distribution of work. If one motor tends to overrun it will then be given more of the work, and so the resistance will be equalized. Such a device is the coupling of Pouyer-Quertier, generally known as Pouyer's Coupling.

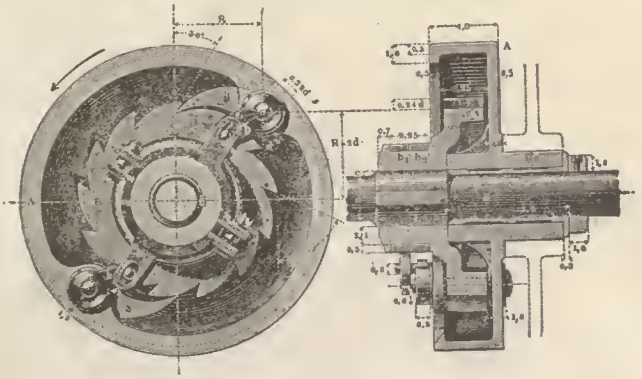

FIG. 453.

This is shown in Fig. 453. In this case the parts are so disposed that the part *A*, which is driven by one source of motive power, is loose on the shaft *B*. This part *A* may have gear teeth upon its circumference, for example, or may have a gear wheel mounted upon its hub, as shown by the dotted lines; the hub being bushed with bronze. Upon the shaft *B* a ratchet wheel is keyed; the pawls *a*, *a*, being upon *A*, engage with the teeth when *A* drives *B*, but if *B* gains upon *A*, or *A* stops while *B* continues to move, the pawls are thrown out of action. The direction of motion is shown by the arrow. The pawls are released by the action of the friction bands b_1 and b_2, which are carried forward by friction upon *B*, whenever *B* gains upon *A*, the levers *b* throwing the pawls *a* out of gear. As soon as the limit of travel of the levers *b*, *b*, is reached, the friction bands b_1, b_2 slip upon *B*, being able to move no faster than *A*. When the speed of *A* increases and gains upon *B*, the pawls are again thrown into gear and *A* is automatically coupled to the shaft. In order that the pawls may not bind upon the ratchet teeth in releasing it is necessary that the angle γ, which the pawl makes with the face of the ratchet tooth, must be less than the complement of the angle of friction; in this case $\gamma = 60°$. Pouyer uses only one friction band and makes both pawls engage at the same time. In the illustration the ratchet wheel is made with an odd number of teeth (13), and the pawls are placed so that a movement of only ½ the pitch will cause the parts to become engaged. The above proportion of the angle of the teeth is of importance, as otherwise the points of the teeth are apt to be broken. The pawls also should be of hardened steel.

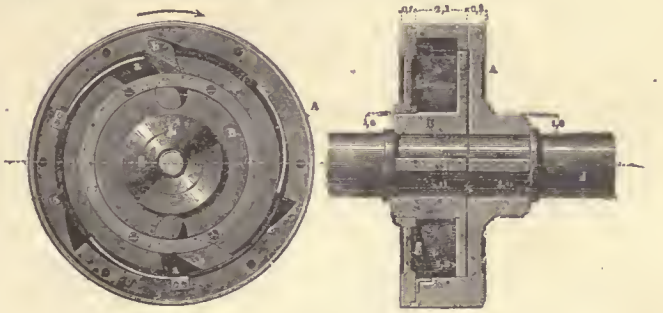

FIG. 454.

In Germany Uhlhorn's Coupling is used for similar service, as shown in Fig. 454. Here *A* is the part connected to the motor, and *B* is fast to the driven shaft. *A* is an internal ratchet wheel into which the pawls *b* enter. The springs *a* serve to insure the entrance of the pawls into the teeth, which engagement continues so long as *a* drives *B*. If the speed of *A* is retarded, the pawls are retracted as shown in the lower part of the figure. In this case the springs act to keep them out of gear, being the reverse action to that of an ordinary ratchet gear.

The pawls are fitted with half-journals (see § 95), and are held in place by a plate ring, as shown. Uhlhorn originally used only two ratchet teeth in *A*, but increased the number afterwards to four, so that the parts would engage in a movement of one-fourth a revolution. It is better to use an odd number, as three, and by proper spacing of the pawls the greatest play will be one-half a space, or one-sixth a revolution with three teeth, as in the case of Pouyer's Couplings. *B* may be the driving part instead of the driven, but in that case the direction of the arrow must be reversed.

CHAPTER XI.

SIMPLE LEVERS.

§ 159.

JOURNALS FOR LEVERS.

In machine design a simple lever, or rocker arm, is a lever arm which is mounted upon an axle or shaft, at the end, about which it moves, and carries a journal upon the other end. For the proportion of the journal see Chapter V. The forms which

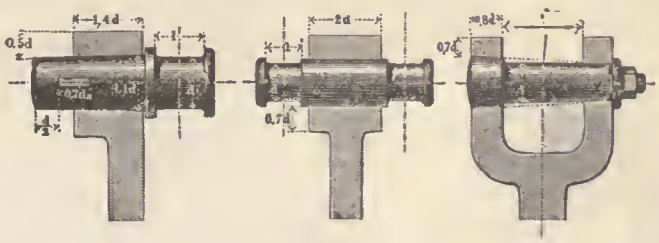

FIG. 455.

may be given to such journals are shown in Fig. 455, and are single overhung, double, or forked. The manner of securing the pin in the hub or the lever is most important. The pin should not be driven in up to the shoulder on the taper, but sufficient space left to insure that the fit is tight in the taper. This clearance is shown plainly in the figure. The same result

* Dingler's Polyt. Journal, Vol. 149, p. 22.
† Dingler's Polyt. Journal, Vol. 153, p. 251.
‡ German Patent, 16952.
§ Zeitschr. d. Vereins d. Ing., Vol. V, 1861, p. 301.
‖ Engineer, 1868, July, p. 64.
¶ German Patent, 7205.
** In the U. S. by the Yale & Towne Mfg. Co.

may be attained by counter-sinking the collar into the hub on the lever. In the case of double overhanging pins, care should be taken that the load is equally divided between the two sides, so that the pressure upon each pin shall be equal to ½ P. In the fork-ended lever the fit on both ends of the pin should be portions of the same cone.

Example 1. For $P = 4400$ lbs., we have from the table in § 90 for alternating pressure and wrought iron journal, the diameter $d = 0.027\sqrt{4400} = 1.8''$, and the length the same. For steel, we have $d = 0.024\sqrt{4400} = 1.6''$, and the length $l = 1.3 \times 1.6 = 2.08''$. For a forked lever, a wrought iron pin with the same load the diameter, according to (98) would be $d = 0.0185\sqrt{4400} = 1.2''$, and the length $l = 3.5 \times 1.2'' = 4.2''$.

All levers are not subjected to alternating pressure, but have the pressure constantly in one direction, as for example, the beams of single-acting pumping engines, etc. In such cases larger journals are needed.

Example 2. A wrought iron journal for a forked lever, under constant pressure of 4400 lbs., according to formula (98), should have a diameter $d = 0.0212\sqrt{4400} = 1.4''$, and length $l = 3d = 4.2''$. If the material had been cast iron we should have had $d = 0.29\sqrt{4400} = 1.92''$, say 2", and $l = 6''$. For steel we have $d = 0.0185\sqrt{4400} = 1.2''$, $l = 4d = 4.8''$.

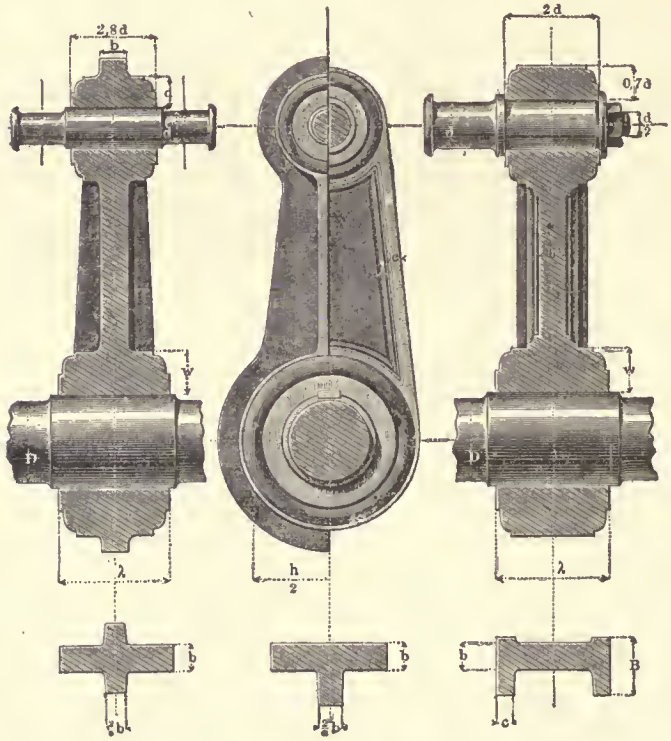

FIG. 456.

§ 160.

CAST IRON ROCK ARMS.

Rock arms may be either of cast or wrought iron. The hubs for wrought iron arms are given in the preceding illustrations, and in Fig. 456 are given some proportions for the various parts of cast iron arms. A fork-ended arm is shown below, among the walking beams, or if the fork hub is on the main axle, see the rules already given under Axles, Chapter VIII.

§ 161.

ROCK ARM SHAFTS.

The axle upon which a rock arm works is usually subject both to bending and torsional stresses. The methods of calculation for all important cases are given in Chapters VIII and IX. The case which occurs most commonly is the overhung rock arm at the end of a shaft, and this is here given a special examination.

If we have a, the distance between two planes normal to the axis, and passing through the middle of the pin and the middle of the bearing on the shaft, Fig. 457, there is an ideal bending moment with a lever arm R, acting upon the bearing of the shaft, for a load P on the pin equal to

$$(M_b)_i = Pa' = P\left(\frac{3}{8}a + \frac{5}{8}\sqrt{R^2 + a^2}\right) \quad . \quad . \quad . \quad . \quad (150)$$

See § 150.

The lever arm a' is readily obtained graphically, as is shown in the illustration. For its numerical determination we have,

$$\left.\begin{array}{l} \text{if } R > a \\ \quad a' = 0.625\,a + 0.6\,R \\ \text{and if } R < a \\ \quad a' = 0.957\,a + 0.25\,R \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

The lever hub must be made strong enough when the shaft is only subject to torsion, or when it is also subject to bending.

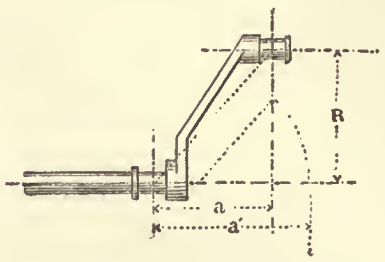

FIG. 457.

For wrought iron shafts wrought iron levers should be used, and for cast iron shafts cast iron levers.

Let:

w = thickness of metal of hub,

λ = length of hub,

D = the shaft diameter for the statical moment PR of a lever of the same resistance, see (133) and (134).

for

$$\left.\begin{array}{lccc} \dfrac{w}{\lambda} = & \dfrac{1}{2} & \dfrac{1}{25} & \dfrac{1}{3} \\ \dfrac{w}{D} = & 0.45 & 0.42 & 0.40 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (152)$$

If a lever is to be fitted to a shaft of greater diameter than D, we first determine the imaginary value of D, and insert it in (152). The same method is adopted if a cast iron lever is to be used with a wrought iron shaft, and vice versa. The shape for cast iron levers is given above, in Fig. 456.

Example 1. If the lever of Example 1, § 159 is made of wrought iron, and is 24 inches long, its statical moment $PR = 24 \times 4400 = 105600$ inch pounds. This gives, from (131) $D = 0.091\sqrt[3]{105600} = 4.3''$, and if we take $\frac{w}{\lambda} = \frac{1}{2}$, we have from (152), $w = 0.45 \times 4.3'' = 1.93''$. $\lambda = 1.93'' \times 2 = 3.86''$, say $3\frac{7}{8}''$.

The hub may also be calculated of such dimensions as to be strong enough to be forced on cold, and thus obtain sufficient friction to hold without the use of a key (see § 65, formula 66). The friction Q of the hub upon the shaft must then be $\geq \left(\frac{PR}{\frac{1}{2}D'}\right)$ in which D' is the diameter of the shaft at the point where the hub is fitted.

Example 2.—In the case of the same lever as the preceding example

$$D' = D \text{ and } \frac{PR}{\frac{1}{2}D} = \frac{105600}{2.15} = 49116$$

We may then take $Q = 50{,}000$ and let $l = \lambda = 3\frac{1}{8}''$ and $S_s = 10{,}650$, and substituting in formula (66), we get:

$$\frac{w}{D} = \tfrac{1}{2}\sqrt{\frac{\pi \times 4.3 \times 3.875 \times 0.2 \times 10{,}650 + 50{,}000}{\pi \times 4.3 \times 3.875 \times 0.2 \times 10{,}650 - 50{,}000}} - 1$$

$$\frac{w}{D} = \tfrac{1}{2}\sqrt{\frac{161500}{61500}} - 1 = \tfrac{1}{2} \times 0.63 = 0.31$$

The key is used as an extra precaution for security.

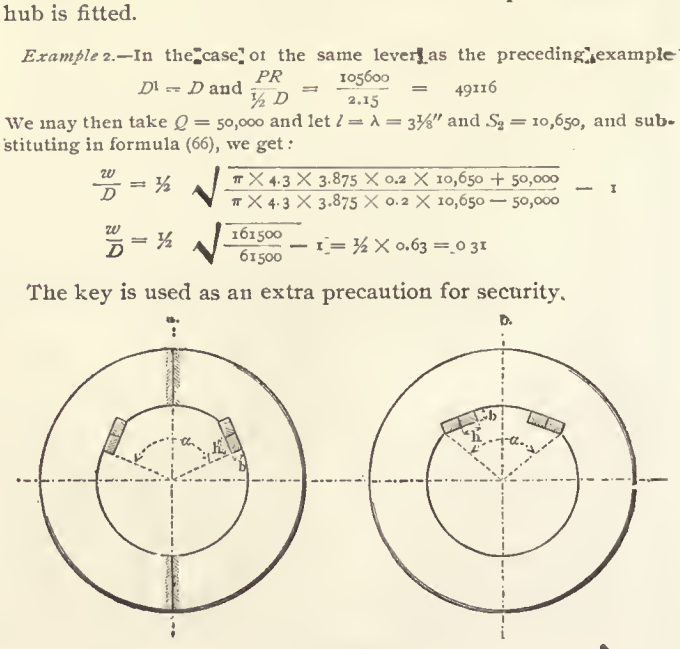

FIG. 458.

A special method of keying, especially adapted for the hubs of levers and wheels, has been designed by engineer Peters. It consists of two parallel systems of keys, as shown in Fig. 458.

The taper of the keys is $\frac{1}{20}$. The arrangement shown at (*a*) is preferable, as it weakens the hub less than (*b*). The angle α may be taken $= 135°$, the thickness of keys $b = \frac{1}{15} D'$, and mean width $h = 2b$.

The form (*a*) is especially suited for hubs which are made in two parts.

Those hubs which are upon shafts subjected to bending, are considered under the heading of Combined Levers, in Chapter XIII.

§ 162.

LEVER ARMS OF RECTANGULAR SECTION.

The calculations of the dimensions of simple lever arms of rectangular section are made upon the assumption that the force

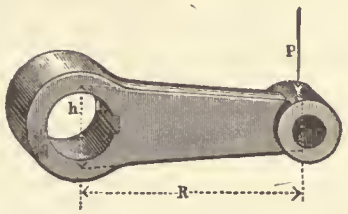

FIG. 459.

P acts in a plane, passing through the middle of the arm, Fig. 459, and in a direction normal to the arm.

If we let

h = width of the arm at the axis,
b = thickness of the arm at the axis,
S = the maximum permissible stress,

$$b = 6\,\frac{PR}{S\,h^2}$$

Taking S for wrought iron = 8500, and for cast iron = 4250, we have

for wrought iron.	for cast iron.
$b = 0.00072\,\frac{PR}{h^2}$	$0.00144\,\frac{PR}{h^2}$ . . (153)

These formulæ are adapted for the determination of b, when h has been selected, the latter being most conveniently chosen with regard to the other condition.

Example 1.—Let P=4400 lbs., R=24" for a lever arm of wrought iron, and N=7⅛" we have from (153);

$$b = 0.00072\,\frac{4400 \times 24}{(7.125)^2} = 1\tfrac{1}{2}''.$$

If b is kept constant for the whole length of the arm, the width at the small end may be $0.5h$, while if a constant ratio of $b : h$ is kept, the small end $= \frac{2}{3}h$ (see § 10, Case III and VII).

If the force P does not act in the middle plane, as often occurs, then there must exist a combined bending and twisting stress on the arm. We may then derive a combined stress whose bending moment will give an ideal arm R'.

If the plane in which the force P acts is distant from the middle of the arm by an amount c, we may make approximately, (see § 150):

$$\left.\begin{array}{ll} & R' = \frac{3}{8}\,R + \frac{5}{8}\,\sqrt{R^2 + c^2} \\ \text{or} & R' = 0.975\,R + 0.25\,c \\ \text{if} & R > c, \\ \text{and} & R' = 0.625\,R + 0.6\,c \\ \text{is} & R > c. \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; (155)$$

R' may be determined readily by the graphical method, Fig. 460. The third case shows the method for inclined arms.

Example 2.—In the case of the lever of the preceding example, let C= 15.75". This gives $R > C$ and we have from (154):

$$R' = 0.975 \times 24 + 0.25 \times 15.75 = 23.4 + 3.94 = 27.34''$$

This gives for b,

$$b = 0.00072\,\frac{4400 \times 27.34}{(7.125)^2} = 1.7''$$

Cast iron arms are sometimes made of cruciform section, see Fig. 456, in which case the ribs may be neglected.

§ 163.

LEVER ARMS OF COMBINED SECTION.

The sections shown in Fig. 461 are designed to secure an economy of material. Their dimensions are readily determined by first calculating a corresponding arm of rectangular section, and then transforming it into an I section, or double II shape. If h_0 be the depth and b_0 the breadth of the equivalent rectangular arm, and h and b the corresponding terms to be found, as in Fig. 461, we have

$$\left.\begin{array}{l} \dfrac{b}{b_0} = \dfrac{1}{1+a} \\ \text{in which} \\ a = \left(\dfrac{B}{b} - 1\right)\left[6\,\dfrac{c}{h} - 12\left(\dfrac{c}{h}\right)^2\right] \end{array}\right\} \quad . \; . \; (155)$$

These formulas permit a choice of the ratios $\frac{B}{b}$ and $\frac{c}{h}$, which may be left to the judgment of the designer. In (155) the angle

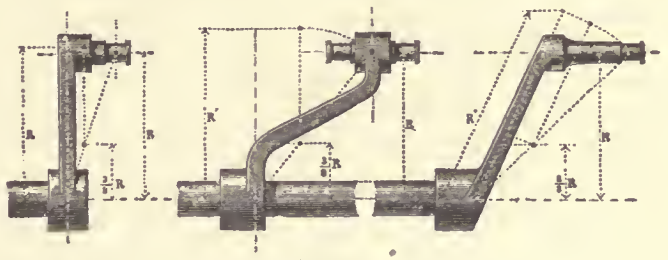

FIG. 460.

irons of the third example in Fig. 461 have been neglected, and may be considered as making up for the weakening of the rivet holes. The following table gives a series of values for (155) which will simplify the calculations materially. The table will also be found useful for other purposes, as all sorts of beams, crane booms, etc.

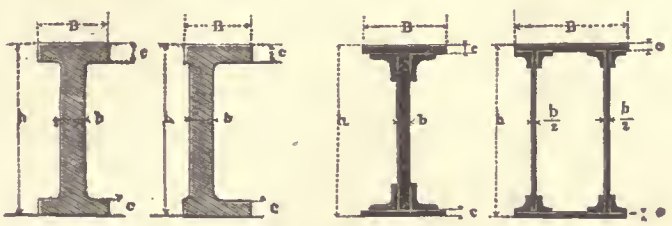

FIG. 461.

§ 161.

TABLE FOR TRANSFORMING ARM SECTIONS.

$\frac{h}{c}$	Values of $\frac{1}{1+a}$									
	$\frac{B}{b} = 2.5$	3	3·5	4	4·5	5	6	7	8	10
6	0.50	0.43	0.38	0.33	0.30	0.27	0.23	0.20	0.18	0.14
7	0.52	0.45	0.40	0.35	0.32	0.29	0.25	0.21	0.19	0.15
8	0.54	0.47	0.42	0.37	0.34	0 31	0.26	0.23	0.20	0.16
9	0.56	0 49	0.44	0.39	0.36	0.33	0.28	0.24	0.22	0.18
10	0.58	0.51	0.46	0.41	0.37	0.34	0.29	0.26	0.23	0.19
11	0.60	0.53	0.48	0.43	0.39	0.36	0.31	0.27	0.24	0.20
12	0.62	0.55	0.50	0.44	0.41	0.37	0.32	0.29	0.26	0.21
14	0.64	0.58	0.52	0.47	0.44	0.40	0.35	0.21	0.28	0.23
16	0.67	0-60	0.55	0.50	0.47	0.43	0.38	0.34	0.30	0.25
18	0.69	0.63	0.57	0.52	0.49	0.46	0.40	0.36	0.33	0.27
20	0.71	0.65	0.60	0.55	0.52	0.48	0.42	0.38	0.34	0.29
22	0.73	0.67	0.62	0.57	0.53	0.50	0.45	0.40	0.37	0.31
24	0.75	0.68	0.64	0.59	0.56	0.52	0.47	0.42	0.38	0.33
27	0.76	0.71	0.66	0.62	0.58	0.55	0.50	0.45	0.41	0.35
30	0.78	0.73	0.68	0.64	0.61	0.57	0.52	0.47	0.43	0.37
33	0.79	0.75	0.70	0.66	0.63	0.60	0.54	0.50	0.45	0.39
36	0.81	0.76	0.72	0.68	0.65	0.61	0.56	0.52	0.48	0.41
40	0.83	0.78	0.74	0.70	0.67	0.64	0.58	0.54	0.50	0.44
45	0.84	0.80	0.76	0.72	0.69	0.66	0.61	0.57	0.53	0.47
50	0.85	0.81	0.78	0.74	0.71	0.68	0.63	0.59	0.56	0.49

Example 1. A lever arm has a length $R = 78.75''$ and the journal pressure at the end = P = 5500 pounds. It is to be of cast iron of double T section with a height $h_0 = 12\frac{5}{8}''$. According to (153) we have for a rectangular section

$$b_0 = 0.00144\,\frac{5500 \times 78.75}{(12.625)^2} = 3.9''.$$

This is also so thick as to be impracticable, and hence the double T section may be compared. Here we may take $c : n = 1 : 12$, $B : b = 4$, and we get from the table $\frac{1}{1+a} = 0.44$ and $b = 0.44\,b_0 = 1.71''$, and the flange

breadth $B = 0.44\ b = 1.71 \times 0.44 = 0.752$, the web thickness $= c = \frac{1}{12} = h = \frac{12.625}{12} = 1.05''$, all of which are practical dimensions. It may be found desirable to have $c = b$, or any reasonable ratio for $B : b$, and $c : h$ be chosen.

Example 2. A wrought iron arm has been found to require $b_0 = 2\frac{3}{8}''$, $h = 12\frac{5}{8}''$. It is desired to make $\frac{b}{b_0} = 0.25$ and in column 10 we find 0.25 opposite $\frac{h}{c} = 16$. Hence $b = 0.57''$ and $B = 10 \times 0.57 = 5.70''$ and $c = \frac{12.625}{16} = 0.8''$.

CHAPTER XII.

CRANKS.

§ 165.

Various Kinds of Cranks.

Cranks are those forms of simple levers which are so arranged that they may, together with their various connections, make entire and repeated revolutions about an axis. These may be divided into the following four classes:

1. Single Overhung Cranks.
2. Return Cranks.
3. Double Cranks, or Cranked Axles.
4. Eccentrics.

These will be briefly treated in succession.

§ 166.

Single Wrought Iron Cranks.

These cranks may be proportioned according to the rules given for simple levers and rocker arms (§ 159 *et seq*). Fig. 462 shows the usual form; the arm tapers to two-thirds its base dimensions both ways, and is made slightly convex on the back.

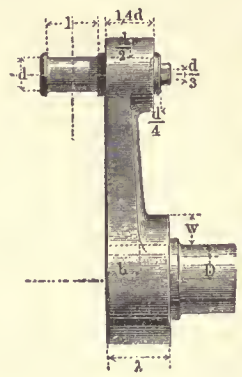

FIG. 462.

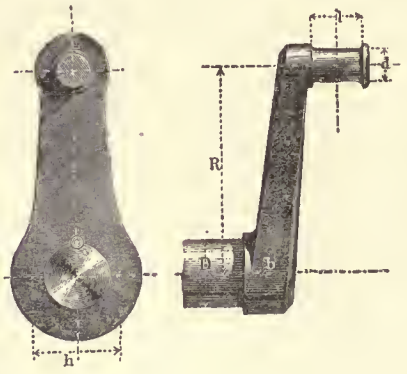

FIG. 463.

The crank-pin is forced or driven in, and secured with a cap bolt. Fig. 463 shows a crank forged in one piece. In this case the width of the arm at the base is determined by the necessary amount of shoulder on the shaft. The proportions of the pin are obtained from the rules in § 159.

§ 167.

Graphostatic Calculations for Single Overhung Crank.

The crank is such an important detail of machine construction that it demands a most careful discussion, hence a graphostatic investigation of the stresses in it is here given.

The Crank Axle.—Having calculated d and l, draw the skeleton diagram of the crank, that is, the neutral axis $A\ B\ C\ D\ E$, Fig. 464, in which $B\ C$ represents the axis of the crank arm, which in this case lies normal to the axis of the shaft, and is placed in its proportional distance from the centre of the crank-pin A, and from the bearing D. Then lay off the force P from a, normal to $E\ a$, choose the pole O of the force polygon (this being best placed upon a line passing through the end of P and parallel to the axis $E\ a$), draw the ray $a\ d\ O$, and line $d\ E$, also the ray $O\ P_1$ parallel to $d\ E$; then $a\ d\ E$ will represent the cord polygon for the bending which P produces upon the axle $a\ C\ E$, and $P\ P_1$ represents the force upon the journal E, and $P_1\ a$ the force upon the journal D. Also make $a\ F$ equal to the crank radius R, draw $F\ G$, and this latter will be the twisting moment (§ 140) which P exerts upon the axis. This moment M_d may be combined with the bending moment M_b, to give for each point an ideal bending moment,

$$M_i = \tfrac{3}{8} M_b + \tfrac{5}{8} \sqrt{M_b^2 + M_d^2} \text{ (see § 45)},$$

from which the polygon curve $c'\ d'\ e'$ and surface of moments $C\ c'\ d'\ e'\ E$ are obtained. From the latter, in combination with the pin diameter d, and ordinate t of the base of the pin, the diameter of the shaft may be obtained according to formula (124).

The Crank Arm.—Prolong $E\ a$ to a_0, and transfer the cord polygon $D\ a\ d$ to the base line $B\ C$, that is, make the angle $a_0\ B\ C =$ the angle $D\ a\ d$, and then will $B\ a_0\ C$ be, with horizontal ordinates, the surface of moments for the bending of the

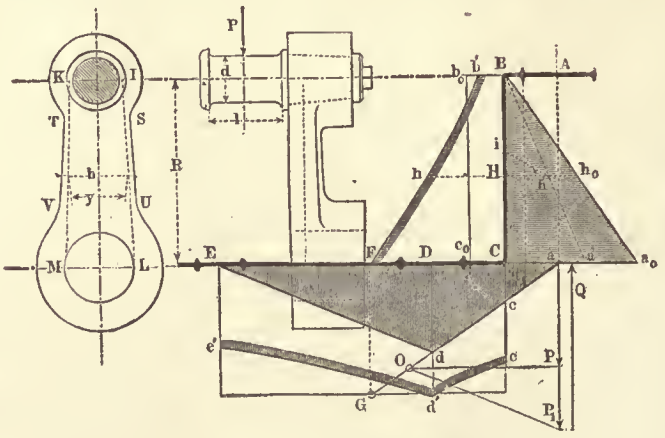

FIG. 464.

crank arm due to the force P. Also make $C\ c_0 = B\ b_0 = C\ c$, then will the horizontal ordinates of the torsion rectangle $B\ b_0\ c_0\ C$ be the moments with which P acts to twist the crank arm about the axis $B\ C$. This moment may again be combined with the bending moment to give an ideal moment as before; ($a_0\ a' = \frac{3}{8}\ a_0\ C$, draw $B\ a'$, make at any point H, the space $H\ i = \frac{5}{8}\ B\ b_0$, and make $H\ h = h_0\ h' + h'\ i$) which gives the surface of moments $B\ b'\ h\ F\ C$ for the crank arm. From this and from the diameter d and ordinate t, we can construct the conoidal form of the arm $I\ K\ L\ M$, according to formula (124). From this, again, the profile $S\ T\ U\ V$ of an arm of rectangular section may be derived, the width h being assumed for any point and the corresponding thickness b obtained from the value y of the conoid, according to the formula:

$$\frac{b}{y} = 0.6 \left(\frac{y}{h}\right)^2 \quad \ldots\ldots\ldots \quad (156)$$

in using which, the second table of numbers at the close of this work will be found useful. If the position of the axis $B\ C$ does not give satisfactory results, the operation must be repeated with a better relation of parts. By proceeding in this manner the dimensions of a crank and axle may be so determined that they will be equal in strength to the pin upon which the power is exerted.

In the preceding diagram the crank arm was taken as normal to the axle. A slight inclination may be neglected, but if the

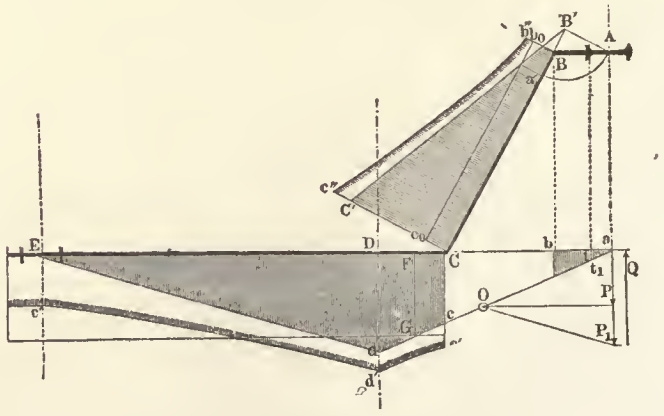

FIG. 465.

angle is greater, as shown in Fig. 460, it should be so considered in the diagram. The procedure is then as follows (Fig. 465): The diagram for the crank shape is constructed as before, the portion under $a\ b$ being used only for the shank $A\ B$ of the crank-pin, and the portion under $C\ E$ being combined as before with the torsion moment, to obtain the surface of moments $C\ c'\ d'\ e'\ E$.

The crank arm is again subjected to bending and twisting; the lever arm is now $B'\ C$, $A\ B'$ being made normal to the axis

$B\,C$ of the crank arm, the bending polygon being a portion of the triangle $C\,B'\,C'$, in which the angle at B' is equal to the angle $d\,a\,D$. The twisting force acts with a lever arm $A\,B'$; its moment is obtained by drawing an ordinate at a' normal to $B\,C$, $B'\,a'$ being taken equal to $B'\,A$. The combination of moments gives the surface of moments $B\,b''\,c''\,C$ in same manner, and of the same use as in the preceding case.

§ 168.

Cast Iron Cranks.

The crank-pin is sometimes made spherical instead of cylindrical; such a one is shown in Fig. 466 on a cast iron crank. The sphere will be of suitable diameter if described from the middle of a normally proportioned overhung crank-pin without making allowance for shoulder. The crank-pin is secured by cold rivetting the end in place, an excellent method and one often used. The **I** formed section can be proportioned by the use of the table in § 164. When h is taken as equal to the hub

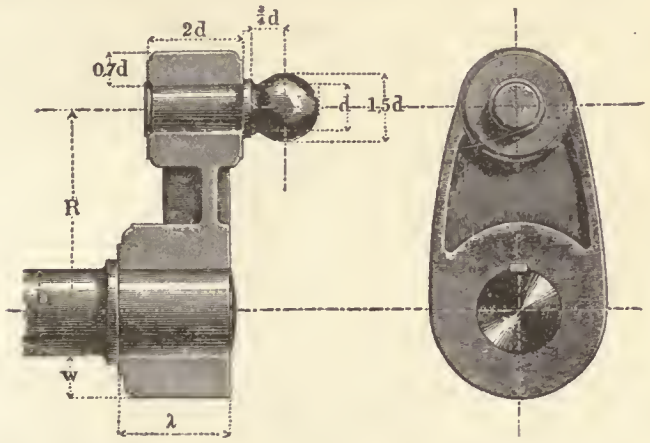

Fig. 466.

diameter, the cross section sometimes works out too light to be suitable for casting, and in such cases it must be increased according to judgment. Sometimes cast iron cranks are made simply by laying out the proper hubs for the shaft and crank-pin, and then joining them by an arm of rectangular section.

If it is desired to employ the graphostatic method, the dimensions may first be determined for a wrought iron crank of rectangular section, and then doubling the depth (see § 162) for cast iron, and obtaining the proportions for **I** formed section according to § 164.

§ 169.

The Return Crank.

A return crank is one which is formed upon the pin of an ordinary overhung crank, returning back toward and having rotation about the same crank shaft as the main crank. Fig. 467 shows a wrought iron return crank otherwise similar in construction to the one shown in Fig. 463. Frequently the return arm is on the same line as the main crank, as shown in the illustration, but in many cases it is differently placed. The arm and pin of the return crank are similar in shape and proportions to an ordinary overhung crank. The arm of the main crank demands no especial consideration, when, as is usually the case, there is but little pressure on the pin of the return crank. The main crank-pin must be determined separately. It is subjected both to bending and to torsion. For this purpose the formula (154) are to be used, remembering that when the return crank is driven by the main crank the moment of the return crank is greatest in the middle of the main crank-pin.

§ 170.

Graphostatic Calculation of the Return Crank.

The graphostatic diagram for a return crank, with both main and return crank inclined to the axis of the crank shaft, is shown in Fig. 468. The skeleton $A\,B\,C\,D\,E\,F\,G\,H\,I$ is first drawn, the dimensions $A\,B\,C\,E$ and $F\,G$ being taken to correspond with those chosen to meet the requirements of the cranks under consideration. The pressure 1 upon the return crank-pin is here taken as opposed to the pressure 2 upon the main crank-pin.

Force polygon.—After choosing a scale for the measurement of the forces, the force polygon (on the right) can be drawn. The line 0 to 1, measured upward, represents the pressure on the return crank-pin; O is the pole chosen on a horizontal line drawn through 0, and the line 1 to 2 represents the pressure on the main crank-pin, measured downwards. Draw the rays 0 O, 1 O, 2 O, also draw the line $a\,d'$ parallel to 1 O, until it intersects at d' the line dropped from D (the line of direction of the

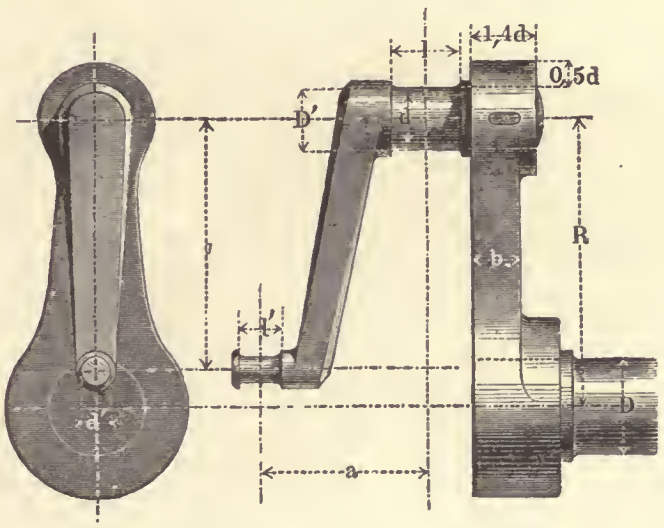

Fig. 467.

force 2); draw $d'\,g$ parallel to 2 O until it intersects a perpendicular through G, the line of the force 3, which we know acts upward, but the magnitude of which is yet undetermined. In order to determine it, as well as the fourth force which acts at H, join g with H, giving $H\,a$ as the closing line which is horizontal because we have chosen the pole O on a horizontal through 0. Now draw in the force polygon O 3 parallel to $H\,g$, then the line 2 to 3 is the third force acting at G upward, and the line 3 to 0 gives the downward force at H. Hence we have the figure $a\,d'\,k\,g\,H$ as the cord polygon of the system of forces. At k is a zero point (see § 132) and for convenience in showing the figure it is preferable to turn the triangle $k\,g\,H$ over to the position $k\,g'\,H$. The cord polygon thus found will be of service in constructing the surface of moments, as will be seen

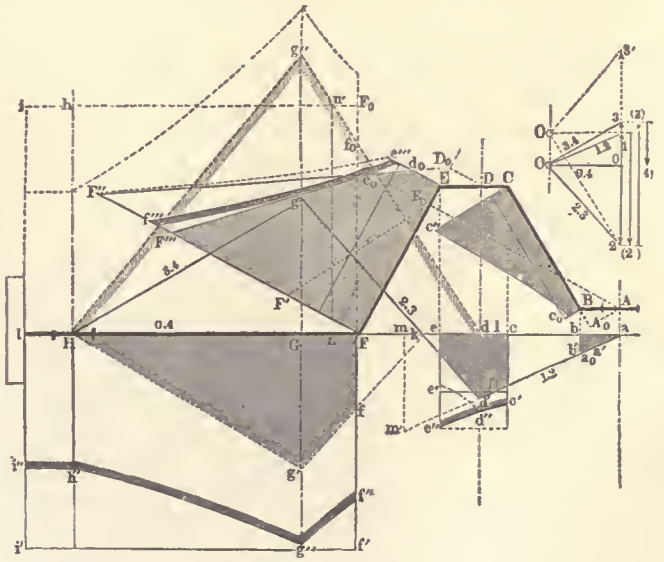

Fig. 468.

later. For the determination of the shank $A\,B$ draw from A on the pressure 1 the triangle $a\,b\,b'$, whose ordinates will serve to determine its dimensions.

Crank-Pin C E D.—This is subject to bending, as shown by the surface of moments $c\,d'\,e$, and to twisting by the force 1 acting as a lever arm $r = C\,c - B\,b$. In order to determine the twisting moment, take $a\,l = r$, and draw the ordinate $l\,l'$, this latter will then be the desired moment, and the corresponding surface a rectangle on $c\,e$. Combining this, as before, with the trapezoid $c\,d'\,e$ gives the surface of moments $c\,c'\,d''\,e'\,e$. Should it occur that the only pressure acting is that upon the return

crank-pin, the surface will be modified as follows: prolong the line $a\,d'$ to m', and taking this bending polygon, obtain the corresponding surface of moments $c'\,d''\,e$, from which the crank pin $C\,D\,E$ can be proportioned. The minimum length l of the crank-pin must be that due to the pressure 2, as given before, for overhung journals.

Axle F G H I.—This is subjected to bending according to the polygon $F\,f\,g'\,H$, and also to torsion by the moment of the force 2 less that of the force 1. In order to find the first, we choose in the force polygon a second pole O', upon a horizontal passing through the starting point of the force 2, returning the same pole distance. Draw 2 O' and make $d\,g''$ parallel to it, make $d\,n = C\,c = R$, and we have in the ordinate $n\,n'$ the desired twisting moment. Make the abscissa of the ordinate at $a' = A\,a = R - r$, and this ordinate will then be the moment with which the force 1 twists the arm backward. Taking this from $n\,n'$ gives the height $F f'$ of the torsion rectangle $F\,I\,i'\,f'$ which we may combine with the bending surface in the manner already given, and thus obtain the surface of moments $F f''\,g''\,h''\,i''\,I$. Should the case occur in which the force 1 becomes zero, as is the case at some points in steam engines when the return crank operates the valve motion, we have for a bending surface $F f_0\,g''\,H$, and for a torsiou surface $F F_0\,i\,I$, which gives a surface of greater ordinates to be used. Such a case is given in unlettered dotted outline shown upon the base $F\,I$. It is assumed that the portion $H\,I$ is subjected only to the action of a torsion couple, hence the polygon there becomes a rectangle.

Return Crank Arm B C.—This is subjected to torsion by the force 1, with an arm $A\,A_0$ perpendicular to $C\,B$ prolonged (its moment being equal to the ordinate at a_0), and to bending by the arm $A_0\,C$, whose polygon is a triangle on $C\,A_0$ and angle at A_0 equal to $l\,a\,a_0$. The reduced surface is shown at $C\,B\,c_0\,c'$.

Main Crank Arm E F.—This is subjected to bending forwards with a moment surface $D_0\,F\,F''$, the angle at D_0 being equal to $e\,d\,g''$, and to forward twisting with an arm $D\,D_0$, which is perpendicular to $F\,E$ prolonged; it is also subjected to backward bending by the force 1, with a surface $E_0\,F\,F'$, and backward twisting by the arm $A\,E_0$ normal to $F\,E$. The combined bending moments give the surface $E\,d_0\,e_0\,F'''\,F$, and the combined twisting moments the rectangular shown upon $E\,F$, the combination of both resulting in the final surface $E\,e'''\,f'''\,F$. Should the force 1 become zero the figure will be increased to that shown by the dotted lines.

§ 171.

The Simple Crank Axle.

Crank axles may be divided into simple and multiple cranks. A simple crank axle is shown in Fig. 469.

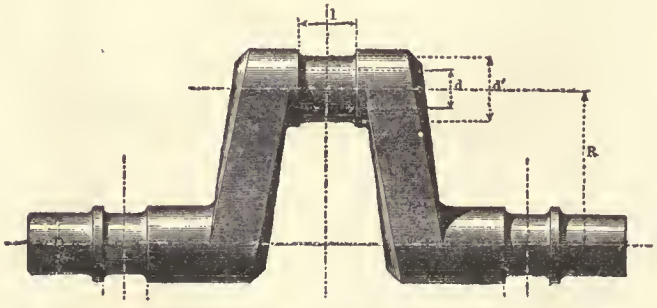

Fig. 469.

The analytical discussion of such a crank axle is such a complicated matter, and the practical results are so readily obtained with all needful accuracy by the graphostatic method, that the latter is only given here. In Fig. 470 is shown a skeleton diagram $A\,B\,C\,D\,E\,F\,G\,H$ of a crank axle with both arms inclined.

If we make the value of the force P, which acts upon the crank pin, equal to Q when it acts in the direction $K\,M$, it will be equal to $\frac{Q}{\cos \alpha}$ when the connecting rod is in any inclined position $K\,L$; α being the angle of the rod with the axis $K\,M$. For a constant force Q the pressure P will be a maximum when $K\,L$ acts normal to $L\,M$, and this is so nearly the same as the value for the vertical position $M\,L_1$ of the crank, or $\frac{Q}{\cos \alpha_1}$, that this latter may be taken for the graphical examination without a closer determination. The force at M is equal to P in magnitude and also parallel in direction, and at K is a normal pressure, which is $N = Q \tan \alpha$ and is a maximum for the position $K_1\,L_1\,M$. Hence we may safely assume that the moments with which the crank arms and the axle are bent attain the maximum at the same time, and are those due to the force P. In the example the crank pin is at E, at B and H are bearings, at A is a couple by which the shaft is subjected to torsion due to the force P acting with a lever arm R. This problem is very similar to the preceding, the portion $H\,G$ taking the place of the return crank, with the difference that the force at H is variable and indeterminate, but is dependent upon the pressure P at E.

Force Polygon.—In order to make the closing line of the polygon horizontal, draw the line $B\,e'$ to any desired point e' on the normal $E\,e'$, join e' with H; then on any convenient scale draw the force P, from O, in the diagram on the right, and make $o\,O$ parallel to $H\,e'$, 1 O parallel to $B\,e'$ and O 2 normal to P. Then the distance 1 to 2 is the upward force P_2 acting at B, and 2 to 0 the force P_3 at H, O_2 being the pole distance.

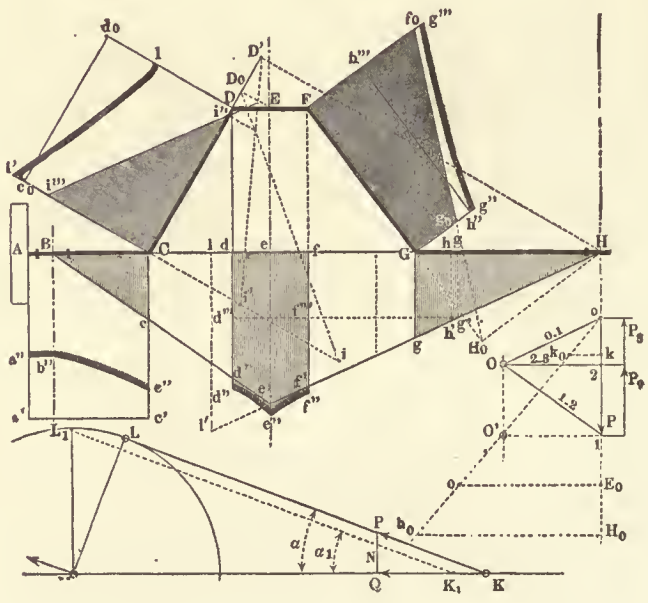

Fig. 470.

Axle Shank H G.—This is subjected to bending by the force P_3 at H. The triaugle $H\,G\,g$ is the surface of moments, and the ordinates may be used to determine the dimensions of the journal at H.

Axle Shank B C.—The surface of moments for bending is the triangle $B\,C\,c$. In addition to the bending is the twisting moment $P\,R$; in order to determine this make O' 1 normal to P and equal to O 2, and also make $E_0\,e_0$ parallel to O_2 and equal to R, then $o\,E_0$ is the desired moment, which laid off at $C\,c'$ and $A\,a'$ aud combined with $B\,C\,c$ in the manner already described, gives the surface of moments $A\,B\,C\,c''\,b''\,a''$.

Crank Pin D E F.—The surface for bending moments is the figure $d\,f\,f'\,e'\,d'$. For twisting we have the force P_3 at H, with a lever arm of $E\,e = R$. Make $H\,g = E\,e = R$ and the ordinate $g\,g'$ is the desired moment, which transferred to $f'''\,d'''$ and combined with the preceding surface gives the surface $d\,f\,f''\,e''\,d''$. The greatest ordinate $e\,e''$ should be used if the pin is to be cylindrical.

Crank Arm G F.—Draw $E\,D_0$ parallel to $H\,D'$ normal to $C\,D$. We then have forward bending by the force P at D_0; backward bending by P_3 acting at D'. The cord polygons for these are, the triaugle $D_0\,C\,i$ (with $C\,i = o\,H_0$ in the force polygon, where $H_0\,h_0 = C\,D_0$), and $D'\,C'\,i$; which when combined give the surface $C\,i''\,i'''$ for the beuding of the arm $D\,C$. We also have a forward twisting from the force P with the arm $E\,D = k\,k_0$ in the force polygon, and the moment $o\,k$ acting backward from the force P_3 with a lever arm $H\,D' = H\,l$ in the cord polygon and a moment $l\,l'$. The difference between these moments laid off at $D\,d_0$ and $C\,c_0$ and the resulting torsion rectangle combined with the bending triangle gives the surface $C\,D\,I'\,I$, so that all five portions of the diagram now have their moment surfaces determined. The method of using these for the determination of dimensions is the same as before.

The figures show clearly the various stresses at the respective portions of the crank and throw light upon the manner in which breakages occur.

If both crank arms are normal to the axis, the solution is greatly simplified, and the diagram assumes the form given in Fig. 471. In this we have again $A\,B\,C\,D\,E\,F\,G\,H$ as the skeleton, and at A a torsion couple whose moment is equal to $P\,R$.

Force Polygon.—In this case the altitude $e\,e'$ of the triangle $B\,e'\,H$ is taken as the measure of the force P. $B\,b''$ is made equal to $e\,e'$, $b''\,O$ drawn parallel to $e'\,H$, and $O\,b$ made normal to $B\,b''$, thus giving $b''\,b$ as the force P_3 at H, $b\,B$ that at B, and $O\,b$ is the corresponding pole distance.

Axle Shank H G.—This is only subjected to bending, and the surface of moments is $H\,G\,g$.

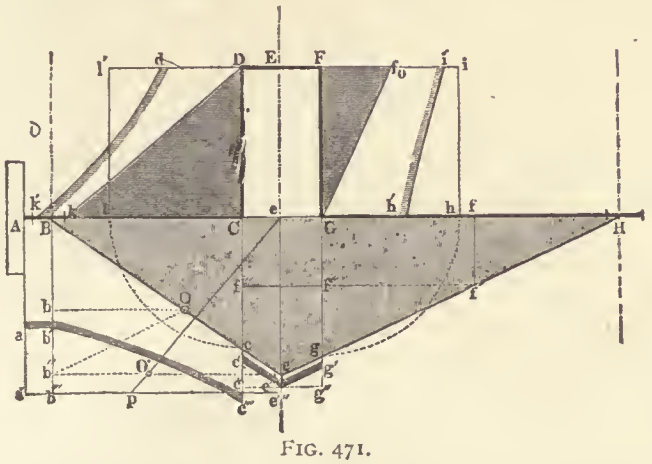

FIG. 471.

Axle Shank A B C.—This is subjected to bending, as indicated by the triangle $B\,C\,c$, and also to torsion by a moment $P\,R$. Make $e'\,O'$ parallel to $C\,B$ and equal to the pole distance $b\,O$, draw $e'''\,p$ parallel to $e'\,O'$ and equal in length to $E\,e = R$, then $e\,e'''$ is the desired twisting moment, giving for $A\,C$ the torsion rectangle whose altitude $A'\,a' = B\,b''' = e\,e'''$. The combination of bending and torsion moments gives the moment surface $A\,B\,C\,c'''\,b'\,a$.

Crank Pin D E F.—This is subjected to bending according to the surface of moments $C\,G\,g\,c$, and to torsion by the force P_3 at H, with a lever arm $R = C\,D = H\,f$, and a moment $f\,f' = G\,f'' = C\,f'''$. By combining the twisting and bending moments the surface $C\,G\,g'\,e''\,c'$ is obtained, and for cylindrical crank pins the rectangle of a height $G\,g'' = C\,c'' = e\,e''$ is to be substituted for the irregular outline.

Crank Arm F. G.—This is subjected to bending by the force P_3 acting at G. The surface of moments is $G\,F\,f_0$, the angle at G being equal to $f\,H\,f'$; it is subject to torsion by the same force acting with a lever arm $H\,G$, giving a moment $G\,g = G\,h = F\,i$. The combination of twisting and bending moments gives the surface $F\,G\,h'\,i'$.

Crank Arm C D.—Here we have bending with the force P_3 and an already known moment $e\,e''' = C\,k$ at C. Twisting is due to the moment $C\,c = C\,l = D\,l'$. For the combined moments these give the surface $C\,D\,d\,k'$.

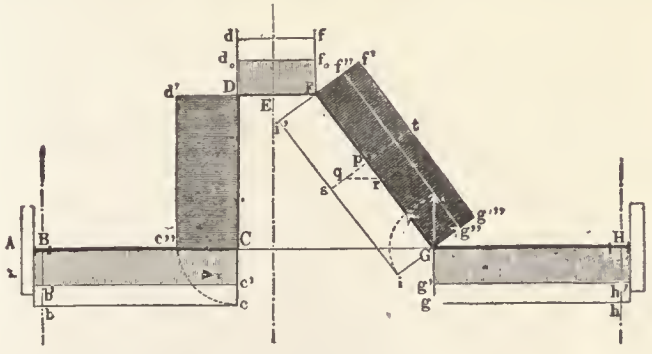

FIG. 472.

For the same given distances of E from B and H the torsion stresses on the crank arms are greater for arms normal to the axis than for inclined arms, so that in the former case heavier arms are required. The torsion in the crank arms grows less and less the nearer the points C and G approach B and H, which is a point to be considered in the interest of economy of material. It is also to be noted that the total length of crank axle $F\,G\,H$ or $D\,C\,B$ is less for inclined arms than for right-angled cranks.

In many cases a crank axle is so situated that it is subjected to torsion at either one end or the other. In such cases the diagram should be constructed for both sets of conditions, and laid upon each other, the greater value in all cases being taken. Of course, care must be taken to use the same pole distance and same scale for measuring forces in both cases. An example of such a case is found in the paddle engines made by Penn, with oscillating cylinders, the air pump being worked from the middle of the crank pin. The conditions in this case are somewhat different from the preceding, and may be examined with the help of the following diagram (Fig. 472):

Here we have the skeleton $A\,B\,C\,D\,E\,F\,G\,H$, and not taking into account the force at E, the force couple gives by means of the cord and force polygon the moment values $B\,b = C\,c = G\,g = H\,h$, from which the following results are obtained:

Axle Shank A B C.—Pure torsion, which, converted into an equivalent bending moment, gives $B\,b' = C\,c' = \frac{5}{8}\,B\,b$ (see IV., § 16, when $M_b = 0$).

Axle Shank G H.—This is the same as the preceding, and $H\,h' = G\,g' = C\,c'$.

Crank Pin D E F.—We have here the same twisting moment as in the axle shanks $D\,d = F\,f = B\,b$ and $D\,d_0 = F\,f_0 = B\,b'$.

Crank Arm C D.—We have in this portion a bending moment of the magnitude $C\,c'' = D\,d' = C\,c$, of which the plane stands normal to the plane of the surface of the crank arm. The surface of moments is in this case equal to a rectangle of the height $B\,b = C\,c$.

Crank Arm F G.—In this case we have both torsion and bending. The couple is decomposed at G into two parts, one acting normal to the axis of the crank arm, and the other in the direction of the arm. The first gives the torsion rectangle $G\,F\,f''\,g''$, the latter the bending rectangle $F\,G\,i\,i'$, which combined give the moment surface $F\,G\,g'''\,f'''$, in which we again have $p\,q = \frac{5}{8}\,G\,i$, $p\,r = \frac{3}{8}\,G\,g''$, $p\,t = G\,g'' = q\,s + q\,r$.

Thus far we have proceeded as though there were no force acting at E. When such exists, however, first determine the bending and twisting moments as shown in Fig. 472, add or subtract, according to direction, the twisting moments, taking into account the position of the planes of bending action, and finally combine the bending and twisting moments so found, according to the method of Case IV., § 16. The amount of work which this investigation requires of the drawing-room of any machine-shop is small compared with the importance of a thorough determination of all the stresses which act upon such a piece of work as a crank shaft forging.

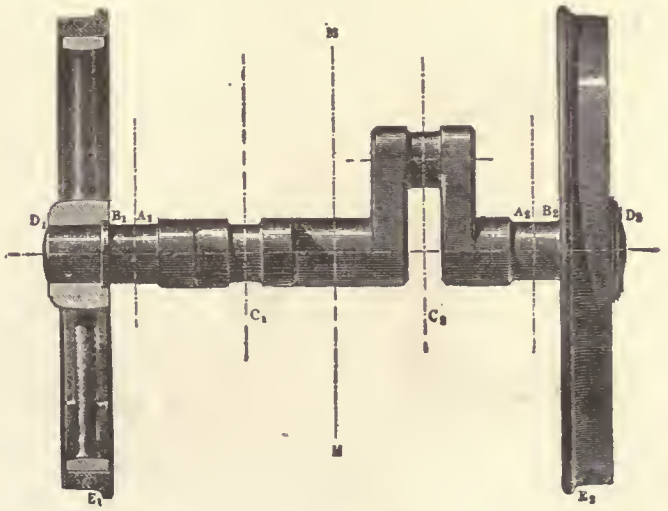

FIG. 473.

§ 172.

MULTIPLE CRANK SHAFTS, LOCOMOTIVE AXLES.

One of the most important forms of crank axles made of wrought iron or steel is that used for locomotive engines. As an example of this subject, the crank axle for an inside connected locomotive is given in Fig. 473. In drawing the diagram of moments it is necessary to take into account the diameter of the driving-wheels, as will be shown in Fig. 474. C_1 and C_2 are centres of the steam cylinders, A_1 and A_2 are the journals, and $B_1\,D_1$ and $B_2\,D_2$ are the hubs of the respective driving-wheels. The cranks at C_1 and C_2 are placed at right angles with each other, taking the position which the axle shows in Fig. 473. An inspection of the figure shows three distinct loads acting upon the axle: 1, the pressure in the vertical plane due to the weight of the locomotive and to the lateral action upon the wheel

flanges; 2, the horizontal pressure of the piston against the crank C_2 opposed by a corresponding adhesion at the circumference of the driving-wheels; 3, the oblique pressure of the connecting rod acting upon the crank C_1. Other small pressures, such as those due to the eccentrics, etc., may be neglected.

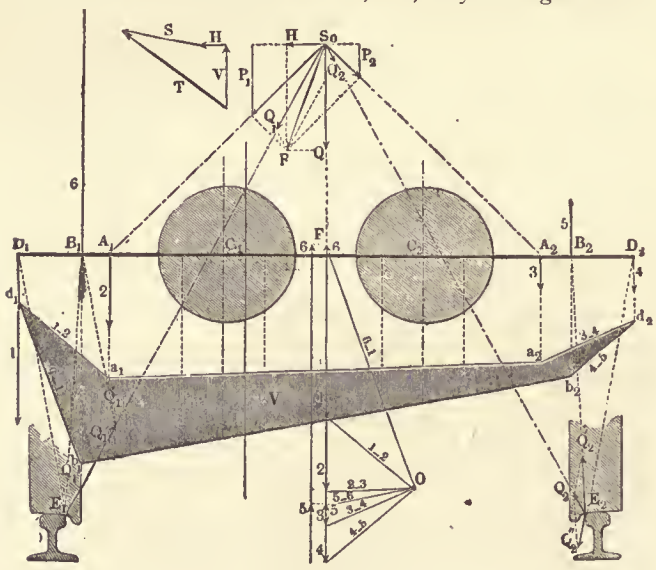

FIG. 474.

Forces and Moments in the Vertical Plane.—Fig. 474. From the point S_0 of the height of the centre of gravity of the locomotive lay off the force Q, to represent that portion of the weight which is borne by the axle under consideration. The oscillations and action of centrifugal force upon curves also produces a horizontal force H, which may be taken as equal to 0.4 Q. The resultant R of the two forces Q and H is the load upon the axle. This may be decomposed into the pressures P_1 and P_2 upon the journal at A_1 and A_2, and into the pressures Q_1 and Q_2 upon the wheels at E_1 and E_2, which pressures, with their reactions, produce the stresses on the axle. The forces Q_1 and Q_2 can be decomposed into two others referred to the wheel hubs $B_1 D_1$ and $B_2 D_2$. This gives six vertical pressures acting to bend the axle, viz.: 1, 2, 3 and 4 acting downward at D_1, A_1, A_2 and D_2, and 5 and 6 acting upward at B_2 and B_1. From these forces, by choosing any desired pole distance, the force polygon F, 4, O may be constructed, and also the cord polygon or surface of moments $d_1 a_1 a_2 d_2 b_2 b_1$, and this surface gives by its ordinates the proportional bending moments in the vertical plane for each point in the axle; this entire surface is designated by the letter V.

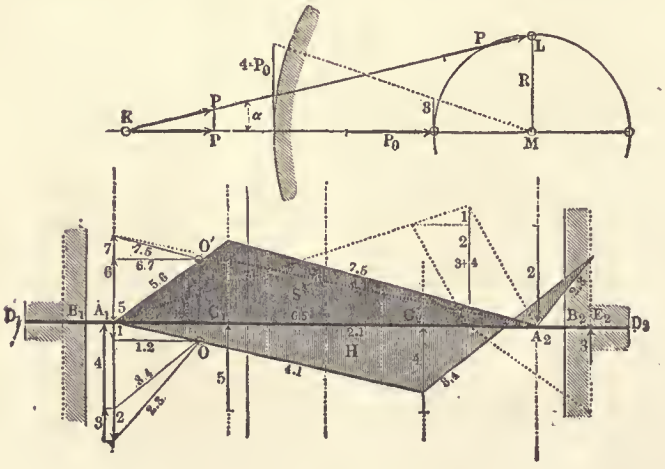

FIG. 475.

Forces and Moments in the Horizontal Plane.—Fig. 475. As already shown in a preceding paragraph, the pressure P on the crank pin for the position $L M$ of the crank is somewhat greater than the pressure P_0 on the piston; its moment of rotation about the shaft is $\frac{P_0}{\cos \alpha} . R \cos \alpha$, which $= P_0 R$, so that upon the assumption that the wheel on the left slips on the rail, the other one must oppose a resistance whose moment equals $P_0 R$ and the frictional resistance 3 at $E_2 = P_0 \frac{R}{r}$. Combining this force 3 at E_2 and also the force $4 = P_0$, and the resistances 1 and 2 at the journals, we are enabled to construct the force polygon $A_1\ 2\ O$ and the corresponding cord polygon H for the horizontal forces, as shown in the light sectional portion of the diagram. The forces 1 and 2 are found by taking the position of the resultant of the two forces 3 and 4, as shown in the figure, and decomposing their sum into the portions which would go respectively to A_1 and A_2, as shown by the construction given in the dotted lines.

Forces and Moments in the Inclined Plane of the Connecting Rod.—The force $Q = 5$ acts at C_1, making an angle with the horizontal equal to $M K L$. As shown in the illustration, this may be decomposed into the two opposing forces 6 and 7 at A_1 and A_2, and by taking the same pole distance as before to construct the force polygon we obtain the cord polygon S, shown by the dark section lining, and giving the surface of moments for bending in the inclined plane of the connecting rod.

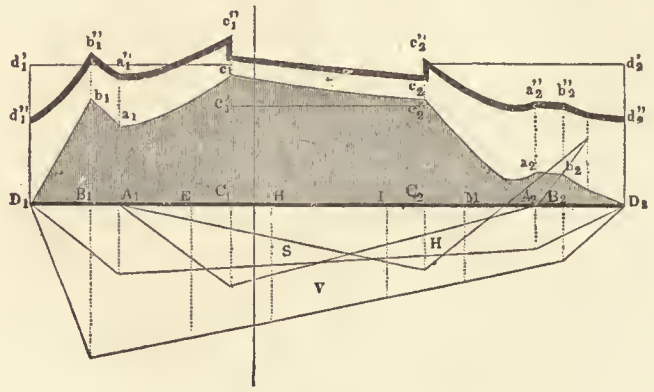

FIG. 476.

Combination of the Three Preceding Cord Polygons for Bending of the Axle.—Fig. 476. Since the three preceding sets of forces are acting at the same time to produce bending in the axle, it is necessary to combine the diagrams in order to obtain the final result. For this purpose we can treat the respective ordinates in the same manner as if they were forces, as in § 44. Taking the successive points upon the axle, we construct the corresponding ordinate polygons, whose closing lines give the resulting moment both in direction and magnitude. One of these ordinate polygons is shown in the upper portion of Fig. 474, to the left; it belongs to the point C_1. The vertical ordinate V in this case acts upward, the horizontal ordinate H continues toward the left, and the inclined ordinate S also continues to the left, thus giving the resultant T as the line joining the origin of V with the termination of S. We thus obtain for the entire axle the surface of moments $D_2 D_1 a_1 c_1 c_2 a_2 b_2$, which gives the proportion of bending stresses of the axle, as distinguished from those of the crank arms.

The Torsional Moments for the Axle.—The position of crank described above and selected for this investigation gives a torsional moment only upon the crank to the left, and also one of the magnitude PR upon the axle extending to the point D_2. If both cranks stand at an angle of 45° with the horizontal, there will be produced in both end shanks $C_1 D_1$ and $C_2 D_2$ moments equal to $\sqrt{2}\ PR$, or about 1.4 PR. Under these circumstances the moments at the ends become $D_1 d_1' = D_2 d_2'$, while in the body of the shaft $C_1 C_2$ we have the moment $C_1 c_1' = C_2 c' = PR$, always keeping the scale of forces and the pole distance the same in all of the diagrams. It must be remembered that in this position of the cranks the bending moments are somewhat different from those shown in the preceding diagrams.

Combination of Bending and Twisting Moments.—The bending and twisting moments can now be combined according to the formula of § 45, and thus the surface of moments $D_2 D_1 d_1 b_1 \ldots d_2''$ obtained, by the help of which the shanks $C_1 D_1$ and $C_2 D_2$ and body of the axle $C_1 C_2$ can be proportioned, after the diameter for any one of the ordinates, as, for example, that at $B_1 b_1$, has been determined. The half of the diagram which gives the greatest ordinates should be used for both halves of the axle.

Crank Pin at C_1.—The two crank pins are treated separately in Figs. 477 and 478, since the moments can be laid out more conveniently in that way. For the pin $F G$ at C_1 we have, in addition to the bending moments obtained from Fig. 476, and shown by the surface $F G c_1$, the combined forces on the left, up to the point E, acting to twist the pin. The resultant of these forces is yet to be found. The vertical forces are those shown at 1, 2 and 6 of Fig. 474, their algebraic sum being shown at I, in Fig. 477. The horizontal force acting backwards is II, represented as 1, in Fig. 475. The inclined force acting downwards

and backwards, shown at *III*, corresponds to the force 6, of Fig. 475. The closing line (not shown) from *III* to C_1 would give the resultant, and its horizontal component *IV* acts to twist the crank pin *FG*, with a lever arm $EF = R$. In the force polygon (above, on the left) we take *a O* to be the pole distance, as before; lay off *IV* downward from *O*, draw *a IV e*, make $af = R$; then will *fe*, perpendicular from *f*, be the twisting moment Ff'. Combining this with the surface of bending moments $FG c_1$, we obtain the final surface $FG c_1'$.

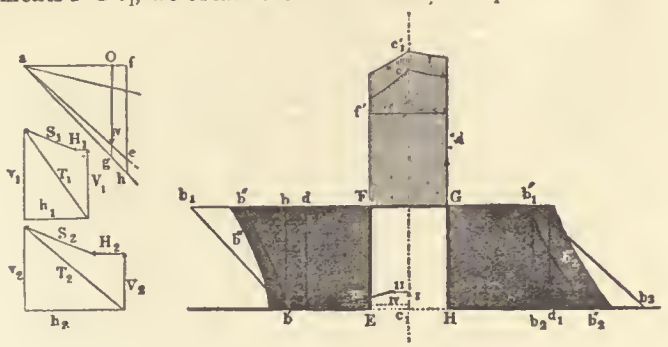

FIG. 477.

Crank Arm E F.—The ordinate polygon $V_1 H_1 S_1 T_1$ (on the left) is constructed for the point *E*. The horizontal component h_1 of the resultant T_1 acts to twist the arm *E F*, $Fd = h_1$; the vertical component v_1 acts to produce a bending of the arm in the plane of the diagram, $Fb = v_1$; also the force *IV* acting at *E* tends to bend the arm normal to the plane of the diagram, with a moment $b b_1 = Fd_1$ at *F*. The combination of the bending moments gives the surface $E F b' b''$, which, with the torsion rectangle *E F d*, gives the final surface $E F b'''$.

Crank Arm G H.—The ordinate polygon $V_2 H_2 S_2 T_2$ is constructed for the point *H*. The horizontal component h_2 acts to twist the arm *G H*, $Hd_1 = h_2$; the vertical component v_2 shows the bending in the plane of the diagram, $G b_1 = v_2$; also, the force *P* bends the arm normal to the plane of the diagram with a moment $PR = fh$, of the force polygon above, on the left, in which $Og = P$, $af = R$. Again, make $b_2' b_3 = fh$. The combination of the bending moments gives the surface $G H b_1' b_2''$, and the combined bending and twisting moments give the final surface $G H b_2'''$.

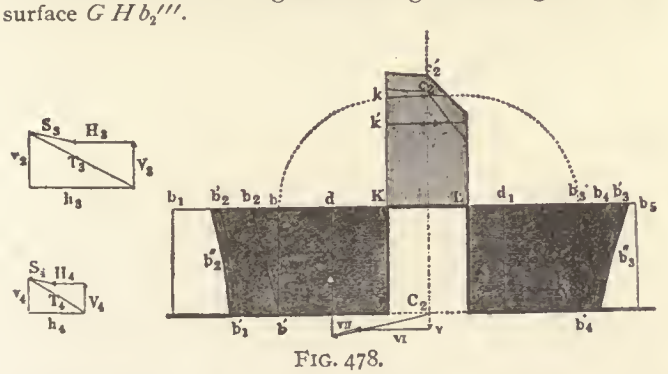

FIG. 478.

Crank Pin K L.—Fig. 478. This crank pin is subjected to the beuding moments which act between *M* and *J*, and indicated by the surface $K L c_2$, obtained from Fig. 476. The collected forces which act on the left of C_2 tend to twist the pin. The resultant of the forces 3, 4 and 5, Fig. 474, shown at *V* in Fig. 476, acts downward, the resultant (difference) of the forces 2 and 3, Fig. 475, and shown at *VI*, acts horizontally backward, and the force 7, of Fig. 475, shown at *VII*, acts inclined backwards. The vertical component of the force polygon *V, VI, VII*, acts to produce twisting at *M*, remembering that the crank *JK* is taken in the horizontal position. The moment of this vertical component has the magnitude $k k'$. Also we have acting to twist the pin the couple shown on the left (as discussed in connection with Fig. 472) with a moment already determined and shown at $C_1 c_1'$ in Fig. 476, and here laid off at $K k$, from which, since the previously determined twisting moment $k k'$ acts in the opposite direction, we must subtract $k k'$, giving finally for the crank pin *K L* the twisting moment $K k'$, which, when combined with the bending moment, gives the surface $K L c_2'$.

Crank Arm J K.—This is subjected to twisting by the moment $Kd =$ the vertical component v_2 of the ordinate polygou $V_3 H_3 S_3 T_3$. For bending in the vertical plane we have the moment $Kl = Kk$, as already shown in Fig. 472; also in the same manner and directiou by the vertical component of the forces *V, VI* and *VII* with the moments $b b_2$ at *K* (see the diagram of these moments in the upper left portion of Fig. 477). It is subject to bending in the horizontal plane by the horizontal compouent h_3 of the ordinate polygon, the moment being $b b_1$. The combination of bending moments gives the surface $J K b_1' b_2'$, and the final combination with the twisting moment $K d$ gives the surface $J K b_2''$.

Crank Arm L M.—The twisting moment is $L d_1 =$ the vertical component v_4 of the ordinate polygon for the point *M*. The bending moment $L b_3 = K k$, also $b_3 b_4$ due to the vertical force at *M*, and also the bending moment $b_3 b_5 =$ the horizontal component h_4 of the ordinate polygon. The combination of bending moments gives the surface $M L b_3'$, and the final combination with the twisting moment gives the surface $M L b_3''$.

Of the four crank arms, *J K* is subjected to the greatest stress at the pin, and *G H* at the axle. In practice, therefore, the surfaces $J K b_2''$ and $G H b_2'''$ should be drawn upon each other and the greatest ordinate used. The resulting dimensions, with possibly slight modifications, should then be used for all four arms.

Although the construction of such a graphostatic diagram of moments involves some labor, the result is most satisfactory, since by assuming a stress of say $\frac{2}{3}$ the modulus of working stress (about 17,500 lbs. for wrought iron, 25,000 lbs. for steel) the design can be properly proportioned without further care.

The calculations for locomotive axles with outside cranks is similar to the preceding, although the diagrams are necessarily somewhat different, although laid out in the same general manner.

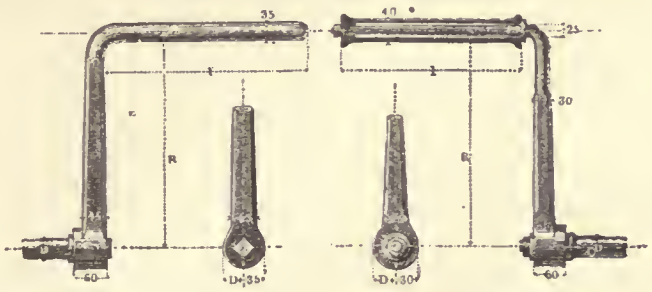

FIG. 479. FIG. 480.

§ 173.

HAND CRANKS.

The chief peculiarity in a hand crank lies in the adaptation of the crank pin to be operated by hand. In Fig. 479 is shown a crank for two men, and in Fig. 480 for one man. The dimensions for the parts indicated by the letters are as follows:

	For 2 men.	For 1 man.
$R =$	14″ to 18″	12″ to 16″
$l' =$	16″ to 19″	12″ to 13″
$D -$	$1\frac{1}{2}$″ to $1\frac{3}{4}$″	$1\frac{1}{4}$″ to $1\frac{1}{2}$″

The other dimensions figured in the illustrations are in millimetres. When placed at opposite ends of the same shaft, hand cranks should be set at 120° with each other.

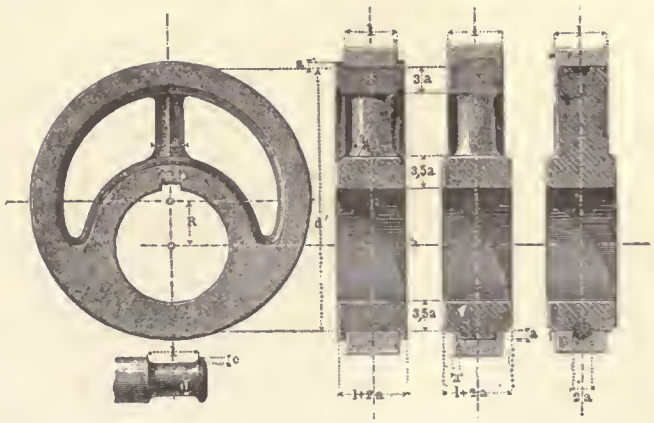

FIG. 481. FIG. 482. FIG. 483. FIG. 484.

§ 174.

ECCENTRICS.

An eccentric is nothing more than a crank in which (if the crank arm is *R* and the shaft diameter *D*) the crank pin diameter *d'* is made so great that it exceeds $D + 2R$, or is greater than the shaft and twice the throw. The simpler forms of eccentric construction are shown iu the illustrations. The most prac-

tical of these is that shown in Fig. 483, the flanges on the strap, as shown in the section, serving to retain the oil and insure good lubrication.

The breadth of the eccentric (properly the length of pin l) is the same as that of the equivalent overhung journal subjected to the same pressure; for the depth of flange a we have

$$a = 1.5\,e = 0.07\,l + 0.2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$

from which the other dimensions can be determined as in the illustrations.

For some forms of shafts with multiple cranks or other obstructions the eccentrics cannot be made as shown above, but must be in halves, bolted together.

CHAPTER XIII.

COMBINED LEVERS.

§ 175.

Various Kinds of Combined Levers.

Two simple levers with the same hub form what is termed a Combined Lever. When both arms have a common centre line they form a Beam, or so-called Walking Beam; and when they form an angle with each other they are called an Angle Beam,

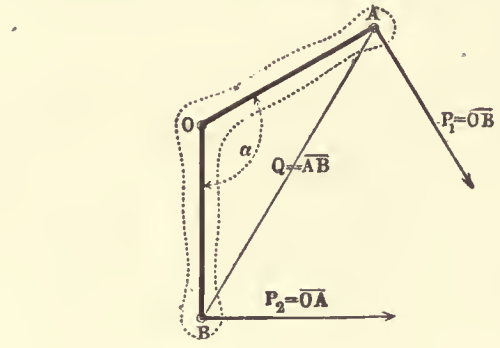

Fig. 485.

or frequently a Bell Crank. The pressure Q, upon the axle of an angle lever $A\,O\,B$, Fig. 485, is determined by the relation

$$Q = \sqrt{P_1^2 + P_2^2 - 2\,P_1\,P_2 \cos a}$$

if P_1 is the force acting at A, and P_2 that at B, both acting at right angles to their respective arms; a being the angle between the arms. This may be shown graphically by making $P_1 = O\,B$ and $P_2 = O\,A$, when Q will $= A\,B$, the third side of the triangle. If the forces P_1 and P_2 do not act at right angles to the arms, the triangle must be constructed by drawing lines from O, normal to the directions of the forces.

The variety of combined levers is very great, and only a few of the principal forms are here given.

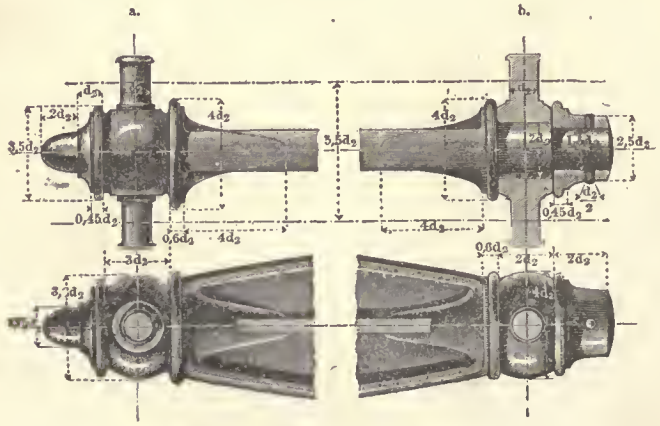

Fig. 486.

§ 176.

Walking Beams.

One of the principal forms of combined levers is the walking beam, for use in some forms of steam engine. These are usually made of cast iron, with journals and pins similar to those given in Fig. 456; and other forms of journals are also shown in the following figures.

Fig. 486 a shows an ornamented beam-end, with the pin keyed fast. Fig. 486 b shows a beam-end with a bored cross-head and pins combined, fitted on the turned end of the beam and secured by the pinned collar shown. This construction requires careful fitting, and is somewhat expensive.

Fig. 487 a. This is a fork journal; the fit is made with a very slight taper, secured by cap bolt and large washer at one end. The pin is kept from turning by a projection under the head, let into the boss on the beam.

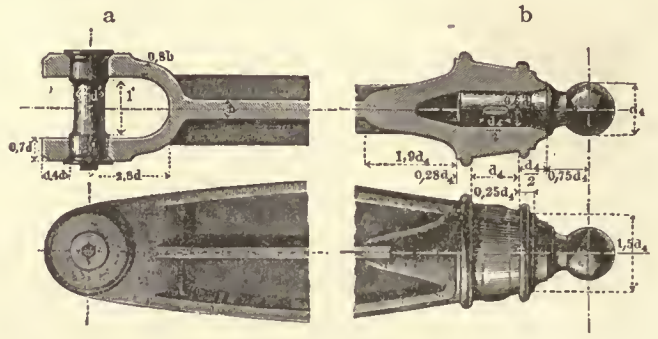

Fig. 487.

Fig. 487 b. This is a spherical bearing with its shank driven into the end of the beam and keyed fast, this form giving great freedom of motion to the connecting rod.

The diameters of pins are determined as already given in § 90. The load is to be considered as acting continuously or intermittently, according as the engine is single or double acting.

Fig. 488 shows a form of beam which has been extensively used. In order to secure lateral stiffness, the beam centre should not be made too short. A good proportion is that given in the figure, in which the distance between centres of bearings is made equal to $6\,d + \frac{1}{10}\,A$. The distance between centres of journals for the ends of the beam is made from $4.6\,d_2$ to $5.5\,d_2$;

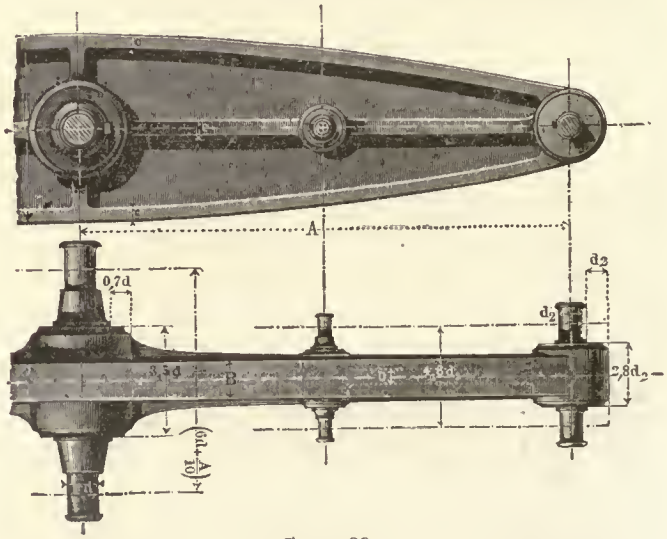

Fig. 488.

d_2 being the journal diameter, as shown. The depth h of beam in the middle must not be made less than

$$h = 4\,d + \frac{A}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (158)$$

in which d is the diameter of the beam centre, and A the half length of the beam. If the two arms are of unequal length their mean should be taken.*

The curved outline of such beams is drawn according to the methods given in § 142, starting from the crown of the beam to the hub for the pins at the ends. The ribs in the middle of the beam are given the same thickness, c, as the flange at the edges, and the breadth of flange is shown in the plan at B (see § 163).

Another form of beam is shown in Fig. 489. This is made double, and in such case each half is calculated separately. In Fig. 490 is shown a section of such a double beam in which the parts are somewhat widely separated. The two plates are firmly bolted together, the bolts passing through tubular stays, as shown, and the parallel motion rods are hung between the two parts of the beam.

* In the United States much greater depth is given to beams of this sort, sometimes 2 to 2½ times that given by the formula. Skeleton beams with cast-iron centres and wrought-iron bands are also much used.

A beam of somewhat unusual form is shown in Fig 491, being a portion of the hydraulic riveting machine of Mackay & McGeorge, built by Rigg.* The beam centre is at *A*, the rivet die at *B*, the hydraulic pressure is exerted by small and large cylinders at *D* and *C* respectively. The water pressure is taken from an accumulator and discharged into an outlet pipe placed somewhat higher than *D*. By means of a suitably arranged valve

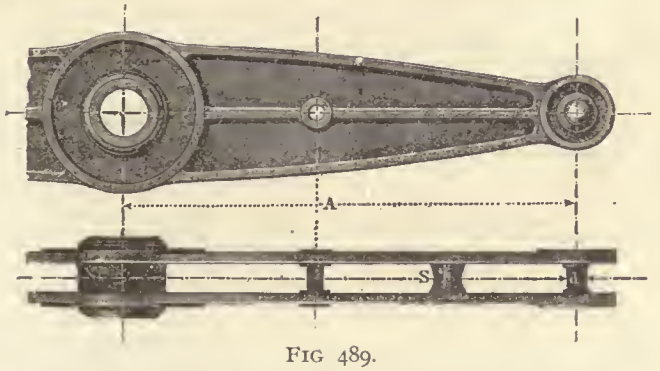

FIG 489.

gear the high pressure water is first exerted upon the small cylinder, and water from the discharge pipe delivered to the large cylinder, thus closing the die upon the rivet at *B*. Then the high pressure water is also delivered to the large cylinder, making a still greater pressure upon the rivet, with practically

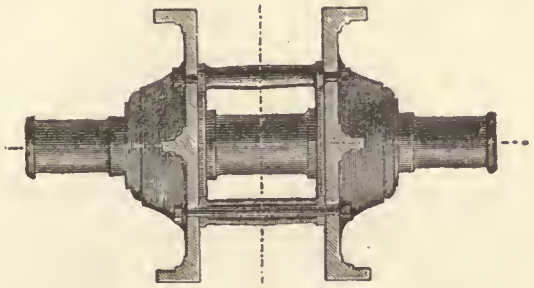

FIG. 490.

no expenditure of water, as that cylinder is already filled. The pressure upon the rivet is 60 tons. The beam is made of a section of uniform resistance (see § 9). At *E* is a short shear for cutting beams, angle iron, etc. The distance *B C* is 12 feet.

Wrought iron beams are not uncommon, and for moderate

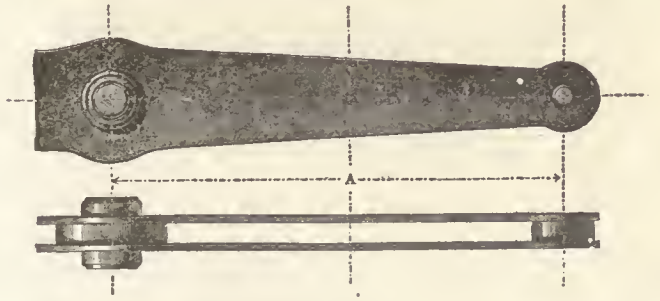

FIG. 492.

loads and dimensions are conveniently made in the double form, as shown in Fig. 492. The depth *h* in the middle may be taken at 0.8 times the value given by formula (158). For larger beams of wrought iron, the girder form shown in Fig. 491 is to be preferred.

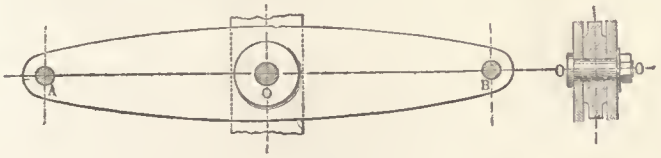

FIG. 493.

Another form of beam is the equalizing lever, used to distribute the weight among the springs (see Figs. 102 and 103, § 41). In Fig. 493 is shown a lever of wrought iron for a heavy engine (the Prussian standard freight engine). The length *A B* is 1180 mm. = 46½″, and the connections at *A*, *O* and *B* are made with

* See *Engineering*, March, 1875, p. 223.

bolts. Fig. 494 shows the form used on American locomotives. The example is from a passenger engine, and extends between

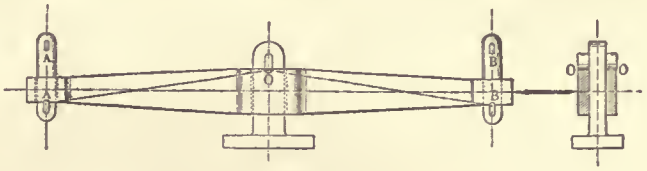

FIG. 494.

the springs of the driving-wheels, being 7½ feet long. At *O*, *A* and *B* are half journals, and the connections at *A* and *B* are not rigid. The bearings are not on a straight line, as in the German form, but the variation is trifling.

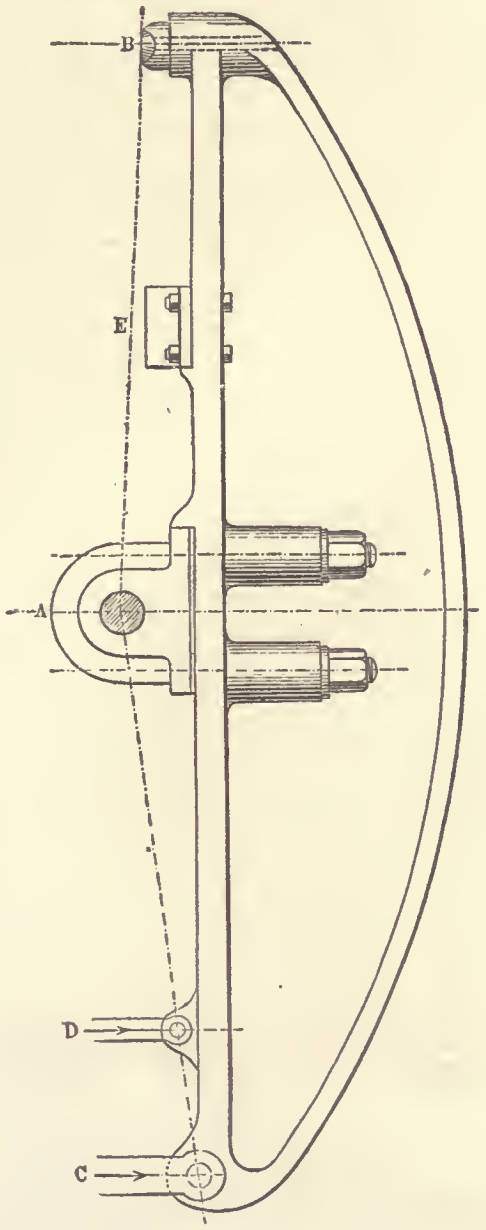

FIG. 491.

§ 177.

SCALE BEAMS.

In scale beams the bearings are usually made upon knife edges (see § 95), generally with an angle of 60°. A special form is here given, Fig. 495, which may serve as an example, showing the main supporting beam of a bridge-scale, in triangular form. In the construction *a*, the main bearings are at *O O*; the bearings *A A* form a double journal analogous to Fig. 476; at *B* is the end journal, here set in a cast-iron head. In the form shown at *b*, we have two separate bearings at *O O*, the parts being held together by a bolt *C*.*

* For similar examples see E. Brauer's "Konstruktion der Waage" (Scale Construction), Weimar, Voss, 1880.

Scale beams should show very little deflection under their load. They are therefore made very deep in proportion to their total section, and the stresses taken at 4250, 8500 and 14,220 lbs. respectively for cast-iron, wrought-iron and steel.

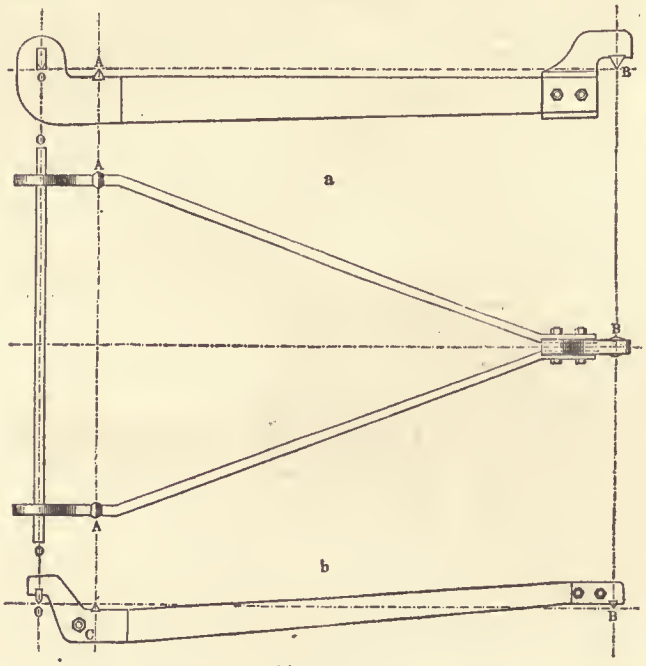

FIG. 495.

CHAPTER XIV.

CONNECTING RODS.

§ 178.

VARIOUS PARTS OF CONNECTING RODS.

Connecting rods are used in various forms for transmitting the motion of various reciprocating parts of machines to levers, beams or cranks, or *vice versa*. It is necessary to consider separately the ends or heads which contain the bearings for the crank and cross-head pins, from the body of the rod. The dimensions and proportions of the ends are governed, to a greater or less extent, by the dimensions of the bearings, the latter being either forked, overhung or necked, and their size determined by the pressure to which they are subjected.

§ 179.

CONNECTIONS FOR OVERHUNG CRANK PINS.

The strap and key connection shown in Fig. 496 is widely used. The boxes are surrounded and drawn together by the

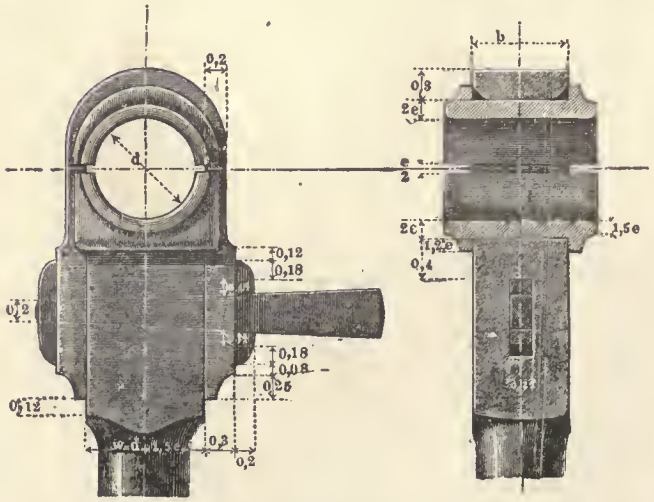

FIG. 496.

strap and key, and by driving up the latter they may be closed together to take up wear. In determining the dimensions, the boxes and their surrounding parts will be considered separately, as in the case with other bearings. The unit or modulus for the boxes is

$$e = 0.07\,d + 0.118'' \quad \ldots\ldots\ldots \quad (159)$$

being the same as used or other bearings, d being the diameter of pin.

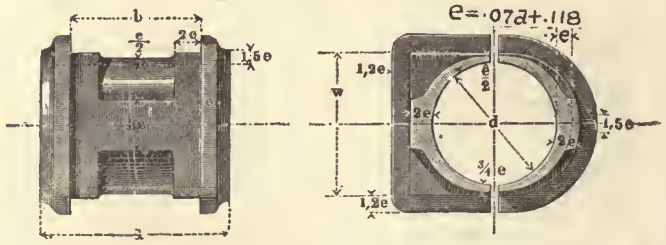

FIG. 497.

Fig. 497 shows two views of the brasses, the dimensions of the other parts being based on the following modulus:

$$d_1 = 0.0267\sqrt{P} + 0.2'' \quad \ldots\ldots\ldots \quad (160)$$

The breadth b may be made equal to $0.8\,d_1$, or if the length of the journal is made equal to its diameter b becomes $= d - 2e$.

Example: If $P = 7920$ pounds alternating load, we have from (93) $d = 2\frac{3}{8}''$, l also $= 2\frac{3}{8}''$, and according to (166)

$$d_1 = 0.0267\sqrt{7920} + 0.2''$$
$$= (0.0267 \times 88.9) + 0.2'' = 2.57'', \text{ say } 2\tfrac{9}{16}''$$

We also have $e = (0.07 \times 2.375) + 0.181 = 0.3''$. Also $b = l - 2e = 2.375 - 0.6 = 1.77$. Applying the value of d_1 to Fig. 496, we have the thickness of strap $= 2.57 \times 0.2 = 0.514$ on the sides, and $2.57 \times 0.3 = 0.76$ on the end; also the thickness of key $= 2.57 \times 0.22 = 0.56''$, and the other dimensions in a similar manner.

The key must be given much less taper when it is used without a set screw, as in the illustration, than when a set screw is used. In the former case a total taper of $\frac{1}{12}$ is used, and in the latter $\frac{1}{6}$ is safe.

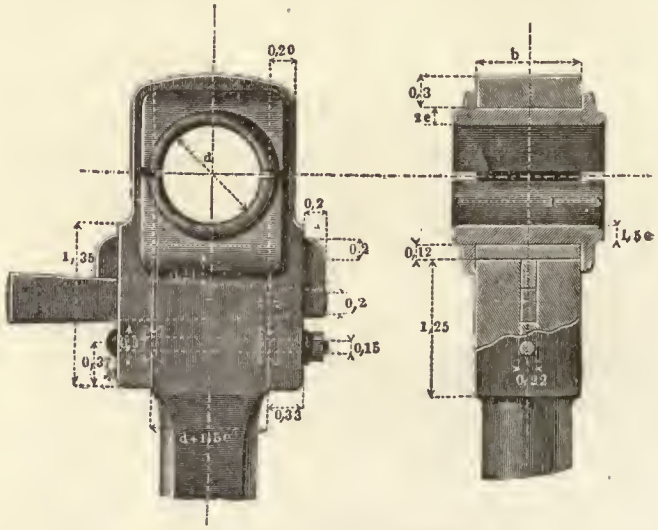

FIG. 498.

The boxes are best made to bear closely together instead of being set open, as shown in the figure, and better practice in this respect is shown in Figs. 499 and 500. In this case the boxes must be filed off to permit them to be closed up for wear.

An objection to the form of strap end just shown is that the continual keying up of the boxes tends to shorten the rod. The reverse action takes place with Sharp's strap end, Fig. 498, the action of keying up tending to lengthen the rod.

In Fig. 499 is shown a capped end of solid bronze, as made by Penn. The two halves are fitted closely together, so that the joint must be filed out to take up for wear, or else a number of thin slips of copper may be inserted in the joints and removed one at a time, as may be found necessary. The diameter δ of the bolts must be made, so that they shall not be less, measured at the base of the thread, than the value given by formula (84). For V thread this is given by making

$$\delta = 0.0142\sqrt{\frac{P}{2}}$$

and if square thread bolts are used they should be made slightly larger. The stress on the material with these sizes will then be between 7000 and 8000 pounds, which is not excessive. (Compare Example 2, § 182.)

The nuts of these bolts are fitted with Penn's locking device, Fig. 243. For rods of large dimensions, such as are used on heavy marine engines, the boxes are cored out in order to secure economy of material.

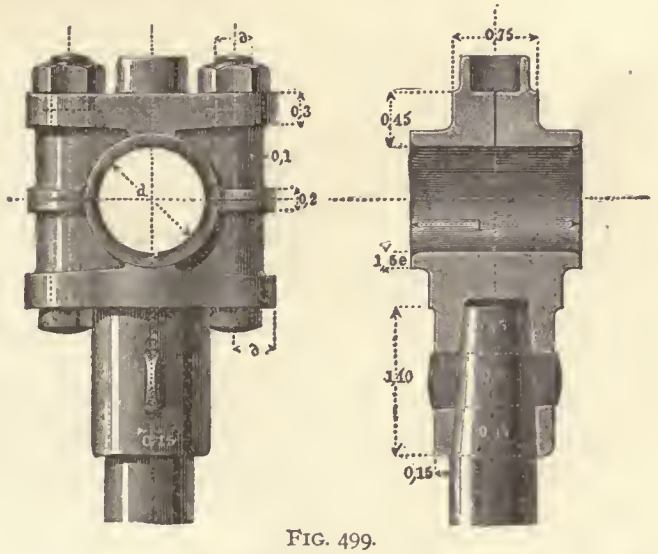

FIG. 499.

In Fig. 500 is shown a rod with a solid end, and is a very excellent form, and with proper tools, not too expensive to construct. The boxes are made of bronze, lined with white metal and turned on the outside. The movable box is fitted with a wrought iron pressure block, which receives and transmits the pressure of the key. The boxes are provided with small projections, which engage with corresponding recesses and prevent them from turning.

The key-slot in the rod is made with semi-circular ends, partly because the machine which forms the slot leaves it in that condition, and also because this shape weakens the section

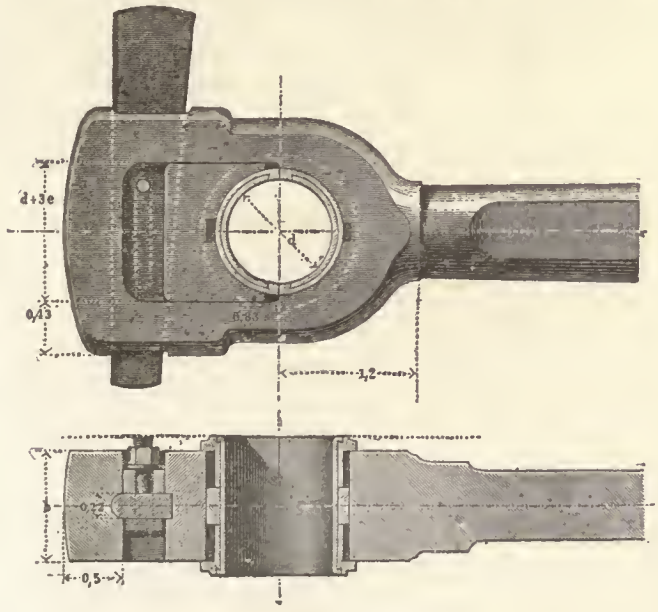

FIG. 500.

of the rod less. The key itself is made flat on the side which bears on the pressure block, in order that liners may be introduced when necessary. The key is secured by the method shown in Fig. 201. It will be noticed that the nut is set so deep in the recess that a socket wrench is required to turn it. This is done in order that nothing may project beyond the dotted clearance line.

In Fig. 501 is shown another solid rod end, much used on locomotive engines. The boxes are made without flanges on the back, so that they can readily be removed after taking out the key. In this case there is no pressure block, but the box upon which the key acts is given instead a thickness of $3e$ instead of $2e$. The method of securing the key is the same as before. An oil box is formed on the upper side, and covered with a brass lid attached by screws. The hole in the oil cup, shown in the plan, is fitted with a tube and wick.

In both of these forms the tendency of the wear is to shorten the length of the rod, and if the reverse is desired, the key may be placed behind the other box. These rod ends are designed so that as much of the work as possible may be performed by the lathe, and sharp angles and corners have everywhere been avoided, as they tend to weaken the material.

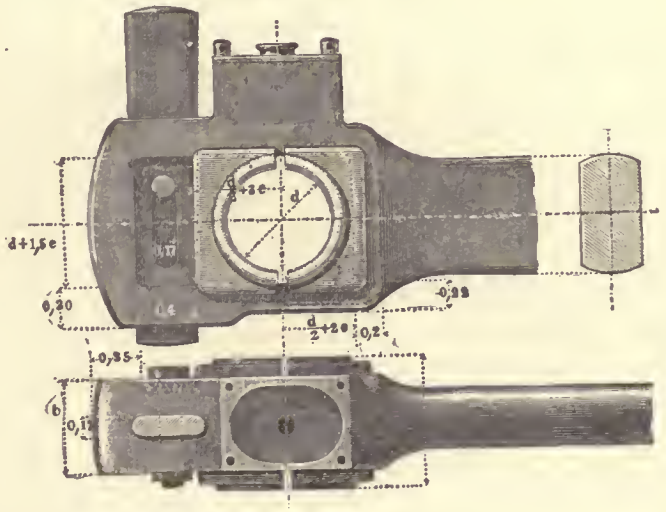

FIG. 501.

A third form of solid end is shown in Fig. 502, and is designed by Krauss, of Munich. It is intended to be made of steel, and is very compact and simple. The key is made in two parts, and

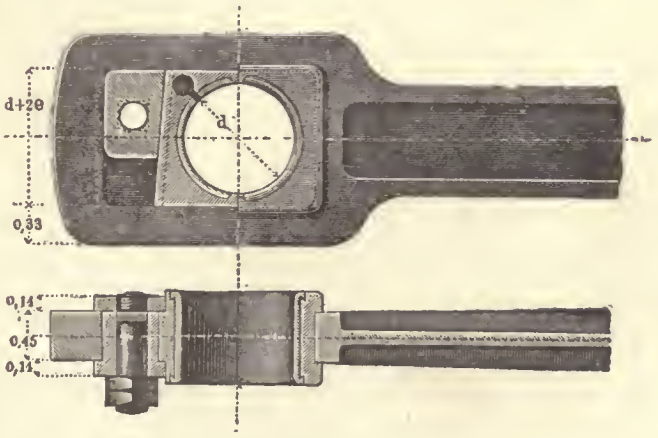

FIG. 502.

is combined with its own locking device. The boxes are made of wrought iron lined with white metal, and an oil chamber is formed in the one shown on the left.

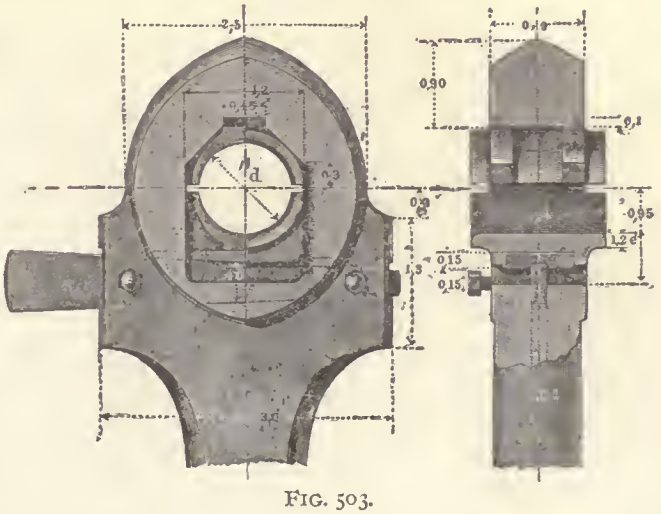

FIG. 503.

Fig. 503 shows an end of cast iron, also made solid, and with the key acting to take up the wear from below, much as in the design of Sharp, Fig. 498. Cast iron rods were formerly much used on the parallel motion connections of beam engines.

§ 180.

Stub Ends for Fork Journals.

Fork journals designed according to the method previously given, are made much smaller in diameter than the corresponding overhung journals. On this account the breadth b' cannot be determined in the same proportion to the diameter of pin d, as with overhung pins, as the proportion will vary somewhat for various conditions. In order to take this difference into account, we may take for such rod ends, instead of the modulus given in (160), the following:

$$\frac{d'_1}{d_1} + \sqrt{\frac{b}{b'}}\sqrt{\frac{d'}{d}} \quad \ldots\ldots\ldots \quad (161)$$

in which b is the breadth correspouding to the length of the normal pin, and d_1 its modulus, calculated according to (160). This enables us to use the proportions of all the preceding examples for fork journals as readily as for overhung crank pins. The thickness of metal e in the boxes may be made the same as before, using in every case the actual diameter d' of the journal. The formula (161) assumes the same material to be used

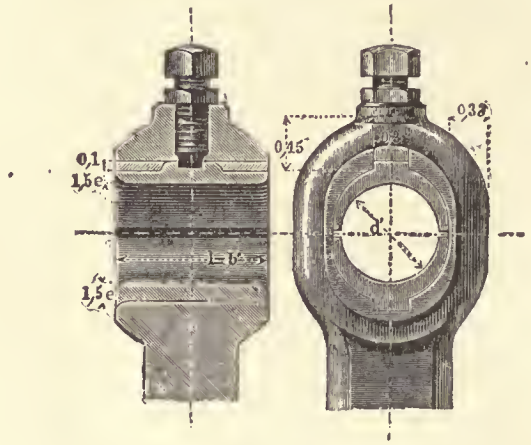

Fig. 504.

in both cases, and gives the rod end for the fork journal approximately the same strength as one proportioned for a normal pin. It is not, however, possible to make an empirical formula cover every case, and some examples will be found much heavier, such as, for example, would give a modulus of

$$\frac{d_1}{d_1} = \left(\frac{b}{b'} \cdot \frac{d'}{d}\right)^{\frac{3}{4}}$$

One of the portions whose dimensions will not bear much reduction is the key, since it is subject to shearing action and its limited surface must not be subjected to too great pressure.

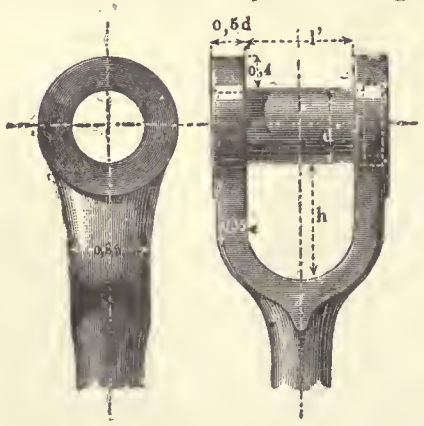

Fig. 505.

For this reason the dimensions of the key should in no case be made less than those given for stub ends for overhung pins.

Example. Given an alternating pressure of 7920 pounds, let it be required to design a strap end similar to Fig. 496. We have for an overhung pin according to (93), $d = 2\frac{3}{8}''$, and for a fork journal, according to (98), $d'' = 1\frac{1}{16}''$, and the length l'' of the latter $= 2\,d'' = 3\frac{3}{8}''$. We then give the strap the proper breadth for the overhung pin, that is, $d - 2\,e$, or $2.375 - 0.58 = 1.795$, say $1\frac{13}{16}''$. We then have, for the moduli, respectively; for the overhung pin, according to (160), $d_1 = 2.56''$, and for the fork journal, according to (161), $d_1' = 2.56\sqrt{\frac{b}{b'} \times \frac{d''}{d}} = 2.56\sqrt{\frac{1.6875}{2.375}} = 2.16$. From this we get for the thickness of the strap $2.16 \times 0.2 = 0.432''$, say $\frac{7}{16}''$, on the sides; and at the end, $1.16 \times 0.3 = 0.648$, or nearly $\frac{11}{16}''$. They may be made the same as for an overhung pin, giving a thickness of $2.56 \times 0.22 = 0.56''$, or $\frac{9}{16}''$, and a depth of $2.56 \times 0.2 = 0.512$ at the small end.

In Fig. 504 is shown a solid wrought iron end, suitable for forked journals as made at Seraing. In the plan shown in Fig. 505 the journal and fork are formed in the rod end, and the bearing is made in the crosshead, as shown later in Fig. 540, § 189. Such rod ends have been used for locomotives by Polonceau, and for marine engines by Humphrey. In these cases the values of b' and h, must be chosen to suit the space at the disposal of the designer in each instance.

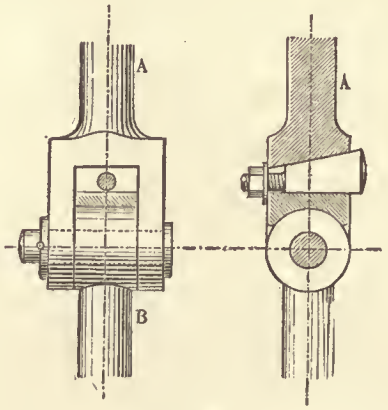

Fig. 506.

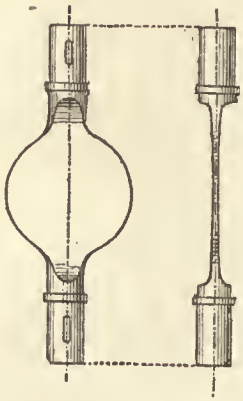

Fig. 507.

For fork bearings in which there is but little angular movement, as, for example, in valve-rod connections, the form shown in Fig. 506 may be used. In this case the key and block press upon the half bearing of the outer part of the portion B, as shown. Such connections are sometimes also made by using a flexible steel plate, as shown in Fig. 507; and this form may be called a plate liuk. This has been used in some forms of locomotives and in the old style Langen Gas Engine.

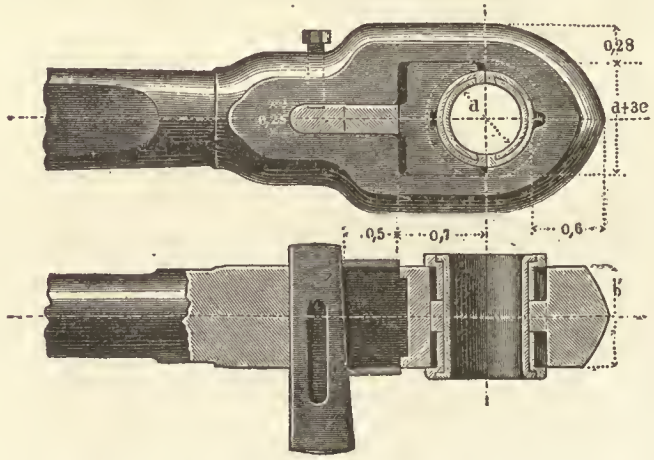

Fig. 508.

In Fig. 508 is given an end for a fork journal, such as would be a suitable one for the cross-head end of the rod showu in Fig. 500. The boxes are made cyliudrical and fitted with a wrought-iron pressure block. The pressure of the key is transmitted to the block by a bronze intermediate, but this arrangement of key involves much clearance space. The method of securing the key is that shown in Fig. 200, and the whole design is well worked out.

Another form for the cross-head end of a rod is shown in Fig. 509, and this is well suited to be used with the form given in Fig. 501, on the crank end, for locomotive use. The key is set up by turning the screw; the latter can also be secured at every sixth of a revolutiou, by means of the arraugement of pin and washer as shown in Fig. 237.

§ 181.

Connections for Enlarged or Neck Journals.

As shown in § 92, there is uo definite relatiou between the diameter d' of a neck journal and the diameter d of the corresponding overhung journal; hence it is impracticable to use the rules dependent upon the length of the overhung jourual, which have beeu given in the discussions of return cranks, crank axles and eccentrics. It is, however, necessary to devise some method of proportioning the rod ends for such cranks, and for this purpose we may use the figures given for overhung crank pins, by making a modification in the modulus according to formula (161). In such cases we must remember to use the value of d' in determining the unit e for the proportions of the boxes.

Example.--Suppose, instead of the fork journal of the preceding example, we have a neck journal of a diameter $d' = 4\frac{3}{4}''$, and length $l' = 3\frac{1}{2}''$, with a stub end like that shown in Fig. 496. We have $d_1 = 2.56''$, $b = 1.8125''$, $d = 2.375$. We may make the value of b' the same as for the corresponding overhung crank, or $b' = b$, and we then obtain from (161) $d_1' = d_1 \sqrt{\frac{4.75}{2.375}} = 2.56 \times 1.414 = 3.62''$, say $3\frac{5}{8}''$. For the boxes we have $e = 0.07 \times 4.75 + 0.125 = 0.45''$, say $\frac{7}{16}''$.

In the following examples are given modern designs for rod

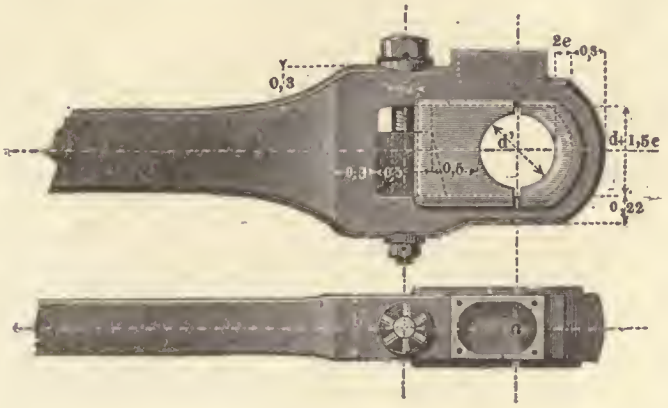

FIG. 509.

ends for neck journals, and others may be obtained by modifications of the preceding forms.

Fig. 510 shows a solid end connection for a spherical journal. The sphere in this case is made 1.5 times the diameter of the

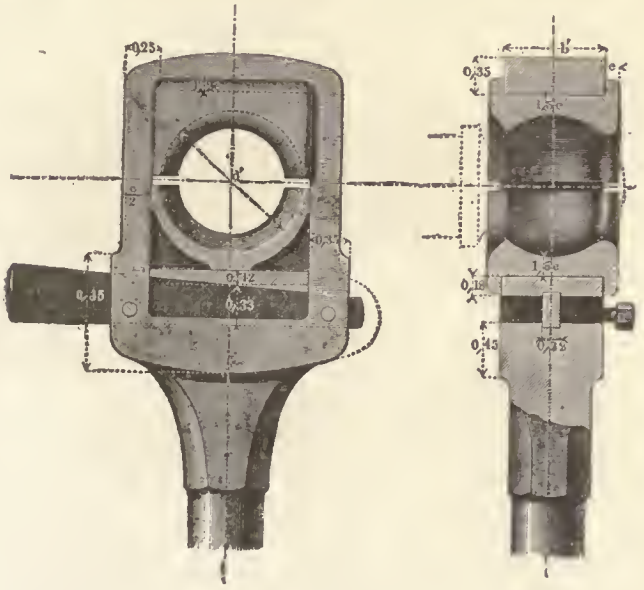

FIG. 510.

corresponding cylindrical journal, and an example of this form may be seen on the beam in Fig. 487*b*. This gives $\frac{d'}{d} = 1.5$; and if, as before, we make $b' = b$, we have $d'_1 = d_1 \sqrt{1.5} = 1.225\, d_1$. If, again, $d = 2.375''$, we have $d' = 3.56''$, $d_1 = 2.56''$, and $d'_1 = 2.56 \times 1.225 = 3.13''$, say $3\frac{1}{8}''$. The boxes are made without side flanges, so that they can be removed by backing out the key. The key may be arranged to be fitted above or below the boxes, as may be desired. When used upon locomotive engines, this form is sometimes strengthened as indicated by the dotted lines.

For the connections of crank axles, return cranks and similar situations it is necessary to use a form of rod end which can be opened. The following forms are designed for this purpose, being made with blocks which are firmly bolted in place, but readily removable.

Fig. 511 shows a form similar to Fig. 500. The block is fitted between two shoulders and also secured by two through bolts. Fig. 512 shows a design by Krauss, in the same style as Fig. 502, and used with it on a locomotive connection. The block is here made of bronze, and also forms one-half of the bearing; it is held in place by a through bolt, which is omitted in the drawing. A cross-section is shown above, the offsets serving to keep the block from twisting on the bolt. The gap between the boxes is filled with slips of copper. The rod and bolt are both made of steel.

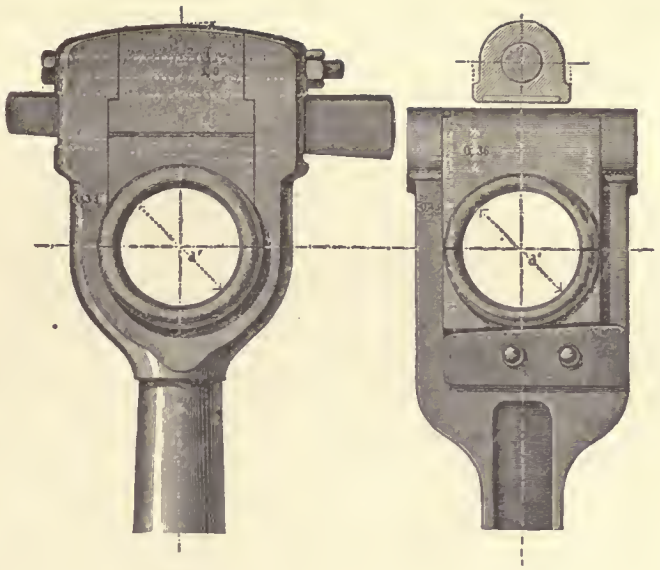

FIG. 511. FIG. 512.

Figs. 513 and 514 show two forms of eccentric straps, both intended to be made of bronze. The breadth b' is equal to l, the length of the corresponding cast-iron journal (see § 92). If $d = 1\frac{9}{16}''$, $d_1 = 1.8''$, $l = b = 2.375''$, we have, if $d' = 15.75$, $b' = l = 2.375''$, $d'_1 = 1.8 \sqrt{\frac{15.75}{1.5625}} = 5.71''$. The diameter, δ, of the bolts of these eccentric straps is determined from the following:

$$\delta = 0.33\, d_1 + 0.06\, d_1' \quad \ldots\ldots\ldots \quad (162)$$

in which d_1' is the modulus for a neck journal and d_1 the mod-

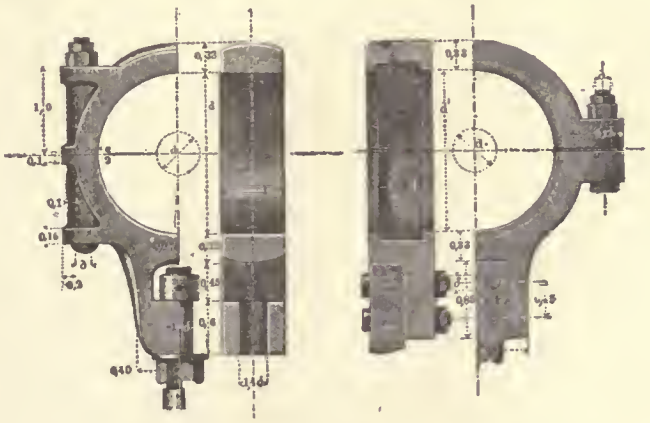

FIG. 513. FIG. 514.

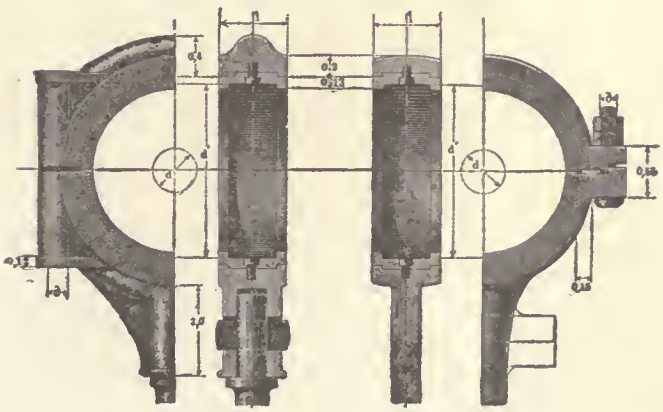

FIG. 515. FIG. 516.

ulus for the corresponding overhung pin. If we take the values above given, $d_1' = 5.71''$ and $d_1 = 1.8''$, we get

$$\delta = 0.33 \times 1.8 + 0.06 \times 5.71 = 0.9866'', \text{ say } 1''.$$

If we make $d' = d$ and $d_1' = d_1$, we obtain from (162) the same dimensions as on a capped and bolted rod end.

In Fig. 515 is shown a design for a cast-iron strap, with brouze lining, although this latter may be omitted. The eccentric rod is secured by means of a key, and if two ecceutrics are placed close side by side, the keys should be placed at 45° from the position shown.

Fig. 516. This is a wrought-iron strap, also lined with bronze. In this, as in the preceding example, the joints betweeu the two halves of the bronze lining are close, and those of the strap are open, and by filing the ends the halves may be closed together to provide for wear. Instead of forging the rod in oue piece with the lower strap, it may be made with a T head and bolted fast, as shown by the dotted lines.

Example. The eccentric straps on the engines of the "Arizona," 6600 H. P., by John Elder & Co., of Glasgow, are made as in Fig. 515, but with the rods attached by T heads, as described above. The diameter of eccentrics $d' = 54''$, the breadth $l = 5''$, and the shaft diameter $= 22\frac{1}{2}''$.

§ 182.

Round Connecting Rods.

The body of a connecting rod may be made of wrought iron, cast iron, steel, or even wood. In the latter case it is usually only subject to tension.

If the rod is of circular cross section, of diameter D, and the force of tension be P, we have the following relations:

$$\left.\begin{array}{ll} \text{Wrought Iron} & \dfrac{D}{\sqrt{P}} = 0.0148 \\ \text{Steel} & \dfrac{D}{\sqrt{P}} = 0.0117 \\ \text{Cast Iron} & \dfrac{D}{\sqrt{P}} = 0.0212 \\ \text{Oak} & \dfrac{D}{\sqrt{P}} = 0.0578 \end{array}\right\} \quad \ldots\ldots \quad (163)$$

These give stresses of 5600, 9500, 2800 and 400 pounds respectively, or above two-thirds the value given for ordinary conditious.

These formulæ may also be used for short rods which are subjected to compression, but if the length L, of the rod is so great as to permit bending, the diameter must be made somewhat greater. From an examination of case II, § 16, and also § 127, we should not permit P to be greater than $\frac{\pi^2 J E}{L^2}$, in which J is the moment of inertia of the cross section of the rod, and E the modulus of elasticity of the material employed. In order to determine how small P must be, or rather how large the co-efficient of safety m, must be taken so that we shall have $P = \frac{1}{m} \cdot \frac{\pi^2 J E}{L^2}$, there are various conditions to be considered; the requiremeuts being almost as varied as in the case of columns.

Leaving then the value of m, to be subsequently determined, we have $J = \frac{\pi}{64} D^4$ and $E = 28{,}400{,}000$, for wrought iron and steel, 14,200,000 for cast iron, and 1,562,000 for oak, and hence the following formulæ for the diameter of rod.

$$\left.\begin{array}{ll} \text{Wrought Iron or Steel} & D = 0.0164 \sqrt[4]{m}\ \sqrt{L\sqrt{P}} \\ \text{Cast Iron} & D = 0.0195 \sqrt[4]{m}\ \sqrt{L\sqrt{P}} \\ \text{Wood} & D = 0.034 \sqrt[4]{m}\ \sqrt{L\sqrt{P}} \end{array}\right\} \quad . \; . \quad (164)$$

We have for

$m =$	1.5	2	3	4	6	8	10	15	20	25	30	40	50	60
$\sqrt[4]{m} =$	1.11	1.19	1.32	1.41	1.56	1.68	1.78	1.97	2.11	2.24	2.34	2.51	2.66	2.78

If we represent the entire co-efficient of $\sqrt{L\sqrt{P}}$ by C we may write for the above formulæ

$$\frac{D}{\sqrt{P}} = C \sqrt{\frac{L}{\sqrt{P}}}$$

and may then determine values for C according to the degree of security required. As already stated, there is a wide variety of values of m to be deduced from practice. For statiouary engines of moderate size we find m, very high, often 50 to 60. These however are not to be taken as standards because they are rarely designed for economy of material, but rather for perfection of action. For medium and large stationary engines we find m from 5 to 25, probably averaging about 20. If the rod is made with a forked end, and two bearings, its lateral stiffness is thereby increased, aud m may be made as low as 4. If $m = 20$ we have for wrought irou or steel, $C = 0.0346$.

Example 1.—For a wrought iron connecting rod 118.11" long, acting under a pressure of 31,680 pounds, taking $m = 20$, and $C = 0.0346$ we have a diameter $D = 0.0346 \sqrt[4]{118.11\sqrt{31680}} = 5''$.

This gives the diameter in the middle; it may be somewhat reduced at the ends, these latter being made of a diameter $= 0.7\ D$, giving a cycloidal sinoide as in Fig. 5, formula (23). The euds of the rod should be worked off into the body in such a manner as not to make too abrupt a change of cross section. This becomes more important in high speed engines. In the case of locomotives there is sometimes a marked bending action upon the rod, there being a so-called "whip action" at every revolution of the crank, dependent upon the rotative velocity

Fig. 517.

and the weight of the rod. This action also occurs in a lesser degree in slower runniug engines, and is greatest at a point betweeu the middle and the crank end of the rod. For this reason it is sometimes thought desirable to make the greatest diameter of rod, not at the middle, but somewhat nearer the crank end, as shown in Fig. 517.

For moderate pistou speeds this point need hardly be considered as it is amply provided for iu the co-efficient of security, but for high speeds and heavy ends it should be given due consideration. In the high speed type of engines such as the Porter Allen, tlie greatest strength of rod will be found at the crank pin end. At the same time, as will be seen, the value of m, for high speed locomotive engines, is usually made small.

For marine engines, m is usually takeu quite high, viz.: 30, 40, 60 or even 80, and the ratio $\frac{D}{\sqrt{P}}$ proportionally smaller. Iu such engines the rod is generally made proportional to the cylinder diameter, being about 0.085 to 0.095 times the bore. It must be remembered that in marine engines the stresses due to flexure of the hull, and general lack of rigidity, demand a higher co-efficient of security than for stationary engines.

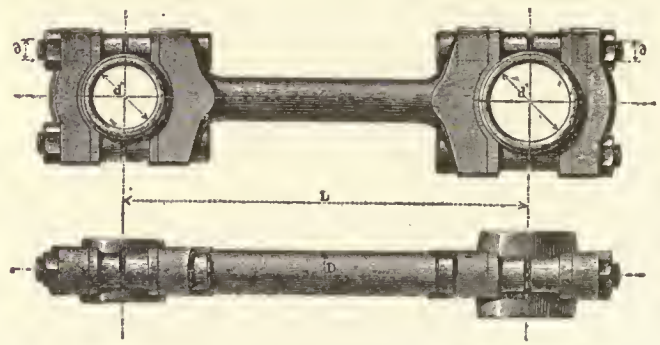

Fig. 518.

In Fig. 518 is shown a rod for a screw propeller engine. The body of this rod is truly cylindrical, and the ends are similar to that shown in Fig. 500.

Example 2.—Let $P = 94{,}600$ lbs. $L = 60''$. Taking, as before, $m = 20$ we have

$$\frac{D}{\sqrt{P}} = 0.0346 \sqrt{\frac{L}{\sqrt{P}}}$$

$$D = 0.0346 \sqrt{94{,}600} \sqrt{\frac{60}{\sqrt{94{,}600}}} = 4.67''.$$

In a similar case, executed by Maudslay, the rod was made 6" in diameter, which corresponds to a value $m = 54.7$. The diameter δ, of the bolts in this case was 3", and according to the rule given for Fig. 499, they should be $3\frac{3}{8}''$.

§ 183.

Rods of Rectangular Section.

If it is desired to make the body of the rod rectangular in cross section, it is first necessary to determine the diameter for circular section by the methods of the preceding section, and then determine the equivalent rectangular section.

Let:
h, be the larger side of the rectangle,
b, be the shorter side,
δ, the diameter of the equivalent circular section at the same point; then for a given value of h, we have:

$$\frac{b}{\delta} = \sqrt[3]{\frac{3\,\pi\,\delta}{16\,h}} = 0.84\,\sqrt[3]{\frac{\delta}{h}} \quad \ldots\ldots\ldots\ldots \quad (165)$$

and for a given value of b:

$$\frac{h}{\delta} = \frac{3\,\pi}{16}\left(\frac{\delta}{b}\right)^3 = 0.59\left(\frac{\delta}{b}\right)^3 \quad \ldots\ldots\ldots \quad (166)$$

and for a given ratio $\frac{b}{h}$:

$$\frac{b}{\delta} = \sqrt[4]{\frac{3\,\pi}{16}\,\frac{b}{h}} = 0.88\,\sqrt[4]{\frac{b}{h}} \quad \ldots\ldots\ldots\ldots \quad (167)$$

from which we deduce the following table:

$\frac{h}{\delta}$	$\frac{b}{\delta}$	$\frac{h}{\delta}$	$\frac{b}{\delta}$	$\frac{h}{b}$	$\frac{b}{\delta}$
1.0	0.84	1.6	0.72	1.0	0.88
1.1	0.81	1.7	0.70	1.25	0.83
1.2	0.79	1.8	0.69	1.50	0.79
1.3	0.77	2.0	0.67	1.75	0.76
1.4	0.75	2.2	0.65	2.00	0.74
1.5	0.73	2.4	0.63	2.5	0.74

If it is desired to calculate the rectaugular section directly, without refereuce to the equivalent circular section, we proceed

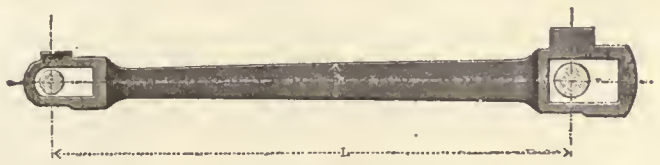

FIG. 519.

as before, using the least moment of inertia of the section, $J = \frac{1}{12}\,h\,b^3$, and thus obtain for wrought iron or steel:
for any given value of b:

$$h = 0.0000000425\ m\ \frac{P\,L^2}{b^3} \quad \ldots\ldots\ldots\ldots \quad (168)$$

for any given value of h:

$$b = 0.0002\ \sqrt[3]{m}\ \sqrt{\frac{P\,L^2}{h}} \quad \ldots\ldots\ldots\ldots \quad (169)$$

and for any given ratio of h, to b:

$$h = 0.0144\ \sqrt[4]{m}\ \sqrt{\left(\frac{h}{b}\right)^3}\ \sqrt{L\,\sqrt{P}} \quad \ldots\ldots \quad (170)$$

For the last formula we have, when:

$$\frac{h}{b} = 1.5 \quad 1.6 \quad 1.7 \quad 1.8 \quad 1.9 \quad 2.0 \quad 2.1 \quad 2.2 \quad 2.3 \quad 2.4 \quad 2.5$$

$$\sqrt[4]{\left(\frac{h}{b}\right)^3} = 1.36\ 1.42\ 1.49\ 1.55\ 1.62\ 1.68\ 1.74\ 1.80\ 1.87\ 1.93\ 1.99$$

The most important application of flat connecting rods is upon locomotive engines. In this case the co-efficient of security is taken very low, *i. e.*, the rod is made as light as possible, in order that the "stored velocity" may be kept small, aud the "whip" actiou reduced.

An examination of practical examples shows values of m, from 2 to 1.5, takeu at the middle of the rod. At the cross

FIG. 520.

head end the depth is reduced to 0.8, to 0.7 that at the middle, and the depth at the crank end is that due to the taper thus indicated. An example of such a rod is shown in Fig. 519.

Example 2.—Given in a locomotive $P = 28{,}600$ lbs. $L = 72''$ $\frac{h}{b} = 2.5$. We have, if $m = 1.5$, according to § 182, $\sqrt[4]{m} = 1.1$ hence $h = 0.0144 \times 1.1 \times 1.99\ \sqrt{72\,\sqrt{28{,}600}} = 3.5''$ and $b = 3.5 \times 0.4 = 1.40$ say $1\frac{7}{16}''$.

The "whip" action before referred to, is much more powerful in the parallel rods of locomotive engines than in the main connecting rods. Such a parallel rod, or side coupling rod is shown in Fig. 520. The keys for the boxes at each end of the rod are placed on the same side of the boxes, so that their action will not affect the distance between centres, providing

FIG. 521.

the wear is alike upon both ends, and for this reason it is desirable also that both pins should be of the same length. (See § 92.) In determining the cross section of such rods, it is to be assumed that the resistauce offered is the same for both wheels. This means, that for two coupled wheels, one-half the total driving force is exerted upon each; for three wheels, two-thirds the total force is exerted upon the first rod, and one-third upon the second. At the same time it must not be forgotteu that under certain circumstances one of the wheels may slip. For this reason it is advisable to take a somewhat larger value for m, than for the driving rods. It is, therefore, not advisable to make m, less than 2, and if possible it should be greater, at least for two coupled wheels. If this is done there need be no fear that the rod will be excessively strained through slippage of wheels.

Example 3.—The locomotive of the preceding example has two pairs of coupled driving wheels. We have for the force transmitted through the coupling rod, $P = \frac{28{,}600}{2} = 14{,}300$ lbs. The length $L = 8$ ft. 4 in. $= 100''$, and we will take the ratio $\frac{n}{b} = 2.5$ as before. Taking $m = 2$, we have from (170) $h = 0.0144 \times 1.19 \times 1.99\ \sqrt{100\,\sqrt{14{,}300}} = 3.73''$ say $3\frac{3}{4}''$. This gives for b, $3.75 \times 0.4 = 1\frac{1}{2}''$. This corresponds closely with the proportions used on Borsig's locomotives. Otber examples in practice give values of m, as 1.9, 2.11, 2.8, etc.

A rod of mixed section, passing from circular into rectangular, is shown in Fig. 521, being the very elegaut connecting rod of the Porter-Allen engine. Iu the illustration $L = 5$ feet.

§ 184.

CHANNELED AND RIBBED CONNECTING RODS.

Cast iron connecting rods are often made of cruciform or ribbed section, much in the same mauner as axles. In such

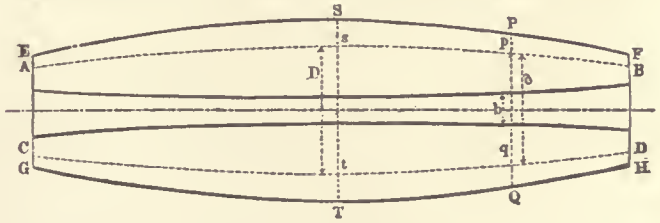

FIG. 522.

cases it is best to determine an ideal round rod, according to Fig. 5, from which the desired section can be derived.

For any given case, let:
δ = the diameter of the ideal rod,
n, and b, the width, and thickness, respectively,
then for any selected value of b,

$$\frac{\delta}{h} = \frac{b}{h}\,\sqrt[4]{\frac{16}{3\,\pi}}\,\sqrt{\left(\frac{b}{h}\right)^3 + \frac{h}{b} - 1} \quad \ldots\ldots\ldots \quad (171)$$

from which we get the following table:

$\frac{\delta}{h}$	$\frac{b}{h}$	$\frac{\delta}{h}$	$\frac{b}{h}$	$\frac{\delta}{b}$	$\frac{b}{h}$	$\frac{\delta}{h}$	$\frac{b}{h}$	$\frac{\delta}{h}$	$\frac{b}{h}$
0.643	0.10	0.700	0.14	0.748	0.18	0.816	0.25	0.901	0.36
0.653	0.11	0.714	0.15	0.758	0.19	0.831	0.27	0.928	0.40
0.673	0.12	0.724	0.16	0.768	0 20	0.855	0.30	0.958	0.45
0.690	0.13	0 736	0.17	0.789	0.22	0.872	0.33	0.987	0.50

Example 1.—In Fig. 522, let $A\,B\,C\,D$, be the ideal round rod from which to construct a cruciform section; $E\,F\,G\,H$, is the width selected for the ribs, the ratio of $\frac{S\,T}{s\,t}$ being, for example 1.5 We theu have $\frac{\delta}{h} = \frac{1}{1.5} = 0.667$, and this value in column 1, of the table gives for $\frac{b}{h}$, something between 11, and 12. This gives $b = 0.12\,h = 0\,12\,S\,T$. If $P\,Q = 1.4\,p\,q$, we have $\frac{\delta}{h} = 0.7$ and in columns 3 and 4 we fiud $b = 0.14\,P\,Q$.

For constructive reasons the I section is preferred for locomotive rods. Such a rod is shown in Fig. 523. This is made with a slight swell in the middle, but the scale of the drawing is too small to allow it to appear.

FIG. 523.

Such rods are either made with straight or rounded profile, as shown in Fig. 524. Neglecting the rounding we have for the least moment of inertia of the section,

$$J = \tfrac{1}{12}\,(2\,c\,B^3 + (h - 2\,c)\,b^3)$$

For conveuience of calculation we may, as in § 163, assume a

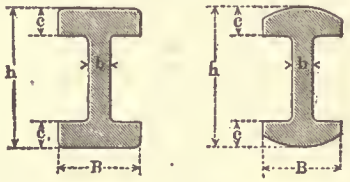

FIG. 524.

rectangnlar section of a height h, and breadth b_o, and then have

$$\frac{b_o}{b} = \sqrt[3]{1 + 2\,\frac{c}{h}\left[\left(\frac{B}{b}\right)^3 - 1\right]} \quad \ldots\ldots \quad (172)$$

from which, when the ratios $\frac{c}{h}$, and $\frac{B}{b}$ are given, the numerical values can be readily deduced.

Example 2.—A coupling rod of I section, on a locomotive engine built by Krauss & Co., has the following dimensions: $h = 3.149''$, $b = 0.39''$, $B = 1.85''$, $c = 0.6$, $L = 96.45$, and $P = 10{,}890$ lbs. To determine the degree of security m, we substitute these values in (172) and obtain:

$$b_o = \sqrt[3]{1 + 2\,\frac{0.6}{3.149}\,(4.7^3 - 1)} = 1.325''.$$

We then have from (168)

$$m = \frac{6_o{}^3\,h}{0.0000000\,425\,P\,L^2} =$$

$$= \frac{(1.325)^3 \times 3.149}{0.0000000425 \times 10{,}890 \times (96.5)^2} = 1.70$$

The completed rod weighed only 125 pounds.

§ 185.

FORMS OF CAST AND WROUGHT IRON RODS.

In Figures 525 and 526, are shown comparative forms for a round connecting rod of wrought iron, and a cast iron rod of cruciform section. In the case of the cast iron rod, the flnted

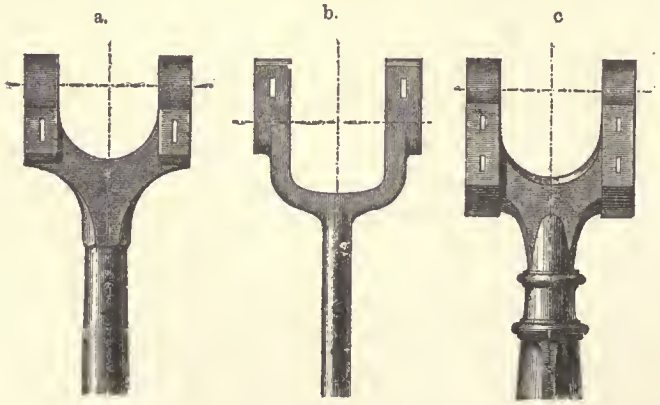

FIG. 527.

portion terminates in collars near each end, the lower part at the crank end being made of flat rectangular section, enough longer than the crank arm to insnre the necessary clearance.

In Fig. 527 are shown some special forms for forked ends. Fig. 527 *a*, is a very short fork, Fig. 527 *b*, is for a flat wrought iron rod, and Fig. 527 *c*, is suitable for a long rod of cast iron. The boxes on these rods may be well secured by strap and key as in Sharp's pattern, Fig. 498. In some cases connecting rods are made in the form of trussed frames, and the form of the ends are governed by the form of cross head used. The latter will be considered in the following chapter.

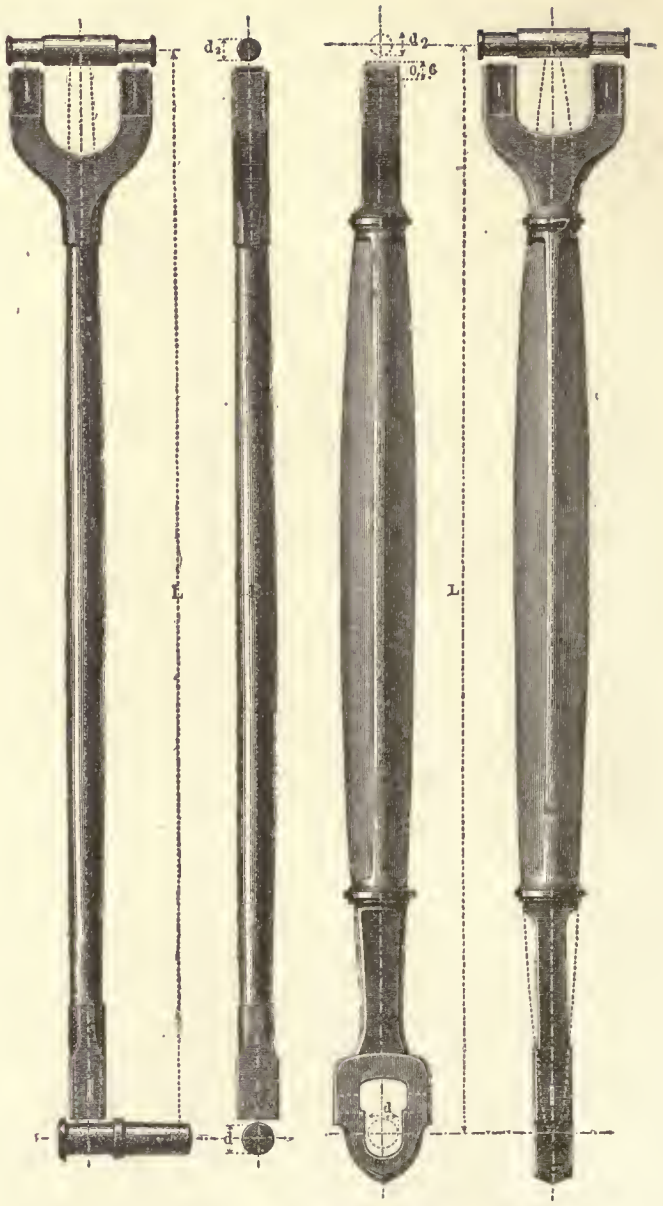

FIG. 525. FIG. 526.

CHAPTER XV.

CROSS HEADS.

§ 186.

VARIOUS KINDS OF CROSS HEADS.

A cross head is that portion of a machine which makes the connection between the vibrating rod and the piston rod or other piece having a rectilinear motion. Cross heads are made with various kinds of journals, either overhnng, forked or double;

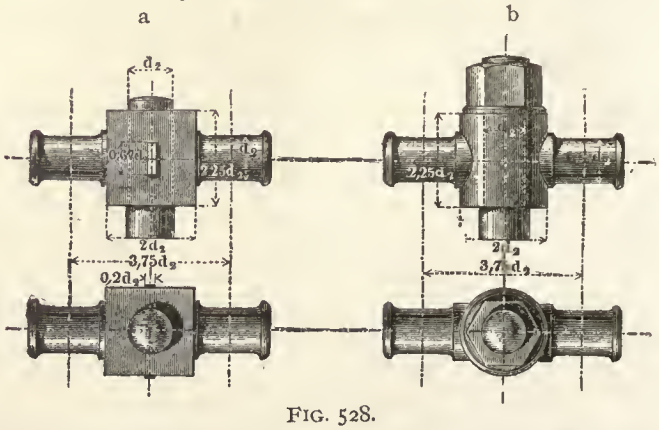

FIG. 528.

and in this respect are similar to the ends of levers, the difference being that the path is curved in the one case and straight

in the other. The path of a cross head is generally determiued either by some form of parallel motion, or by guides, or in some cases only by the piston or other rod to which it may be attached. This gives the following classificatiou :

1. Free Cross Heads,
2. Cross Heads for Link Guides,
3. Cross Heads for Slidiug Guides ;

and this classification will be observed in the followiug discussion.

§ 187.

FREE CROSS HEADS.

In Fig. 528, *a* and *b*, are shown two forms suitable for small free cross heads. These are made with double journals of wrought iron. The diameter of the piston rod iu tbe cross

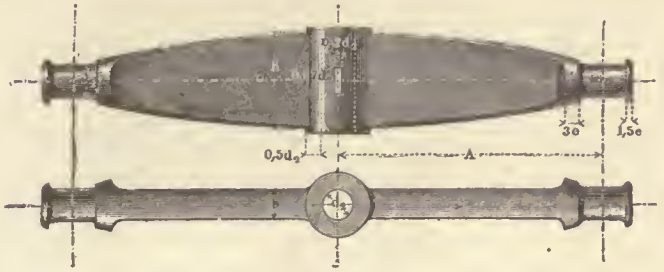

FIG. 529.

head should not be less than d_2. A modification of this form is shown iu Fig. 529. Satisfactory proportions will be obtained by making the height h in the middle equal to

$$h = 2.5\, d_2 + \frac{A}{14} \quad \ldots\ldots\ldots \quad (173)$$

in which A is the length of arm ; also for the thickness b, which is uuiform,

$$b = 0.00035\, \frac{P\,A}{p^2} \quad \ldots\ldots\ldots \quad (174)$$

The curve of the profile may be made as shown in § 142.

Example 1. Given the load $P = 8800$ lbs., and the length of arm $A = 15.75''$ for a cross head, as in Fig. 529. According to the table of § 90, we have $d_2 = 0.027\sqrt{P} = 0.027\sqrt{4400} = 1.85''$.

We have from (173)

$$h = 2.5 \times 1.85'' + \frac{15.75}{14} = 5.75'',$$

and from (174)

$$b = 0.00035\, \frac{8800 \times 15.75}{(5.75)^2} = 1.47'', \text{ say } 1\tfrac{1}{2}''.$$

The other dimensions as given in the figure are: Hub thickness $0.5\, d_2 = 0.5 \times 1.85'' = 0.925''$, say $1''$; depth of key $= 0.67 \times 1.85 = 1\tfrac{1}{4}''$; thickness of key $= 0.2 \times 1.87'' = \tfrac{3}{8}''$.

Example 2. The engine of the steamship "La Plata" has steam cylinder $103''$ diameter, with a maximum steam pressure of 26 pounds per square inch, giving a total pressure of about 217,000 pouuds on the piston rod. The length A is $68''$, and in the executed engine the builder, Napier, has made $k = 28''$, $b = 7''$, $d_2 = 10''$, the length of journal $= 15''$, these latter agreeing closely with those obtained from § 91. The hub length was made $= 30''$, and hub thickness $5''$, with a bore of $10''$. Accoiding to the above formulæ, we get $d_2 = 8.75''$, $h = 27''$, $b = 7\tfrac{1}{4}''$.

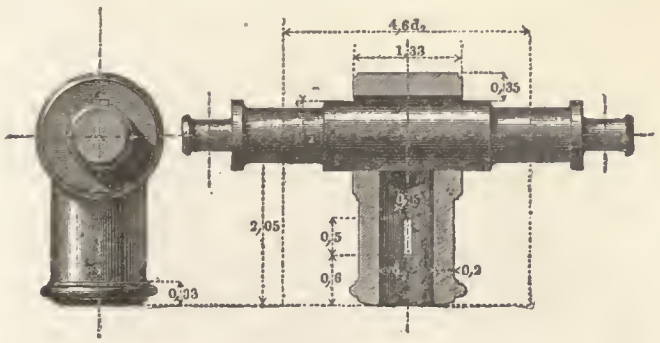

FIG. 530.

§ 188.

CROSS HEADS FOR LINK CONNECTIONS.

Cross heads which are intended to be guided by a system of linkages or parallel motions are made with a pair of link journals in additiou to the journals for the connecting rod, and the former are generally made as prolongations of the latter.

In Fig. 530 is showu a wrought iron cross head for use upon a beam engine in connection with a Watt parallel motion. The unit upon which the dimensions are based is

$$d_1 = 0.026\sqrt{P} + 0.2 \quad \ldots\ldots\ldots \quad (175)$$

in which P is the total load on the cross head. The same modulus serves for the simple proportions of the following cross heads. The load P_3 upon the link journals can be determined from the load P_2 of the rod journals by the following relations :

$$\frac{P_3}{P_2} = \frac{\sin\,\alpha}{\cos\,\beta} \quad \ldots\ldots\ldots \quad (176)$$

iu which α is the greatest angle which the connecting rod makes with the axis of the piston rod, and β the angle which the link

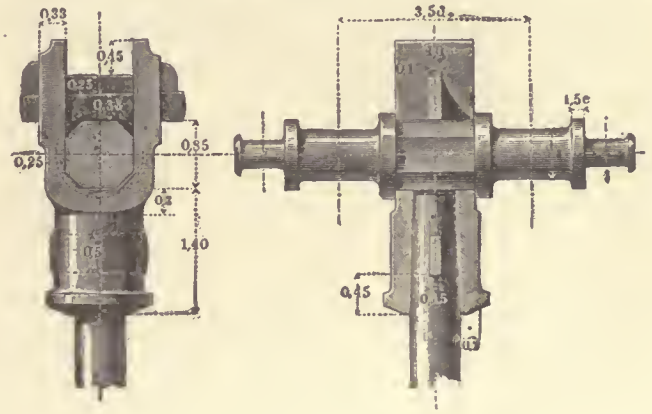

FIG. 531.

makes with a uormal to the axis of the piston rod when α is a maximum, the latter positiou beiug determined most readily from the drawing.

Example. If the angle α at its maximum is 20°, and the corresponding value of $\beta = 15°$, we have

$$\frac{\sin\,\alpha}{\cos\,\beta} = \frac{0.3420}{0.9659} = 0.35,$$

hence $P_3 = 0.35\, P_2$.

When the connecting rod acts directly upon a crank the angle α is usually 20° or more, but when the connection is to a beam it is seldom greater than 10°.

Another form of wrought iron cross head for link connections is shown in Fig. 531. This form is especially convenient when occasion requires that the piston rod be disconnected readily, and is especially adapted for direct-acting steam engines.

§ 189.

CROSS HEADS FOR GUIDES.

Cross heads for use with guide bars are made in many varied forms for steam engines and pumps. The form is modified to a great extent by the number and arrangement of the guide bars.

Fig. 532 shows a much used form of cross head for four guide bars. If the engine runs constantly in the same direction, and the pressure upon the pistou acts always in the direction of its

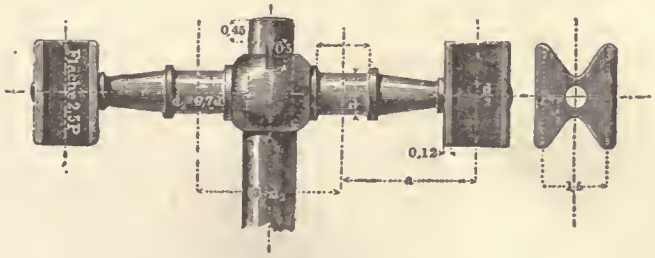

FIG. 532.

motion or in the opposite directiou, the pressure will be almost entirely coufiued to one pair of guide surfaces, the other pair ouly coming into action in the case of extraneous forces. If the pressure acts sometimes with the direction of motion and sometimes against it, the result will be to cause the pressure on the slides to alternate. In most steam eugines the pressure changes not only iu direction but in magnitude, especially near the end of the stroke. The slides should be made of a softer material than the guide bars in orde1 that the greater wear may come upon those parts which are most easily replaced. In order to reduce wear it is also desirable that the surface of each slide should uot be less than $2.5\, P$; P being the total pressure on the piston in kilogrammes, and the area thus obtained being iu square millimetres. This is about equivalent to $0.0018\, P$; P being the total pressure in pounds, and the area giveu in square inches. Many use double this area, or $0.0036\, P$, with correspouding reduced wear ou the parts. The pressure on the sur-

face of the slides, with the ordinary ratio of connecting rod to crank arm, will then be about 120 pounds per square inch in the first case and about 60 pounds in the second.

If we represent the superficial pressure, rubbing velocity and coefficient of friction for slide and crank pin respectively by p_1, p_2, v_1, v_2, f_1, f_2, we have for the lineal wear per second: $U_1 = \mu_1 p_1 v_1 f_1$ and $U_2 = \mu_2 p_2 v_2 f_2$, in which μ_1 and μ_2 are coefficients due to the materials used. Some of these values vary at differ-

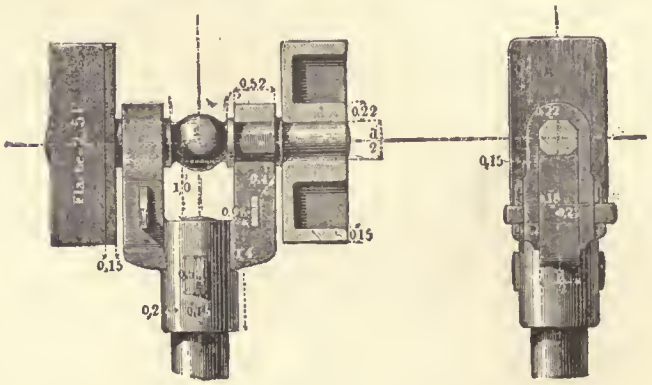

FIG. 533.

ent portions of the stroke. If, however, we take them at the same instant, we have the ratio of wear for that point,

$$\frac{U_1}{U_2} = \frac{\mu_1 p_1 f_1 v_1}{\mu_2 p_2 f_2 v_2}$$

The point of maximum wear upon guides is near the middle of the stroke, where $v_1 = \frac{2 \pi R n}{60 \times 12}$ and $v_2 = \frac{\pi d n}{60 \times 12}$.

Taking the values of μ and U the same in both cases, we obtain, by substitution in the preceding equation,

$$\frac{p_1}{p_2} = \frac{d}{2R}$$

which gives an average ratio of about $\frac{1}{12}$, and taking p_2 at 1420 pounds gives about 120 pounds for p_1. If we consider the pressure on the pin to be alternating and that on the slides continu-

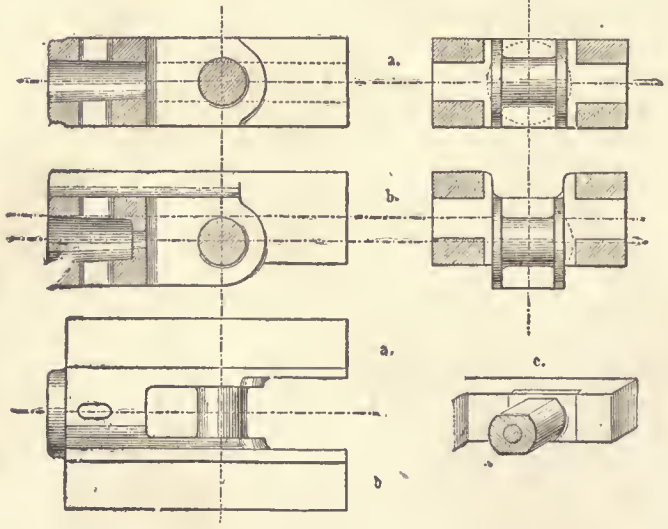

FIG. 534.

uous, p_2 becomes only 710, making p_1 about 60 pounds. If the ratio of connecting rod to crank arm is unusually small, the pressure Q on the slides at mid-stroke should be calculated, and it may be taken as $Q = \frac{P \times R}{L}$.

The cross head shown in Fig. 533 is arranged for a fork journal, the latter being also in this case made spherical. The fork, which is keyed to the piston rod, is intended to be made of wrought iron; should it be made, instead, of cast iron, the thickness of the metal about the hub should be increased to $0.28\,d_1$, and its length to $1.75\,d_1$. This form permits the slides to be brought closer together than in the preceding design.

A very simple form of cross head for four-bar guides is used on many American locomotives, as shown at *a* and *b*, Fig. 534. For constructive reasons, to obtain the necessary clearance, this form is sometimes made as at *b*, with the middle plane of the guides above the axis of the piston rod. The cross head is of cast iron, with the pin cast in, and finished by special machinery. A similar form of cross head to that shown at *a* is used on the Porter-Allen engine, except that a steel pin is inserted as shown at *c*. The flattening of the top and bottom of the pin serves to assist in the distribution of the lubricant.*

The area of slides in America is about that given by the foregoing rule.

Example. A wood-burning passenger engine has cylinder 16″ diameter, at 110 pounds pressure, giving $P = 22{,}110$ lbs. The surface of each slide measured 79 square inches, or about $22{,}110 \times 0.0036 = 79.59$ sq. in.

The forms of cross head shown are generally fitted with slides of white metal or bronze, and in some instances bearing surfaces of glass have given good results.

There is one form of marine engine which requires a special form of cross head. This is the so-called back-acting engine, in which the crank shaft is placed between the cylinder and the cross head, and there are two piston rods, passing above and

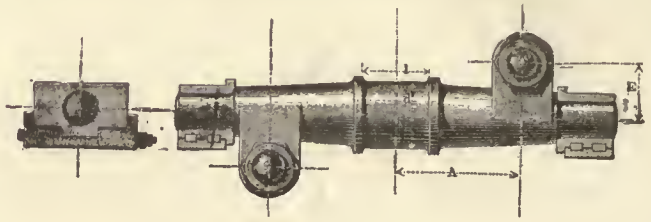

FIG. 535.

below the shaft. There have been many varieties of this type constructed. In Fig. 535 is shown a design by Maudslay. The body of the cross head is formed like an axle, with two projecting bosses for the attachment of the piston rods. The distance E is governed by the diameter of the crank shaft, and A by the clearance space required for the crank arms. In this design the slides are placed outside of the piston rods; other builders, as Ravenhill, place them between the rods and the journal d', where, as will be seen, there is sufficient room. The lower portion of the slides are made of bronze and fitted with adjusting keys. The dimensions of the body are obtained by considering it as an axle, remembering that the forces act to produce twisting with the arm E as well as bending with the arm A. The length l' is to be taken in connection with the diameter d', so as to keep the pressure on the journal within practical limits. English practice in such construction gives pressures ranging from 800 to 1800 pounds per square inch. The diameter δ of the threaded ends of the rods is the same as given for Fig. 499.

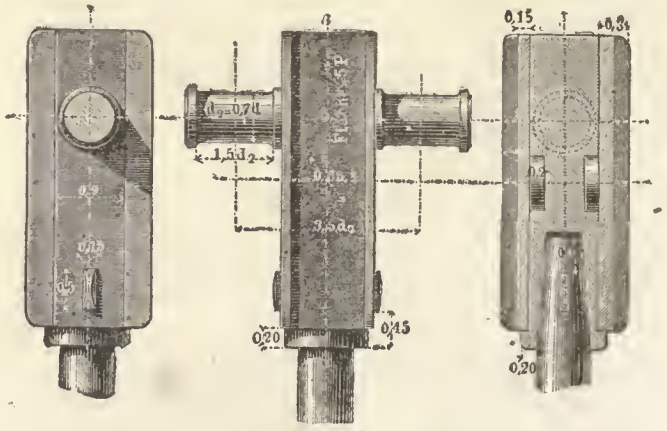

FIG. 536.

In Fig. 536 is shown Stephenson's cross head. Here the guides are brought so close together that each pair merge into one, and there are but two guide bars. The middle piece, of wrought iron, is made with two journals, for a forked connecting rod. The slides are best made of bronze, the area being as before $= 0.0036\,P$, except in the case of locomotives, where the limited space often causes it to be reduced to $0.0018\,P$.

Another design for double guide bars is that of Borsig, shown in Fig. 537. This contains a fork journal, whose projected area $l' \times d'$ should not be made too small. Sometimes this is made so small that the pressure reaches 3000 to 4000 pounds, and hot bearings and cut boxes are apt to follow. Judgment in this respect is most important for all bearings. The slides are made of cast iron, with bronze shoes, which are packed out with thin slips of copper or zinc.

* This is also done on a horizontal engine built by Brown, of Winterthur See *Engineering*, Jan., 1880, p. 70.

Fig. 538 shows a noteworthy form of cross head used on the Western Railway of France. The body is of wrought iron, the slides and piston rod connection are of steel. The manner in

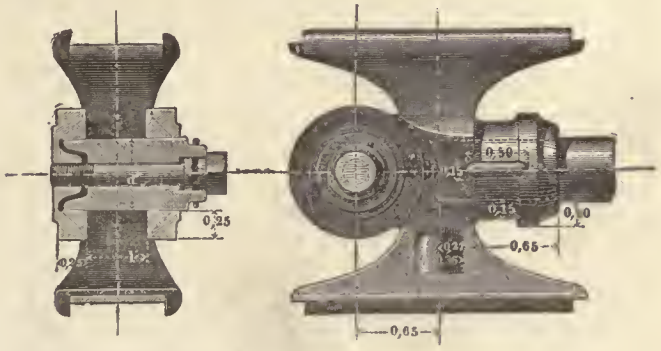

FIG. 537.

which the reverse taper of the rod is secured, by meaus of a key and conical shell of steel, is of peculiar interest. Since this is a special construction a few dimensions are given in the figure (in millimetres). The area of the slides does not appear to be

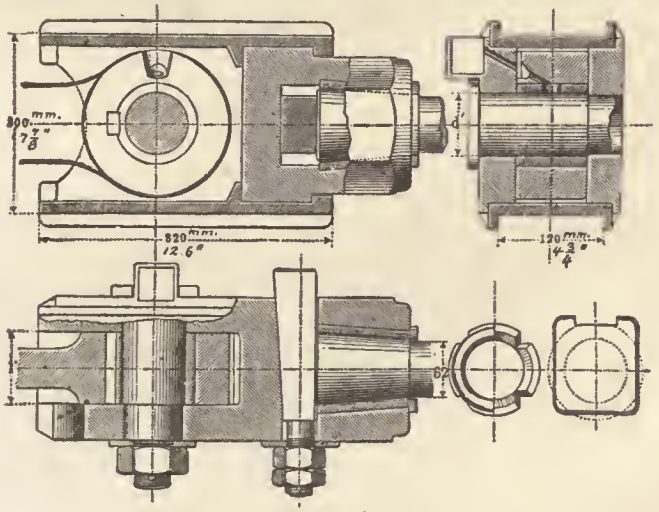

FIG. 538.

large. The rather complicated form of the head of the pin is shown in the lower right haud corner of the illustration.

Fig. 539 shows a cross head of the so-called "slipper" type, for siugle guide bar. This is well adapted for situations in which the direction of rotation is constant and the pressure always downward. In order to provide for possible lifting forces, and to meet the reverse action of compressiou and inertia, the beveled shoes are used, although a square shoulder is to be preferred. The area of slide should uot be less than 0.0036 P, preferably more.*

Another form of cross head for single guide is given iu Fig. 540. This is from a marine engine by Humphreys, Tennant & Co., and is intended to serve for pressure in either direction. In this case the bearing is in the cross head, aud the pin is intended

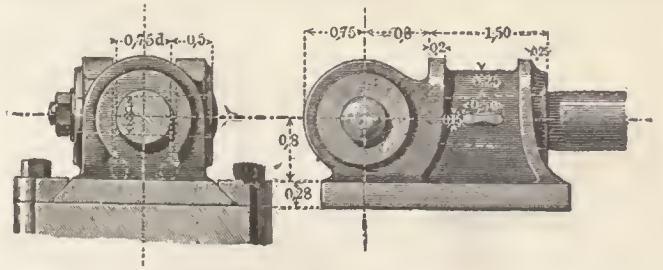

FIG. 539.

to be fast in the connecting rod, being attached as in Fig. 537. The wear on the bearing is taken up by the removal of thin slips of copper, originally placed in the vertical joint; and wear upon the guide, by the insertion of similar slips between the cross head and slide. The whole construction is applicable to many situations. The middle portion is in this case made of brouze, but may be of cast irou, when the beariug is lined with white metal. The modulus for the dimensions is the same as in formula (160), and the bolt diameter δ, as in Fig. 499, using for d the diameter of the equivalent normal wrought iron overhung pin.

A somewhat similar cross head has been designed by Napier for use with the horizontal back-acting marine engine, Fig. 541. This is intended to be used with a forked connecting rod. The

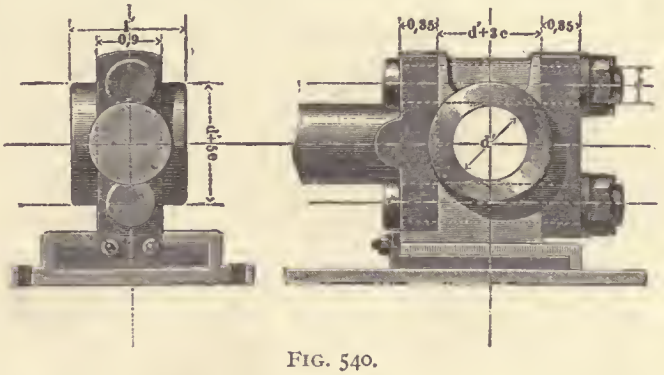

FIG. 540.

middle block is made of cast iron, and the distance B is kept as small as possible, in order to reduce the size and weight. The depth of arm h is determined as in Case I or II, § 6. The bolts, whose diameter δ is calculated as for Fig. 499, are secured by jam nuts.

Iu Fig. 542 is another excellent design by Maudslay for similar service. This is for an ordinary connecting rod, as in Fig. 518. The pin is formed in the crooked wrought iron piece which also forms the arms. The thickness b' of the latter is determined from the corresponding moment after having selected the depth h, which in this case is made equal to d'. The value

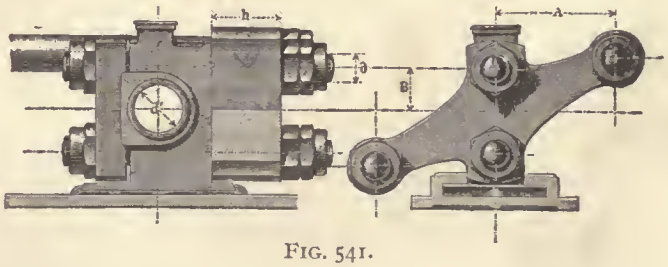

FIG. 541.

of d' is calculated as in the case of an axle. The screw diameter δ' is calculated as before, aud should be made fully as large as the formula gives. The small lug ou the lower part of the right arm is for the attachment of the pump rod. Such attachments are frequently made to the cross heads of marine eugines, of which this is a good example. On the left the slide is shown in section. This is cast of brouze, with the channels shown

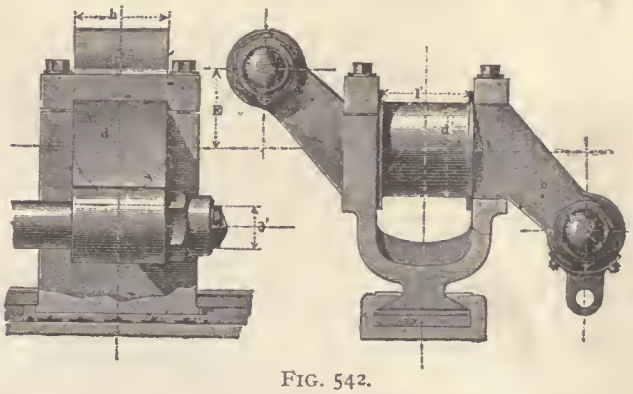

FIG. 542.

filled with white metal. The small shoe on the right, which is secured by screws, can be removed, so that slips of thin copper cau be iuserted to take up for wear. These last two cross heads, although uuusual iu appearance, show how a difficult constructive problem can be solved completely, aud may be regarded as types.

§ 190.

GUIDES AND GUIDE BARS.

Guides are made of wrought irou, cast iron or steel. If the eutire pressure comes upou one guide, as iu the designs just described, aud the guide is supported ouly at the ends, which are

* For a similar cross head, designed by Stroudley, for locomotive service, see *Engineering*, Feb., 1867, p. 65.

separated by a distance $= s_1 + s_2$, it must be calculated to resist beuding. Taking the crank at right angles to the guide, as the most unfavorable position, and calling the pressure Q, and the distances of the two points of support from the centre of the

FIG. 543.

cross head as s_1 and s_2, Fig. 543, we have the bending moment of the bar $= Q \frac{s_1 s_2}{s_1 + s_2}$, aud for the relation betweeu the depth and width of bar:

$$h = \sqrt{\frac{6}{S} \cdot \frac{Q}{b} \cdot \frac{s_1 s_2}{s_1 + s_2}} \quad \ldots\ldots \quad (177)$$

The permissible value of stress S for wrought iron or steel should be small, say 7000 pounds, in order that but little deflection shall occur. Any springing is especially hurtful in this case, since it prevents the entire surface of the slides from bearing fairly, and thus causes greatly increased pressure upou the

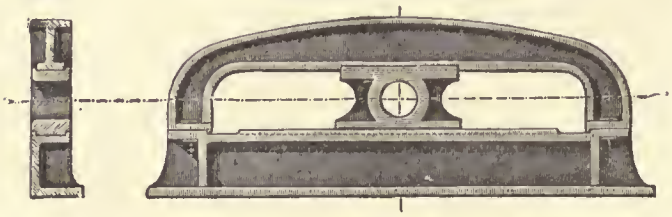

FIG. 544.

points which are in contact. Deflections of $\frac{1}{16}''$ or more are sometimes found, with corresponding irregular wear upon the slides. This subject can be thoroughly investigated graphically by taking the various positions of the load.

In Fig. 544 is shown a form of cast irou guides, inteuded to receive pressure only upon the lower guide. This is only subject to compression, and hence very little deflection can occur.

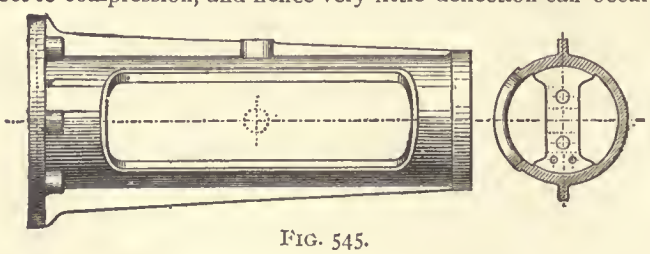

FIG. 545.

The sectional view on the left shows the disposition of the material, and it will be noticed that the flanges on the cross head are arranged so as to retain the oil. The upper guide is bolted to the lower, and should the motion be reversed, throwing the pressure on the upper guide, the bolts must be made proportionally stronger.

A form of guides which is coming more and more into use for stationary engines is that shown in Fig. 545. Here the flat

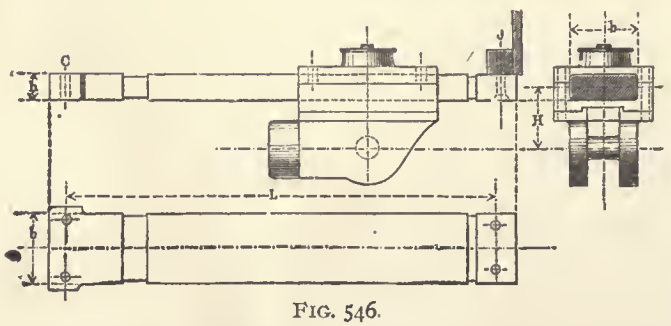

FIG. 546.

guide surfaces are replaced by portious of a cylinder. An especial advantage of this construction lies in the possibility of boring the guide surfaces in exact alignment with the cylinder. Any twisting of the cross head is prevented by the connecting rod and crank pin, or, if necessary, a tongue on the lower slide may fit into a groove in the guide.

The cross head for such guides may be similar to Fig. 537, the lower guide being adjusted by a key.

The single guide bar has beeu used in locomotive practice, Fig. 546, which was shown both on American and Belgian engines at the Paris Exposition of 1878. The guide is bolted to the cylinder at C, and to the yoke at J. The cross head is a simple modification of the form in Fig. 534*b*. Engineer J. J. Birckel has shown that there is a heavy lateral stress on such a guide bar, due to the necessary end play in the driving axles, and a wide bar is therefore necessary. He makes the width $b = 2\frac{2}{3} h$, and makes

$$h = \text{Const} \sqrt[3]{\frac{G H^2}{Q L^2}}$$

in which G is the weight of the parts subject to lateral vibration, Q the normal component of the piston pressure, L the length of guide bar, and H the distance from centre of bar to centre of rod. In the case of a cyliuder 18″ diameter at 100 lbs. steam pressure, $G = 8800$ lbs., $L = 51.2$ and $H = 7.5''$, the values obtained are: $b = 8''$, $h = 3''$.

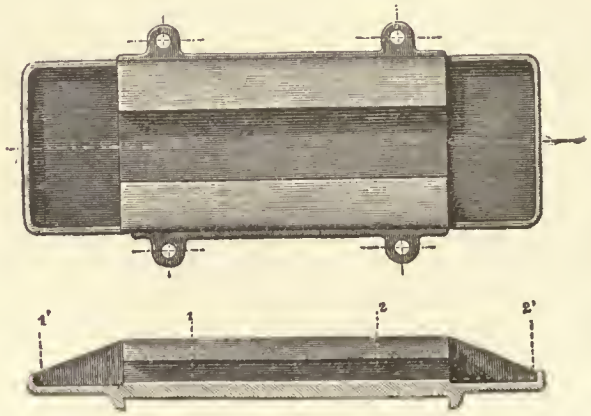

FIG. 547.

Fig. 547 is a cast iron guide for horizontal marine engine, suitable for a cross head such as is shown in Fig. 540. This is especially arranged to retain the lubricating oil, and as the cross head moves between the positions $1' - 1$ and $2 - 2'$, every stroke, it dips in the oil at each end and carries it over the guide.

Example. The steamship "Arizona" is fitted with single guide bars and automatic lubrication. The pressure on one slide is 64,000 lbs., the area being $47'' \times 27'' = 1269$ sq. in., or a pressure of about 50 pounds per inch.

CHAPTER XVI.

FRICTION WHEELS.

§ 191.

CLASSIFICATION OF WHEELS.

Wheels are used in many varied ways to transmit motion in machine construction. They may be divided into two great classes:

1. Friction wheels,
2. Gear wheels,

accordiug as they transmit motion by frictional contact, or by the engagement of gear teeth.

Each of these classes may again be divided into:

(*a*) Direct acting, and
(*b*) Indirect acting wheels,

according as the force is transmitted directly from one wheel to another, or indirectly, by means of belt, cord, chain, or similar device. This gives four divisions for consideration, as follows:

I. Direct Acting Friction Wheels, or friction gearing, pure and simple.
II. Direct Acting Tooth Gearing, otherwise called simply, gearing.
III. Indirect Acting Friction Wheels, such as Pulleys, Cord Wheels, &c.
IV. Indirect Acting Tooth Gearing, such as Chain Wheels.

The first three forms exhibit the greatest variety, and will be given the first consideration.

The relative position of the axes has a most important influence upon the form of a pair of wheels. The positions may be grouped as follows:

1. The axes geometrically coincide,
2. They are parallel,
3. They intersect, at au angle,
4. They are at an angle, by pass without intersecting.

This gives four groups under each of the preceding main divisions.

§ 192.

The Two Applications of Friction Wheels.

Direct acting friction wheels may be used to accomplish either one of two different functions and their construction varies according to the use to which they are put.

The first application is that in which the wheels are pressed together with sufficient force to prevent the surfaces from slipping upon each other, under which circumstances the motion of one wheel will be transmitted to the other.

The second application is that in which the so-called rolling friction is so small that the wheels, when interposed between two surfaces which are relatively in motion, act to reduce the otherwise injurious frictional resistance.

Hence we see that friction wheels may be used:

(*a*) To transmit motion, and

(*b*) To reduce resistance.

The first application includes what may be called driving friction wheels, or commonly simple friction wheels, and the second application includes all the various forms of friction rollers, roller bearings, ball bearings, and the like. The two kinds have also been termed friction wheels and *anti-friction* wheels.

§ 193.

Friction Wheels for Parallel Axes.

The surfaces of a pair of friction wheels in contact are almost always of circular curvature, and when a pair of such wheels roll freely upon each other the number of revolutions will bear an inverse relation to the radii of the respective circles. This

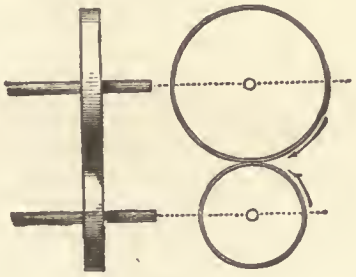

Fig. 548.

ratio is called the velocity ratio of the wheels. If we call the revolutions per minute of each wheel n for the driver and n_1 for the driven wheel; and the corresponding radii R and R_1, we have for the velocity ratio:

$$\frac{n_1}{n} = \frac{R}{R_1} \quad \ldots\ldots\ldots \quad (178)$$

Friction wheels for parallel axes are made with cylindrical surfaces. Fig. 548. In order that there shall be no slipping between the surfaces we must have a pressure Q, which, to transmit a force P, at the periphery of the wheels, must not be less than

$$Q = \frac{P}{f} \quad \ldots\ldots\ldots \quad (179)$$

f being the co-efficient of friction.

The value of f for various materials may be taken as follows:

For Iron on Iron	0.10 to 0.30
" Wood on Iron	0.10 to 0.60
" Wood on Wood	0.40 to 0.60

Friction driving is often very simple and practically effective It had been almost neglected for general uses, when it was very successfully applied in various forms of saw mill machinery. This was especially the case in the lumber regions of America.*

The best results are obtained in practice from surfaces of wood on iron, the wooden surface being preferably the driver, so that any stoppage on starting shall not wear hollows in the softer material.† The rim is built up in such a manner as to place the grain of the wood as nearly as possible in the direction of the circumference. The best wood for the purpose is maple, but linden, poplar and pine have been used with good results. Great care must be taken to make the wheels truly cylindrical, and they should be keyed upon their axles and finished while running in their own proper bearings. Under these conditions a wheel of maple can transmit a circumferential force of about 28 pounds per inch of face width, or from 15 to 20 pounds for the other woods above mentioned.

This gives for maple face:

$$b = \frac{P}{28} = \frac{1180\,(HP)}{v} \quad \ldots\ldots\ldots \quad (180)$$

and a width 1½ to 2 times greater for the other woods, HP being the horse power transmitted, and v the circumferential velocity in feet per minute. Substituting for v its equivalent value, $\frac{2\pi R n}{12}$ we have

$$b = \frac{2414}{R} \cdot \frac{HP}{n} \quad \ldots\ldots\ldots \quad (181)$$

Such wheels are made in practice up to 6 feet in diameter and 30 inches face, transmitting upwards of 60 horse power.

According to the experiments of Wicklin, the coefficient of friction is about 0.30 to 0.32, from which the pressure of contact must be $Q = 3\frac{1}{3}\,P$. The ease with which these wheels can be thrown out of gear is a very convenient feature.

Example 1. Let 10 H. P. be required to be transmitted by friction wheels, the speed of shaft being 80 revolutions per minute, and a circumferential velocity of 1180 feet per minute given. We get from (180) $b = \frac{1180}{1180} \cdot 10 = 10''$ face, and from (181) $R = \frac{2414 \times 10}{10 \times 80} = 30''$. If the driven shaft is run 100 revolutions per minute, the radius of its wheel will be $R_1 = 30'' \times 0.8 = 24''$.

Example 2. Required to transmit 1 H. P., the given value being $n = 90$, $n_1 = 75$, $R = 12''$, $R = 13.66''$. From (181) we have

$$b = \frac{2414}{12 \times 90} = 2\tfrac{1}{4}''.$$

If pine is used, this should be doubled, giving $b = 4\frac{1}{2}''$.

The method of construction of these wheels is as follows: For large wheels, 4 to 10 feet in diameter, the rims are made from 6 to 7 inches deep, built up of wooden segments 1¼ in. to 2 in. thick, forming ⅛ to $\frac{1}{10}$ the circumference, and so placed that the direction of the fibre shall follow the circumference of

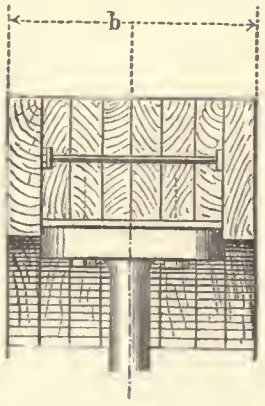

Fig. 549.

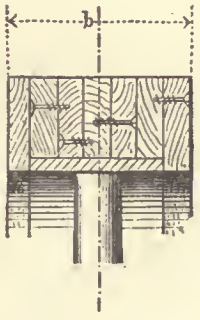

Fig. 550.

the wheel as nearly as possible. These segments are firmly clamped together and secured by bolts or nails. The actual face is made about 2 in. narrower than the working face b. This rim is then securely fastened to the arms, which are very strong and made with feet or pads which are mortised into the rim and both keyed and bolted fast. The number of arms varies from 6 to 8, and for very wide faces two sets are used; see Fig. 549. An additional ring of wood is then put on each side, bringing the width up to the full value of b, and these outer segments are deeper than the others, so that the ends of the keys are entirely covered; the completed wheel is then turned and finished in place, as before stated.

Smaller wheels are built upon iron drums, the segments being screwed together and clamped between the outer rims, Fig. 550. Projections on the iron rim, let into wood, prevent the latter from turning. The total thickness of rim is about 4 in. Care must be taken that the wood is thoroughly dry.

The driven wheel of iron is made similar to a belt pulley, but with a much stronger rim and more and heavier arms; when a wider face than 16 in. to 18 in., double arms are used. Both wooden and iron wheels should be carefully balanced, in order to avoid vibration.

An important and ingenious use of friction wheels is in connection with a drop hammer, the wheels being used to raise the drop. Merrill's drop hammer, Fig. 551, is operated by two iron friction wheels A and C, which together act upon the oak plank B, to which the hammer drop is attached. The roller A is the driven one, and its shaft runs in eccentric bearings on each side, which are operated by levers D and press the parts to-

* See Wicklin, "Frictional Gearing," *Sci. Am.*, vol. 26, p. 227; also Appleton's "Cyclopædia of Mechanics," vol. 2, p. 36; also Cooper's "Use of Belting," p. 288.

† Surfaces of compressed paper against iron are now in general use.—*Trans*

gether. When the parts are in the position shown, the plank and hammer are raised, and when the lever D is lifted, the wheels separate and the hammer is allowed to drop. In some

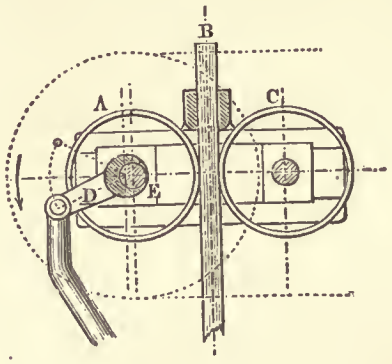

FIG. 551.

similar designs both rollers are driven, as in the hammer of Hotchkiss and Stiles,* and also in the so-called "Precision Hammer," of Hasse & Co., of Berlin.†

§ 194.

FRICTION WHEELS FOR INCLINED AXES.

When the axes are inclined to each other, the surfaces of the wheels, unless they are very narrow, become portions of cones, with a common apex at the intersection of the axes. Fig. 552. Each pair of circles in the surfaces then roll together as if cylindrical. Wheels of this sort may be constructed in a similar

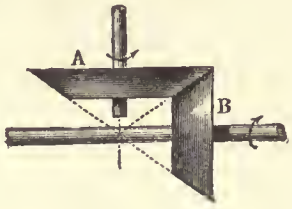

FIG. 552.

manner to those described in the preceding section. In Fig. 553 are shown, at a and b, two sizes of conical wooden friction wheels. The outer disk is placed with the fibres in a radial direction, but the others have the grain of the wood arranged as nearly as possible circumferentially. These disks should be most carefully fitted, glued and bolted together. Especially important is it that conical surfaces should be turned to the cor-

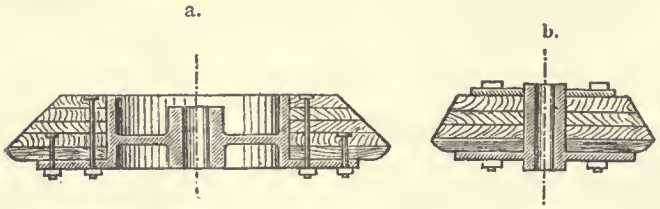

FIG. 553.

rect angle. The pressure is applied from the end of one of the two shafts in such a manner that the force may be applied or removed at the thrust bearing.

The most extensive application of friction driving, both with cylindrical and conical surfaces, is found in locomotive engines. The high pressures necessarily used compel in this case the use of iron or steel tires. The force Q here exceeds 6 tons.‡

In some cases a combination of one conical wheel and one narrow wheel with rounded edge, as in Fig. 554, may be used for the transmission of small powers. In this case both wheels are made of iron. The pressure is easily applied to the disk wheel B, and the mechanism is so arranged that it can be shifted along its axis, so that a variable speed motion is obtained. It must be noted that in this form the surfaces in contact are necessarily very limited, and hence it is desirable, as in the case of friction couplings, to have the diameters as large as possible, and the linear velocity high, in order that the driving force may be kept as small as practicable. The most convenient modification of this form is that in which the angle B of the cone is made 180°, when we obtain a pair of friction disks, Fig. 555.

The velocity ratio, when A is the driver and B the driven, and x is the distance from the axis of a, is expressed by:

$$\frac{n_1}{n}=\frac{x \sin \beta}{r}, \text{ which} = \frac{n_1}{n}=\frac{x}{r} \quad \ldots\ldots\ldots \quad (182)$$

when $\beta = 180°$. The change of velocity is expressed by the line $O N$. If B is the driver and A driven, we have

$$\frac{n'_1}{n}=\frac{r}{x \sin \beta}, \text{ which } =\frac{n'_1}{n}=\frac{r}{x} \quad \ldots\ldots\ldots \quad (183)$$

when $\beta = 180$; n being the number of revolutions of B. These are the equations of an equilateral hyperbola; see Fig. 555. When the value of x approaches near zero, the driving of A by B becomes impracticable.*

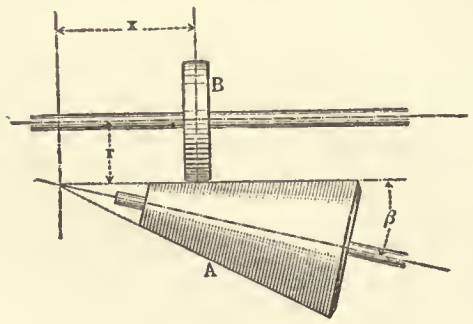

FIG. 554.

In Fig. 556 is shown a form of variable speed gear in which one disk is placed between two others. The disks A_1 and A_2 revolve with the same velocity in opposite directions, and the driven disk B is placed between. The velocity ratio can be varied from 0 to $\frac{R}{r}$ proportional to x.† The pressure is applied at the ends of both horizontal shafts. This arrange-

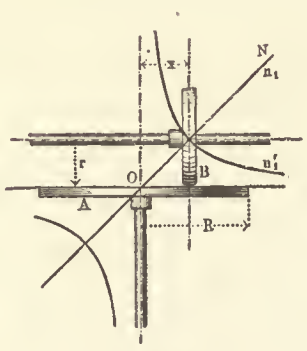

FIG. 555.

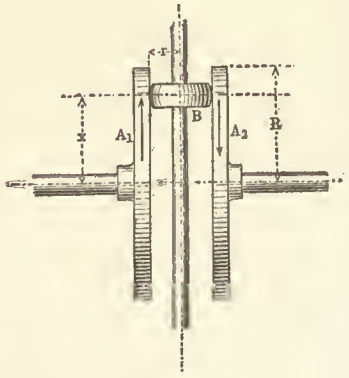

FIG. 556.

ment has been used for driving centrifugal machines, and more recently for potters' wheels, the control over the speed being especially useful in the latter case, the position of the variable disk being controlled by a treadle.

Another arrangement of disk friction wheels to produce a variable speed is that of Rupp, shown in 557. A is the driver, B the driven, and C the intermediate, the latter being adjustable on its axis. The variation is between the limits

$$\frac{a-R}{R} \text{ and } \frac{R}{a-R}$$

according to the relation

$$\frac{n_1}{n}=\frac{x}{a-x} \quad \ldots\ldots\ldots \quad (184)$$

which gives the equilateral hyperbola shown in Fig. 557, intersecting the axis of ordinates when $x = 0$. Rupp recommends especially that the intermediate wheel be made of a number of

* See Appleton's "Cyclopædia of Mechanics," vol. 2, p. 85.

† German Patent 2685. In this hammer the lower part of the plank is reduced, and the whole design very ingeniously worked out.

‡ The surfaces in contact are sensibly flattened. Krauss' experiments showed that with a pressure of 12,000 pounds, a steel tire on an iron rail gave a surface of contact of 0.309 sq. in., and with a pressure of 8250 pounds, a surface of 0.24 sq. in. In the Fontaine locomotive the pressure of contact was about 8 tons on each wheel.

* In the variable speed gear of Lecœur (German Patent 17,078) a loose disk is filled in the centre of A, so that if B approaches too near the centre the motion ceases.

† See Berliner Verhandlung, 1866, p. 39. This arrangement has been used especially for regulating the speed of cotton-spinning machinery.

thin disks, all loose upon the shaft. This does uot appear to be advantageous in view of formula (184), since there is a different ratio for each disk, aud hence some of them must slip.

A similar device is that of Barnhurst, Fig. 558, in which the disk is placed between two cones.*

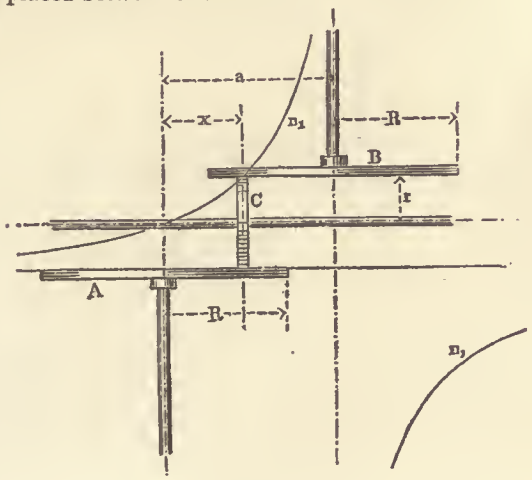

FIG. 557.

By making two of the disks fast on one shaft, and placing the driving wheel between them, with sufficient clearance to enable either to be brought in contact with the driver, the driven shaft may be operated in either direction or allowed to remain

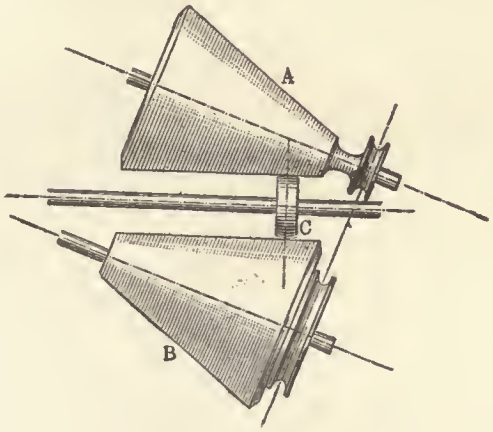

FIG. 558.

at rest, Fig. 559. A_1 A_2 are the driven, and B the driver. This is ingeniously applied in Cheret's Press, in which the screw of the press is on the axis of B, aud is turued in either direction by the friction wheels.

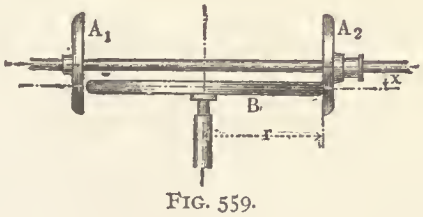

FIG. 559.

§ 195.

FRICTION WHEELS WITH INCLINED AXES NOT INTERSECTING.

In the case of friction wheels whose axes are rigidly held, and, while inclined, do not intersect each other, there is always more or less lateral slipping. The figures which, under these conditions, exert a maximum amount of rolling action and a minimum of slipping are a pair of hyperboloids of revolution (see § 218). If, however, the axes are so arranged as to permit longitudinal motion, either with the bearings or in them, the wheels will be relieved from slipping. Such an arrangement, by Robertson, is shown in Fig. 560.† The disk A acts upon a cylinder B, the axis of which makes a small angle with that of A. When the disk A is revolved, it rolls a helical path upon the cylinder, and also moves in the direction of its axis. The angle α corresponds to the angle of the screw thread. Robertson has applied this device as a feed motion to a wood lathe. This ar-

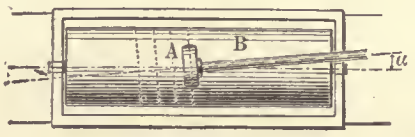

FIG. 560.

rangemeut may also be reversed, A being held in its bearings, and B, with its bearings, permitted to travel.

The same principle may be used with cones on disks, but these devices appear to possess limited practical application.

Friction wheels, the axes of which coincide, are the same as friction couplings.

§ 196.

WEDGE FRICTION WHEELS.

Wedge friction wheels are those in which the cross section of the rim is wedge-shaped. They were designed in Italy by Minotto and in England by Robertson, aud hence are known by both names; in both cases being applied to wheels with parallel axes. Two forms of rim section are given in Fig. 561. In this case the radial pressure Q is much less than with cylindrical wheels, and for any wedge angle θ it is equal to

$$Q = P\left(\frac{\sin\frac{\theta}{2} + f\cos\frac{\theta}{2}}{f}\right) \quad . \quad . \quad . \quad . \quad . \quad (185)$$

A disadvantage of this form is the fact that true rolling action only takes place in one cylindrical section through each rim, and hence there is much hurtful friction from the slippage at other points; this defect becomes less as the ratio of the wedge depths k, k_1 to the radii R, R_1 diminishes.* In order that the ratio $\frac{k}{R}$ and $\frac{k_1}{R_1}$ may be kept as small as possible without reducing the surface of contact, the rim is made with multiple grooves, as in the form on the right. The angle θ is generally made = 30°, although Robertson used much smaller angles.

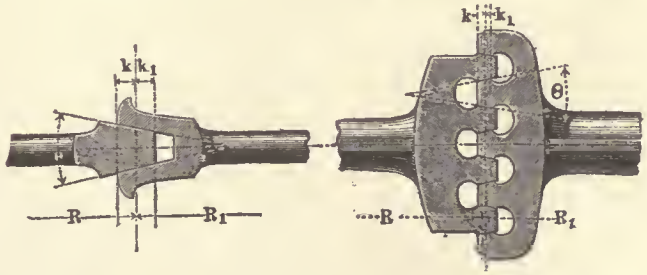

FIG. 561.

These wheels grow warm and wear rapidly when operated continuously at high speeds. Minotto has also made especial efforts to design bevel wedge friction wheels; he uses only one groove, and adjusts the position so that wedge profile shall always act at the same point. Robertson makes the grooves nonadjustable, as in spur wheels. Wedge friction driving has been proposed for locomotive driving, and models made on this plan have ascended steep grades; the wear in this case comes mainly upon the track.

Wedge friction wheels have been used in America for many years on winding engines; and they are especially useful in driving ship's windlasses, on account of the ease with which they can be thrown in and out of gear.† More recently wedge friction wheels have been used by Gwynne and also by Weber in Berlin, at high speeds, and apparently with good endurance,

* See *Engineer*, June, 1880, p. 404; also H. König, German Patent No. 9365.
† See *Engineer*, 1867, p. 410, in which many interesting designs by Robertson are given.

* Hausen, in *Dingler's Journal*, vol. 137, 1855, p. 1, shows that the actual rolling circle is always on that portion of the wedge surface towards the driving-wheel, and changes its position when the driver becomes the driven. See also Ad. Ernst, in Zeitschr. d. V. deutscher Ingenieure, xxvi, p. 243.

† H. D. Andrews' steam windlasses are made with wedge gear of from 4 to 12 grooves. The diameters of the friction wheels are as follows:

H. P.	Slow speed.	Fast speed.	Drum. Diam.	Drum. Length.
5	4—30″	8—26″	6″	27″
8	4—30″	8—26″	8″	27″
10	6—36″	12—30″	8″	30″
15	6—36″	12—30″	8″	30″

driving centrifugal pumps at 700 revolutions per minute. These wheels are with single groove and wedge, the wedge being of curved profile, and hence acting somewhat like the adjustable device of Minotto.*

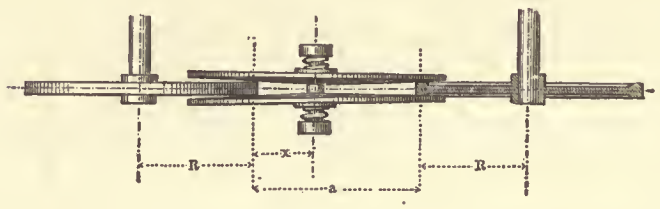

FIG. 562.

Single-groove friction wheels have also been used in America for mill gearing.

Sellers has devised an ingenious form of wedge friction gear for changing the rate of feed on engine lathes. This is composed, Fig. 562, of two simple disks and a pair of very obtuse cone plates, the latter being pressed together by springs. The axis of the cone plates is movable, thus giving change of speeds. The ratio of change is similar to Rupp's gearing, formula (184).

§ 197.

SPECIAL APPLICATIONS OF FRICTION WHEELS.

The previously stated condition of wedge friction wheels, that there is but one line at which rolling action takes place, and that slipping occurs at all other points of contact, is utilized in various methods in machine design, as for example, in rolling mill machinery.

In this case a third piece is driven, compressed and altered in form between two friction rolling members. The rolls and the metal may be considered as a train of friction gearing. In the case of a plate mill, the plate may be considered as a pair of friction wheels of infinitely great radii; this is also the case in rolling bars. In a tire mill one surface is an internal and one an external wheel, of variable radius. The three-high mill may be similarly compared to a train of friction gears.

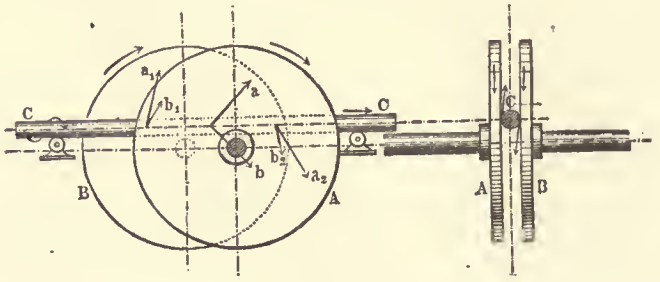

FIG. 563.

A very interesting application is that referred to in § 148, as in use at the Kirkstall Forge, and shown in Fig. 563. *A* and *B* are plane friction disks. The round bar *C* passes between them, slightly above the centre and partly rolling, partly sliding, receives both an endlong motion and a motion of revolution upon its axis. The disks revolve in the same direction, and of the opposed forces which tend to cause revolution of the bar those which act in the portion of the disks between their axes, *i. e.*, between the vertical dotted lines in the figure, preponderate, and determine the direction in which the round bar revolves. The horizontal components of the sliding forces at all portions of the disks, act to carry the bar forward, so that it receives a combined spiral motion and is at the same time rolled and straightened. The earlier method of rolling round bars was by means of semicircular grooves, but this does not give either as round or as straight a product. Many similar examples in rolling mill machinery will be found, resembling friction driving gear.

In the same way, various forms of grinding mills are made upon the principle of friction combinations, as in the case of the Bogardus mills, with flat grinding disks, and also in the case of grinding rollers, Fig. 564. Here the round trough *A* revolves, and in it act the rollers B_1, B_2, and the width of face of the rollers compels a sliding action, forward on the outer edge and backward on the inner. The trough may be stationary and the shaft *a*, carrying the rollers, revolve. Rollers with inclined axes are also used for grinding, and a similar device has been made for straightening rouud rods.

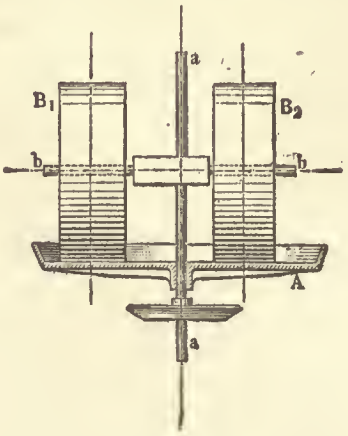

FIG. 564.

§ 198.

ROLLER BEARINGS.

Roller bearings, sometimes called anti-friction rollers, may be used in either of two forms:

(*a*), in such manner that the rollers are carried in their own bearings, the latter receiving the load;

(*b*), or in such a manner that the rollers are placed between two moving surfaces and act with a rolling motion upon both of them.

Roller bearings are used in connection with surfaces which are flat, round, or even spiral. Examples of rollers upon cylindrical surfaces are given in Fig. 565, in which *a* and *b* are forms used on pillar cranes, and b_1 is the more general form of *b*. Rollers are also used in axle bearings, and in heavy pulley blocks, where indeed the sheaves themselves are a form of friction roller.

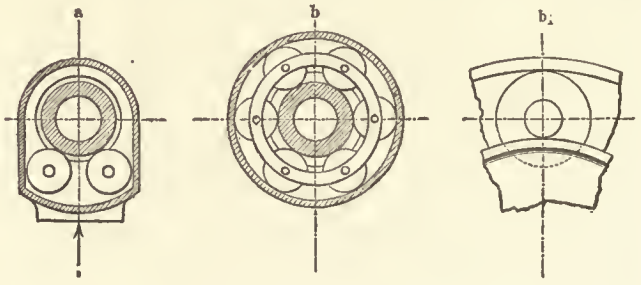

FIG. 565.

A form of roller bearing which is subject to very heavy loads is that used to carry the ends of bridge beams and trusses, to provide for expansion and contraction. These are made either with round rollers, as at *a*, Fig. 566, or with double segmeuts, as at *b*.

For round, solid rollers, the load may approximately be investigated as follows:—Let l be the length, r the radius of each roller, and P the load. This load will be carried by a surface of a width b, included in the angle (measured at the centre of the roller) $\beta = 2\phi$. We have for the relation of these elements:

$$P = E\, l\, r \frac{\beta^3}{48} \text{ and } S = \frac{E}{16}\beta^2$$

E being the modulus of elasticity, and S the fibre stress upon the material.

Also:

$$S = 0.83 \sqrt[3]{E} \sqrt[3]{\left(\frac{P}{lr}\right)^3}$$

and

$$\frac{P}{lr} = \frac{4}{3}\sqrt{\frac{S^3}{E}}$$

It will be seen that for any given material the relation $\frac{P}{lr}$ can be so made as to keep the stress within practicable limits.

* See *Engineering*, 1868, pp. 502, 593, and 1869, p. 353. Engineer Brauer, assistant in the Royal Technical High School, has attempted to adapt the principle of the Weston Clutch (§ 157) to friction wheels. The wheels are made of a number of thin plates, with rubber washers between them, and a slight axial pressure is sufficient to cause them to grasp each other with much friction. A description will be found in Berlin Verhandlung, 1877, p. 295.

These may be chosen as follows, both surfaces being of the same material:

	Cast Iron.	*Wrought Iron.*	*Steel (hardened)*
$E =$	14,220,000	28,440,000	42,660,000
$\frac{P}{l\,r} =$	425 to 500	340 to 400	1000 to 1400
$S =$	11,000 to 12,000	11,000 to 13,500	25,000 to 32,000

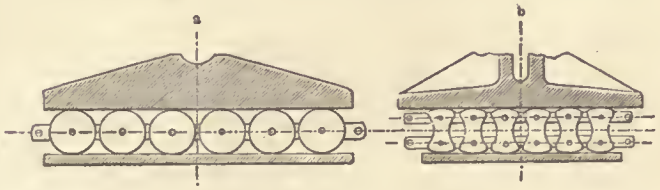

FIG. 566.

Example 1. The bridge over the Elbe at Hohnstorf has spans of 330 feet. The bearings are made of cast iron of the form shown at *b*. The pressure is 792,000 pounds on six rollers, the dimensions of the latter being, $l = 53''$, $r = 4{,}125''$.

We have therefore $\frac{P}{r\,l} = \frac{132{,}000}{53 \times 4.125} = 603.8$,

hence $\beta = \sqrt[3]{\frac{48}{14{,}220{,}000}}\ \sqrt[3]{603.8} = 0.126$.

This gives for the breadth b of the contact surface under this load,

$b = \beta\, r = 4.125 \times 0.126 = 0.522$, and $S = \frac{E}{16}\,\beta^2 = \frac{14{,}220{,}000}{16}\,(0.126)^2 = 14{,}280$ lbs.

Example 2. Bridge over the Rhine at Wesel; span 125.7 feet, rollers and bearings of hardened steel. The load is 770,000 pounds on six rollers, as shown at *a*, and $l = 27\ 75''$, $r = 3.875''$. These values give $\frac{P}{r\,l} = 1193$; $\beta = 0.11$ and $b = 0.43''$, and $S = 32{,}450$ lbs.

Example 3 Clifton Bridge at Niagara. The load of 171,600 lbs. is carried upon 11 steel rollers, on bearings of the same material, their dimensions being $l = 6.3''$, $r = 0.6''$. This gives a high value for $\frac{P}{l\,r} = 4127$; $\beta = 0.17$, hence $b = 0.102$, and $S = 74{,}210$ lbs.

Ball bearings are frequently used instead of cylindrical bearings, and for some forms of journals are most convenient, although the bearing surfaces being only points, they are not so well adapted for heavy pressures.

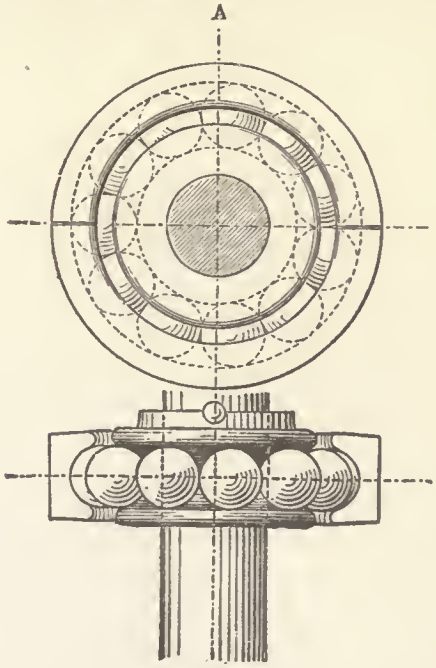

FIG. 567.

A form of roller bearing used in agricultural machinery is that of Cambon, shown in Fig. 567. The steel ring, with semicircular groove is secured to the shaft, and in the groove, or globoid ring, the steel balls, 09 to 13 in number, are placed. These are held in place by a corresponding external ring, made in halves. The outer ring is held in the journal box. Cambon uses balls of $\frac{5}{8}''$ to $1''$ in diameter, which are rolled in a mill of similar construction to the bearing.

It may be remarked that when roller bearings are used for car or wagon wheels we have a combination of friction wheels, since the wheels themselves are properly friction rollers and the introduction of rollers into the axle bearing makes the latter device what may be termed friction wheels of the second order.

Another system of the higher order is the very ingenious arrangement of planet rollers of Mechwart. In this apparatus, which seems to have been so completely conceived by the inventor as to be incapable of further improvement, friction rollers are utilized to the fullest possible extent. The system is indicated in the diagram Fig. 568. R, R_1, R_2, are the rollers of a roller-mill. The axis a, of the roller R, is carried in a stationary bearing; the axes b and c are carried upon liuks $b_1\ a_1$ and $b_2\ a_2$, so that they may be moved to or from a, or may be pressed against the latter. The length of these links is governed by a screw adjustment. The rollers are pressed together by the ring-roller R_3, acting upon the planet roller r_3 and rollers r_1 and r_2, the latter being loose upon the axes of the main rolls R_1 and R_2. The planet roller r_3 acts against the roller r, which is fast on the axis of R. If it is desired to exert greater pressure upon the rollers, the roller R_1 is forced towards R_3 by means of the lever combination $a_1\ b_1\ c_1\ d_1$, the lever $d_1\ c_1$ being held in position by a ratchet section, the position being changed as the rollers wear. The ring roller R_3 reduces very greatly the wear upon the journals of the grinding rollers, as it converts the greater part of their journal friction into rolling friction. In order to equalize the effect of the weight of the upper roller, the lower roller R_2 is counterbalanced by a weight, which, acting through the system of levers $c_2\ d_2\ a_2$ exerts an upward pressure equal to the combined weight of R_1 and R_2.

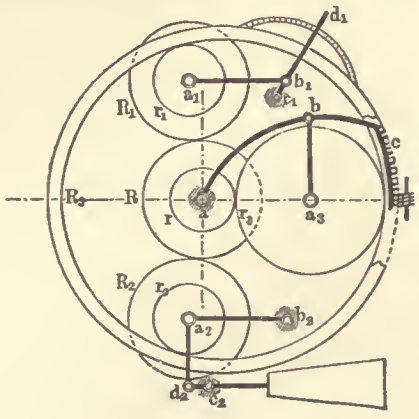

FIG. 568.

The above described apparatus is fitted to both ends of the rollers. In order to provide for any slight inequality in diameter between the opposite ends of the rollers, another adjustment is provided. This consists of the lever $a\ b\ c$, to which the planet roller is suspended by the link $b\ a_3$. This permits the planet roller to be forced into the narrower space between r and R_3, by means of the worm and worm sector shown at c. The ring R_3 is a continuous steel forging, aud the rollers are chilled castings. The rollers R, R_1, R_2 are geared together, the gears having double spiral teeth, as shown in § 222.

Friction rollers are sometimes used in connection with toothed gearing, an example of which will be seen later in Fig. 589. Whitworth used rollers in the place of a nut in his screw planing machine, and they are used in worm gearing by Bourdon, and the higher form of worm, the globoid (see § 224) by Jensen, and by Hawkins. Many applications are also found among instruments of precision, notably Atwood's Machine, and Amsler's Planimeter.

CHAPTER XVII.

TOOTHED GEARING.

§ 199.

Classification of Gear Wheels.

The relative position of the axes of gear wheels governs their general form, although not to so great an extent as in the case of friction wheels. This is due to the fact that the geometric shapes, which in the case of friction wheels form the actual surfaces, are only theoretically used in the case of gear wheels as forms upon which to design the teeth.

Gear wheels for parallel axes are called spur gears; their form is based on the cylinder. Wheels for inclined intersecting axes are based upon cones, and are termed conical gears, or more commonly bevel gears. For inclined, non-intersecting axes, the

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base form is the hyperboloid, which name is also given to the gears. For many applications of inclined axes the teeth are made spiral, giving the various forms of spiral gears and worm gears.

If the motion is to be transmitted at a uniform rate, the base figures are solids of revolution (cylinders, cones, hyperboloids); the wheels themselves being round, while if the motion is not to be transmitted uniformly the outlines will be irregular. In the following discussion only round gear wheels will be considered.

A. THE CONSTRUCTION OF SPUR TEETH.

§ 200.

General Considerations.

The form of gear teeth may be so chosen that all gears of the same pitch will work together. Wheels of this sort are called interchangeable, while wheels which are not so made will run only in pairs.

In each pair of round wheels there are two circles, struck from the centres of the wheels, which have at each moment the same linear velocity, and are called in general the ratio circles. The particular ratio circles for a pair of spur gear wheels are called their pitch circles. Upon these circumferences are laid out the pitch divisions, *i. e.*, the spacings from centre to centre of the teeth.

The teeth themselves are prismatic in shape, the base of the prism being the outline of the tooth. The portion of the tooth which projects beyond the pitch cylinder is called the point of the tooth, and that portion within the pitch cylinder is the base. The surfaces of the point are called the faces of the tooth, and the surfaces of the base, the flanks.

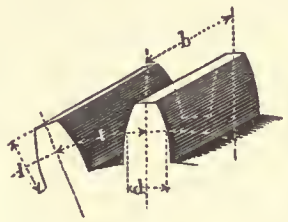

Fig. 569.

In spur gear teeth we also have, Fig. 569, the length l, the breadth on face b, the tooth thickness d, the pitch being indicated by t, the two latter being measured on the curve of the pitch circle.

All the teeth in one and the same wheel are made of the same thickness and same spacing, so that any tooth will fit into any space. It follows from this that, the spaces being made of suitable size to receive the teeth, that the inverse ratio of the number of revolutions n and n_1 of a pair of wheels is equal to the direct ratio of the respective numbers of teeth Z, and Z_1, or:

$$\frac{n_1}{n} = \frac{Z}{Z_1} \quad \ldots\ldots\ldots\ldots \quad (186)$$

This statement is equally true for circular and non-circular wheels. It also holds good if the thickness of the teeth is different at different portions of the circumference, providing only that care is taken that the spaces in the smaller gear come around to meet their proper teeth each revolution. If, therefore, under these conditions we have the number of teeth given for any case, we may consider the above relation as *the fundamental formula of the transmission of motion by toothed gearing*. This is rather to be considered as an inevitable principle of construction rather than a fine geometrical distinction. It depends upon the primitive form of gear construction which has been in use for centuries in the Orient, where no other care is taken in the proportioning of gears except that they are large enough and that the pin teeth are sufficiently strong.

No general principle can be laid down for the form of the flanks of teeth. For round wheels, the ratio of the angular velocities, *i. e.*, that of the differentials of the simultaneous angles of rotation ω_1 and ω must equal the ratio $\frac{Z}{Z_1}$. This affects the flanks as being those surfaces upon which the ratio $\frac{d\,\omega_1}{d\,\omega}$ depends.

The form of the teeth is of great importance. Especially necessary is it that the division of the pitch shall be accurately made; errors in the shape of the flanks are even less injurious than errors in the dividing. Accurate spacing can only be accomplished by the use of suitable gear-cutting machinery, and such machines are now in general use.*

* The machines most extensively in use in Germany are those of the Berlin Anhalt Maschinenfabrik, in Berlin, and the Maschinenbauanstalt of Briegleb, Hansen & Co., in Gotha.

Accuracy in spacing is of especial importance in the change gears of a lathe, as any error in a gear produces a corresponding defect in the screw which is being cut. Such defects are still more apparent if the lathe is used for cutting spiral gears (see § 221, below). The smooth motion which such spiral gears are intended to produce may thus be prevented by irregular cutting. The choice of tooth outline to be adopted, either for the entire product of an establishment, or for any class of work, can only be made after a careful consideration of all the conditions upon which so much depends. These considerations will be taken up and discussed in the following sections:

§ 201.

Pitch Radius. Circumferential Division.

For any pitch t, and number of teeth Z for a round wheel we have for the radius R of the pitch circle:

$$\frac{R}{t} = \frac{Z}{2\pi} = 0.15916\,Z \quad \ldots\ldots\ldots \quad (187)$$

which gives, according to formula (186)

$$\frac{R}{R_1} = \frac{n_1}{n} \quad \ldots\ldots\ldots\ldots \quad (188)$$

The radius obtained from formula (187) is never a whole number, because π is an irrational number, so that R will always contain a fraction if the pitch is a whole number. The following table will facilitate the computation in such cases. If the irrational feature is to be kept out of the value of R, the length of the pitch divisions must not be made whole numbers, but fractions or multiples of π, and this method is used in many establishments. If we call the pitch $= t$, we have under this plan:

$$R = \frac{Z}{2}\left(\frac{t}{\pi}\right) \quad \ldots\ldots\ldots\ldots \quad (189)$$

This corresponds to the so-called "diametrical pitch" system of England and America.

Example. Suppose a wheel of 24 teeth and a pitch of 6×3.14 millimetres, we have according to (187), for the radius R, of its pitch circle, $R = \frac{24}{2} \times 6 = 72$ mm, and if we have in English units a pitch of $\frac{3}{16} \times 3.14$ for a wheel of 30 teeth, we have according to (189), $R = \frac{30}{2} \times 3 = \frac{45}{16} = 2\frac{13}{16}$

A convenient instrument in this connection is a circumference scale. This consists of a prismatic rule of wood or metal upon which, for the metric system, a length of 314 millimetres is laid off, and on a parallel line the same distance is divided into 100 equal parts. Corresponding points on the two scales will then have to each other the ratio $1 : \pi$. This scale is also useful for the rectification of circles and circular arcs. Similar scales may be prepared upon sixteenths, tenths or any subdivision of the inch.

In the following discussion both methods will be used, namely: that in which the pitch is taken in rational numbers, thus making the radius irrational; and that in which the pitch is made rational in units of the circumference scale, and hence the radius becomes rational. The following table is not to be confounded with that of Donkin, * made according to the expression,

$$\frac{r}{t} = \frac{1}{2\,sin\left(\frac{180^\circ}{Z}\right)}$$

which gives the radius of the circumscribing circle of a regular polygon of Z sides, each having a length equal to t. This latter radius differs from the radius R above referred to for small values of Z, and confusion in this respect has given rise to numerous errors.

§ 202.

Table of Radii of Pitch Circles.

Examples in the use of the following table. (Note. This table was calculated for use with the metric system, in which the pitch is generally taken in millimetres. It may, however, be used equally well in English units, by taking the pitch in sixteenths, in order to make the divisions sufficiently small.)

Example 1. A wheel of 63 teeth, and $1\frac{7}{8}$" pitch is to be made; required, the radius of the pitch circle. The pitch is here 30 sixteenths, and we have at the intersection of the columns for 60 and 3, the number 10.03, hence $R = 10.03 \times t = 10.03 \times 30 = 300.9$ sixteenths or 18.8" giving a diameter of 37.6".

The table may also be used to determine the number of teeth when the pitch is chosen and the radius given.

* See Salzenburg's Vortäge, p 93, and others.

Example 2. Given a wheel of 40 inches radius and 1.6″ pitch. This gives $\frac{R}{t} = \frac{40}{1.6} = 25$. The nearest value to this in the table is 24.99; at the intersection of 150, and 7, and hence 157 is the number of teeth.

When the radius and number of teeth are given the table may be used to find the pitch.

Example 3. Given $R = 15\frac{3}{4}''$ $Z = 54$. We find in the table at the intersection of 50 and 4, the value of $\frac{R}{t} = 8.59$. We then have $t = \frac{R}{8.59} = 1.83''$.

Z	0	1	2	3	4	5	6	7	8	9
0	0.00	0.159	0.318	0.477	0.637	0.796	0.955	1.114	1.273	1.432
10	1.59	1.75	1.91	2.07	2.23	2.39	2.55	2.71	2.86	3.02
20	3.18	3.34	3.50	3.66	3.82	3.98	4.14	4.30	4.46	4.62
30	4.77	4.93	5.09	5.25	5.41	5.57	5.73	5.89	6.05	6.21
40	6.37	6.53	6.68	6.84	7.00	7.16	7.32	7.48	7.64	7.80
50	7.96	8.12	8.28	8.44	8.59	8.75	8.91	9.07	9.23	9.39
60	9.55	9.71	9.87	10.03	10.19	10.35	10.50	10.66	10.82	10.98
70	11.14	11.30	11.46	11.62	11.78	11.94	12.10	12.25	12.41	12.57
80	12.73	12.89	13.05	13.21	13.37	13.53	13.69	13.85	14.01	14.16
90	14.32	14.48	14.64	14.80	14.96	15.12	15.28	15.44	15.60	15.76
100	15.92	16.07	16.23	16.39	16.55	16.71	16.87	17.03	17.19	17.35
110	17.51	17.67	17.83	17.98	18.14	18.30	18.46	18.62	18.78	18.94
120	19.10	19.26	19.42	19.58	19.73	19.89	20.05	20.21	20.37	20.53
130	20.69	20.85	21.01	21.17	21.33	21.49	21.65	21.80	21.96	22.12
140	22.28	22.44	22.60	22.76	22.92	23.08	23.24	23.40	23.55	23.71
150	23.87	24.03	24.19	24.35	24.51	24.67	24.83	24.99	25.15	25.31
160	25.46	25.62	25.78	25.94	26.10	26.26	26.42	26.58	26.74	26.90
170	27.06	27.21	27.37	27.53	27.69	27.85	28.01	28.17	28.33	28.49
180	28.65	28.81	28.97	29.13	29.28	29.44	29.60	29.76	29.92	30.08
190	30.24	30.40	30.56	30.72	30.88	31.04	31.19	31.35	31.51	31.67
200	31.83	31.99	32.15	32.31	32.47	32.63	32.79	32.95	33.10	33.26
210	33.42	33.58	33.74	33.90	34.06	34.22	34.38	34.54	34.70	34.85
220	35.01	35.17	35.33	35.49	35.65	35.81	35.97	36.13	36.29	36.45
230	36.61	36.76	36.92	37.08	37.24	37.40	37.56	37.72	37.88	38.04
240	38.20	38.36	38.51	38.67	38.83	38.99	39.15	39.31	39.47	39.63
250	39.79	39,95	40.11	40.27	40.42	40.58	40.74	40.90	41.06	41.22
260	41.38	41.54	41.70	41.86	42.02	42.18	42.34	42.49	42.65	42.81
270	42.97	43.13	43.29	43.45	43.61	43.77	43.93	44.09	44.25	44.40
280	44.56	44.72	44.88	45.04	45.20	45.36	45.52	45.68	45.84	46.00
290	46.15	46.31	46.47	46.63	46.79	46.95	47.11	47.27	47.43	47.59

§ 203.

General Solution of Tooth Outlines.

In a pair of gear wheels, the two tooth outlines which work together lie in a section at right angles to the axes of the wheels and in the plane of this section the construction and action of the teeth is to be considered. The so-called general solution of tooth outlines is that by which, if a form of tooth be given for one wheel, the proper form of tooth for the other wheel may be drawn so that the motion will be transmitted with a uniform velocity ratio. Several such solutions will be given.

I. *The Author's First Solution.* Fig. 570. Given the tooth

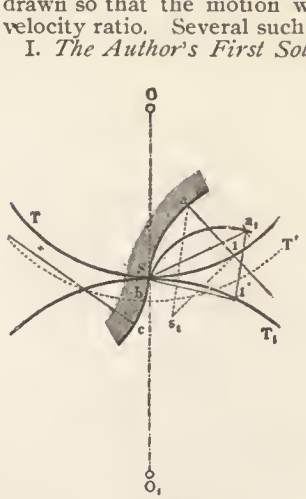

Fig. 570.

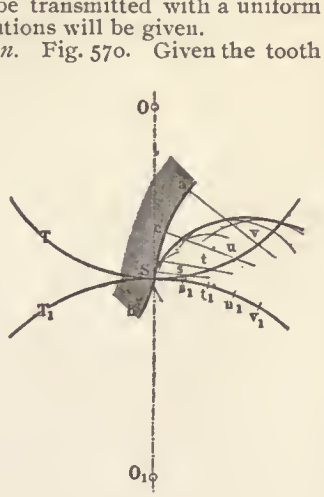

Fig. 571.

profile $a\ S\ b\ c$, also the pitch circle T, of the wheel O, and the pitch circle T_1 of the wheel O_1; required the tooth curve $a_1\ S$ for the wheel O_1.

Place the given curve so that the point S, where it crosses the pitch circle, lies on the line joining the centres $O\ O_1$, thus making S, a point common to both profiles. In order to find a second point a_1, which shall work in contact with a point a, draw $a\ 1$ normal to the given curve at a, make the arc $S\ 1' =$ arc $S\ 1$, and the distance $1\ s_1 = S\ 1'$, and $S\ s_1 = 1'\ 1$. Then with S as a centre strike an arc with a radius $S_1\ a$, and from $1'$, an arc with a radius $1\ a$, and the intersection of these arcs will be the desired point a_1 of the required curve. For such points as c, where the normal to the curve does not intersect the pitch circle the given pitch circles cannot be used, therefore if these points are required the pitch circles must be transposed (exaggerated in the figure). The curve thus found sometimes assumes an impracticable form without being geometrically incorrect.

II. *Abridged Solution.* (*Poncelet.*) Fig. 571. Mark off on the pitch circle T_1, the points $s_1, t_1, u_1 v_1 \ldots\ldots\ldots$, which roll into contact with points $s, t, u, v, \ldots\ldots$ of the given circle T, draw from s_1, t_1, u_1, &c., arcs with radii respectively equal in length to the normals to the given tooth outline $v\ a$, $u\ c$, etc., then will a curve drawn tangent to these arcs be the required outline. The points s, t, u, v, should be taken close together. If the lengths of the normals $v\ a$, $u\ c$, etc., are taken backward from the points s_1, t_1, u_1, &c., instead of forward, the outline for an internal gear tooth will be obtained for the wheel $O_1\ T_1$.

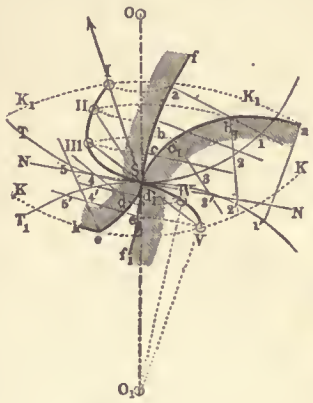

Fig. 572.

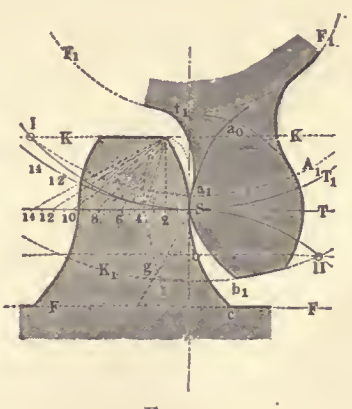

Fig. 573.

III. *The Author's Second Solution.* Fig. 572. The tooth outline $a\ b\ c\ S\ d\ e$ is given, and its pitch circle T, also the pitch circle T_1. Draw the normals $a\ 1$, $b\ 2$, $c\ 3$, &c., also draw from O, as a centre, arcs through a, b, c, &c., and make $S\ I = a\ 1$, $S\ II = b\ 2$, $S\ III = c\ 3$, &c., and draw the curve I, II, III, S, IV, V, &c.; this curve will be the path of the point of contact of the teeth, and may be called the Line of Action.

IV. *Theoretical Profile of the Flank.* Fig. 573. In order to obtain the necessary strength it is frequently desirable to make the root of the tooth as thick as can be done without interfering with the path of the face of the corresponding tooth of the other gear. This path may be determined in the following manner. Let $a\ S\ b$ be the profile of the tooth for the wheel T, $a_1\ S_1\ b_1$ that for the wheel T_1, $a_1\ a_0$ the prolongation of the flank outline for the latter tooth, and I S II the line of action between the limits of the outside diameter circles K and K_1. Lay off from S, on both pitch circles the corresponding spaces $S\ 1$, $1\ 2$, $2\ 3$, &c., $S\ 1'$, $1'\ 2'$, $2'\ 3'$, &c. Take in the dividers successively $S\ a$, $1\ a$, $2\ a$, $3\ a$, &c., and describe arcs from $1'$, $2'$, $3'$, &c., and the envelope of these arcs will give the path $a\ a_1\ g$, or so-called theoretical profile of the flank. The actual profile of the flank $a_1 f_1$ is drawn tangent to the theoretical curve, to the point where it crosses the clearance circle F_1. The theoretical curve is a prolonged or abridged cycloidal curve (see § 205). In the figure, in which T is a straight line, or rack, the curve is an abridged evolute.

§ 204.

The Action of Gear Teeth.

In solution III, of the preceding section, reference was made to the *line of action* * of a pair of gear wheels, and this line bears an important relation to the theory of the action of gear wheels.

The line of action intersects the pitch circle at the same point as the tooth profile and cuts the latter at right angles, so that the tangent $N\ N$ of the line of action (Fig. 572) is normal to the tooth profile. Each point of action corresponds to a point of contact of the teeth and also to a point of contact of each of the pitch circles; so that, for example, the point II of the line of action corresponds to the point 2 on T, and $2'$ on T'. That portion of the pitch circle between the pitch point of the line of

* First discussed in Moll & Reuleaux's "Konstruktionslehre für den Maschinenbau."

action and the initial point of contact is called the rolling arc for the given point. For example $S\,2$ is the rolling arc on T, for the point II, and $S\,2'$ on T_1, for the same point.

The sum of the rolling arcs between the two extreme points (arc $1\,S + S\,5$, or arc $1'\,S + S\,5'$) is called the arc of action, and its length indicates the duration of the action of the given pair of teeth, which is easily determined graphically. It depends upon the length of that portion of the line of action which it is desired to use. This is usually taken between the limits of the circles of the outside and the base of the teeth, which gives in Fig. 572 the line of action VI.

For any wheel of given tooth outline and pitch diameter there is but one line of action, and for a given line of action but one tooth profile. This latter can only be determined from the line of action when the rolling arcs for the pitch point of the line of action are also given.

For cycloidal teeth the rolling arc is also the line of action and for this reason the geometrical discussion is much simplified. In order that a pair of gear wheels should work properly together, their lines of action should correspond and their rolling arcs be of equal length for homologous points of action. By conforming to these conditions any number of gear wheels may be made to operate with a given wheel. Such wheels are said to be interchangeable or series wheels, since the common line of action is symmetrically disposed on each side of the pitch circle, as well as on each side of a radial line passing through its pitch point.

The ray drawn from the pitch point through any point on the line of action (as $S\,I$, in Fig. 572) gives the direction of the pressure between the teeth for that point.

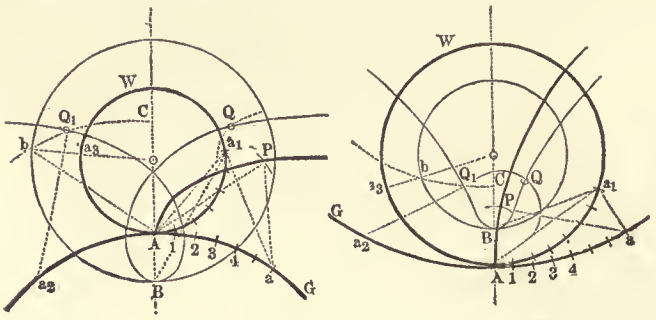

FIG. 574.

§ 205.

THE CYCLOIDAL CURVES.

For the generation of tooth outlines for gears to be used interchangeably in series, the cycloidal curves or those produced by rolling circles are the best. When one circle rolls upon another in the same plane without sliding, each point in any radius describes a curve which is called either a common, extended or abridged cycloid, according as the point is situated on the circumference of the circle, or on a radial line without or within the circumference.

The stationary circle is the base circle of the curve, and its radius will be here indicated by R; and the radius of the rolling circle by r. If we consider either radius negative when it lies within the other circle, and negative when it lies without, we may distinguish the five kinds of cycloidal curves whose radii have the relation R, and r, in the following manner:

Base Circle.	Rolling Circle.	Corresponding Curve Name.
$+R$	$+r$	Epicycloid.
$+\infty$	$+r$	Orthocycloid.*
$-R$	$+r$	Hypocycloid.
$+R$	$\pm\infty$	Evolute of circle.
$+R$	$-r$	Pericycloid.

The following properties are common to all five curves:

1. *The normal to any element of the curve passes through the corresponding point of contact of the generating and base circles.*

2. *The centre of curvature of any element of the curve is at the intersection of the normal with a right line which joins the starting point of the curve with the centre of the base circle.* For the extended, or abridged cycloidal curves the starting point is taken on the radius prolonged of the curve element, at right angles to a normal to the curve at the point of contact of the rolling circles.

*The author gives this name to the common cycloid because the latter term properly includes the whole class.

Upon the first property depends the suitability of the cycloidal curves for use as tooth outlines, and in the second lies the practicability of approximating them by circular arcs.

§ 206.

THE GENERATION OF CYCLOIDAL CURVES.

I. *Exact Solution.* Fig. 574. G is the base circle, W the rolling circle, A the starting point of the curve. Lay off from A, on G and W, small arcs of uniform spacing, and let a, and a_1 be two of the corresponding points of division. From A, with radius a, a_1, strike an arc, and from a, with the chord $A\,a_1$, another arc, and the intersection of the two arcs at P, will be a point in the curve.

This solution, which is shown in Fig. 574, both for external and internal rolling, holds good for all five curves.

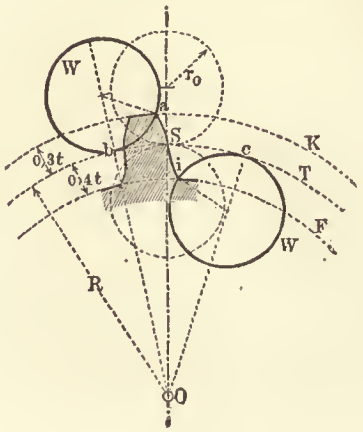

FIG. 575.

II. *Abridged Solution.* From the points 1, 2, 3, . . . a, with radii equal to the corresponding chords of the rolling circle, strike arcs, which arcs will include the entire curve with sufficient accuracy if the points of division be taken sufficiently close together.

In order to draw the extended or abridged curve, starting say at B, determine first a point P, on the ordinary curve, then draw from a, with a radius $a_1\,B$, an arc, and from P, another with radius $A\,B$, and the two arcs will intersect in a point Q, of the curve.

Or, draw through a_3 a radius $a_3\,b$ in the rolling circle and through b, an arc $b\,C$, concentric with the base circle, and make $a_2\,Q_1 = A\,b$, then will Q_1 be the point in the curve for the rolling of the arc $A\,a_3$ upon $A\,a_2$.

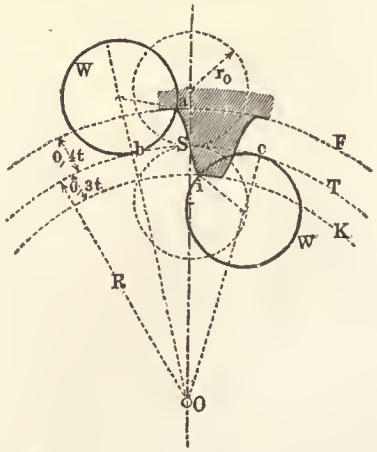

FIG. 576.

§ 207.

THE GENERATION OF INTERCHANGEABLE TEETH.

The tooth profile for interchangeable gears is generated in a similar manner, both for external and internal gears, by using a rolling circle of constant diameter for each pitch.

I. *External Teeth.* Fig. 575. Given the number of teeth Z, and pitch t, or ratio $\frac{t}{\pi}$ of the wheel. Make $O\,S = R = \frac{Z\,t}{2\,\pi}$ $= \frac{1}{2}\,Z\left(\frac{t}{\pi}\right)$, and the radius r_0 of the rolling circle $W = 0.875\,t$

$= 2.75 \frac{t}{\pi}$; draw the outside circle of the teeth K, with a radius $= R + 0.3\,t$, and the inside circle F, with radius $= R - 0.4\,t$, and make the thickness of tooth $= \frac{19}{40} t$. Arc $S b$ = arc $a b$; arc $S c$ = arc $i c$. $S a$, the face curve, is generated by the rolling of W upon T; $S i$, the flank curve, by the rolling of W in T. For pinions of eleven teeth, $S i$ becomes a straight line and radial. Pinions with as few as seven teeth can be made to work on this system, for although the flanks are undercut, they are still within the limits of the theoretical flank profile (see § 203, and Fig. 573, where a seven tooth pinion is shown with a rack tooth). The backlash is $\frac{1}{10} t$.

II. *Internal Teeth.* Fig. 576. The generation of internal teeth is similar to the preceding. The radius of base circle is $-R$, and the length of tooth above and below the pitch circle is $0.3\,t$, and $0.4\,t$, as before; $r_o = 0.875\,t = 2.75 \frac{t}{\pi}$, and the thickness of tooth $= \frac{19}{40} t$. The flank $S a$ is generated by rolling W upon T, and the face $S i$, by rolling W inside of T.

In the case of a rack $R = \infty$, $S a$ and $S i$ then become similar portions of the common orthocycloid (see Fig. 573).

In teeth of this form the line of action coincides with the rolling circles, the portion included being = arc $b a$ + the corresponding arc $b_1 a_1$ of the opposing wheel, when both are external gears, and + the arc $c i$ for an internal gear working with a spur gear. The duration of action e, varies between 1.22 and 1.60.

§ 208.

TOOTH OUTLINES OF CIRCULAR ARCS.

Instead of using the exact tooth outlines as generated by the rolling circles, two circular arcs may be used as a close approximation (see § 205).

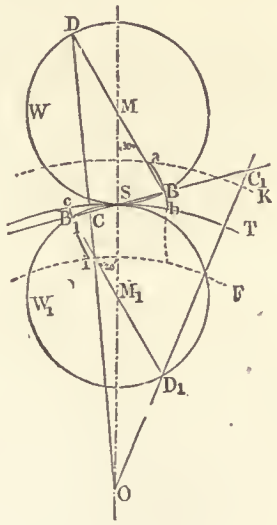

FIG. 577.

Fig. 577. Draw the pitch circle T, and outer and inner circles K and F, as before, also the centres M, and M_1 of the rolling circles W and W_1, which latter are in contact with each other and with the pitch circle at S. Draw the diameters $B\,M\,D$ and $B_1\,M_1\,D_1$ in such manner that the angle $B\,M\,S$ = angle $B_1\,M_1\,S_1 = 30°$, join B and B_1 with the prolonged line $C_1\,B\,S\,B_1$, and draw through D and D_1 the lines $O\,D$ and $O\,D_1\,C_1$; then will the intersections at C and C_1 with $B_1\,C\,S\,C_1$ be the required centres of curvature for the arcs $a\,B\,b$, and $c\,B_1\,i$.

Through C and C_1 draw circles with O as a centre, and on these circles the centres for all the teeth will be found, the arc $a\,B\,b$ being struck from C, and $c\,B_1\,i$ from C_1.

The radii of curvature ρ may be calculated from the following formula:

$$\left.\begin{aligned} \frac{\rho}{t} &= 0.45 \frac{2Z \pm 11}{Z \pm 11} \text{ and} \\ \frac{\rho}{\left(\frac{t}{\pi}\right)} &= 1.42 \frac{2Z \pm 11}{Z \pm 11} \end{aligned}\right\} \quad \ldots\ldots \quad (190)$$

The plus sign gives the radius $C\,B$, for the face (ρ_a) and the minus sign gives the flank radius $C_1\,B_1$, (ρ_i). The flank should be joined to the bottom of the space by a small circular fillet.

Example 1.—Given $Z = 63$, $t = 1.3125''$, we have for the radius for the face of the teeth:

$$\rho_a = 1.3125 \times 0.45 \frac{126 + 11}{63 + 11} = 1.093''.$$

For the flank radius we have:

$$\rho_i = 1.3125 \times 0.45 \frac{126 - 11}{63 - 11} = 1.306''.$$

Example 2.—Given $Z = 11$, $\frac{t}{\pi} = 0.4$. We have:

$$\rho_a = 0.4 \times 1.42 \frac{22 + 11}{11 + 11} = 0.85'',$$

Also,

$$\rho = 0.4 \times 1.42 \frac{22 - 11}{11 - 11} = \infty$$

hence the flanks are straight radial lines.

Example 3.—Given $Z = 7$, $t = 2''$. We have:

$$\rho_a = 2 \times 0.4 \frac{14 + 11}{7 + 11} = 1.12'',$$

Also,

$$\rho_i = 2 \times 0.4 \frac{14 - 11}{7 - 11} = -0.6'.$$

The negative sign indicates the undercut flank. This is shown in Fig. 573

It is better to use the exact method given in § 207, for wheels with fewer than fifteen teeth, as the approximation becomes less accurate for the lower numbers.

§ 209.

EVOLUTE TEETH FOR INTERCHANGEABLE GEARS.

Gear teeth may be given the evolute form, which curve is developed by unwrapping a line from a base circle, which is concentric with, and bears a definite relation to the pitch circle.

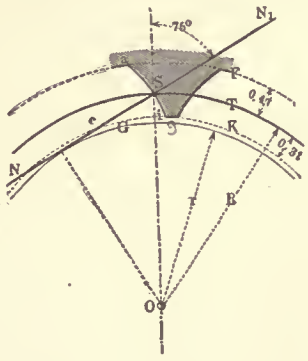

FIG. 578.

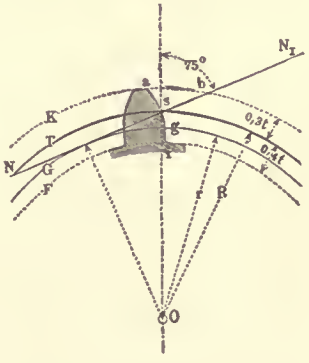

FIG. 579.

External and Internal Teeth. Fig. 578 and Fig. 579. Given the number of teeth Z, and pitch t, or ratio $\frac{t}{\pi}$ for the required wheel. Make $O\,S = R = \frac{Z\,t}{2\,\pi} = \frac{1}{2} Z \left(\frac{t}{\pi}\right)$ and draw the outer and inner circles, giving the distances $f = 0.4\,t$, $k = 0.3\,t$ above and below the pitch circle, also make the thickness of the tooth $= \frac{19}{40} t$.

Draw the line $N\,S\,N_1$ at an angle of 75° with $O\,S$, and it will

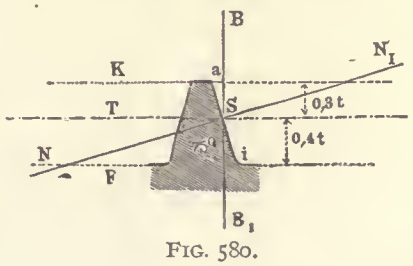

FIG. 580.

be tangent to the base circle G, the radius of which $= r = 0.966\,R = 0.154\,Z\,t$, $= 0.483\,Z \left(\frac{t}{\pi}\right)$. If now we unwrap the line $N\,S$ upon the circle G, from S outward to a, and inward to g, the path $a\,S\,g$ of the point S will be the required tooth outline, which for wheels of fewer than 55 teeth may be prolonged by a radial line to reach the bottom circle.

The line of action is the straight line $N\,N_1$; and extends from $S\,b$ to $S\,b_1$ on the other gear, or in the internal gear to $S\,c$. To determine the duration of contact e the pitch t can be carried to

the base circle by drawing radii, and the length measured. For two equal wheels of 14 teeth, e is only a little greater than unity; it varies between 1 and 2.5.

Rack Teeth. Fig. 580. The profile $a\,S\,i$ is straight and makes an angle of 75° with the pitch line T. The angle 75° can readily be laid off by using the drawing triangles of 45° and 30° together.

For low numbered pinions the base circle closely approaches the pitch circle. This sometimes introduces an error into the

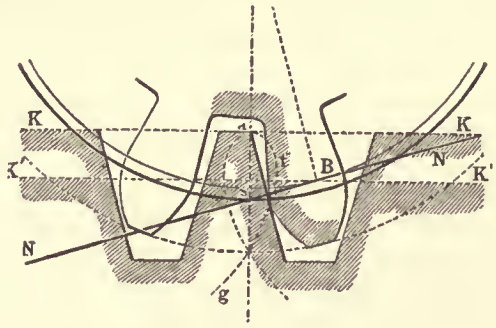

FIG. 581.

action. If the portion $S\,B$, of the line $N\,N_1$, which lies between the pitch and base circles, Fig. 581, is shorter than the length of face of the opposing tooth, the point a will interfere with the flank of the pinion tooth, as shown in the path $a\,f\,g$. (See also Fig. 573.) In order to avoid this, the tooth to which the point a belongs must not extend above the line $K'\,K'$.

This exists for teeth made in the manner given, when $Z \geq 28$. Another method of avoiding this difficulty is to round off the tooth at a, and this is more frequently adopted in practice. An important application of evolute teeth is shown in § 222.

§ 210.

PIN TEETH.

Teeth with radial flanks can always be generated by making the inner rolling circle for each wheel equal in diameter to one-half the pitch circle. This will give radial flanks and curved

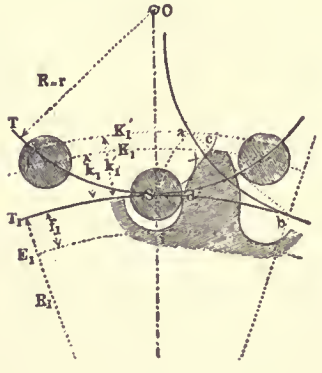

FIG. 582.

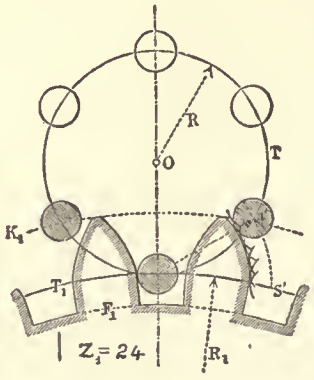

FIG. 583.

faces to both gears, but wheels made on this system are not interchangeable, and are therefore not practical for general machine construction. Such teeth are still much used by watchmakers on account of the ease with which they may be fitted by filing.

If the diameter of the rolling circle is made greater than the radius of the pitch circle a form of tooth is obtained which is practicable, but which is comparatively little used.

If, in a single pair of wheels, the rolling circle be taken for one wheel equal to the pitch circle, of the other wheel, we obtain for the teeth of the wheel upon which the rolling is done, an outline of cycloidal form, while the teeth of the other wheel become mere points. In practice these points are the centres about which pins are described and such gears are called pin-tooth gears.

External Pin-tooth Gearing. Fig. 582. The pins are circular in section and in diameter equal to $\frac{19}{40}\,t$; the tooth profile for the wheel R_1 is then a curve parallel to the path $S\,a$, described by rolling the circle T on T_1. The arc $S\,b = a\,b$, and circles of the diameter of the pin, struck from successive points of the path $S\,a$, will outline the tooth profile $c\,d$, the flank $d\,i$ being a circular quadrant. The curve of action $S\,I$ is limited by the outer circle K_1' at I, and is in all cases greater than t, generally not less than 1.1 t. This gives the limit of tooth length k_1' and also determines k_1. If it is desired to construct the actual line of action, the method of case III, § 203, may be employed.

Fig. 583 shows a pinion of six pins gearing into a wheel of 24 teeth. The diameter of the pins is here made $= \frac{t}{3}$. The flanks of the 24 tooth wheel are made radial with square corners in order to permit ready filing and finishing.

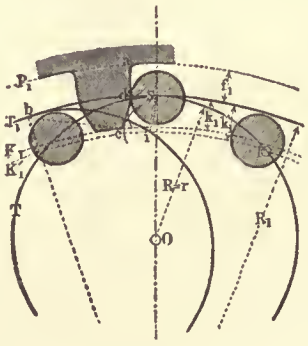

FIG. 584.

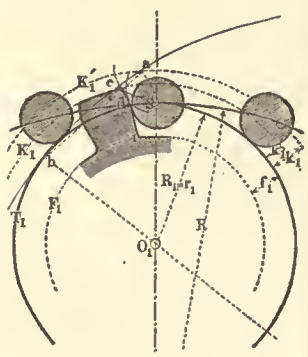

FIG. 585.

Internal Pin-tooth Gearing. Fig. 584. This is similar to the preceding. The tooth profile $c\,d$ is a parallel to the curve $S\,i$, generated by rolling T in T_1, the arc $S\,b = i\,b$. $S\,I$ is the line of action and is made equal to, or greater than 1.1 t. The flank $d\,a$ is made radial.

In Fig. 585 the pinion is made with the pin teeth and the spur teeth are on the internal gear. The profile $c\,d$ is parallel to the curve $S\,a$, generated by rolling T upon T_1; the arc $S\,b = a\,b$, $S\,I$ is the line of action, as above, and is made equal to, or greater than 1.1 t; the flank $d\,i$ is made radial.

If in Fig. 584 we make the radius R_1 infinitely great, we obtain a rack, and the tooth profile is a curve parallel to the common cycloid. If we make R, in Fig. 585, infinitely great, we obtain a common form of rack, with pin teeth.

Pin teeth have the practical advantage that they may readily be turned in the lathe. They are especially adapted for situations where they are exposed to the weather, as in sluices, swing bridges, wind-mills, etc. In such cases the pins are often made of round bar iron, without being turned.

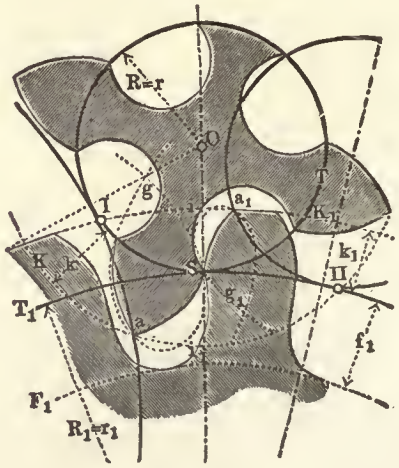

FIG. 586.

Double Pin Gearing. Fig. 586. If two gears on this system are run together, one gear may be made with very few teeth, and hence a great difference in velocity ratio obtained, with a minimum distance between centres. In this case both pitch circles become rolling circles. $S\,a$, the pinion face, is generated by rolling T_1 on T, the action extending on $S\,I$ for the point S on the wheel T. $S\,a_1$, the gear tooth face, is generated by rolling T on T_1, the action extending on the line $S\,II$, for the point S, on the wheel T_1. $S\,i$, the flank profile, is made to conform to the theoretical profile $S\,a_1\,g_1$ (see case IV, § 203), and the other flank is made in a similar manner from the theoretical profile $S\,a\,g$. Such gears are sometimes used in hoisting machinery.

§ 211.

Disc Wheels with Pin Teeth.

It is not an essential requirement that the tooth profile shall be in the immediate line of the pitch circles, as it can be placed within or without to a greater or less extent. In such cases a tooth system is obtained in which the teeth of one wheel pass almost or entirely arouud those of the other wheel, and hence there can be no so-called bottom circle to the latter teeth. Such wheels are so constructed that the teeth are placed upon the side or face of a disc, or shield, and are called disc wheels, or "shield gearing." *

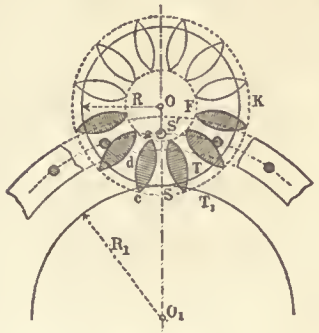

Fig. 587.

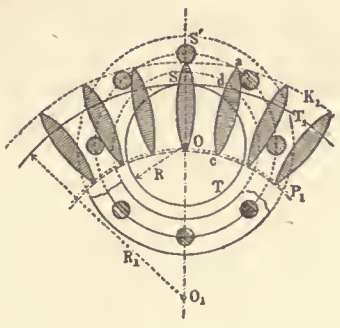

Fig. 588.

For such wheels pin teeth are well adapted. Fig. 587 shows a pair of such wheels arranged for external action, and Fig. 588 for internal action. One wheel of each pair is fitted with round pin teeth, and the other has, in the first case, a tooth profile parallel to an extended epicycloid, and in the second case parallel to an extended hypocycloid.

A peculiar form of disc gearing is shown in Fig. 589. In this case $R = \frac{1}{2} R_1$, $Z = 2$, $Z_1 = 4$. the round pins being on R. The flanks of R_1 are entirely within the pitch circle, and become straight lines parallel to the straight line hypocycloid $S\,i$. The arc of action is about 2 t, and the backlash can be reduced almost to zero, the teeth on R being made as rollers.

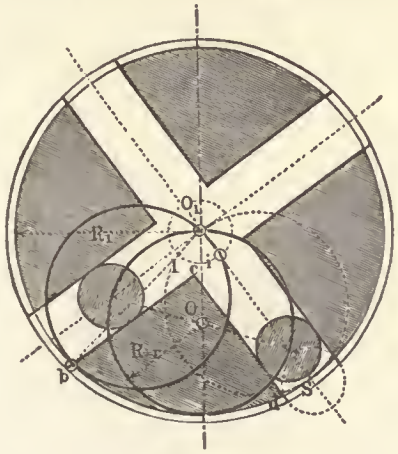

Fig. 589.

If the distance between centres $O\ O_1$ of a pair of wheels for internal action remains constant, and the radius is increased, they will overlap entirely, and the pitch circles will cease to appear as an element in the construction. The wheels will have equal angular velocity and revolve in the same direction.

Such a pair of disc wheels is shown in Fig. 590. Both wheels are made with pin roller teeth, the sum of the pin radii being equal to the distance $O\ O_1$. The pius are shown of equal diameters, although they may be unequal, as shown in the dotted lines. Such wheels may be called Parallel Gears, as two radii which are parallel iu one position remain parallel at all times.†

A second form of parallel gears is shown iu Fig. 591. The curve $a\ b\ c$ is a circular arc, of radius $d\ a$, which includes four segments of the lenticular shaped pins for the wheel O_1.

If the pair of parallel gears of Fig. 590 are placed on opposite sides of an axis $A\ A_1$ uormal to two adjoiniug pins aud parallel to $O\ O_1$, the actiou of the wheels will be correct. In Fig. 592 is shown such a pair of right angle wheels.

* Called *Scudi Dentati* iu Zonca's Teatro di Machine, Padua, 1621.

† This form of gearing was described and named by the author in Berlin Verhandlung, 1875, p. 294.

Such gear wheels have been described more than once,* but are rarely used; they are well adapted to transmit motion to the hands of large tower clocks.

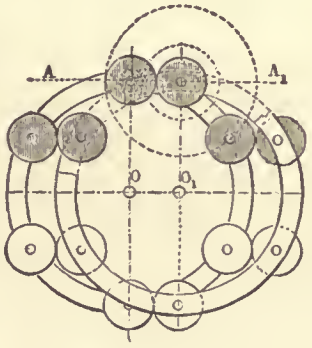

Fig. 590.

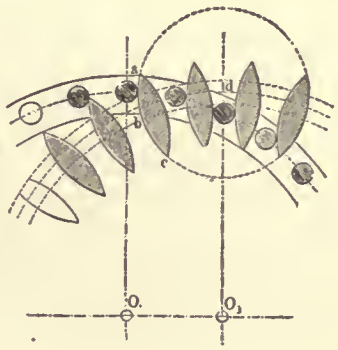

Fig. 591.

§ 212.

Mixed Tooth Outlines. Thumb Teeth.

By combining the preceding forms of teeth, practical shapes may often be made for special service. The two following examples will illustrate:

Mixed Outline. Fig. 593. For the low numbered pinions sometimes used in hoisting machinery, it is important that the

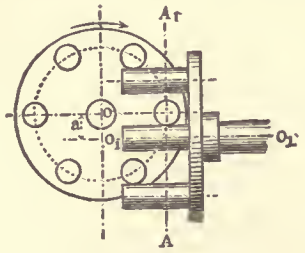

Fig. 592.

piniou teeth shall not be too much undercut, so as to avoid difficulty in making the gears. It is desirable that the flanks on the pinion should be radial. In order to obtain sufficient duration of action, which for a three tooth pinion should not be less than 1.15 t, the face curves of the teeth should be prolouged

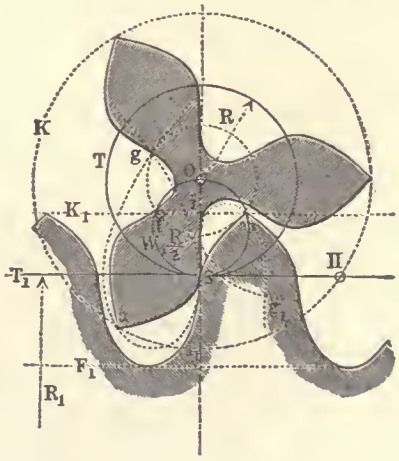

Fig. 593.

until they intersect. The curve $S\ a$ is an arc of an evolute formed by unwrapping the pitch line T_1 from the circle T; $S\,i$ is the radial flank, obtained by rolling the circle W of radius $= \frac{1}{2} R$ in T; $S\ a_1\ g_1$ is the theoretical profile for the tooth space for the wheel T.

$S\ a$ acts with the point S of the rack tooth over the path $S\,II$. $S\ a_1$ is a cycloidal curve generated by rolling W ou T_1, and acts over the path $S\,I$ with the flank $S\,i$ of the wheel T.

* See Tom Richards' Aide-memoire. 1848, I, p. 656. Willis' Principles of Mechanism, 1851, p. 145, Laboulaye, Cinematique, 1854, p. 275.

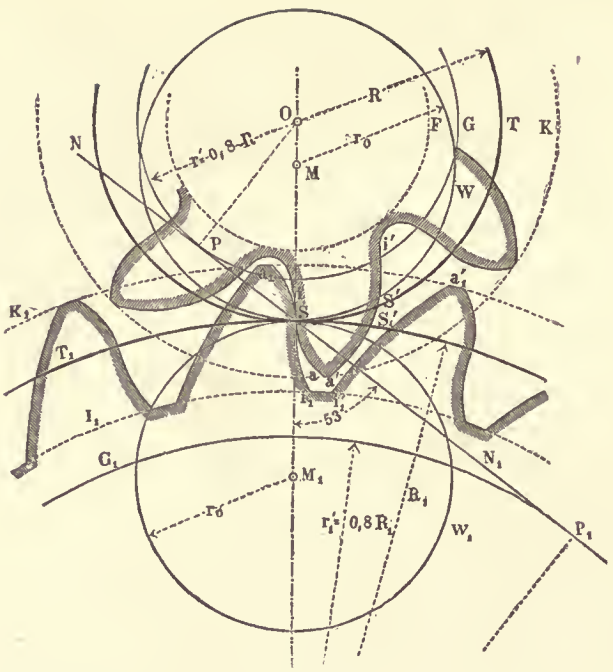

FIG. 594.

Thumb-shaped Teeth. By combiniug the evolute and epicycloid, using the two curves for opposite sides of the same tooth a profile of great strength is obtained. This form is of especial service for heavy driving wheu the motion is constantly iu the same directiou.* From the peculiar form these have been called thumb-shaped teeth. The following proportions will be found suitable for cases in ordinary practice.

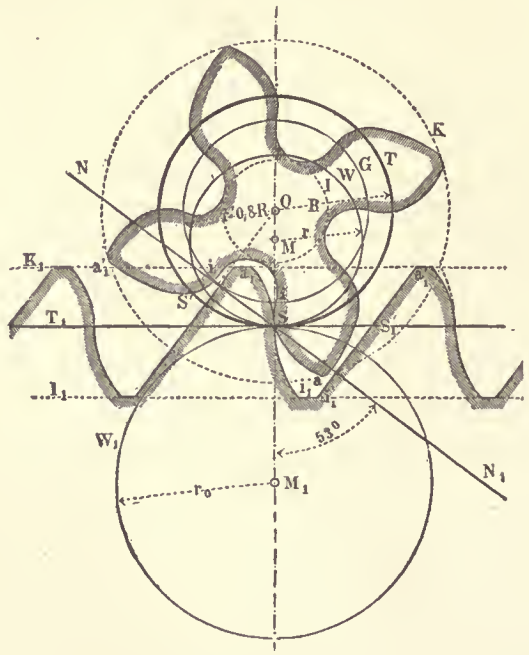

FIG. 595.

Fig. 594. *Spur Gearing with Thumb-shaped Teeth.* $a\,S\,i$ and $a_1\,S\,i_1$ are profiles formed of epicycloidal curves, according to the description iu § 207, in which $r_0 = 0.875\ t$ or $2.75\ \frac{t}{\pi}$.

$a'\,S'\,i'$ aud $a_1'\,S_1'\,i_1$ are evolute curves developed from base circles with radii $r' = 0.8\ R$, and $r_1' = 0.8\ R_1$, giving au angle of 53° (more accurately 53° 8′). For wheels of less than fifteen teeth, as in the seven toothed pinion shown in Fig. 594, the flanks must be modified as showu iu § 203, to avoid interference.

In Fig. 595 is shown a four-toothed pinion on this system, working with a rack. $S\,a$ and $S\,i_1$ are made as before with $r_0 = 0.875\ t$ and $S\,i$ and $S\,a_1$ with $r = \frac{3}{4}\ R$; the evolute curves being generated as before with an angle of 53°.

* This form of mixed outline has been described by Willis in 1851; it was revived by Gee in 1876 and used in practice; he made the angle α greater than here given, viz. 68°.

The rack teeth are made straight on the oue side, as already shown for rack teeth on the evolute system. Applications for teeth of this form are given in § 226.

§ 213.

Tooth Friction in Spur Gearing.

The frictiou of spur gear teeth is mainly dependent upon the form of the tooth outline, and may be investigated by cousidering the form, extent and position of the line of action. In most cases the friction is proportional to the duration of action ε. A coefficient, dependent upon the position of the line of action may be determined from ε, and may be taken $= \frac{1}{2}$, when the arc of action is equally divided on both sides of the central position; as in the case of epicycloidal teeth; aud $= 1$, when, as in many cases, such as pin tooth gearing, the arc of action is entirely on one side of the centre; while for evolute teeth it may be taken $= \frac{3}{4}$, that being about midway between the two preceding forms. The tooth friction is also greatly dependent upon the number of teeth in both wheels, being proportional to their harmonic mean, and it dimiuishes rapidly as the number of teeth is increased.

If we make the coefficient of friction $= f$ and take the number of teeth as Z, and Z_1, we have for the percentage of loss p_r iu tooth friction:

$$\left.\begin{array}{l} \textit{a. Epicycloidal Teeth.} \\ p_r = \pi f \left(\frac{1}{Z} \pm \frac{1}{Z_1} \right) \frac{\varepsilon}{2} \\ \textit{b. Evolute Teeth.} \\ p_r = \pi f \left(\frac{1}{Z} \pm \frac{1}{Z_1} \right) \frac{3}{4}\,\varepsilon \\ \textit{c. Pin Teeth.*} \\ p_r = \pi f \left(\frac{1}{Z} \pm \frac{1}{Z_1} \right) \varepsilon \end{array}\right\} \quad \ldots\ (191)$$

The value of the coefficient of friction f is in no case small, even when the teeth are well lubricated, on account of the usual high pressures; a usual value may be taken, $f = 0.15$, while for new and dry wheels it reaches 0.20 to 0.25 and even higher.

The minus sign in the formula is to be used when one of the wheels (Z_1) is an internal gear.

Example 1. In a pair of epicycloidal gears, of seven teeth, the value of $\epsilon = 1.225$. Taking $f = 0.15$ we have according to (191 *a*) for the loss by tooth friction:

$$p_r = 3.14 \times 0.15 \times \frac{2}{7} \times \frac{1.225}{2} = 0.0824,$$ or about 8¼ per cent.

Example 2. *Epicycloidal Teeth.* $Z = Z_1 = 40$, $\epsilon = 1.44$ and we get:

$$p_r = 3.14 \times 0.15 \times \frac{2}{40} \times \frac{1.44}{2} = 0.0169,$$ or about 1.7 per cent.

Example 3. *Epicycloidal Teeth.* $Z = 7$, $Z_1 = -60$ (internal gear). $\epsilon = 1.40$ and we get:

$$p_r = 3.14 \times 0.15 \left(\tfrac{1}{7} - \tfrac{1}{60}\right) \frac{1.40}{2} = 4.2$$ per cent.

Example 4. *Epicycloidal Teeth.* $Z = 7$, $Z_1 = \infty$ (rack). $\epsilon = 1.37$

$$p_r = 3.14 = 0.15 \left(\tfrac{1}{7} + 0\right) \frac{1.37}{2} = 4.6$$ per cent.

Example 5. *Pin-tooth Gearing.* $Z = 6$, $Z_1 = 40$. We have, as determined by construction, as in Fig. 583, $\epsilon = 1.166$. Hence we get from (191 *c*):

$$p_r = 3.14 \times 0.15 \left(\tfrac{1}{6} + \tfrac{1}{40}\right) \times 1.66 = 2.6$$ per cent.

Example 6. *Evolute Teeth.* $Z = Z_1 = 40$. $\epsilon = 1.92$. We have from (191 *b*):

$$p_r = 3.14 \times 0.15 \times \frac{2}{40} \times \frac{3 \times 1.92}{4} = 3.4$$ per ceut., or double that in Examp. 2.

It will be seen that the tooth friction is least with epicycloidal teeth and greatest for piu gearing; evolute teeth being midway between.

The wear upon gear teeth is affected by other considerations besides that of the coefficient of friction, the pressure of the teeth upon each other, aud the relative rubbiug movement of various portions of the profile also eutering into the problem. The wear is therefore not constant for a constant pressure, and it is an error to assume, as is sometimes done, that the form of evolute teeth is unaltered by wear. These teeth usually show the greatest proportional alteration by wear, since the flank of the tooth below the pitch circle has a very much less rubbiug movement than the portion of the opposing tooth which rubs against it and heuce the wear is unequal.

* Approximately.

The effect of this may frequently be observed in practice, where the smaller of a pair of evolute gear wheels will be noticed to be worn into deep hollows below the pitch circle.

The conclusions given above about the percentage of loss may also be determined geometrically in the following manner:

Take the two portions of the tooth profiles which work together and divide each by the chord of the corresponding portion of the line of action, multiply each result by the ratio of the length of its portion of the line of action to the entire length of the line of action, and then multiply the sum of the two quotients by the coefficient of friction.

The result will be the percentage of loss, pr. The chord referred to becomes the line of action itself in the case of evolute teeth. This method serves also for pin teeth, and is very useful for the designer, as the data can all be taken off the drawing with the dividers.

§ 214.

General Remarks on the Foregoing Methods.

Each of the preceding methods possesses its merits and disadvantages.

Epicycloidal Teeth. These possess the great advantage that they will work together in any series with as few as seven teeth, while for evolute teeth the lowest in series is 14 teeth, and in no case fewer than 11. The loss from tooth friction is a minimum with this form, and the wear less injurious to the shape of the tooth. The minor objections which have been raised are that the double curve increases the difficulty of construction, and that any variation of the distance between centre causes imperfect action to follow.

Evolute Teeth. The advantages of this form are that the simple shape is readily made and that any variation of the distance between centres does not affect the action.

Against these must be set the fact that for low numbered pinions the flanks must be altered to avoid interference, or the tops of the teeth must be taken off. The fact that the distance between centres may vary is rather an objection in many cases, as the arc of action is reduced, and in transmission of heavy power the shocks upon the teeth are liable to be increased.

Evolute teeth are well suited for interchangeable gears, if low numbered pinions are not required (30 teeth being the minimum), and where but small power is to be transmitted they are excellently adapted. For wheels which run only in pairs, and hence for bevel gears, this form is excellent, since it is so readily made. See § 222.

Pin tooth gearing and the mixed outlines are only used for special work, such as in hoisting machinery and the like, and in such cases the wheels are often made of wrought iron or steel.

Disc wheels have a very limited application, but in some special forms of mechanism they are very useful, and will be discussed further. See Chapter XVIII.

B. CONICAL GEAR WHEELS.

§ 215.

General Considerations.

In the case of conical gear wheels, or as they are generally termed, Bevel Gears, the working circles of a pair of gears which run together, lie on the surfaces of a pair of cones, the apex of each cone being at the intersection of the axes of rotation. In such case the pitch circles are taken at the p base circles of the respective cones, as SD, and SE, Fig. 596. The length of the teeth is measured on the supplementary cone, to each base cone, SB being the supplementary cone for SD, and SC that for SE, BC being at right angles to AS. The length of teeth is laid off on SB and SC, and the width of face on SA; the tooth thickness being spaced off on the pitch circle and all the teeth converging to the point A.

The respective radii SD and SE of the two cones are found by dividing the angle a of the axes, in such a manner that the perpendiculars SD and SE let fall from S to the axes, bear the same ratio to each other as do the numbers of teeth, or inversely as the number of revolutions; thus $SD : SE = Z : Z_1 = n_1 : n$. There are, therefore, two solutions possible, according as the pitch line SA is taken within the angle a, or in its supplement; or what is the same thing, according to which angle is taken as the angle of the axes. The difference between the two consists in the fact that for a constant direction of revolution of the driving shaft the driven gear revolves in one direction for the first solution and in the opposite direction for the second solution. One of the solutions gives an interual gear, when $n_1 : n < \cos a$.

If bevel gears are required to interchange (see § 200) they must not only be of the same pitch, but must also have the same length of contact line, AS, Fig. 596. Since these conditions are very infrequent, it follows that bevel gears are generally only made to work in pairs. In practice it is found that a variation of less than 5 per cent. in the length of the contact line may be neglected. Gears of the same pitch and same angle of

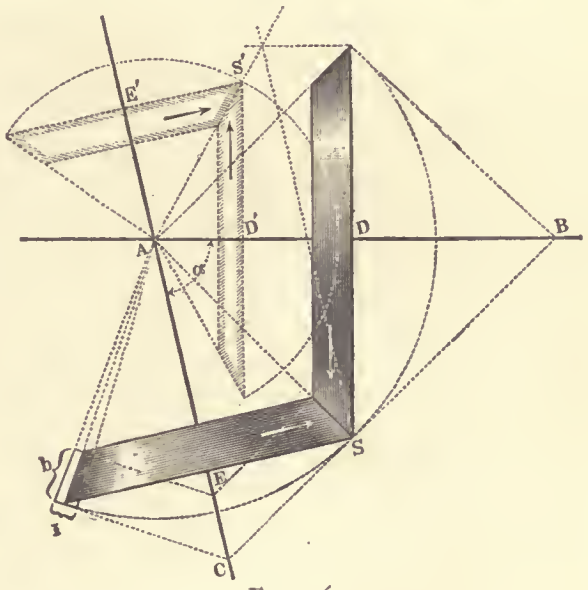

FIG. 596.

axes, but with a small variation of contact line, are called "bastard gears." A pair of right angled bevel gears of 80 and 45 teeth, might be altered in practice, if required, into bastard gears of 80 (1 ± 0.05), *i. e.*, 84 to 76 teeth, which would work with the other gear of 45 teeth.

§ 216.

Construction Circles for Bevel Gears.

The geometrical figures which are formed by one cone rolling upon another, require that both cones should have a common apex. The surface thus developed is called a spherical cycloid. Of these there are five particular forms, as with the plane cycloids, the latter being really those for a cone with an apex angle of 180°. The spherical cycloid is very similar in form to the plane cycloid, as are also the corresponding evolutes; the branches of the curves assuming a zig-zag form.*

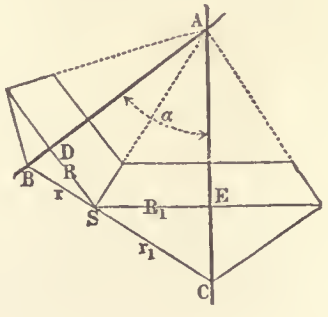

FIG. 597.

The use of the spherical cycloid for the formation of bevel gear teeth would involve many difficulties. In order to construct such teeth, it is therefore common to use the method (first devised by Tredgold) of auxiliary circles, based upon the supplementary cones, and enabling the teeth to be laid out in a similar manner to those of spur gears. The auxiliary circles for the bevel gears R and R_1, Fig. 597, are those of the spur gears having the same pitch, their radii being respectively r and r_1, the elements BS and CS of the supplementary cones.

For any given angle a between the axes, the radius r, and number of teeth 3, for the auxiliary circle can be determined

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*See Berliner Verhandlung. 1876, pp. 321, 449, Reuleaux, Development of the Spherical Cycloid.

from the radii R and R_1, and tooth numbers Z and Z_1, by the following formula:

$$\left.\begin{aligned} \frac{r}{R} &= \frac{\sqrt{R^2 + R_1^2 + 2\,R\,R_1 \cos a}}{R_1 + R\cos a}, \\ \frac{z}{Z} &= \frac{\sqrt{Z^2 + Z_1^2 + 2\,Z\,Z_1 \cos a}}{Z_1 + Z\cos a} \end{aligned}\right\} \quad \ldots \quad (192)$$

If the axes are at right angles, we have

$$\left.\begin{aligned} \frac{r}{R} &= \frac{\sqrt{R^2 + R_1^2}}{R_1}, \quad \frac{z}{Z} = \frac{\sqrt{Z^2 + Z_1^2}}{Z_1} \\ \frac{r}{r_1} &= \left(\frac{n_1}{n}\right)^2 \end{aligned}\right\}$$

Example.—A pair of bevel gears have 30 and 50 teeth, and an angle between axes $a = 60°$, hence $\cos a = \frac{1}{2}$, and we have for the auxiliary circle of the 30 tooth gear: $z = 30\,\frac{\sqrt{30^2 + 50^2 + 2 \cdot 30 \cdot 50 \cdot .05}}{50 + 30 \cdot 0.5} = 6\,\frac{\sqrt{4900}}{13} = 32.3$, say 32.

For the 50 tooth gear we have also: $z_1 = 50\,\frac{\sqrt{4900}}{30 + 50 \cdot 0.5} = 64$.

From these numbers and the given pitch, the auxiliary circles can be laid off and the teeth drawn.

Low tooth numbers are not available for bevel gears, since the errors which are involved in the method of auxiliary circles become disproportionately great. By using not fewer than 24 teeth for the bevel gear, a minimum of 28 for the auxiliary circle is obtained, and the evolute system can be used to advantage. This form of tooth is best adapted for this purpose, on account of its simplicity of form, notwithstanding the minor defects which have already been noticed.

The loss from tooth friction in bevel gears is approximately equal to that of their corresponding auxiliary gears.

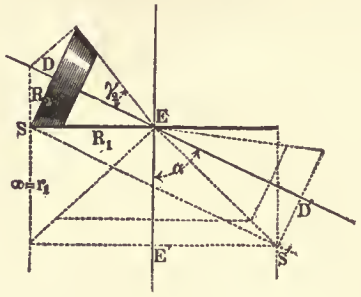


FIG. 598.

§ 217.

THE PLANE GEAR WHEEL.

Internally toothed bevel gears are not used, on account of the practical difficulties involved in their construction. There is, however, an interesting form of gear wheel which lies intermediate between the external and internal forms. If the numerical ratio between a pair of bevel gears is $= \cos a$, one of the solutions for the base cone gives for the latter a plane surface, $S\,E$, Fig. 598.

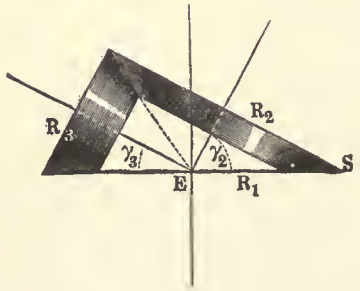


FIG. 599.

The supplementary cone in this case becomes a cylinder, and the radius of the construction circle becomes infinitely great, hence the tooth outlines are similar to those used for rack teeth. If the evolute system is used the teeth are very simple, and the plane gear in some cases becomes a very convenient form of construction.

As already stated, the ratio is

$$\frac{R_2}{R_1} = \cos a \quad \ldots \quad (193)$$

from which, if for example $a = 60°$, we have $\frac{R_2}{R_1} = \frac{1}{2}$. If the angular relation of the axes is given it follows that but one velocity ratio can be obtained. This is determined from the angle γ_2, which is one-half the apex angle of the cone R_2, and from the ratio $\frac{R_2}{R_1} = \sin \gamma_2$.

It is sometimes very convenient to arrange a plane gear so that it may work with both of a pair of bevel wheels. This is shown in Fig. 599, in which the gears R_2, R_3 have the semi-apex angles γ_2, γ_3, and have their axes at right angles. We then have:

$$\frac{R_2}{R_3} = \tan \gamma_2 = \cot \gamma_3,$$

from which we obtain the following values:

$\frac{R_2}{R_3} = \tan \gamma_2 =$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{4}{3}$	2	3	4
$\gamma_2 =$	14°	18°30′	26°40′	36°50′	45°	53°10′	63°20′	71°30	76°
$\frac{R_2}{R_1} = \sin \gamma_2 =$	0.242	0.317	0.449	0.600	0.707	0.800	0.894	0.948	0.970

Either of the wheels R_2, R_3, can be used with the plane gear R_1 if the number of teeth have the ratio given by the value of $\sin \gamma_2$. Although this limits its application, yet the plane gear is frequently found very useful for angular transmissions.*

C. HYPERBOLOIDAL GEAR WHEELS.

§ 218.

BASE FIGURES FOR HYPERBOLOIDAL WHEELS.

Hyperboloidal wheels are used to transmit motion between inclined, non-intersecting axes. The figures upon which they are based are hyperboloids of revolution having a common generatrix. These may be determined in the following manner.

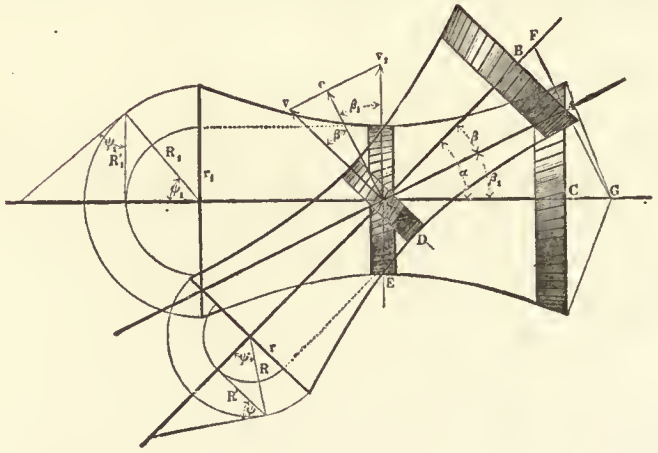
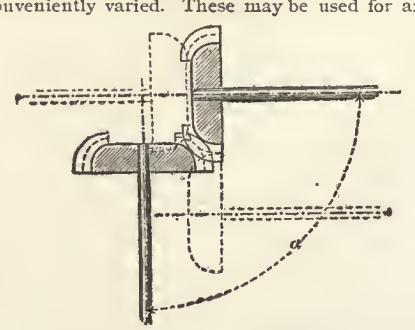


FIG. 600.

In Fig. 600 is given a projection normal to the line of shortest distance between the two axes. The angle a is divided into two parts β and β_1, in such a manner that the perpendiculars let fall from any point A, of the line $S\,A$, upon the two axes, shall be inversely proportional to the revolutions of the gears. $S\,A$ is then the contact line of the hyperboloids; $A\,B = R'$ and $A\,C$

*The so called "Universal Gears" of Prof. Beylich, introduced in 1866, should be considered as a variety of conical gears in which the angle of the axes may be conveniently varied. These may be used for axes of angles



varying from 0° to 180°. As shown in the illustration, these wheels are formed of globoids of the III Class (see § 224), the meridians forming the teeth and spaces. They have found but limited application. A model of these gears is in the kinematic cabinet of the Royal Technical High School.

$= R'_1$, are projections of the radii of the hyperboloids intersecting at A. We have

$$\frac{R'}{R_1'} = \frac{\sin\beta}{\sin\beta_1} = \frac{n_1}{n} = \frac{Z}{Z_1} \quad \ldots\ldots\ldots \quad (194)$$

The actual radii R and R_1 are yet to be determined, as well as the radii $S\,D = r$, and $S\,E = r_1$ of the gorge circles.

For the latter we have:

$$\frac{r}{r_1} = \frac{\tan\beta}{\tan\beta_1} = \frac{\frac{n_1}{n} + \cos\alpha}{\frac{n}{n_1} + \cos\alpha} \quad \ldots\ldots \quad (195)$$

that is, r and r_1 have the same relation to each other as the portions $A\,F$ and $A\,G$ of a perpendicular to the line of contact. If we call the shortest perpendicular distance between the axes $= a$, we have:

$$\left.\begin{aligned} \frac{r}{a} &= \frac{1 + \frac{n}{n_1}\cos\alpha}{1 + 2\,\frac{n}{n_1}\cos\alpha + \left(\frac{n}{n_1}\right)^2} \\ \frac{r_1}{a} &= \frac{1 + \frac{n_1}{n}\cos\alpha}{1 + 2\,\frac{n_1}{n}\cos\alpha + \left(\frac{n_1}{n}\right)^2} \end{aligned}\right\} \quad \ldots\ldots \quad (196)$$

The radii R and R_1 are hypotenuses for the triangles whose sides are R' and r, R_1' and r_1 (see the left of the figure) or:

$$\left.\begin{aligned} R &= \sqrt{R'^2 + r^2} \\ R_1 &= \sqrt{R'^2_1 + r_1^2} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (197)$$

R' and R_1' being determined as above, when the distance $S\,A = l$ is given. For the angles β and β_1 we have the general expressions:

$$\left.\begin{aligned} \tan\beta &= \frac{\sin\alpha}{\frac{n}{n_1} + \cos\alpha} \\ \tan\beta_1 &= \frac{\sin\alpha}{\frac{n_1}{n} + \cos\alpha} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (198)$$

As in the case of bevel gears, two solutions are possible according as the angle α, or its supplement, is taken in determining the line of contact $S\,A$, Fig. 601. The choice of solution

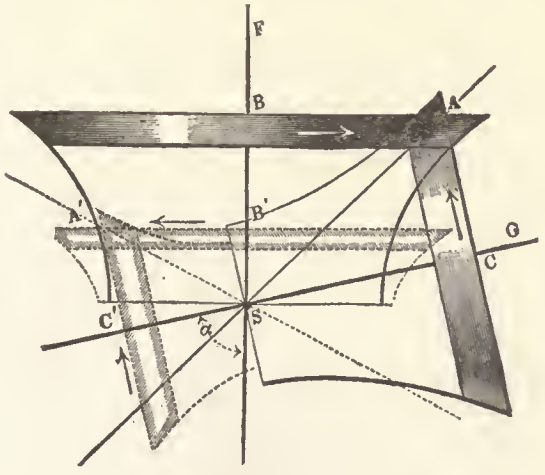


FIG. 601.

governs the direction of rotation of the driven gear, and one of the solutions renders it practicable to make an internal gear; although this construction has been little used, and has but little practical value.

If the angle of the axes $\alpha = 90°$ we have

$$\frac{r}{r_1} = \tan^2\beta = \left(\frac{n_1}{n}\right)^2 \quad \ldots\ldots\ldots \quad (199)$$

also:

$$\left.\begin{aligned} \frac{r}{a} &= \frac{n_1^2}{n^2 + n_1^2}, \\ \frac{r_1}{a} &= \frac{n^2}{n^2 + n_1^2}, \\ \tan\beta &= \frac{n_1}{n}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (200)$$

In the construction of the wheels, corresponding zones are chosen on the two hyperboloids. If the distance between the axes is small, the zones lying in the gorge circles are generally unsuitable, but when the distance is greater they may be used and the figures approximated by truncated cones.

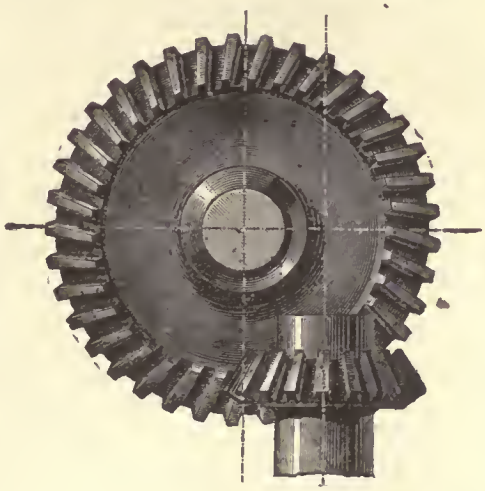


FIG. 602.

Example 1. $\alpha = 40°$, $\frac{n_1}{n} = \frac{1}{2}$, (see Example 1, in § 221), $a = 4''$.

We have $\frac{R'}{R_1'} = \frac{1}{2}$,

$$\frac{r}{r_1} = \frac{0.5 + \cos 40°}{2 + \cos 40°} = \frac{1.266}{2.766} = 0.4577,$$

$$\frac{a}{r} = \frac{1 + 2\cos 40°}{1 + 2 \times 2\cos 40'' + 4} = \frac{2.532}{8.064} = 0.31398,$$

$r = 0.31398 \times 4 = 1.256''$,

$r_1 = 4 - 1.256 = 2.744''$.

Also $\tan\beta = \frac{\sin 40°}{2 + \cos 40°} = \frac{0.6428}{2.766} = 0.232393 = \tan 13°\,5'$, and

$\beta_1 = 40° - \beta = 26°\,55'$.

If we take $S\,A = l = 8''$ we have $R' = l \sin 13°\,5' = 8 \times 0.226368 = 1.81''$,

$R_1' = 8 \sin 26°\,55' = 8 \times 0.452634 = 3.62''$; finally

$R = \sqrt{(1.81)^2 + (1.256)^2} = 2.2''$ and

$R_1 = \sqrt{(3.62)^2 + (2.744)^2} = 4.54''$.

Example 2. $\alpha = 90°$, $\frac{n_1}{n} = \frac{5}{9}$ or say the number of teeth $Z = 36$, and $Z_1 = 20$; $a = 0.75''$. We have from (197):

$$\frac{r}{r_1} = \left(\frac{9}{5}\right)^2 = \frac{81}{25} = 3.24, \text{ and from (200)}$$

$$r = \frac{a \times 9^2}{5^2 + 9^2} = \frac{0\,75 \times 81}{106} = 0.573'',$$

$$\text{and } r_1 = \frac{0.573}{3.24} = 0.177''.$$

For β, we have $\tan\beta = \frac{n_1}{n} = 1.8$, hence $\beta = 60°\,57'$, and $\beta_1 = 29°\,3'$.

If we make $R = 2''$, we have from (197):

$$R' = \sqrt{R^2 - r^2} = \sqrt{2^2 - 0.573^2} = 1.916'',$$

and hence R_1', according to (194) is $= \frac{3}{2} R_1' = 1.063''$, hence

$$R_1 = \sqrt{1.063^2 + 0.177^2} = 1.078''.$$

The appearance of such a pair of gears is shown in Fig. 602. According to the table in § 202 the pitch for the larger gear is: $t = \frac{R}{5.73} = \frac{2}{5.73} = 0.35''$, and for the smaller gear $t_1 = \frac{1.078}{3.18} = 0.339''$.

Example 3. $\alpha = 90°$, $\frac{n_1}{n} = 1$, $\beta = 45°$, $r = r_1$, $R = R_1$. In this case the hyperboloids become similar (see Example 4, § 221.)

Example 4. In the special case in which $\frac{n_1}{n} = \cos\alpha$, and the position of the contact line, which is determined by β, lies in the supplement to α, so that $\frac{n_1}{n} = \cos\alpha$, the base figures become, the one a normal cone and the other a plane hyperboloid, see Fig. 603. This construction is similar to the preceding forms of plane and bevel gears, and may be conveniently used to work with a train of common bevel gears, although but few practical applications occur, partially owing to the fact that the prolonged axis of the bevel

gear passes through the plane gear. For $\alpha = 60°$, $\frac{n_1}{n} = -\frac{1}{2} = -\cos 60°$ we obtain the plane gear. We have $\tan \beta = \frac{1}{3}\sqrt{3}$, $\beta = 30°$, $\tan \beta_1 = \infty$, $\beta_1 = 90°$. Also $\frac{R'}{R_1'} = \frac{\sin 30°}{\sin 90°} = 0.5$; $r = o$, $r_1 = a$, $R = R'$, $R_1 = \sqrt{R'^2 + a^2} = \sqrt{4R^2 + a^2}$.

If $\frac{n_1}{n}$ be negative and less than $\cos \alpha$ we obtain an hyperboloidal internal gear.

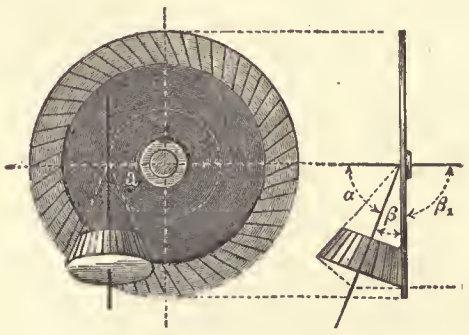


FIG. 603.

Rack teeth may also be constructed to work with hyperboloidal gears. In this case the teeth of the rack are inclined while the pinion becomes an ordinary cylindrical spur gear, since in order to satisfy equation (195) with $r_1 = \infty$, the angle $\beta = o$, and $\beta_1 = \alpha$, see Fig. 604. Applications of this construction may be found in various machine tools.

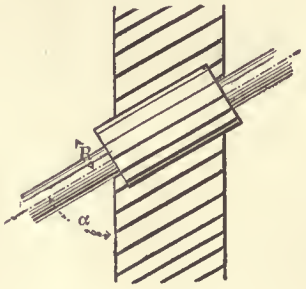


FIG. 604.

§ 219.

TEETH FOR HYPERBOLOIDAL GEARS.

The construction of the exact forms for the teeth of hyperboloidal gears is a very difficult operation, and in practice an approximation is used similar to that employed for bevel gears. The method adopted is to determine the supplementary cone to the hyperboloid used, and as in the case of bevel gears, use the corresponding construction circle.

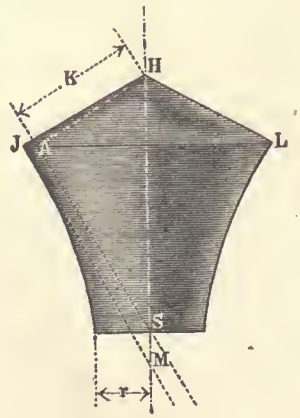


FIG. 605.

The apex H (Fig. 605) is determined by drawing $A\,H$ perpendicular to the generatrix $S\,A$, which, as before, is taken parallel to the plane of the drawing. The teeth will be formed with sufficient accuracy if two construction hyperboloids are taken with the same angle of contact as the base hyperboloids, according to the conditions in (198) and (199), and the teeth are formed on the surfaces, which are described by the edges of the construction hyperboloids upon the base hyperboloids.*

* See Herrmann's Weisbach's Mechanics, II. ed., III, 1, p. 418 *et seq.*

If it is desired to approximate to the hyperboloidal zone by the use of a conical surface, the apex must be determined. In this case the generatrix $S\,A$ is rotated about the axis $H\,S$ until A falls on the poiut J of the circumference, when the new projection of the generatrix will pass through the apex M of the cone.

The tooth friction of hyperboloidal gears is necessarily great. This will be considered later, in connection with the speed of the rubbing surfaces, which is similar to that of the spiral gears which are tangent at the gorge circles (see § 220.)

D. SPIRAL GEARS.

§ 220.

CYLINDRICAL SPIRAL GEARS.

Cylindrical spiral gears may be used in the same manner as hyperboloidal gears for the transmission of motion between inclined axes, and in some cases possess advantages over the latter. There are a number of useful variations of spiral gears.

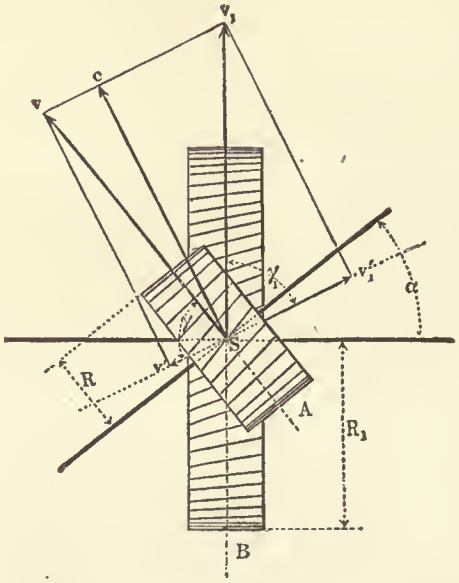


FIG. 606.

In Fig. 606 is shown a pair of wheels, A and B, both with left hand spirals and corresponding tooth profiles. The pitch angles γ and γ_1 are so chosen that at the point of contact the pitch cylinders have a common tangent, so that if a be the angle of inclination of the axes, $\gamma + \gamma_1 + a = 180°$. If we indicate by v and v_1 the circumferential velocity in the direction of the tangent and normal respectively, we have:

$$\frac{v_1}{v} = \frac{\sin \gamma}{\sin \gamma_1} = \text{whence } \frac{n_1}{n} = \frac{R \sin \gamma}{R_1 \sin \gamma_1} = \frac{Z}{Z_1} \quad . \quad . \quad . \quad (201)$$

The normal pitches, $\bar{t} = t \sin \gamma$, and $\bar{t}_1 = t_1 \sin \gamma_1$ must be equal to each other, whence $\frac{t}{t_1} = \frac{\sin \gamma_1}{\sin \gamma}$.

As indicated by the components of velocity v' and v_1', there is an end long sliding action of the teeth upon each other, with a velocity:

$$c' = v' + v_1' = c\,(\cot \gamma + \cot \gamma_1) \quad . \quad . \quad . \quad . \quad . \quad (202)$$

This sliding consumes power and causes wear, and will be at a minimum when v' and v_1' are equally great, that is when $\gamma = \gamma_1$.

With regard to the choice of γ and γ_1 the conditions may be so taken that the position of the coinciding tangents of the two spirals shall be slightly before or slightly after the actual line of contact, but as close as may be possible. This is similar to the position of the line of contact of hyperboloidal gears (§ 218) and may be stated as follows:

$$\frac{R}{R_1} = \frac{\cot \gamma}{\cot \gamma_1} = \frac{\frac{n_1}{n} + \cos a}{\frac{n}{n_1} + \cos a} \quad . \quad . \quad . \quad . \quad (203)$$

as also

$$\cot \gamma = \frac{\sin a}{\frac{n}{n_1} + \cos a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (204)$$

For $a = 90°$ we have $\cot \gamma = \frac{n_1}{n}$. Such spiral wheels, when the teeth are well made, transmit motion very smoothly, but the surface of working contact is very small. When the axes are at right angles and the wheels the same size, it is often inconvenient to use spiral gears on account of the large size required.

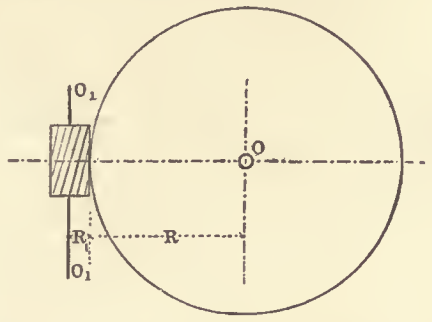


FIG. 607.

Example. Fig. 607. Let $\frac{n_1}{n} = 3$, and $a = 90°$. We have from (203) $\frac{R}{R_1} = \left(\frac{n_1}{n}\right)^2 = 9$ and from (204) $\cot \gamma = \frac{n_1}{n} = 3$, whence $\gamma = 18° 26'$, and $\gamma_1 = 71° 34'$. The sliding velocity is $c' = c\,(3 + 0.333) = 3\frac{1}{3}\,c$. The small value of the angle γ makes it undesirable to use the smaller gear as the driver. These objectionable features are of increasing importance and for example, $\frac{n_1}{n} = 5$, and $\frac{n_1}{n} = 10$, we get $\frac{R}{R_1} = 25$, and 100, and γ about $11\frac{1}{2}°$ and $5\frac{3}{4}°$. The difficulty of cutting the teeth on the lathe also increases, as may readily be seen.

§ 221.

APPROXIMATELY CYLINDRICAL SPIRAL GEARS.

If, of the preceding conditions, only those of formulæ (201) and (203) are strictly observed, the difficulties of construction are much reduced and at the same time satisfactory wheels obtained.

Three methods may be employed: (*a*) a slight modification from the correct spiral form may be given to both wheels, (*b*) one gear may be made a true spiral, and the variation all thrown

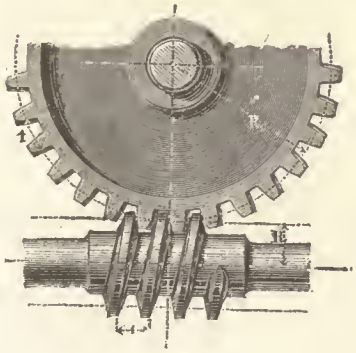


FIG. 608.

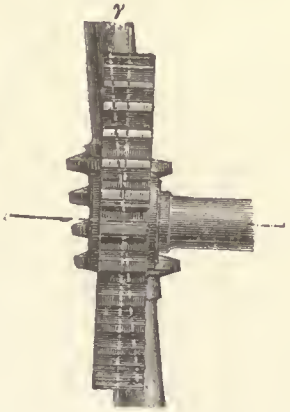


FIG. 609.

into the other gear, or (*c*) the wear which is at first caused by running the approximate forms together may be disregarded until the parts have worn themselves into smooth action. From these reasons a widely varying practice in the construction of spiral gears will be found. One of the most important applications is that of the worm and worm wheel, Fig. 608. In this case $a = 90°$ and $Z = 1$, the teeth of the wheel R_1 being inclined at an angle γ, with the edge of the wheel, whence $\tan \gamma = \frac{t}{2\pi R}$ $= 0.15916\,\frac{t}{R}$. In the arrangement shown in Fig. 609, we have $a = 90 - \gamma$ and the teeth on R_1 are made parallel to the axis. The pitch of the screw is here made $= \frac{t_1}{\cos \gamma}$ for a pitch t_1 of the wheel. The velocity ratio of transmission, according to the fundamental formula (186) is $n_1 : n = Z : Z_1$, or this case it equals $\frac{1}{Z_1}$.*

* In the illustration $Z_1 = 30$, which in (203) for a true spiral would require $R_1 = 900\,R$, and $\gamma = 88.1°$.

In many cases the worm is made a true spiral and the consequent wear disregarded, but in more careful work the method (*b*) is adopted and the worm wheel cut with a hob, which makes the proper modification in the shape of the teeth.

The friction between the worm and teeth of the worm wheel is very great, as the thread slides entirely across the teeth. We have for the coefficient of friction f, for the ratio between the actual force P' and a force P acting at the same lever arm on the screw, but free from frictional resistance, approximately:

$$\frac{P'}{P} = \frac{\frac{1 + f \,.\, 2\pi R}{t}}{\frac{1 - f\,t}{2\pi R}}$$

For $f = 0.16$ we have practically

$$\frac{P'}{P} = 1 + \frac{R}{t} \quad \ldots\ldots\ldots \quad (205)$$

It follows that to obtain the minimum of frictional loss, $\frac{R}{t}$ must be made as small as practicable.

Morin gives the rule $R = 3\,t$, which makes $\frac{P'}{P} = 4$; Redtenbacher makes $R = 1.6\,t$, whence $\frac{P'}{P} = 2.6$. If we make $R = t$, we get $\frac{P'}{P} = 2$, and this is as low as $\frac{R}{t}$ can well be made. In this case it will be seen that a higher efficiency than 50 per cent. cannot be obtained, and it is also apparent that the worm must be the driver, since the resistance of friction would just balance the reverse driving action. The ordinary tooth friction and the journal friction must of course be added.

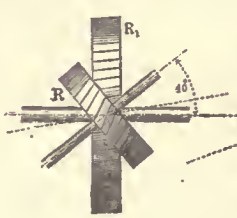


FIG. 610.

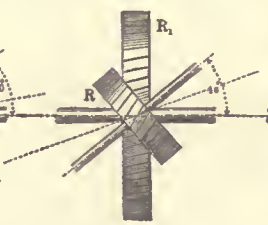


FIG. 611.

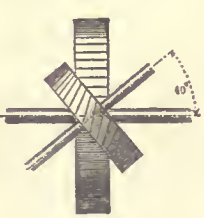


FIG. 612.

The tooth outlines for both worm and wheel are the same as for a rack and gear wheel, taken on a longitudinal section through the axis of the worm. The evolute tooth is especially applicable, and Z_1 must not be less than 28 (§ 209). The surface of contact is theoretically only a mathematical point, but in practice there is a small flattened surface of contact, and if a larger surface is desired the wheel must be cut with a hob of the same form as the worm which is to work with it.

Wheels which have a contact bearing of a point only, may be called precision-gears, as distinguished from power-transmitting gears. The difference, however, cannot be sharply maintained, for as already shown, worm gearing is used for the transmission of both large and small forces.

The possible variations of the pitch angle permit a great variety of spiral gear combinations, as the following examples show:

Example 1. Given $\frac{n_1}{n} = \frac{1}{2}$, the perpendicular distance between axes $a = R + R_1$, and the angle between axes $a = 40°$. If we make $\gamma = 60°$, we have from (§ 220) $\gamma_1 = 180 - 40 - 60 = 80°$ (see Fig. 610), and from (201) $\frac{R}{R_1} = \frac{\sin \gamma_1\, n_1}{\sin \gamma\, n}$ $= \frac{1}{2} \cdot \frac{\sin 80°}{\sin 69°} = \frac{0.5 \times 0.9848}{0.8660} = 0.5686$, from which R and R_1 may be readily determined. If we make $a = 4''$ we have

$$R_1 = \frac{a}{1 + \frac{R}{R_1}} = \frac{4}{1.5686} = 2.55''$$

and $R = 1.45''$. For $Z = 20$, $Z_1 = 40$, the normal pitch $\tau = t \sin \gamma = \frac{2\pi R \sin \gamma}{Z}$ $= \frac{2 \times \pi \times 1.45 \times 0.866}{20} = 0.272 \times 1.45 = 0.394''$.

The circumferential pitch $t = \frac{\tau}{\sin \gamma} = \frac{0.394}{0.866} = 0.454''$, $t_1 = \frac{0.394}{0.9848} = 0.400''$.

The sliding velocity c', according to (202) $= c\,(\cot 60° + \cot 80°) = c\,(0.5774 + 0.1763) = 0.7537$.

Example 2. In order to make c' a minimum, we may make $\gamma = \gamma_1 = \frac{180 - a}{2} = \frac{180 - 40}{2} = 70°$, see Fig. 611. We then have $\frac{R}{R_1} = \frac{1}{2}$, $R_1 = 2.666$, $R = 1.333$, $\tau = \frac{2 \times \pi \times 1.333 \times 0.9397}{20} = 0.394''$, $t = t_1 = \frac{0.394}{0.9397} = 0.419$, and $c' = 2 \cot 70° \times c = 0.728\,c$. It will be seen that the value of c' in Example 1 approached very closely to the minimum.

Example 3. If so desired we may make $\gamma = 90°$, when one wheel will become an ordinary spur gear, Fig. 612, and we have $\gamma_1 = 180 - 40 - 90 = 50°$. $\frac{R}{R_1} = 0.5 \times 0.7660 = 0.383$, $R_1 = 2.89''$, $R = 1.11''$, $\tau = 0.348'$, $t = \tau$, $t_1 = 0.454''$, $c' = 0.8391\ c$.

If instead of a, the normal pitch τ is given, as is generally the case with hobbed worm wheels, we choose γ and γ_1 and then have $R \sin \gamma = \frac{Z\tau}{2\pi}$, whence:

$$R = \frac{Z\tau}{2\pi \sin \gamma},\quad R_1 = \frac{Z_1 \tau}{2\pi \sin \gamma_1} \quad \ldots\ldots\ldots (206)$$

Both R and τ may be given, when γ must be determined, and we have:

$$\sin \gamma = \frac{Z\tau}{2\pi R} \quad \ldots\ldots\ldots\ldots (207)$$

FIG. 613.

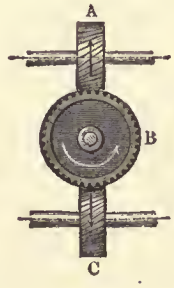


FIG. 614.

FIG. 615.

The following examples illustrate a variety of cases:

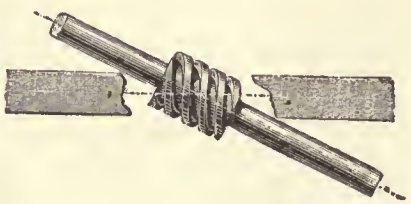
Example 4. $\alpha = 90°$, $Z = Z_1$. The sliding to be a minimum, hence $\gamma = \gamma_1 = \frac{180 - 90}{2} = 45°$. The two wheels are similar, both being left hand or as in Fig. 613, both right hand. The sliding velocity is $c' = 2 \operatorname{col} 45° \times c = 2c$.

Example 5. In the arrangement shown in Fig. 614 there is added to the right angled pair A B, a third wheel C, also right angled, when the wheels A and C will revolve in opposite directions. The middle gear B reverses the motion, as in the case of bevel gears.

Example 6. When $\alpha = 0$, the axes are parallel and a pair of spur gears with spiral teeth is obtained, this form being called Hooke's or White's gearing, Fig. 615. γ and γ_1 together include 180°, and one gear is left, and the other right hand. In this case the teeth are formed in true spirals. In this case the sliding velocity $c' = o$. For the wear on this form of gear see § 222. When $\alpha = o$ and $\gamma = o$ the wheels become spur gears.

FIG. 616.

FIG. 617.



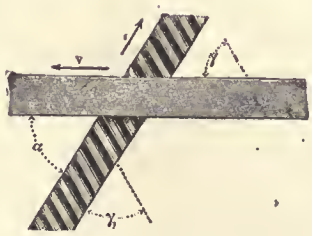


FIG. 618.

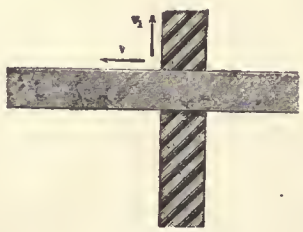


FIG. 619.

If the other limit of spiral gears is reached some noteworthy forms are obtained.

Example 7. $\alpha = 90°$, $\gamma = 10°$, $\gamma_1 = 80°$, $R_1 = \infty$. This gives a rack and screw, Fig. 616. If $\gamma_1 = 90°$ and the teeth normal, $\gamma = 10°$ and $\alpha = 80°$, and the teeth of the rack correspond to the section of a nut. In the Sellers planing machine the rack teeth are placed at such an angle that the lateral pressure just balances the opposing tooth friction.

Example 8. $R = R_1 = \infty$. This gives two racks, sliding in each other, Fig. 618. We have, as before, $v_1 : v = \sin \gamma : \sin \gamma_1$. If $\alpha = 90°$, as in Fig. 619, and $\gamma = \gamma_1 = 45°$, we have $v = v_1$. This construction is used in some forms of boring machinery for cannon, and in screw cutting machines.

Example 9. $\alpha = 90°$, $\gamma_1 = 90°$, also $\gamma = o$, both radii of indefinite magnitude. Fig. 620. This is the so-called revolving rack, used on governors and similar apparatus in which endlong motion is to be transmitted from a revolving piece. The velocity ratio of A to $B = o$.

Example 10. The worm, or endless screw, as already stated, is a form of spiral gear wheel. These are two special forms of worm gear which although seldom used, are of interest. There are the forms of internal gearing shown in Figs. 621 and 622. In the former the worm wheel is the internal gear, while the latter shows an internal worm, with external or spur worm-wheel.

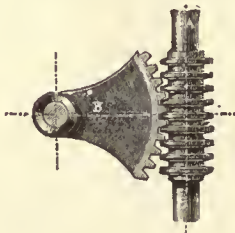


FIG. 620.

FIG. 621.

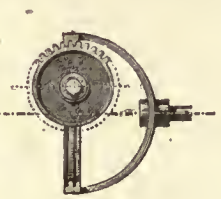


FIG. 622.

§ 222.

SPIRAL GEAR TEETH AND THEIR FRICTION.

Spiral gears are cut in a similar manner to screws, the tool being carried in the slide rest of an engine lathe, and set at the proper angle. The pitch of the screw thread is:

$$s = 2\pi R \tan \gamma,$$

and the travel of the rest is effected by proper change gears, according to the selected values of γ and γ_1.*

The tooth outline to be used is determined according to the radius of curvature of the supplementary spiral, that is, to that at right angles to the spiral to be cut. The radii of curvature r and r_1 to be used are:

$$r = \frac{R}{\sin^2 \gamma},\ r_1 = \frac{R_1}{\sin^2 \gamma_1} \quad \ldots\ldots (208)$$

These give the radii for the construction circles to be used with the pitch τ; the shape of the tool with which the teeth are cut is then determined.

Example 1. For the wheels of the first example in the preceding section, we have:

$$r = \frac{1.45}{\sin^2 60°} = 1.93'',\ r_1 = \frac{2.55}{\sin^2 80°} = 2.58''.$$

If it is preferred to determine r, graphically from formula (208) the method given in § 29 may be employed.

The frictional resistance of spiral gearing is often a matter of much importance. If the frictional resistance is assumed to be zero, we have for the relation of the force P applied to the driving wheel, to the force Q delivered by the driven wheel:

$$\frac{P}{Q} = \frac{\sin \gamma}{\sin \gamma_1} \quad \ldots\ldots\ldots (209)$$

The ordinary tooth friction, which is the same as that of the construction gears (see § 213) to which must be added the friction due to the sliding of the teeth, whenever a is greater than zero. The value of the latter friction is governed by the sliding velocity c'. For the calculation of the loss of useful effect we may use the formula:

$$\frac{P'}{P} = \frac{\sin \gamma_1}{\sin \gamma} \cdot \frac{\sin (\gamma + \phi)}{\sin (\gamma_1 - \phi)} \quad \ldots\ldots (210)$$

in which ϕ is the angle of friction for the coefficient f, whence $\tan \phi = f$. For $f = 0.16$ we have $\phi = 9°$.

Example 2. For the wheels in the preceding example we have

$$\frac{P'}{P} = \frac{\sin 80° \sin 69°}{\sin 60° \sin 71°} = \frac{0.9848 \times 0.9336}{0.8660 \times 0.9455} = 1.12.$$

To this must be added the ordinary friction of the equivalent spur gears. Another source of loss is that due to the lateral forces K and K_1, acting in the direction of the axes. For these we have

$$\frac{K}{P'} = \cot (\gamma + \phi),\quad \frac{K_1}{Q} = \cot (\gamma_1 - \phi) \quad \ldots (211)$$

Example 3. For the preceding gears we have $K = P' \cot 69° = 0.3839\ P'$, $K_1 = Q \cot 71° = 0.3443\ Q$ from which values, in connection with the known dimensions of the journals the corresponding resistances can be determined.

When $\alpha = o$, that is, for parallel axes, the sliding action of the teeth is zero, and the value of P' in (210) is the same as P; hence spiral gears for parallel axes work without the tooth friction due to lateral sliding, the ordinary tooth friction alone remaining, as well as the forces K and K_1.

* Brocot's Tables will be found of service in arranging change gears. (Calcul des Rouages par Approximation. Paris, 1862).

The tooth friction may be reduced to a very small amount by reducing the bearing surface of the teeth of one gear to a point of contact, or practically to a knife edge. Such gears (devised by Hooke) are only of use for purposes of precision, but in some cases are found serviceable.*

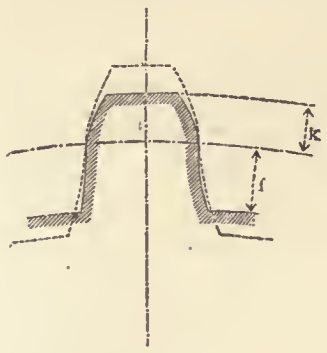


FIG. 623.

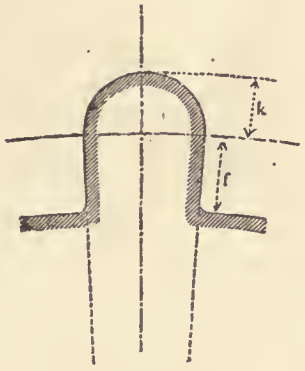


FIG. 624.

Instead of the edge bearing, a rounded surface may be used, with its highest part corresponding to the lineal bearing as already shown by Hooke and by Willis. The tooth outlines for both gears are determined as usual, and then oue or both profiles are redrawn withiu the original curves, Fig. 623, and the modified outlines used to form the tooth spiral; teeth so constructed running nearly free from friction. In such cases the length of flank f, and face k may be reduced as shown. Such forms are more properly to be considered as screw thread profiles than as gear teeth. Willis has shown that in both gears the flanks may be made radial and the crown of the teeth semicircular, Fig. 624. Since such teeth are weakest at the base, it is preferable to use a modified form of the evolute tooth, Fig. 625. This may be approximated to by using a circular arc of smaller radius than $B\,S = R \cos a$, the centre B' being taken on the normal $N\,N$, through the point of contact.

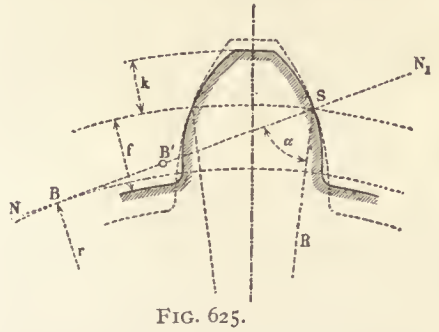


FIG. 625.

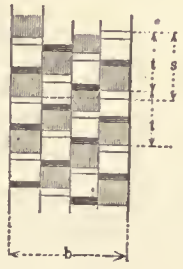


FIG. 626.

A similar form to the preceding gears is the so-called step-gearing, Fig. 626, frequently used in planing machines (by Shanks, Collier and others). The tooth profiles may be modified as above, to reduce friction, but the gradation s should be as great or greater than the pitch t. Fewer than four sections should not be used.

An objection to the use of spiral gears is the axial pressure K, this, however, can be eliminated by the use of double gears of opposite inclination. Such gears have been known for a long time (White, 1808) and for moderate service, have beeu frequently used, as iu spinning machinery, tower clocks, etc., and more recently they have been applied to heavy work, notably for rolling mill gearing, both in Germany and America.

The pinions used in rolling mill work are made with 9 to 16 teeth, with pitch diameters from 4″ to 24″ and over. Evolute teeth are used, with a base angle from 62 to 69°. The face length of the teeth is made about 0.22 t.

If the evolute curve is accurately made, the tooth contact is practically the same as with ordinary spur gears, and the surfaces of contact can readily be discerned, extending diagonally across the teeth. When such a surface of wear is visible, of course the teeth are not free from friction. Fig. 627 shows a cast steel pinion of ten teeth, for rolling mill service. This gear is cast in one piece with its shaft and coupling ends, although in many cases the shaft is made separately.

* These gears have been used in physical apparatus by Bréguet for speeds exceeding 2000, or according to Haton, as high as 8000 revolutions per second or 480,000 per minute.

The space s between teeth at the middle of the gear, is called in the Westphalian shops the "spring" of the teeth. If it is desired to approximate to the frictionless action of the teeth, this "spring" must be slightly greater than the pitch.

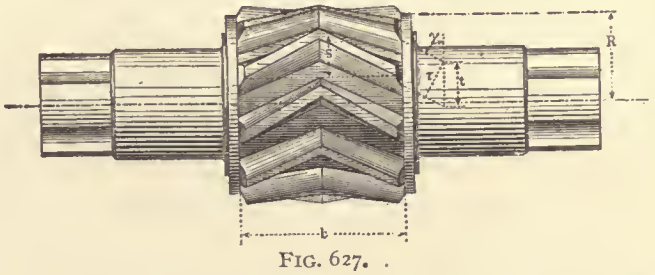


FIG. 627.

For very large transmissions the gears may be made in two parts. Fig. 628 shows a pair of such gears for a reversing rolling mill by the Hagener Steel Works. The pitch diameter is 43.3″, the pitch 8½″, the face of each gear 20″, and the total weight 24,200 pounds. The teeth are made with double reverse angles on each gear, so that the conditions are the same when running in either direction, and the whole is a masterpiece of machine work in steel.

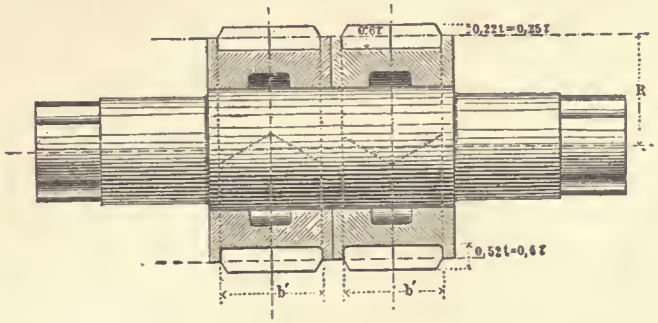


FIG. 628.

§ 223.

SPIRAL BEVEL GEARS.

Spirally formed teeth are sometimes used on bevel gears, and in this case the distance a, between the axes becomes zero, while the angle α remains to be given. For the curvature of the teeth it is best to use a conical spiral of constant pitch, the projection of which on the base of the cone is an Archimedean spiral. Frequent applications of such wheels are to be found in spinning machinery, and they are operated successfully at quite high velocities.*

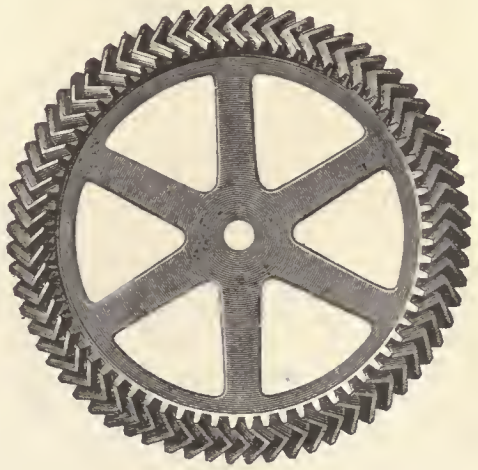


FIG. 629.

The same varieties may be made in bevel, as in spur gears, and in Fig. 629 is shown a reverse spiral bevel gear of cast iron, as made by Jackson & Co., at Manchester. Similar gears are made of cast steel by Asthöver & Co., at Annen in Westphalia. Stepped teeth are also used in bevel gears, and in Fig. 630 is shown such a wheel by A. Piat fils, of Paris.

* For a machine for the correct construction of the teeth of spiral bevel gears, see Genie Industriel, Vol. XII, p. 255.

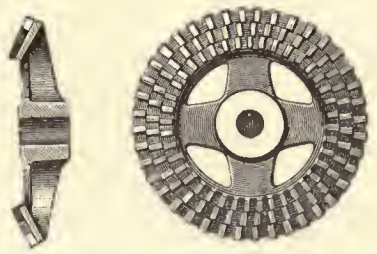


FIG. 630.

§ 224.

GLOBOID SPIRAL GEARS.

If a circle is revolved about an axis $A\,A_1$ coinciding with one of its diameters, and at the same time a radius $C\,S$ is moved about the centre C, with an angular velocity proportioual to that of the circle itself, the circle will generate a sphere aud the point of the radius which is at the surface of the sphere will trace a form of spiral curve. This may be called a spherical spiral,* aud adjoining lines of the spiral on the same meridian are equidistant.

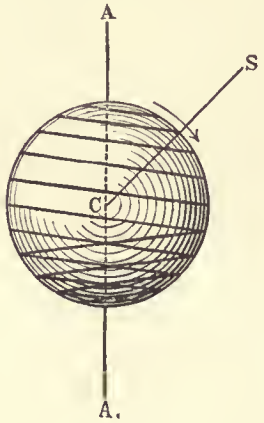


FIG. 631.

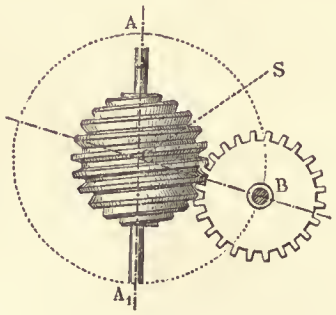


FIG. 632.

If the radius $C\,S$ passes the axis of rotation, the uew spiral will iutersect the one previously traced, as at A_1. Instead of a mere radial line, may be substituted a point which at the same time traces the outline of a tooth space, so that a spherical screw thread is generated with which a spur gear will engage at any point, Fig. 632. If the axes A and B are maintained in their proper positions, the spiral wheu driven, will operate the gear in the same manner as a worm and worm wheel, § 221.

The practical value of this especial form is extended by the fact that the axis of rotation need uot coincide with a diameter of the circle. Under these conditions there may occur a number of forms of bodies of revolutiou bearing an affinity to the sphere, and to which the writer has given the general uame of globoids. The corresponding spirals may be called globoid spirals and the resulting gears, globoid spiral gear wheels. Many of these may be made of practical use. (See Fig. 633.)

There are numerous forms of globoids according to the position which the describing circle holds to the principal axis. The axis about which the radius $C\,S$ turns is called the counter-axis. It stands at right angles to the starting position of the describing circle, and either intersects the principal axis, or is inclined to it without cutting it. We have then r, for the radius of the describing circle; a the shortest distance between the axes A and C, c the distance of the centre of the describing circle from the plane of the principal axis, δ the angle which the principal axis makes with the plane of the describing circle, exteuding from 0° to 90°. This gives four classes of globoids, as follows:

I. $a = o$, $c = o$.
II. $a = o$, c chosen at will.
III. a chosen at will, $c = o$.
IV. a and c chosen at will.

A right globoid is one in which $\delta = o$, and when δ is an acute angle we get an inclined globoid.

The first class is represented by the globoid Fig. 634, giving a symmetrical conical sectiou; if $\delta = o$ we obtain the previously described sphere.

* More properly a spherical cycloid, see § 216; its kinematic axoids are normal cones.

The second class gives the iuclined globoid, Fig. 635, with unsymmetrical couical sections, with regard to the equator, the spiral being on the zone mantles. If $\delta = o$ we obtain a symmetrical, cylindrical hollow section of a sphere, Fig. 636. The spiral, when $a = o$, becomes a spherical cycloid. If $\delta = 90°$ the figure becomes a plane cone, or plaue riug, and the curve becomes a plane cycloid.

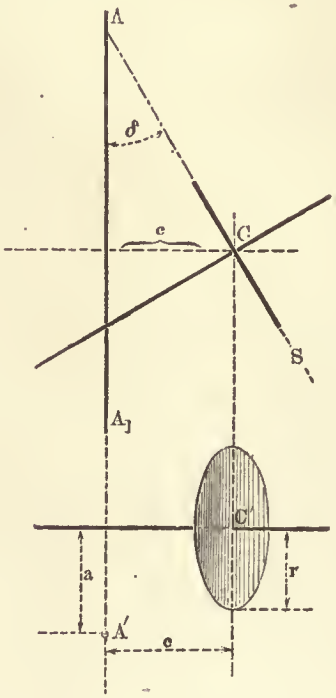


FIG. 633.

We have the third class when $\delta = o$, and $a > r$, giving a so-called cylindrical ring, or right globoid ring, Fig. 637 a, aud when $a < r$, the apple shaped globoid, Fig. 637 b.

If δ is an acute angle, the globoid is flattened, Fig. 638; the globoid of Class I is the limiting case. The spiral curves are globoidal cycloids, which become plane figures when $\delta = 90°$, and the globoid becomes a plane ring or plane cone.

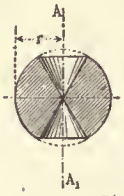


FIG. 634.

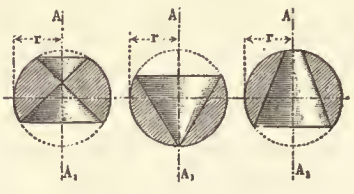


FIG. 635.

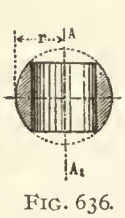


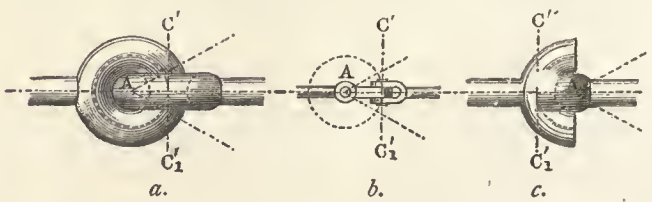
FIG. 636.

The fourth class gives the highest forms, Fig. 639, in which $\delta = o$, and we may have $a > r$, $a = r$, or $a < r$.

The inclined globoids of this class have forms, the limits of which are found in those of the second class, Fig. 635. If $\delta = 90°$ we have again the plane cone or plane ring.*

The practical applications of the globoid spiral gears are varied, and are found mainly in right globoids of classes III and

* Two right globoid rings may unite to form a pair of machine elements when the thickness of one is made equal to the hole in the other, as in Fig. *a*. The two parts then bear the relation to each other of journal and bearing, and are similar to a ball joint. Each of the two elements describes by the relative motion of any point a corresponding path on the other member.



These conditions are approximately found in a pair of chain links. Such a pair may also be considered as a contracted form of universal joint, *A B C*, Fig. *b*, the same relative motion existing between *A* and *C*. The same thing is shown in a fractional form in Fig. *c*, when some method of holding the parts together, such as bands, etc., must be used. This latter resembles closely the ball and socket joints of the human skeleton.

IV. In the valve gear of Stephenson's locomotives, Fig. 640, is found a globoid worm of class III, using the middle part of the globoid apple, Fig. 637 *b*, ($a < r$). In this case the reversing lever B is really a part of an internal gear with a radius $R_1 =$ the radius r of the describing circle.* In this case the internal gear has but a single tooth, although more might be used.

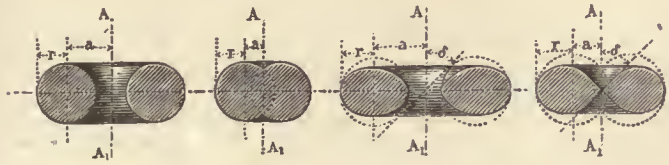


FIG. 637. FIG. 638.

It will be seen that the globoid forms can be used as internal gears. This is shown in Fig. 641, which represents a worm formed as a globoid screw. Its form is practically the same as that of the hole in the right globoid ring, Fig. 637 *a*. The section shown in the figure is of such length that it includes one-fourth of the entire circumference of the worm wheel B, although it could be extended so as to include almost one-half.

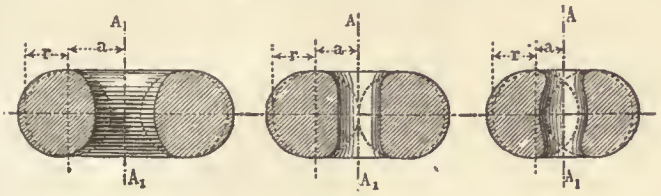


FIG. 639.

The most important point to be considered is the formation of the teeth. R_1 is again made equal to r. Since the globoid is used in the internal form, the two tooth profiles, on r and R_1, fall together. The sliding is in the plane of a normal section through B and $A\ A_1$ and not endlong, and hence the shape of the teeth is absolute.

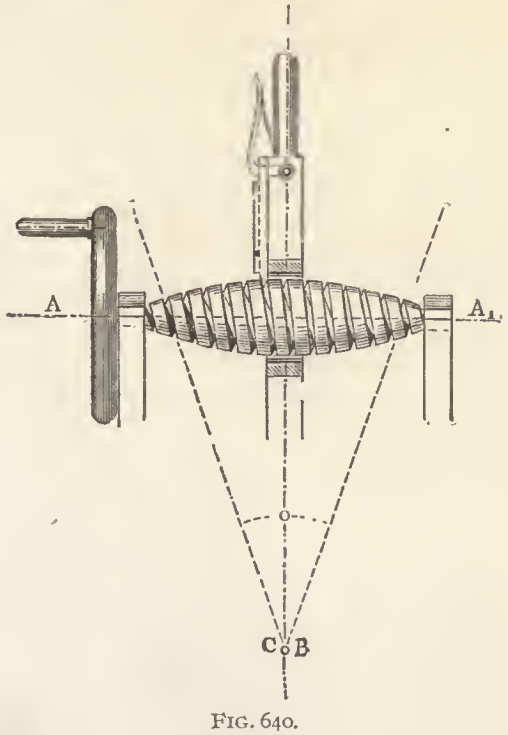


FIG. 640.

(Internal gear tooth, with $R = R_1$). The teeth can be made of straight profile in the worm wheel as well as in the worm.†

The production of the globoid worm in the lathe is not difficult. This form has been frequently used in recent work. The advantages appear to be in the simple form of tooth and in the completeness of the engagement.

* The worm and internal worm-wheel, Fig. 621, is another example of the preceding case.

† This form is described by Smeaton as used in a dividing engine by Hindley, see also Willis. Principles of Mechanism, 1st edition, 1851, p. 163.

An interesting modification is that of Hawkins, Fig. 642*. In this case the wheel B is composed of friction rollers of quite large size and the friction is thereby greatly reduced. Instead of there being only four teeth, as would at first appear, there is in reality an ideal number of teeth, a condition referred to in

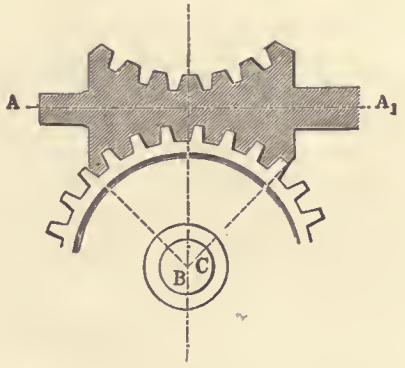


FIG. 641.

the fundamental discussion in § 200. If for every revolution of the globoid screw, one tooth of the wheel engages, there must for each space formed between the rollers be 10 teeth to a quarter revolution, so that instead of 4 teeth in B, there are $4\ (1 + 10) = 44$ teeth.

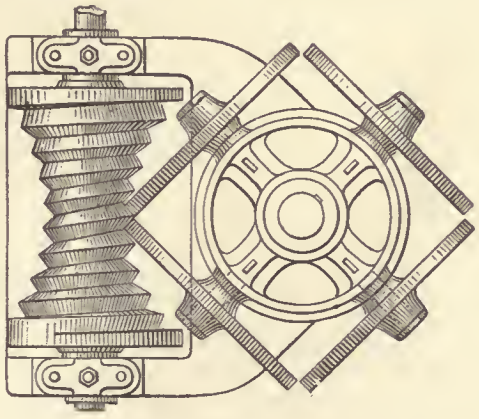


FIG. 642.

The gearing used in Jensen's Winch, Fig. 643, belongs to the globoid class IV, of the form shown in Fig. 639. Usually in this form $a = r$, although sometimes $a < r$, as in Fig. 639 *c*. R_1 is again made $= r$, and the internal globoid form used. The ratio is so chosen that a slow motion can be converted into a

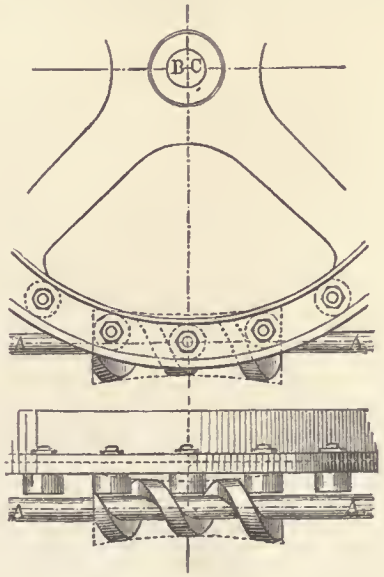


FIG. 643.

fast one, as may also be done with the form shown in Fig. 641 if the pitch of the worm is made sufficiently great. The use of rollers instead of teeth makes a very satisfactory construction.†

* Hawkins' Worm Gearing. Sci. Am. Supplement, No. 104, p. 1648.

† See Uhland's Prakt. Masch. Konstrukteur, also Engineer, Vol. 24, p. 493.

If in the first two classes of globoids the supplementary axis is removed an indefinite distauce, the globoids become plane surfaces, and the globoid screws thereby reach the limit. The limiting case of Class III is the ordinary worm aud worm wheel, and another form is Long's spiral gearing, which also belougs to Class III; a is chosen at will, $c = o$, $\delta = o$. The globoid becomes a plane cone and the globoid screw becomes au Archimedian spiral. If R becomes indefinitely great we obtain a disk with a spiral groove engaging with a rack, the middle section having full tooth contact from top to bottom.* When this is brought into Class IV, we obtain the Archimediau spiral iu its most general form, *i.e.*, the evolute of a circle.

E. CALCULATION OF PITCH AND FACE OF GEARING.

§ 225.

Pitch of Gear Wheels. Tooth Section.

The dimeusious of gear wheels must, for the same pressure on the teeth, be increased to meet shock in proportiou to the increase iu initial velocity. For slow runuing gears this action can be neglected. We may in this respect, therefore, divide gears into two classes, viz.:

Hoisting Gears aud Transmission Gears; and includes under the term hoisting gears all those having a linear velocity at the pitch circle of uot more than 100 feet per uiinute, aud under transmission gears all those running at a higher velocity.

For a pitch t, face b, length of teeth l, and base thickness of tooth h, we have for a tooth pressure P corresponding to a stress S, the general foruiula:

$$b\,t = 6\,\frac{P}{S}\left(\frac{l}{t}\right)\left(\frac{t}{h}\right)^2 \quad \ldots\ldots \quad (212)$$

and for the proportions of length and thickness already adopted we have:

$$b\,t = 16.8\,\frac{P}{S} \quad \ldots\ldots\ldots\ldots \quad (213)$$

This assumes that the resistance of the teeth is proportional to their cross section, which is also equally true for those which have the same ratio of b to t to each other, a condition which is often of much service in practice.

§ 226.

Pitch and Face of Hoisting Gears.

For a hoisting gear of cast iron let:

$(P\,R)$ = the statical uioment of the driving force,
Z = the number of teeth,
R = its previously determined pitch radius, in inches,
t = the pitch,

we have for the given dimeusions:

$$t = 0.23\,\sqrt[3]{\frac{(P\,R)}{Z}},\quad \frac{t}{\pi} = 0.073\,\sqrt[3]{\frac{(P\,R)}{Z}} \quad \ldots\ldots \quad (214)$$

$$t = 0.045\,\sqrt[3]{\frac{(P\,R)}{R}},\quad \frac{t}{\pi} = 0.0145\,\sqrt[3]{\frac{(P\,R)}{R}} \quad \ldots \quad (215)$$

the face b being made

$$b = 2\,t \quad \ldots\ldots\ldots\ldots \quad (216)$$

These are intended to give a fibre stress S of about 4200 pounds. The actual stress is properly somewhat less, because the thickness of the tooth at the base is usually more than $\frac{1}{2}\,t$, as assumed in (213).

Since the value of $\frac{P\,R}{R}$ is the same as the pressure P, we can use (215) iu cases in which P only is given, as for rack teeth.

In discussing the preceding formulæ, consideration must be given to the elements which are usually given or selected iu practice.

Let t' and t be the pitch for two cases respectively, and Z and Z' the number of teeth. Also let S and S' be the stress at the base of the teeth, and let the constant, $6\left(\frac{l}{t}\right)\left(\frac{t}{h}\right)^2$, which in (213) is made equal to 16.8, be called C or C'; we then have, according to (214):

$$t = \sqrt[3]{\frac{2\,\pi\,C(P\,R)\left(\frac{t}{b}\right)}{S\,Z}}$$

whence:

$$\frac{t'}{t} = \sqrt[3]{\frac{C'}{C}\,\frac{S}{S'}\,\frac{Z}{Z'}} \quad \ldots\ldots \quad (217)$$

and for the radii R and R':

$$\frac{R'}{R} = \frac{Z'\,t'}{Z\,t} = \sqrt[3]{\frac{C'}{C}\,\frac{S}{S'}\left(\frac{Z'}{Z}\right)^2} \quad \ldots \quad (218)$$

The value of C depends upon the ratio of the teeth, and upon the value of S for the material used. If we assume the latter to be the same for both cases, the number of the teeth alone remains to be considered. A reduction in the number of teeth increases the pitch, according to (217); and accordiug to (218) reduces the radius.

Example 1. $Z = 11$, $Z' = 7$, hence

$$\frac{t'}{t} = \sqrt[3]{\frac{11}{7}} = \sqrt[3]{1.571} = 1.16;$$

$$b' = 2\,t' = 1.16\,b.$$

$$\text{But } \frac{R'}{R} = \sqrt[3]{\frac{7^2}{11^2}} = \sqrt[3]{\frac{49}{121}} = 0.74,$$

so that the 7 toothed gear will be about ¾ as large as the 11 toothed gear, or a 42 toothed gear for the same case would be about ¾ as large as a 66 toothed gear, aud with 1.16 times greater width of face.

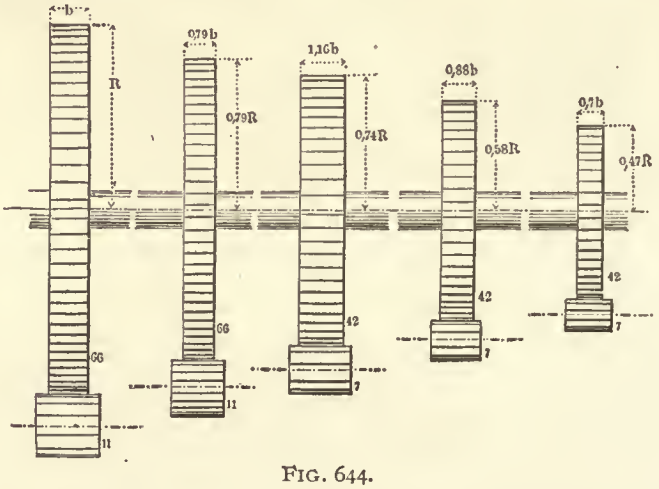


FIG. 644.

The constant C, for a giveu series of gears, should be invariable, and for ordinary spur gears may be taken equal to 16.8, as iu (213). For the so-called "thumb teeth," (§ 212), the coustant may be much smaller, and heuce permit an important reduction iu dimeusions. The value of $\frac{h}{t}$ for wheels of more than ten teeth is not less than 0.7, and introducing this value we get $C' = 8.4$, that is 0.5 C; hence "thumb shaped" profiles are capable of sustaining twice as great a load as the ordiuary form.

Example 2. If, for a given moment $(P\,R)$ the thumb profile is substituted for the ordinary form, without reducing the uumber of teeth, the pitch may be reduced iu the proportiou

$$t' = t\,\sqrt[3]{0.5} = 79\,t,$$

or about 0.8 times, with a proportional reduction in diameter and face.

If, however, the teeth are takeu in the above ratio of 11 : 7, we would have for the pitch,

$$t' = \sqrt[3]{0.5\,\frac{11}{7}} = 0.89\,t,$$

and the radius $R' = R\,\sqrt[3]{0.5\,\frac{7^2}{11^2}} = R\,\sqrt[3]{0.202} = 0.58\,R$.

The influence of the stress S is always important, and it should not be increased above the normal value for the given material, which latter is usually cast iron. An increase of oue-fourth in the permissible stress would reduce the pitch and diameter ouly 7 per cent., but on the other hand it must be remembered that too low a value of S causes an unnecessary increase in the size and weight, not only of the gears but also of the bearings, frame work and other parts of the machine. The value of S used above, viz.: 4200 pounds, has beeu show in practice to give satisfactory results, and there appears to be no good reason for any great variation from it.

When the gears are made of wrought iron, as is sometimes the case, S may be made much higher, and may indeed be taken double, say 8400 pounds. This gives a reduction in t' in the proportion of $t\,\sqrt[3]{0.5} = 0.79\,t$.

* See Civil Engineer and Arch. Journal, July, 1852, also Dingler's Journal, Vol. 125. Weisbach, III, 1st Ed., p. 449, 2d Ed., III, 2, p. 87.

Example 3. For comparison between a wrought iron gear of 7 teeth of thumb shaped outline, with a cast iron gear of 11 teeth of ordinary shape, we have:

$$R' = \sqrt[3]{0.5 \times 0.5 \left(\frac{7^2}{11^2}\right)} = R\sqrt[3]{0.101} = 0.47\ R,$$

$$t' = t\sqrt[3]{\frac{1}{4}\ \frac{11}{7}} = t\sqrt[3]{0.393} = 0.7\ t,$$

and $b' = 0.7\ b$.

In Fig. 644 the five cases given in the last three examples are shown on the same scale, side by side. In order to indicate the fact that the moment $(P\,R)$ is the same in all cases, the shaft diameter has been shown. It will be apparent that there is no definite relation between the diameter of the shaft and the radius of a gear.

The invariability of the moment, which has been maintained in the preceding examples, does not exist of the tooth pressure P upon the driven gear is again transmitted through a second so-called compound gear. If the pinion of a radius R, driving a gear R', compounds by a pinion R_2 on the same shaft into a rack R_2', for example, with a given pressure P, we have from (214)

$$t = \text{Const.}\ \sqrt[3]{\frac{C}{S},\ \frac{P\,R_2}{Z}}$$

whence

$$\frac{t'}{t} = \sqrt[3]{\frac{R_2'}{R_2}\ \frac{C'}{C}\ \frac{S}{S'}\ \frac{Z}{Z'}} \quad \ldots\ldots \quad (219)$$

This gives

$$R' = R\sqrt[3]{\frac{R_2'}{R_2}\cdot\frac{C'}{C}\cdot\frac{S}{S'}\cdot\left(\frac{Z'}{Z}\right)^2}$$

But $R_2 = Z_2\,t$ and $R_2' = Z_2'\,t_2'$, and from formula (215):

$$t_2' = t_2\sqrt{\frac{C'}{C}\cdot\frac{S}{S'}}$$

Hence we get:

$$\frac{t_2'}{t} = \sqrt[3]{\frac{C'}{C}\cdot\frac{S}{S'}\cdot\sqrt{\frac{C'}{C}\cdot\frac{S}{S'}}\cdot\frac{Z_2'}{Z_2}\cdot\frac{Z}{Z'}}$$

By selecting the number of teeth we may make

$$\frac{Z_2'}{Z_2} = \frac{Z'}{Z} \text{ and then obtain:}$$

$$\frac{t'}{t} = \sqrt{\frac{C'}{C}\cdot\frac{S}{S'}} \quad \ldots\ldots\ldots \quad (220)$$

and for the radii:

$$\frac{R'}{R} = \frac{Z'}{Z}\sqrt{\frac{C'}{C}\cdot\frac{S}{S'}} \quad \ldots\ldots\ldots \quad (221)$$

Example 4. A rack with a tooth pressure P, gearing with an 11 toothed pinion, is driven by a larger gear which again engages with an 11 toothed pinion, Fig. 645, the teeth being of the usual shape, and the material cast iron.

This is to be replaced by making all parts of wrought iron, and reducing the number of teeth in the rack pinion to 4, as shown in § 212, all teeth being also altered to the thumb-shaped form. We then have $C' = 0.5\ C$, $S' = 2\ S$, and hence: $t' = \sqrt{\frac{1}{4}} = \frac{1}{2}\ t$, and $R' = R\,\frac{4}{11}\sqrt{\frac{1}{4}} = \frac{2}{11}\ R$.

It will be noticed that in this case the ratio between the larger gear and the pinion on the same shaft is such that in (217) and (218) both are determined for the same moment $(P\,R)$.

Example 5. If, in order still further to reduce the dimensions, steel is used instead of wrought iron, thus permitting a stress of 14,000 pounds, we have $t' = t\sqrt{0.5 \times 0.3} = 0.387\ t$, and $R' = 0.4\cdot\frac{4}{11}\ R = 0.145\ R$, or about $\frac{1}{7}\ R$.

The proportion of the results of the last two examples is shown in Fig. 645. The force P on the teeth of the rack is the same in all three cases.

The statical moment on the main shaft is, however, reduced with the reduction in R', as is consequently that of the intermediate shaft.

The advantages of steel as a material for gear wheels have already been referred to in § 222. Its greater strength enables much lighter wheels to be used for the same service, than with cast iron. For a gear of cast iron and of steel, to act against the same moment, all other things being equal, we have, taking $S' = 14{,}000$, and $S = 4200\,\frac{t'}{t}$, and also $\frac{R'}{R}\sqrt[3]{0.3} =$ about $\frac{2}{3}$ in favor of the steel.

This gives for the ratio of weight $(\frac{2}{3})^3$, that is 0.3, the same as the ratio of S to S', or say three to one. This advantage also exists for transmission gearing, although not to the same extent.

If the velocity ratio in a compound train is comparatively great, it is interesting to note that the most advantageous ratio between gears lies between 1 : 9 and 1 : 10, this giving a minimum of shafts and of teeth.*

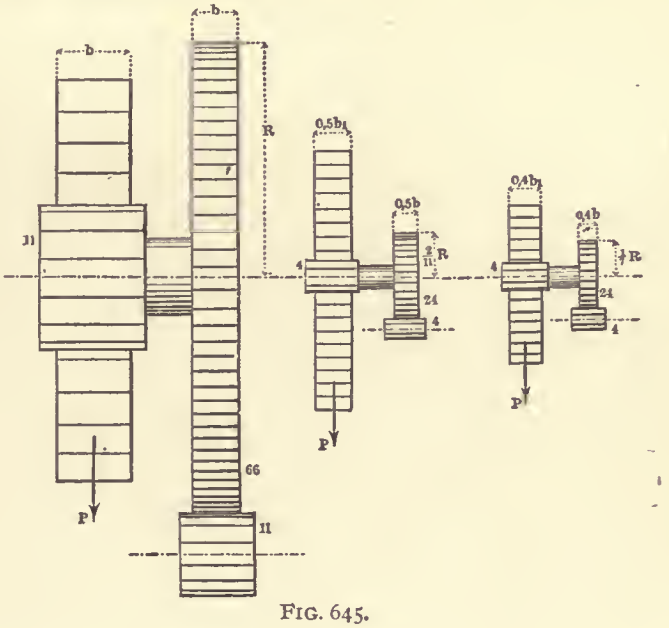


FIG. 645.

§ 227.

TABLE OF CAST IRON HOISTING GEARS.

t	$P=\frac{(PR)}{R}$	$\frac{PR}{Z}$	$\frac{t}{\pi}$	$P=\frac{(PR)}{R}$	$\frac{(PR)}{Z}$
½	127	10	0.15	107	8.67
⅝	200	20	0.20	190	20.56
¾	287	35	0.25	297	40.16
⅞	391	55	0.30	428	69.40
1	511	82	0.35	583	110.20
1¼	798	160	0.40	761	164.50
1½	1150	277	0.45	963	186.00
1¾	1564	440	0.50	1020	320.50
2	2044	658	0.60	1712	555.20
2½	3200	1284	0.70	2330	881.70

Example 1. A force of 100 pounds is exerted on a hand crank of 15 inches radius; what should be the pitch and face of a 10 toothed pinion for the further transmission?

Here we have $\frac{PR}{Z} = \frac{15 \times 100}{10} = 150$, and the nearest value in the table in the third column, is 160, which corresponds to a pitch of 1¼ inches. The face is $= 2\ t = 2\frac{1}{2}$ inches.

Example 2. A rack is to work with a pressure of 2000 pounds on the teeth. This would give a pitch of about 2 inches, or as given in the 4th and 5th columns, a pitch $t : 0.65\ \pi$, which is practically the same, and the width of face $= 2\ t$. If the rack is made of wrought iron, we have $t = 0.707 \times 2 = 1{,}414''$, and the face $= 2.8''$.

§ 228.

PITCH AND FACE OF GEARING FOR TRANSMISSION.

The fibre stress S, which is exerted upon the teeth by the action of a given force P, should be taken smaller for transmission gears as the circumferential velocity v increases, since the

*If ϕ be the total ratio, and k the number of pairs of gears, and the ratio between each pair be $x = \frac{Z}{Z'}$ we have $\phi = x^k$. The total number of teeth in the train, $y = k\,(Z + Z') = k\,Z'\,(1 + x)$. Now $k = \frac{l\,n\,\phi}{l\,n\,x}$, and the product of the number of teeth and the number of pairs gives

$$y\,k = \frac{(l\,n\,\phi)^2\,Z'\,(1+x)}{(l\,n\,x)^2}$$

Differentiating and making the differential coefficient equal to zero we get $l\,n\,x = \frac{2(1+x)}{x}$ which equation is satisfied by $x = 9.19$. For example $\phi = 600$, and the number of teeth in smallest pinion $= 7$. We have the following combinations:

(*a*) $\phi = 20\ 30$, gives $y = 7\ (2 + 20 + 30) = 364$, $y\,k = 728$.

(*b*) $\phi = 4.5.5.6$, gives $y = 7\ (4 + 4 + 5 + 5 + 6) = 168$, $y\,k = 672$.

(*c*) $\phi = 6.10.10$, gives $y = 7\ (3 + 6 + 10 + 10) = 203$, $y\,k = 609$.

The last solution is the best, for although it requires more teeth than (*b*) it has one less pair of gears, and for solution (*a*) the number of teeth, viz.: 210 is inconveniently great.

dynamic action of shock and vibration also increases. For cast iron we may take:

$$S = \frac{9{,}600{,}000}{v + 2164} \quad \ldots\ldots\ldots \quad (222)$$

in which v is the lineal velocity in feet per minute. For steel S may be taken $3\frac{1}{3}$ times, and for wood $\frac{6}{10}$ times the value thus obtained. For cast iron we obtain, for:

Cast Iron:

$v =$	100	200	400	600	800	1000	1500	2000	2500
$S =$	4240	4060	3744	3473	3238	3034	2620	2302	2068

For Steel:

$S =$	14,112	13,520	12,467	11,565	10,782	10,103	8725	7665	6886

And for Wood:

$S =$	2544	2436	2246	2083	1943	1820	1572	1381	1240

The velocity v may be obtained when n and R (the latter in inches) are given, by the following formula:

$$v = \frac{2\,\pi\,R\,n}{12} = 0.5236\,R\,n \quad \ldots\ldots \quad (223)$$

The selection of a proper value for v will be discussed below.

It is also found that the breadth of face b should increase with the increase of P. Tredgold states that the pressure per inch of face, that is $\frac{P}{b}$ should not exceed 400 pounds. This, however, is not to be followed implicitly, since pressures as high as 1400 pounds have been successfully used in practice. It is better, however, to consider the question of wear from the product of $\frac{P}{b}$ into n, which should not exceed a predetermined maximum. It is found that if $\frac{P}{b} \times n$ exceeds 67,000 the wear becomes excessive. In a pair of wheels where the teeth of both are made of iron, the greatest wear comes upon the teeth of the smaller wheel. In this case we may make

$$\frac{P\,n}{b} = \text{not more than } 28{,}000 \quad \ldots\ldots \quad (224)$$

and if possible it should be taken at less than this value. For smaller forces this constant, which we may call the co-efficient of wear and designate as A, may readily be made as low as 12,000, and even 6,000, without obtaining inconvenient dimensions. When the teeth are of wood and iron the wear upon the iron may be neglected, as the wear comes almost entirely upon the wooden teeth. For wooden teeth the value of A should not exceed 28,000, and is better made about 15,000 to 20,000.* It is impossible to give exact values in such constructions, and it must be left to the judgment of the designer as to how far it may be advisable to depart from the values obtained from existing examples.

It must be remembered that the different values of A do not appreciably affect the strength, but rather control the rapidity of wear. When sufficient space is available and a low value can be given to the co-efficient of wear, it is advisable to do so; if this cannot be done, the co-efficient which is selected will give an indication of the proportional amount of wear which may be expected.

In cases where a number of wheels gear into one other wheel, it is better to take, instead of the number of revolutions of the common wheel, the number of tooth contacts, that is the product of the revolutions and number of wheels in the group.

If R is given, as is often the case with water-wheels, fly-wheels, &c., P is also known, and since A can be chosen we have, taking N to be the horse power transmitted:

$$b = \frac{P\,n}{A} = \frac{63{,}000}{A} \cdot \frac{N}{R}$$

hence from (213) for ordinary teeth,

$$t = \frac{16.8\,P}{S\,b} = \frac{16.8\,A}{S\,n}$$

and for thumb shaped teeth,

$$t' = \frac{8.4\,P}{S\,b} = \frac{8.4\,A}{S\,n} \quad \ldots \quad (225)$$

If, however, as occurs in many cases, R is not previously determined, the choice of the number of teeth Z is unrestricted. In such cases we have for the width of face b:

$$b = \frac{396{,}000}{A} \cdot \frac{N}{Z\,t} \quad \ldots\ldots\ldots \quad (226)$$

If we give to A the successive values 30,000, 25,000, 20,000, 15,000, 10,000 and 5,000, we get the following numerical relations:

Common and Thumb Teeth.			*Common Teeth.*	*Thumb Teeth.*
$b = \frac{P\,n}{30{,}000}$	$= 2.1\,\frac{N}{R}$	$= 13.2\,\frac{N}{Z\,t}$	$t = \frac{504{,}000}{n\,S}$	$t' = \frac{252{,}000}{n\,S}$
$b = \frac{P\,n}{25{,}000}$	$= 2.5\,\frac{N}{R}$	$= 15.4\,\frac{N}{Z\,t}$	$t = \frac{420{,}000}{n\,S}$	$t' = \frac{210{,}000}{n\,S}$
$b = \frac{P\,n}{20{,}000}$	$= 3.15\,\frac{N}{R}$	$= 19.8\,\frac{N}{Z\,t}$	$t = \frac{336{,}000}{n\,S}$	$t' = \frac{168{,}000}{n\,S}$
$b = \frac{P\,n}{15{,}000}$	$= 4.2\,\frac{N}{R}$	$= 26.4\,\frac{N}{Z\,t}$	$t = \frac{252{,}000}{n\,S}$	$t' = \frac{126{,}000}{n\,S}$
$b = \frac{P\,n}{10{,}000}$	$= 6.3\,\frac{N}{R}$	$= 39.6\,\frac{N}{Z\,t}$	$t = \frac{168{,}000}{n\,S}$	$t' = \frac{84{,}000}{n\,S}$
$b = \frac{P\,n}{5{,}000}$	$= 12.6\,\frac{N}{R}$	$= 79.2\,\frac{N}{Z\,t}$	$t = \frac{84{,}000}{n\,S}$	$t' = \frac{42{,}000}{n\,S}$

(227)

For transmission gears the minimum number of teeth should not be fewer than 20, in order that the unavoidable errors of construction shall not cause excessive wear; for quick-running gears it is desirable to have still more teeth. The gear wheels on high speed turbines seldom have fewer than 40, and often as many as 80 teeth. When wood and iron teeth are used, the least wear is produced when the wooden teeth are on the driver, because the action begins at the base of the tooth and passes toward the point, while on the driven gear the action is reversed.

If desired a number of teeth Z can be calculated which will give a desired ratio $b : t$. If we combine formulæ (225) and (226) we obtain the useful relation:

$$Z = \frac{396{,}000}{16.8^2\,A^3} \cdot \frac{n^2\,S^2\,N}{\left(\frac{b}{t}\right)} \quad \ldots\ldots\ldots \quad (228)$$

This shows the important influence of A upon Z, and the effect of the number of teeth upon the wear; also the important relation of the tooth profile, since the constant 16.8 (or for thumb teeth 8.4) appears in the second power. It is also seen that Z is dependent on the square of n, and the square of S, other things being constant. These points indicate the methods of obtaining the least stress.

The value of $\frac{b}{t}$ is sometimes made as great as 5. For wider faces and sometimes for narrower, the rim of the gear is made of two adjoining parts.

Example 1.—A water wheel of 60 horse power, 26 feet, 3 inches in diameter, moving with a velocity at the circumference of 256 feet per minute, is to be provided with an internal gear wheel, the pitch circle being 16 inches less radius than that of the water wheel, and gearing into a pinion which is to make 40 revolutions per minute.

We have: $n = \frac{256}{3.14 + 26.25} = 3.1$

and $\frac{n_1}{n} = \frac{40}{3.1}$; also $\checkmark = \frac{256\,(157.5 - 16}{157.5} = 230$ ft per minute. $P = \frac{33000 \times 60}{230} = 8608$ lbs. This gives a permissible stress $S = 4100$ lbs. nearly. We will choose for the smaller wheel $\frac{P\,n}{b} = 25{,}000$, which gives $\frac{P}{b} = \frac{25{,}000}{n_1} = \frac{25{,}000}{40} = 625$, hence $b = \frac{P}{62.5} = \frac{8608}{62.5} = 13\frac{3}{4}''$. We then have from (227) $t = \frac{420{,}000}{40 \times 4100} = 2.56''$. We then have $Z = \frac{2\,\pi\,R}{t} = \frac{2\,\pi\,141.5}{2.56} = 347$. If we make 348 teeth the wheel may be divided into 12 segments of 29 teeth each. For the driven wheel we have $Z_1 = \frac{n_1}{n}\,Z = \frac{3.1}{40} \times 348 = 27$, whence $R_1 = \frac{27 \times 2.56}{2\,\pi} = 11''$.

Example 2.—A turbine water wheel of 100 horse power has a vertical shaft making 96 revolutions per minute, and it is required to drive a horizontal shaft at 144 revolutions, hence a pair of bevel gears are required. We will select wooden and iron teeth, and let the wooden teeth be on the driver. We will assume v to be between 1200 and 1400 feet per minute, which gives $J\,S = 1600$, and make $A = 25{,}000$, also $\frac{l}{t} = 3$. We then have from (228) $Z = \frac{396{,}000}{16.8^2 \times 25000^3} \cdot \frac{96^2 \times 1600^2 \times 100}{3} = 70$. We then have $Z_1 = \frac{96}{144} \cdot 70 = 47$; also $t = \frac{420{,}000}{96 \times 1600} = 2.73''$ say $2\frac{3}{4}''$, $b = 3\,t = 8\frac{1}{4}''$, $v = 1536$ feet per minute.

* See case 10, in § 229 *seq.*

Example 3.—In a given train of gearing, Fig. 646, in which the corresponding wheels of both pairs are of the same size, the force transmitted in each case is inversely as the number of revolutions. In order to have the co-efficient of wear $\frac{Pn}{b}$ alike in both cases it is only necessary to make all the gears of the same face. An example of this kind may be found in the back gearing of many lathes.

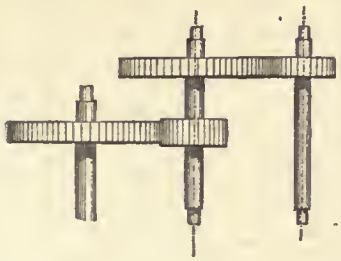


FIG. 646.

Example 4.—Let it be required to construct a pair of durable gears of wooden and iron teeth under the following conditions: $N = 5$, $n = n_1 = 60$, and $\frac{b}{t} = 2$. We may make $v = 500$, which gives, from (222), $S = 2160$, and as great durability is required we will take A as low as 10,000. These values in (228) give:

$$Z = \frac{396,000}{16.8^2 \times 9,000^3} \cdot \frac{60^2 \times 2160^2 \times 5}{2} = 80.8$$

which we may call 80 teeth.

We have from (227)

$$t = \frac{151,200}{60 \times 2160} = 1.167''$$

and

$$b = \frac{396,000}{9000} \cdot \frac{5}{80 \times 1.167} = 2.33''$$

or $2t$, as intended.

Example 5.—Let $N = 40$, $n = 30$, $n_1 = 50$ for a pair of iron gears with teeth of common form, and let $\frac{b}{t} = 2.5$. If we make $v = 300$, $S = 3400$ and we take $A = 25,000$. This gives for the driver gear:

$$Z = \frac{396,000}{16.8^2 \times 25,000^3} \cdot \frac{50^2 \times 3400^2 \times 40}{2.5} = 41.5$$

say 42 teeth, and $Z_1 = \frac{5}{3} Z = 70$,

we have $t = \frac{420000}{50 \times 3400} = 2.47''$

and $b = 2.5\, t = 6.175''$.

If we choose the thumb-shaped teeth, and make $\frac{b}{t} = 3.5$ we get:

$$Z = \frac{396,000}{8.4^2 \times 25,000^2} \cdot \frac{50^2 \times 3400^2 \times 40}{3.5} = 118$$

say 120, and $Z_1 = 200$. $t' = \frac{210,000}{50 \times 3400} = 1.235''$, $b = 4.32$.

This gives smaller teeth, but larger radii than when the common form is used.

When steel is used for gear wheels, special proportions are obtained. It is not too much to say that the value of the co-efficient of wear A should be taken twice as great as for cast iron. The stress S, however, may be taken 3½ times that permissible for cast iron. Taking these points into consideration in formula (228) we see that A would reduce the number of teeth by ⅛, and S would increase it by $\left(\frac{10}{3}\right)^2$, that is, about 11 times, so that the net increase would be $\frac{11}{8}$, if the above values are accepted. It may therefore be laid down as a rule that steel gears should have more teeth for the same service than cast iron gears. The ratio of face to pitch may be made quite large, and in the case of double spiral gears (as Fig. 627) the ratio $\frac{6}{\tau}$ is sometimes made as great as 7 or 8. If the formula for thumb teeth be used, instead of the usual shape, the constant 16.8 will give satisfactory results. The value obtained for the pitch is that for the normal pitch $\tau = t \sin \gamma$, but the width of face is the actual width, as b, in Fig. 627, $2\,b'$ in Fig. 628.

Example 6.—Suppose the wheels given in Example 5 to be made with double spiral teeth of steel. We take $A = 56,000$, and $\frac{b}{\tau} = 6$, also $S = 12,800$. We then get:

$$Z = \frac{396,000}{8.4^2 \times 56000^2} \cdot \frac{50^2 \times 12,800^2 \times 40}{6} = 87$$

We have $\tau = \frac{8.4 \times 56,000}{12,800 \times 50} = 0.74''$,

also $b = \frac{396,000}{56,000} \cdot \frac{40}{87 \times 0.74} = 4.4''$

If we take $Z_1 = 84$, we get $Z = 140$ and $b = 4\frac{1}{2}''$. If $g = 60''$ we have

$$t = \frac{\tau}{\sin 60} = \frac{0.74}{0.866} = 0.854''.$$

We may take $t = 0.875''$, which gives $\tau = 0.866 \times 0.875 = 0.757''$ and

$$\frac{b}{\tau} = \frac{4.5}{0.757} = 5.93, \text{ or nearly } 6.$$

We have then finally $R_1 = 11.6''$, $R = 19.47''$.

§ 229.

EXAMPLES AND COMMENTS.

The following examples taken from actual practice will be of interest: (see Table on following page).

No. 1. From the driving gear of the main steam engine of Fleming's Spinning and Weaving Mill in Bombay. The toothed fly-wheel is the driver, and the teeth are shrouded, as shown in Fig. 651. The coefficient of wear for the driven gear seems high, and does not indicate long endurance.

No. 2. A toothed fly wheel engaging with a pair of equal spur gears; 300 horse-power transmitted by each gear, making a total of 600 horse-power. The value for $\frac{Pn}{b}$ must therefore be multiplied by 2; see last column of the table.

No. 3. This is from the air compressor for the atmospheric railway of St. Germain (now abandoned.) $\frac{Pn}{b}$ is evidently too high, as would probably have become apparent had the gears continued in operation.

No. 4. $\frac{P}{b}$ is very high, but the small number of revolutions keeps the value of $\frac{Pn}{b}$ within reasonable limits.

Nos. 5 and 6. These are from the great water wheel at Greenock. The pressure at the rim is great, but the teeth have worn well in practice, as might have been predicted from the moderate values of $\frac{Pn}{b}$ The value of the latter is almost the same for No. 6 as for No. 5, hence the wear should be about the same for both gears.

No. 7. The teeth in the smaller gear are thinner than those of the large fly-wheel, hence the two values for S. Probably the larger wheel was originally made with wooden teeth.

No. 9. Notwithstanding the high pressure the value of $\frac{Pn}{n}$ is reasonably small. The stress upon the teeth is quite high, as is also the case with No. 4, and lower stresses are to be recommended.

No. 10. This is one of the most noteworthy examples of the whole collection, on account of the very slight wear exhibited. The wooden teeth on the large wheel, (the fly-wheel of the steam engine of the Kelvindale Paper Mill at Glasgow) ran for 26½ years, for 20 hours per day, with a wear upon the teeth, measured at the pitch circle, of only about ⅛ inch. For the first half of this time the engine indicated 84 horse-power, at 38 revolutions. The teeth were lubricated twice a week with talc and graphite. The long endurance is doubtless partially due to the great care which the teeth received, they having been cut upon the wheel in place, but also to the moderate co-efficient of wear.

No. 11. The teeth were found too small in practice, as is indicated by the stress of 3000 pounds; from formula (222) we obtain $S = 1734$ pounds.

No. 12. Two gears with wooden teeth engage with a single pinion on the screw propeller shaft. The teeth are in two sets of 4¾″ width of face each.

No. 13. Very high pressure, which must appear in the wear upon the teeth; apparently it should be difficult to keep them in good condition, owing to the high value of $\frac{P}{b}$.

No. 15. These teeth appear weak, as has been shown by repeated breakages. The wear must be rapid, as indicated by the high value of $\frac{Pn}{b}$.

No. 17. These gears, (designed by Fairbairn) were intended ultimately to transmit double the power at first given, in which case the stress would reach over 4000 pounds, which is admissible but the value of $\frac{Pn}{b}$ would then become rather too high to indicate very great endurance.

No. 20. The value of $\frac{Pn}{b}$ seems too high for the wooden teeth; it is almost too great also for the iron teeth, and it must be remembered that with wooden and iron teeth, the wear comes almost entirely upon the wooden teeth.

No. 22. These gears are from an establishment which has used hyperboloidal gears with much success for power transmission. The angle of the axes is 90°. The use of wooden teeth upon the driver is to be criticised, as tending to increase the liability to wear.

F. THE DIMENSIONS OF GEAR WHEELS.

§ 230.

THE RIM.

The ring of metal upon which the teeth of a gear wheel are placed is called the rim. For cast iron spur gears, the thickness of the rim is given by the formula

$$\delta = 0.4\, t + 0.125'' \ldots\ldots\ldots \quad (229)$$

EXAMPLES OF TRANSMISSION GEARING.

No.	N	n	R	Z	t	b	v	P	S	$\frac{P}{b}$	$\frac{Pn}{b}$	REMARKS.
1	1000	$\frac{36.67}{114.8}$	$\frac{120}{38.25}$	$\frac{144}{46}$	5.25	24	2300	14,000	1877	583	$\frac{21,390}{66,970}$	Iron and Iron. Steam Engine.
2	300	$\frac{25}{100}$	$\frac{146.5}{37}$	$\frac{230}{58}$	4.00	14	1900	5,100	1614	364	$\frac{2 \times .9107}{36,400}$	Iron and Iron.
3	270	$\frac{60}{12}$	$\frac{19.6}{98}$	$\frac{19}{95}$	6.25	20.6	616	14,300	1848	694	$\frac{41,650}{8,330}$	Iron and Iron.
4	240	$\frac{13.3}{44}$	$\frac{110}{33}$	$\frac{208}{68}$	3.125	16	766	10,200	3270	639	$\frac{8,498}{28,110}$	Iron and Iron. Transmission for No. 8.
5	192	$\frac{1.33}{15.14}$	$\frac{400}{35.25}$	$\frac{704}{62}$	3.6	15	280	22,240	7252	1483	$\frac{1,972}{22,450}$	Iron and Iron. Water Wheel.
6	192	$\frac{15.14}{50}$	$\frac{106}{32}$	$\frac{208}{63}$	3.18	15	840	7,425	2275	495	$\frac{7,494}{24,750}$	Iron and Iron. Transmission for No. 5.
7	140	$\frac{30}{55}$	$\frac{58.4}{32}$	$\frac{132}{72}$	2.8	8.6	900	5,000	$\frac{4266}{48.35}$	581	$\frac{17,440}{31,970}$	Iron and Iron. Steam Engine.
8	140	$\frac{30}{54.5}$	$\frac{66.5}{35.75}$	$\frac{138}{76}$	3	13	1045	4,350	3700	335	$\frac{10,040}{18,230}$	Iron and Iron. Steam Engine.
9	120	$\frac{1.51}{13.3}$	$\frac{291}{33}$	$\frac{560}{80}$	3.125	15	240	16,230	5688	1082	$\frac{1,634}{14,390}$	Iron and Iron. Water Wheel.
10	100	$\frac{45}{158.8}$	$\frac{84.5}{24}$	$\frac{176}{50}$	3	10	2000	1,635	924	163	$\frac{7,357}{8,175}$	Wood and Iron. Steam Engine.
11	90	$\frac{26}{80}$	$\frac{85.4}{27.75}$	$\frac{228}{74}$	2.375	5.9	1163	2,500	3000	424	$\frac{11,010}{33,900}$	Wood and Iron. Steam Engine.
12	82.5	$\frac{54}{83}$	$\frac{55.1}{35.8}$	$\frac{114}{74}$	3.1	$\frac{2 \times 4.75}{11.75}$	1558	3,440	1848	362	$\frac{19,540}{2 \times 30,040}$	Wood and Iron. Screw Steamship.
13	50	$\frac{4.0}{7.32}$	$\frac{50.4}{27.5}$	$\frac{96}{52}$	3.25	10.6	104	15,500	7536	1463	$\frac{5,849}{10,700}$	Iron and Iron. Water Wheel.
14	20	$\frac{7.74}{40}$	$\frac{85.4}{16.5}$	$\frac{248}{48}$	2.2	6.3	328	1,980	2420	314	$\frac{2,433}{12,570}$	Iron and Iron. Water Wheel.
BEVEL GEARS.												
15	300	$\frac{93}{50}$	$\frac{24.37}{45.7}$	$\frac{50}{93}$	3.1	13	1187	8200	$\frac{3270}{3697}$	630	$\frac{58,660}{31,540}$	Iron and Iron. Turbine.
16	300	$\frac{100}{111.8}$	$\frac{29.7}{26.7}$	$\frac{55}{49}$	2.7	10	1576	6170	3840	617	$\frac{61,700}{68,980}$	Iron and Iron. Transmission for No. 1.
17	240	$\frac{44}{44}$	42	75	3.5	18	968	8050	2133	447	19,670	Iron and Iron. Transmission for No. 3.
18	200	$\frac{41}{80}$	$\frac{59}{30.1}$	$\frac{98}{50}$	3.8	11.8	1260	5157	2000	437	$\frac{17,920}{34,960}$	Wood and Iron. Turbine.
19	130	$\frac{93}{124}$	$\frac{31.3}{24.8}$	$\frac{80}{60}$	2.4	8	1523	2772	$\frac{2276}{2417}$	346	$\frac{32,220}{42,970}$	Wood and Iron. Turbine.
20	100	$\frac{93}{144.7}$	$\frac{23.4}{15}$	$\frac{70}{45}$	2.1	6.3	1140	2860	$\frac{2985}{3840}$	454	$\frac{42,220}{65,690}$	Wood and Iron. Turbine.
21	50	$\frac{93}{218}$	$\frac{25.6}{10.8}$	$\frac{75}{32}$	2.1	6.3	1236	1313	$\frac{1564}{1848}$	208	$\frac{19,380}{45,430}$	Wood and Iron. Turbine.
HYPERBOLOIDAL GEARS.												
22	16	$\frac{72}{81.6}$	$\frac{21.6}{19}$	$\frac{68}{60}$	$\frac{1.996}{1.993}$	5.9	812	640	$\frac{924}{1250}$	108	$\frac{7,810}{8,851}$	Iron and Wood. Transmission.

See Fig. 647. The rim is thickened in the middle or at one edge to $\frac{6}{5}\delta$, and also stiffened by a rib, and for gears of fine

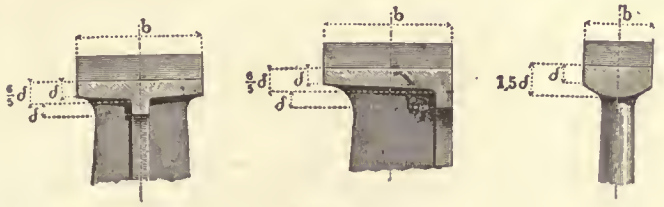


FIG. 647.

pitch the section of the rim is curved, which harmonizes well with arms of oval section. According to (229) a pitch of 1″ would give a rim thickness $\delta = 0.4'' + 0.125'' = 0.525''$ or a little over ½″, and for a pitch of ½″, $\delta = 0.325''$.

For bevel gears of cast iron the rim is made $\frac{6}{5}\delta$ thick at the outer edge, and of the various forms shown in Fig. 648.

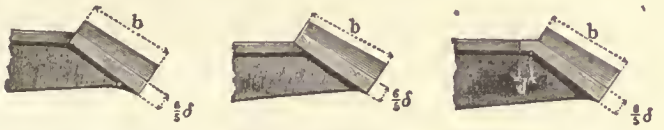


FIG. 648.

For wooden teeth it is necessary to have a deeper and stronger rim, the dimensions being dependent somewhat upon the method of inserting the teeth. The proportions for spur gears

FIG. 649.

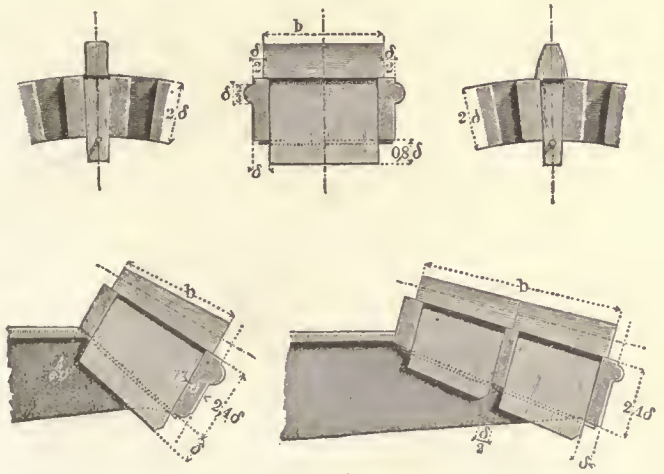


FIG. 650.

are shown in Fig. 649, and for bevel gears in Fig. 650. For very wide faces the wooden teeth are made in two pieces and a stay bar cast in the mortise.

Small pinions are often cast solid, and when subjected to heavy pressures are strengthened by shrouding, as showu in Fig. 651, and sometimes this shrouding is turned dowu to the pitch line.

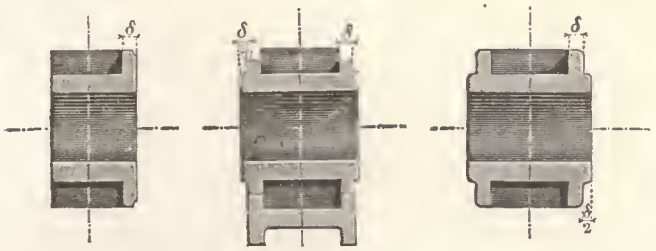


FIG. 651.

For double spiral gears of steel (see § 223) shrouding is to be recommended, and is very generally used. The use of shrouding especially assists in securing good steel castings, for the great shrinkage of the steel, nearly two per cent., tends to produce warped and twisted castings.

Small pinions are sometimes cut from solid wrought iron, in which case the shrouding must be omitted.

§ 231.

THE ARMS OF GEAR WHEELS.

The arms of gear wheels are made according to the following forms, dependent upon the kind of rim used.

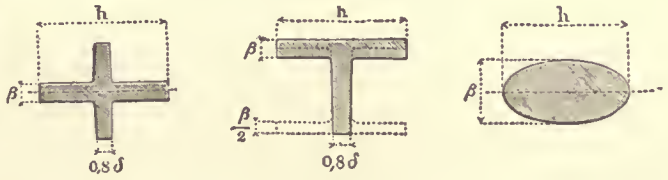


FIG. 652.

Fig. 652. Ribbed sections, which are made sometimes as shown in the dotted lines as may be most convenient in moulding, and oval sections, in which the thickness β of the arm is generally made one-half the width h. A good proportion for the arms is obtained when their number A is made as follows:

$$\left.\begin{aligned} A &= 0.55\sqrt{Z}\sqrt[4]{t} \\ A &= 0.73\sqrt{Z}\sqrt[4]{\frac{t}{\pi}} \end{aligned}\right\} \quad \ldots\ldots \quad (230)$$

From these we obtain the following:

A =	3	4	5	6	7	8	10	12
$Z\sqrt{t}$ =	30	53	83	119	162	211	330	475
$Z\sqrt{\frac{t}{\pi}}$ =	11	23	36	52	71	93	146	209

Example.—For a gear wheel of 50 teeth and 2″ pitch, we have $Z\sqrt{t} = 50\sqrt{2} = 50 \times 1.414 = 70$ and this lies between 53 and 83; being nearer the latter we give the wheel five arms. If the pitch had been ¾″, and the same number of teeth $Z\sqrt{t} = 50\sqrt{0.75} = 50 \times 0.866 = 43.3$ or between three and four arms, the latter number being used in practice.

The width of arm h, in the plane of the wheel is somewhat a matter of judgment, but may suitably be made according to the ratio $h = 2$ to $2.5\,t$, when the thickness β may be obtained from the following formula:

$$\frac{\beta}{b} = 0.07\,\frac{Z}{A}\left(\frac{t}{h}\right)^2 \quad \ldots\ldots\ldots \quad (231)$$

Should this formula give a thickness either too great or too small for convenience in casting, another value for $\frac{h}{t}$ must be taken and the calculation repeated. The following table will assist in this operation.

The taper of the arms may be made as follows: the ribs at the rim are made slightly narrower than the breadth of face b, and at the hub, equal to, or slightly greater than b. For arms of oval section h, may be made equal $2\,t$ at the centre of the wheel, tapering to ⅔ this width at the rim.

§ 232.

TABLE OF GEAR WHEEL ARMS.

$\frac{h}{t}$	Value of $\frac{\beta}{b}$, when								
	$\frac{Z}{A}$=7	9	12	16	20	25	30	35	40
1.50	0.20	0.28	0.37	0.50	0.62	0.78	0.93	1.08	1.24
1.75	0.16	0.21	0.27	0.37	0.46	0.57	0.69	0.80	0.91
2.00	0.12	0.16	0.21	0.28	0.35	0.44	0.53	0.61	0.70
2.25	0.10	0.12	0.17	0.22	0.28	0.35	0.41	0.48	0.55
2.50	0.08	0.10	0.13	0.18	0.22	0.28	0.34	0.39	0.45
2.75	0.06	0.08	0 11	0.15	0.18	0.23	0.28	0.32	0.37
3.00	0.05	0.07	0.09	0.12	0.16	0.19	0.23	0.27	0.31

Example.—Let a wheel have 6 arms, and 120 teeth of 2 inch pitch, the face being 4 inches. If we make $h = 2t$ at the centre of the wheel, we have $\frac{h}{t} = 2$, and $\frac{Z}{A} = 20$, hence we get from the table $\frac{\beta}{b} = 0.35$, and $\beta = 4 \times 0.35 = 1.40''$. If this is considered too thick, we may make $h = 2.25t$, which gives $\beta = 4 \times 0.28 = 1.12''$.

For gears with wooden teeth, and for the iron wheels gearing with them, the dimensions of the arms may be made 0.8 times that given by the preceding rules. If more accurate dimensions are required, the best plan is to determine the pitch of the equivalent iron teeth, and use this value in the calculations.

§ 233.

GEAR WHEEL HUBS.

The hub for a gear wheel generally tapers slightly each way from the arms to the end, the length $L = \frac{5}{4} b$, or somewhat more for wheels of very large diameter, and the thickness of metal about the bore is made $w = 0.4\,h + 0.4''$, in which h is the same as in the preceding section. In cases of much importance reference should be made to formula (66), § 65.

If the wheel is not to be secured by shrinkage the thickness of metal at the ends of the hub may be made $= \frac{3}{4} w$. The key way is cut the entire length of the hub, and for wheels which are subjected to heavy service the metal should be reinforced over the key way. Instead of this, the hub may be strengthened by wrought iron rings, forced on one or both ends. Such rings are usually of rectangular cross section, the thickness being $\frac{1}{2} w$, and add greatly to the strength of the hub. See Chapter III. § 161 to the end.

§ 234.

WEIGHT OF GEAR WHEELS.

The approximate weight G of gear wheels proportioned according to the preceding rules may be obtained from the following:

$$G = 0.0357\, b\, t^2\, (6.25\, Z + 0.04\, Z^2) \quad . \quad . \quad . \quad . \quad (233)$$

The following table will facilitate the application of the formula as it gives the value of $\frac{G}{b\, t^2}$ for the number of teeth which may be given, and the weight can at once be found by multiplyiug the value in the table by $b\, t^2$.

Z	0	2	4	6	8
20	5.04	5.60	6.18	6 77	7.38
30	7.99	8.61	9.24	9.89	10.52
40	11.09	11.90	12.59	13.30	14.02
50	14.74	15.48	16.23	17.00	17.77
60	18.55	19 35	20.15	20.97	21.80
70	22.65	23.50	24.36	25.24	26.12
80	27.02	27.93	28.85	29.79	30.73
90	31.69	32.66	33.63	34.62	35.63
100	36.63	37.67	38.70	39.75	40.81
120	47.40	48.54	49.69	50.85	52.03
140	59.30	60.56	61.82	63.10	64.27
160	72.35	73.73	75.10	76.39	77.90
180	86.54	88.03	89.52	91.02	92.54
200	101.88	103.48	104.98	106.70	108.34
320	118.36	120.08	122.15	123.52	125.27

Example.—For a cast iron gear wheel, proportioned according to the foregoing rules, with 50 teeth, 2" pitch and 4" face, we have $b\, t^2 = 16$, and by the table the multiplier for 50 teeth is 14.74, and the weight $= 16 \times 14.74 = 235.84$ lbs., say 236 pounds. For a gear of 50 teeth, $1\frac{1}{4}$" pitch and $2\frac{1}{2}$" face, we have $b\, t^2 = 3.90625$, which multiplied by 14.74 gives 57.62 pounds.

For bevel gears or for gears with wooden teeth and lighter arms (as given at the end of § 232) the weights will run slightly less than given by the table.

CHAPTER XVIII.

RATCHET GEARING.

§ 235.

CLASSIFICATION OF RATCHET GEARING.

Ratchet gearing may be considered as a modification or extension of wheel gearing. The object of ratchets is to check the action of certain portions of a machine or train of mechanism and so modify an otherwise continuous motion into some intermittent form.

Ratchet gearing may be divided into two main divisions according to the nature of the checking action. When the movement of the checked member is impeded in only one direction we have what may be called a Running Ratchet; and when the movement is checked in both directions, a Stationary Ratchet.

The distinction will be understood by reference to the accompanying illustrations, in which Fig. 653 shows a ratchet wheel and pawl *a b c*, the shape of teeth and pawl permitting motion of the wheel in one direction, and hence forming a Running

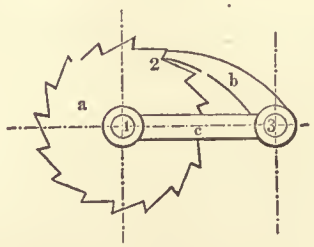


FIG. 653.

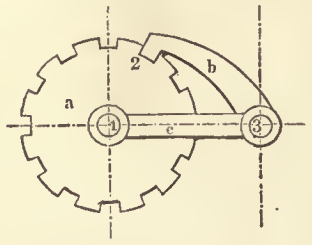


FIG. 654.

Ratchet Gearing, while in Fig. 654 the rectangular notches and pawl for a Stationary Ratchet Gearing. The lifting of the pawl is called the release, and the falling into gear is called the engagement of the ratchet gearing.

If the two members *b* and *c* are held, *a* becomes the intermittent mover, while if *a* be held, the parts *b* and *c* possess the intermittent action; as for example, the sustaining pawl and ratchet wheel of a common hoisting winch in the first case, and the reverse lever and quadrant of a locomotive in the second case.

Ratchet gearing is a portion of constructive mechanism which will repay close investigation. For this purpose the following six groups may be considered:

1. Ratchets pure and simple, such as a ratchet wheel and pawl for the mere prevention of rotation. Examples: the ratchets of a windlass, or of the beam of a loom.

2. Releasing Ratchets; those which act to release members which are under stress, and which by such release are permitted to perform and determinate work. Examples: the pawls which release the drop of a pile driver, the trigger of a gun, or the trip valve gear of some steam engines.

3. Checking Ratchets; those which arrest parts which are already in continuous motion. Example; the safety check ratchets upon elevators, and upon mine hoists.

4. Continuous Ratchets; those in which a combination of pawls acts to drive a member in a given direction with practically a continuous motion. Examples: a ratchet-driven windlass; some forms of counters.

5. Locking Ratchets; those which act to detain certain members in a fixed relation against the action of external forces until released. Examples: some forms of car couplings and of releasing shaft couplings, also the mechanism of locks.

6. Escapements; those forms which permit a member under the action of an impelling force to make a regularly intermittent motion in one direction. Example: the various forms of clock and watch escapements.

By following this classification, the various principal fundamental forms may be briefly examined.

§ 236.

TOOTHED RUNNING RATCHET GEARS.

In running ratchets, the direction of motion which is not checked by the pawl is called the forward motion, and the reverse, the backward motion. The teeth on the ratchet wheel must therefore be so shaped that when the pawl is in engagement the backward motion only must be impeded. It is also important that the form should be so chosen that the first tendency toward a backward movement should act to produce an engagement of the pawl with the teeth.

In determining the form of teeth, Fig. 655, we observe that the most effective point upon the circumference of the wheel for the action of the pawl is that at which the joining line 1.2 of the centre of the wheel 1, with the point of the pawl 2, is at right angles with the pawl radius 3.2. If we describe a circle upon the diameter 1.3, or the distance between centres of wheel and pawl, the intersections 2 and 2′ with the pitch circle of the ratchet wheel will give the two most advantageous points of application. If the point 2 be selected, the attempted reverse movement of the wheel will subject the pawl to compression, while if 2′ be chosen the pawl must be of the hook shape shown, and will be subject to tension. If the teeth of the wheel are to be of straight outline, the flanks should be radial. If a point of

action 2_1 or 2_2, in front or behind 2 or 2′, be chosen, the mechanism will be operative, but less advantageously than when constructed as above, for the lever arm of the force-couple act-

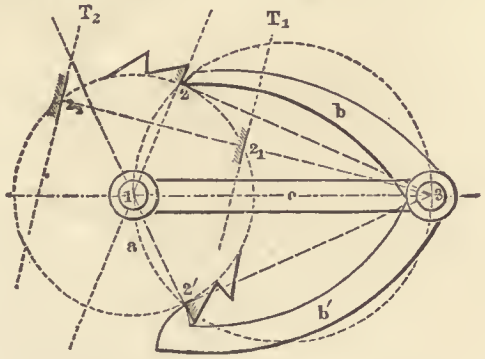


FIG. 655.

ing upon the wheel will be less, and hence the pressure greater. The angle of the flank, which will cause the direction of the force upon the pawl to pass through the axis 3, is found by erecting a perpendicular from 2_1 or 2_2 upon $2_1 . 3$ or $2_2 . 3$.

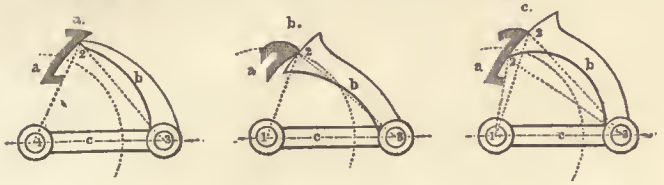


FIG. 656.

It is not necessary to bevel the end of the pawl so that it shall bear in but one point of the tooth, as it is not difficult to shape the tooth profile so that the force P shall pass through the axis 3, when the pawl engages with the tooth. This is accomplished by making the profile of the flank of the tooth a circular arc struck from 3 as a centre, as in Fig. 656 *a*.

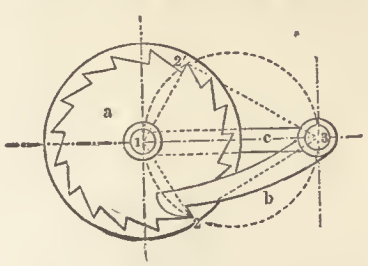


FIG. 657.

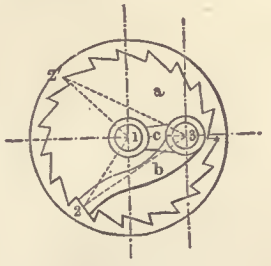


FIG. 658.

The same result will be attained by giving this curve to the end of the pawl, and making the point of the tooth the bearing, as at *b*, or both pawl and tooth may be formed to the curve, as

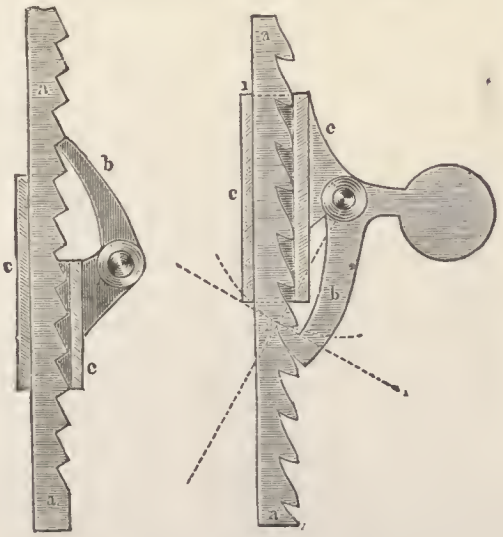


FIG. 659.

at *c*. Since the force which acts upon the pawl has no tendency to cause it to lift out of gear, when constructed as thus described we may call this form of tooth the "dead" ratchet tooth. Other forms of teeth will be considered hereafter.

Internally-toothed ratchet wheels may also be made with the pawls adapted to act either in tension or compression, as at 2 and 2′, Fig. 657. The axis 3 may be within the wheel, Fig. 658, in which case the above given conditions for the best position of the point of action cannot be fulfilled.

If the radius of the ratchet wheel be made infinitely great we have a ratchet rack, Fig. 659, in which *a* is a pawl acting in compression, and *b* a form acting in tension.

An important application of the ratchet rack is shown in Fig. 660, which is the upper portion of the lifting frame for a screw propeller.*

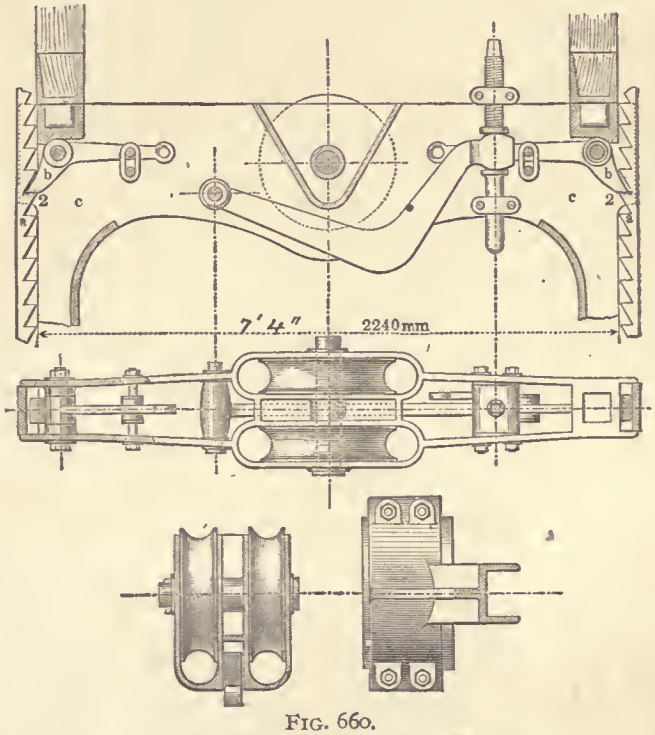


FIG. 660.

The two ratchet racks *a*, which support the frame as it is gradually lifted, are in the middle plane of the ship, being fast to the walls of the propeller well. In order to insure the engagement of the pawls *b b*, they are held in gear by the loop springs of rubber. The frame is raised and lowered by a rope tackle, the sheaves of which are shown, the so-called "*cheese*-coupling" (see § 156), permitting the propeller to be lifted, when its tongue and groove are in the proper vertical position. The pawls are held out of gear by means of lines, during the operation of lowering. The frame and ratchet racks are both made of bronze. The bent lever is another pawl which engages in a notch in a blade of the propeller, and prevents it from revolving during the operation of raising or lowering. There are two wooden struts, the bronze shod ends of which can be seen on each side just above the pawls *b*, their function being to hold the frame firmly in its lowest position, when the propeller is revolving.

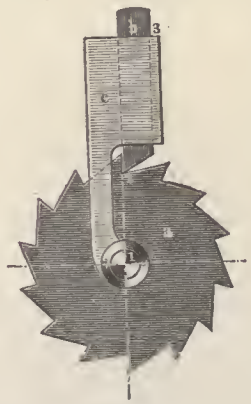


FIG. 661.

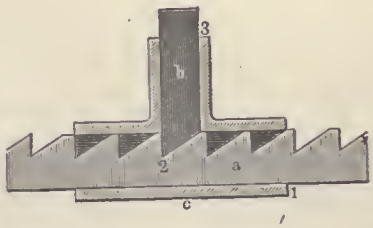


FIG. 662.

Ratchet racks are also used extensively in connection with the hoisting machinery in shafts of mines, etc.

* See Fig. 323, § 117, where one of the bearings for the same propeller is shown.

Instead of giving the ratchet wheel an infiuitely great radius, the arm 2.3 of the pawl may be made infinitely long. This simply means that the motion of the pawl is guided in a straight line, in some form of slide. In Fig. 661 such an arrangement is shown for a ratchet wheel, and in Fig. 662 for a ratchet rack, such forms being not uncommon.

§ 237.

THE THRUST UPON THE PAWL.

The condition that the thrust upon the pawl, in a ratchet gearing, shall pass through the axis of the pawl, is uot always fulfilled, and in some cases it is impracticable to attaiu such a relatiou of the parts. The mutual action of the pawl and ratchet wheel upon each other must therefore always be cousidered. If the flank of the tooth of a spur ratchet wheel (or a tangent to the flank of the outline is curved) does not form a right angle with the plane 2.3 of the pawl, there may exist, under some circumstances, a tendency to force the pawl into the tooth, or in other cases to throw it out of gear.

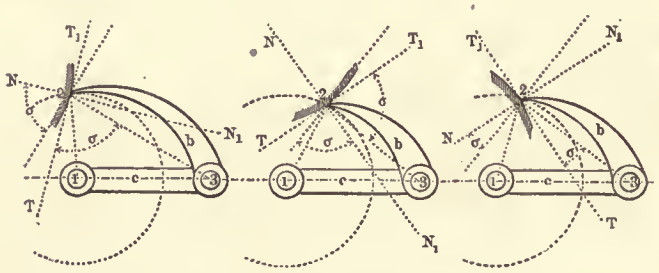


FIG. 663.

In Fig. 663 the various cases are examined. If at the point of contact 2 a normal $N N_1$ to the plane of the tooth flank be drawn, this normal may bear oue of three relations to the triangle 1 . 2 . 3. The "thrust-normal" $N N_1$ may fall without the triangle, or within the triangle, or it may fall upon one of the sides of the triangle.

If it falls upon 2 . 3, the thrust is neutral; if it falls upon 2 . 1, the thrust is zero; that is, there will be no action of the pawl upon the wheel, or *vice versa*,, barring the action of friction.

The angle δ between the line 2. 3 of the pawl and the tangent at 2, which is equal to the augle between the normal to 2 . 3, and the "thrust-normal," is called the angle of thrust. By considering this in connection with the angle of frictiou ϕ various relations are obtained.

On the one part, the force applied will act to alter the position of the pawl, either to or from the centre of the ratchet wheel: on the other part, it will also act to move the ratchet wheel forward aud backward.

These relations are classified for various conditions in the following table, in which a force which acts to force the poiut 2 from 1 is called an "outward" action, and the reverse, an "inward" action.

For the so-called "dead" ratchet tooth $\sigma = 90°$, case 1, hence there is tendency neither to iuward or outward movement. The variations above given are, however, more or less used in practice, and the table will be of service in considering the actiou in such cases. Some examples will be given here, and uumerous others may be found in subsequent illustrations.

In many cases it is desirable that the pawl should be held in engagement with the tooth by the action of the impelling force, as in Fig. 664, this falling uuder the fourth or sixth case. This

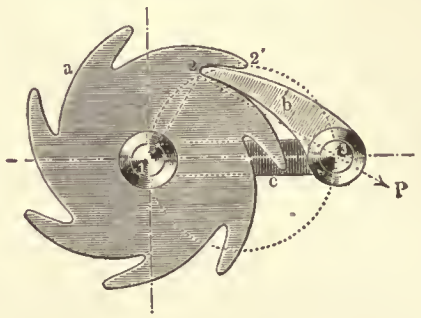


FIG. 664.

form of tooth insures the retentiou of the pawl in place after it has once eutered the tooth, and is sometimes used iu hoisting machinery when heavy loads are to be sustained; au application is also found in Ponyer's Coupling, Fig. 453, in which the secure engagement of the pawls is an important point.

Auother secure form of pawl is shown in Fig. 655.

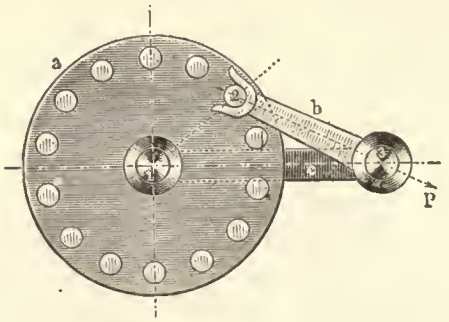


FIG. 665.

In this case the wheel is made with pin-teeth. The pawl has a forked end, the inner flauk tending to produce an inward movement, the outer flauk, outward movement.

In this case, as in the preceding, the wheel must be turned through a small augle before the pawl can be released.

The Thrust Action is:	The Impelling Force:	Outward Movement:	Inward Movement:
	ANGLE OF THRUST $\sigma = 90°$.		
1) neutral.	is without effect.	is without effect.	is without effect.
	ANGLE OF THRUST $\sigma < 90°$ and $> 90° - \phi$.		
2) inward.	is without effect.	produces reverse motion.	produces forward motion.
3) outward.	is without effect.	produces forward motion.	produces reverse motion.
	ANGLE OF THRUST $\sigma < 90° - \phi$ and $> \phi$.		
4) inward.	produces inward movement.	produces reverse motiou.	produced by iupelliug force.
5) outward.	produces outward movement.	produced by impelling force.	produces reverse motion.
	ANGLE OF THRUST $\sigma < \phi$.		
6) inward	produces inward movement.	is without effect.	produced by iupelling force.
7) outward.	produces outward movement.	produced by impelling force.	is without effect.
	ANGLE OF THRUST $\sigma = 0$.		
8) null.	produces inward movement.	produces friction only.	produces friction only.

§ 238.

THE SLIDING FLANKS.

We have discussed the action of the flanks of tooth and pawl which work together during the thrust. It is obvious that greater liberty is permitted in the form of the sliding flanks. It is only necessary that the form shall be such that the forward movement of the ratchet wheel shall lift the pawls properly out of gear. The forms fall under cases 4 to 7. The usual form is the common zig-zag ratchet, but others are also used, as in Figs. 666 and 667, in both of which the teeth are symmetrical.

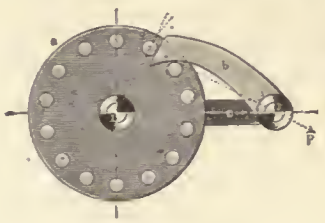


FIG. 666.

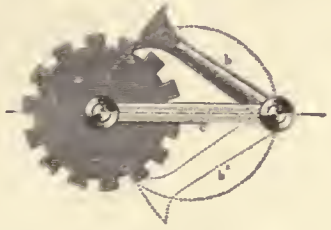


FIG. 667.

If it is desired to have the end of the pawl symmetrical, as in Fig. 667, this may be done, and the pawl may be reversed for a reverse movement as shown in the dotted lines. This form is used on the feed motion of some machine tools.

For some purposes it is desirable to form the thrust flank upon which the impelling force acts, in the same manner as the sliding flank, in which case the pawl must be held in gear by some extraneous force capable of resisting the maximum impelling force which it is desired shall act.

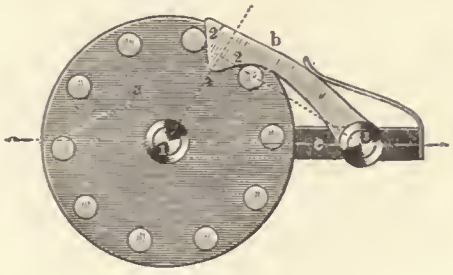


FIG. 668.

Such a form is shown in Fig. 668, which is similar to the nut-locking device shown previously in Fig. 241.

§ 239.

SPRING RATCHETS. QUADRANTS.

The form of ratchet last described possesses an especial property, that is, the action of the spring tends to force the pawl into the space as soon as the point is over the middle of the tooth. This causes the pawl to spring into engagement, hence the name spring ratchet, and this action causes an acceleration of the motion either forwards or backwards as the pawl is forced into the space. Applications of this form are found in repeating watches, in which the wheel is star-shaped, and hence called the star, while the pawl is called the star piu or springer.*

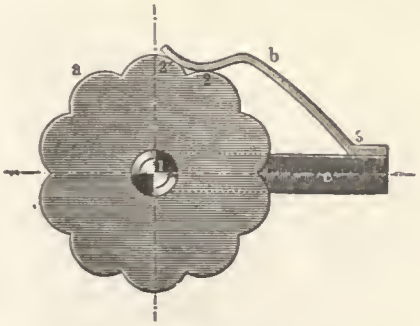


FIG. 669.

A modified form, Fig. 669, is used in Thomas' Calculating Machine. In this case the spring itself acts as the pawl, being attached directly to the arm without joint, forming a plate link. (See § 180.)

* This is shown later among the releasing ratchets.

Instead of using an entire ratchet wheel, a portion only need be made, if the required movement is but small, and in some cases reduced only to a single tooth, as in Fig. 670.

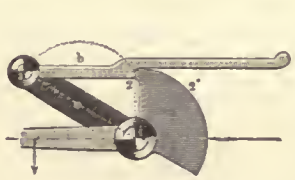


FIG. 670.

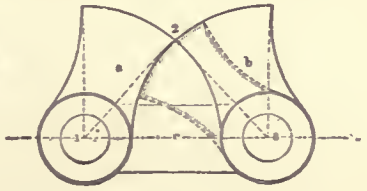


FIG. 671.

Sometimes the two members may be made of similar form, each working alternately upon the other, Fig, 671. Examples of this are found in the valve gear of some Cornish engines. These belong to the so-called "dead" ratchets, and are called, more or less appropriately, quadrants, or sextants.

§ 240.

METHODS OF SECURING PAWLS. SILENT RATCHETS.

The engagement of the pawl with the ratchet wheel is usually secured by the weight of the pawl, sometimes assisted by additional weights, as in Fig. 659. This may also be accomplished by means of a spring. It is desirable to give such springs but little movement, and small frictional resistance. It should therefore be placed near the axis 3, and is best placed in the line 1 . 3, so that 3 . 4,5, shall line in the same straight line, Fig. 672 *a*. If this cannot be conveniently done, it may at least

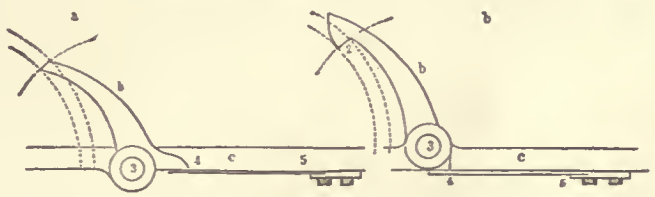


FIG. 672.

be made nearly so, as at *b*. A weak spring with much movement may be seen below in Fig. 680, yet at the same time the line 3 . 4 . 5, is only slightly varied from a straight line. In spinning machinery spiral springs of steel are used, and rubber springs have been used in propeller hoisting frames, Fig. 660.

In devices in which the pawls are sometimes above and sometimes below the wheel the springs are sometimes replaced by using several pawls. This is shown in Fig. 673, which is Wilber's ratchet for use in lawn mowers.

FIG. 673.

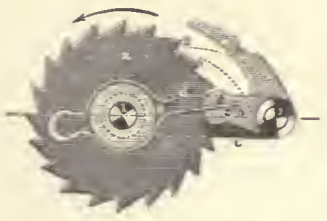


FIG. 674.

Three pawls, with half journal, are here used, and as the axis 1, lies in a horizontal position some one of the pawls is always in engagement by its weight. The movement of the teeth under the pawl, and the dropping of the latter into the spaces produces wear upon the parts, and to avoid this action various devices have been made; these being known as silent ratchets.

A very useful form of silent ratchet is shown in Fig. 674. The pawl is made with a projection 5, which is connected to a friction band *d*, which is carried upon a hub 4 on the ratchet wheel. When the wheel begins to move forwards, the arm 4 . 5 lifts the pawl *b* out of gear. The lift of the pawl is limited by the pins at 5. As the forward motion continues, the band slips upon 4; if reverse movement begins the pawl is at once thrown into gear. This is used in spinning mules, also in Pouyer's coupling, Fig. 453, in which two pawls, each with its own device are used. The principle involved in this device is capable of wide and useful application, as will be seen hereafter.

Another form of silent ratchet is shown in Uhlhorn's coupling, Fig. 454. In this case the pawls *b*, lie close against the flanks of the teeth. They are thrown into gear again by auxiliary ratchets, the spring pawls of which are not silent. These lift the pawls *b*, through a small angle when the engagement is

completed by the self-closiug action of the tooth flanks, Case 4 or 6, § 237.

Ratchet drills, etc., are often made with silent ratchets. Wilber's ratchet, Fig. 673, may be used for this purpose. If it is placed so that the axis, 1, is vertical, the friction of the pawls against the case will lead them into gear in the forward movement and draw them out on the return movement, the friction in this case takiug the place of any operating gear for the ratchets. Various other forms of silent ratchets are iu use.

§ 241.

SPECIAL FORMS OF RATCHET WHEELS.

In spur ratchet gears the axes 1 and 3 of the wheel and pawl lie parallel to each other. These axes, however, may be placed in the same manner as with gear wheels so that they are inclined or intersect each other. A great variety of forms of ratchet gearing may thus be made. The variations do not at first appear as importaut as they really are, but this will appear in the further discussion.

A form of ratchet for inclined axes is the crown ratchet, Fig. 675, which is used in capstans; the wheel, *a*, is stationary, and the arm and pawl, *b* and *c*, revolve.

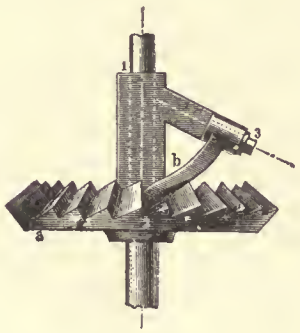


FIG. 675.

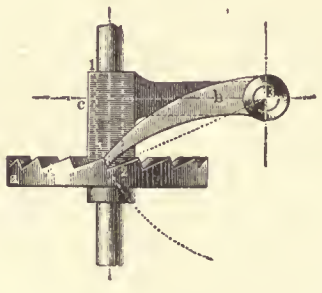


FIG. 676.

The forms shown in Fig. 676 and Fig. 677 are for non-intersecting axes, and use crown wheels also, and hence are called crown ratchets.

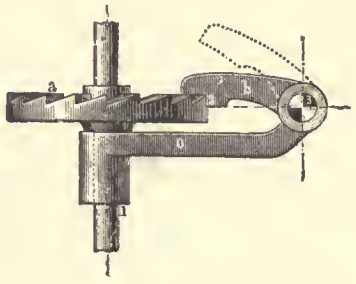


FIG. 677.

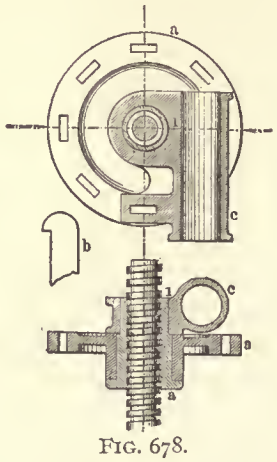


FIG. 678.

By making the wheel, *a*, in the form of a plane wheel, aud substituting a bolt for the pawl, some useful modificatious are made. Fig. 678 shows a form of ratchet used on a wine press, in which the bolt can readily be lifted out and placed in the successive holes as the lever arm is moved backward aud forward.

The ordinary jaw clutch coupliug, Fig. 441, is really only a form of crown ratchet with bolt pawl. The portion on the shaft *A* is the ratchet wheel, and the part fitted to slide on the shaft *B* corresponds to the bolt *b*.

§ 242.

MULTIPLE RATCHETS.

It is frequently desired to construct ratchet gearing so that the minimum limit of movement shall be less than the pitch of the teeth on the wheel. This is accomplished by using two or more pawls acting at correspondiug sub-divisions of the teeth. Such multiple ratchets exhibit a wide variety of forms and find many useful applications, and in mauy cases their true nature is not fully understood.

Fig. 679 is a multiple ratchet of common form, with three pawls, in which the pawls are set a distance from each other

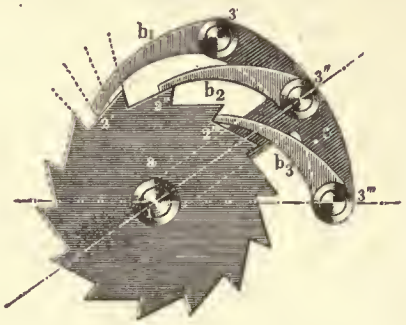


FIG. 679.

equal to $\frac{4}{3}$ of the pitch. From this arrangement the wheel can be moved spaces equal to

$$\frac{1}{3},\ \frac{2}{3},\ 1,\ 1\frac{1}{3},\ 1\frac{2}{3},\ \text{etc.},$$

of the pitch, that is, through $\frac{1}{3}$ the pitch and auy multiples of the same. This is sometimes used in saw mill feed motion, where a fine feed is required with a coarse pitch ratchet.

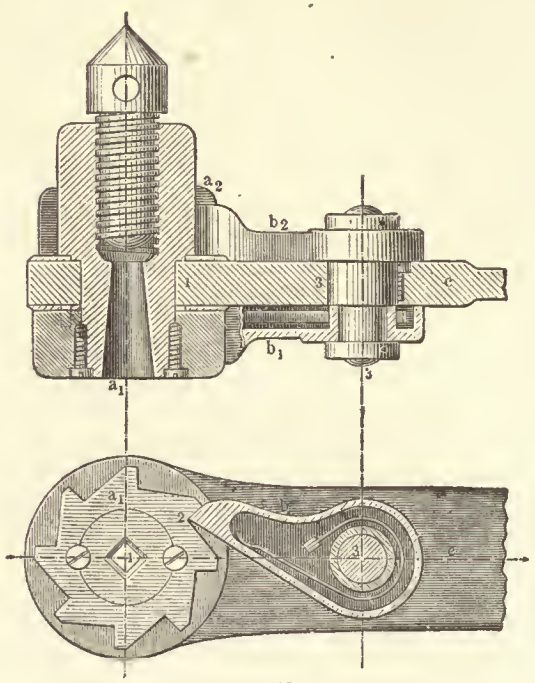


FIG. 680.

A double ratchet is used in Weston's Ratchet Brace, Fig. 680. The pawls b_1 and b_2 are placed one above and one below the arm *c*, and act on the two parts of the double ratchet wheel

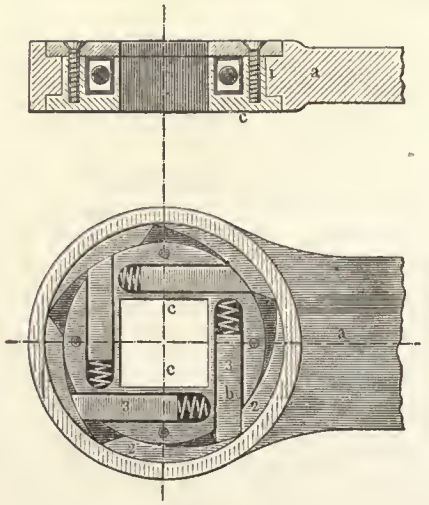


FIG. 681.

a_1, a_2. Another ratchet drill, also by Weston, with four pawls is shown in Fig. 681. This has an internal ratchet wheel with

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five teeth. Double ratchets are also found in Uhlhorn's coupling, Fig. 454, and Pouyer's coupling, Fig. 453.

If it is desired, the pitch may be halved, or divided into any two chosen portions, in which case the pawls may be made in one piece, Figs. 682, 683.

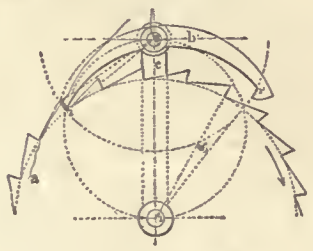


FIG. 682.

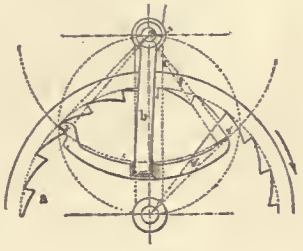


FIG. 683.

In each of these there is one pushing and one pulling pawl upon the axis 3, the pitch being halved and the pawls acting alternately. One form shows a spur wheel, the other an internal wheel. The form of the double pawl has caused this to be called an "anchor" ratchet.

If the wheel is a so-called "face" gear, that is, with the teeth projectiug from the face of a disc, two similar pawls may be used, both pushing or both pulling, and forming the same anchor, Figs. 684, 685.

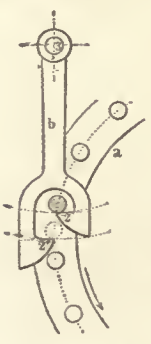


FIG. 684.

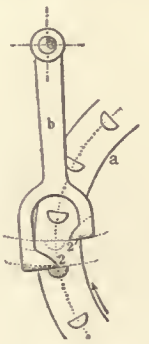


FIG. 685.

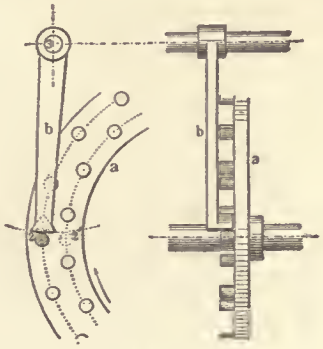


FIG. 686.

If the teeth are set alternately in two concentric rings, the two pawls may be merged into one, as in Fig. 686. This latter form appears to be new.

§ 243.

STEP RATCHETS.

A very instructive form of multiple ratchet gearing is obtaiued by combining more than two pawls into one piece, and arranging two such pawls to work together, aud this form is capable of

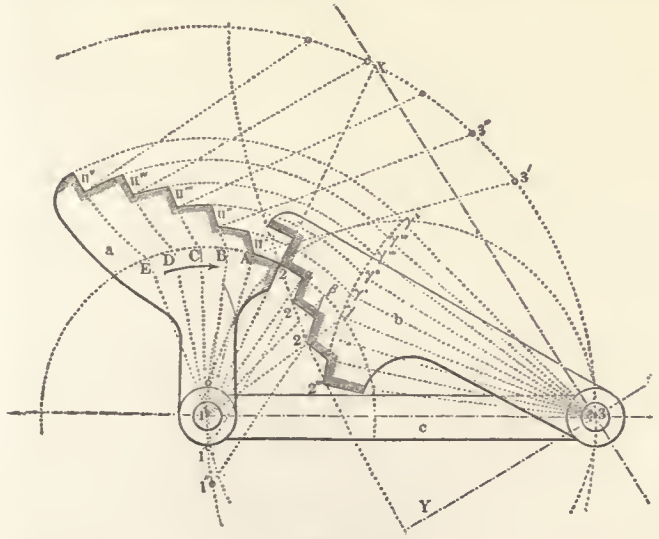


FIG. 687.

very extended application. In the ratchet combination *a b c*, Fig. 687, we have such a combination of two multiple pawls, with "dead" engagement, released by lifting the pawl *b*. The part *a*, which is impelled in the direction of the arrow is thus released, but is arrested again by the shoulder 2′. If the flank *a* 2′ is formed in the arc of a circle from the center 3, a farther lifting of *b* will cause, without resistance, a fresh release of *a*, again arrested at β 2″, and a similar action again for the flank γ 2‴; the points 2, *a*, β, γ all lying on a circle struck from the centre 1. Thus a continuous lifting of *b* will produce three successive advaucces of *a*. The angle of each advance of *a* may be called the augle of advance, and the correspondiug angle of lift of *b* the angle of release. In this case the angles of advance are all made equal to each other, as are also the angles of release. Wheu the position in which 2 is arrested by the flank γ 2‴ is reached, the angle of thrust σ becomes so small that further travel cannot well be obtained. If it is required to provide for still further movement it cau be done by making additional teeth behind 2, as II, II′, III″, etc., which will engage successively with *b* at 2‴. The construction of "dead" form of teeth is clearly shown in the diagram. As before, the angles of advance and release are made uniform. The mechanism as constructed will give nine successive engagements. The ratchet surfaces ou *b* are struck from 2, aud the sliding surfaces on *a* from 1; the flanks on *a* with a radius $3.2''' = 3\gamma$, the flanks on *b* with a radius 1.2.

It is to be noted that the two parts *a* and *b* are interchangeable in their functions, so that when the extreme notch II^{v} of *a* has been reached, *a* may be reversed in movement and *b* follow step by step to its former position.

Such step-ratchets are seldom used in practice, but mauy useful applications are possible.

In Fig. 688 is given a form of step ratchet arranged to give a uniform angle of advance together with uniform drop of the pawl. The pawl *a* is acted upon by the force indicated by the arrow, and teeth are upou a cam-shaped disc.

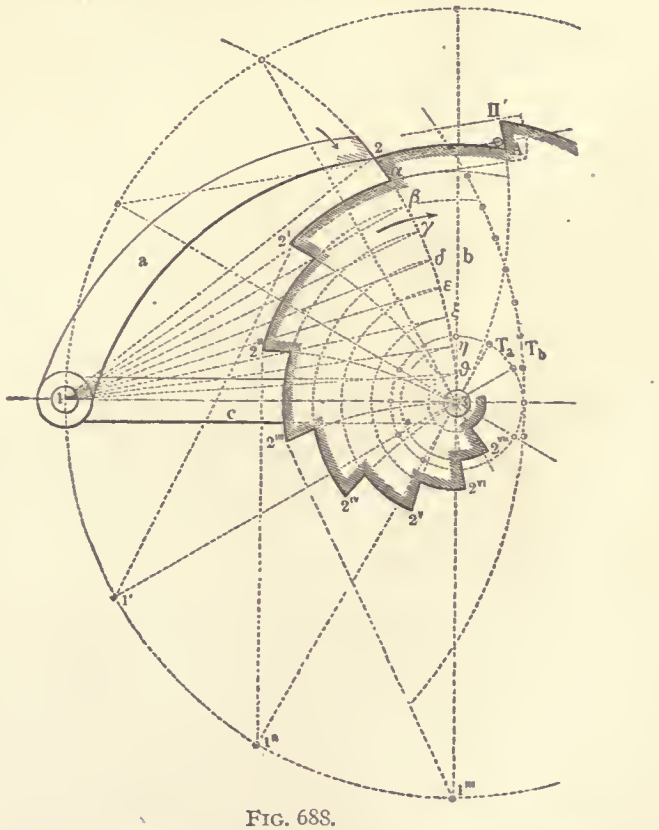


FIG. 688.

An arc with radius 1.2 passes through 3, the angles of release ou *b* are 30°, and the successive angles of drop of *a* are 5°. This form of ratcbet is used in the striking mechanism of repeating watches, and is knowu as a "snail" movement. The arm *a* in this case is frequently made ot the form shown in dotted lines at *A*. The construction of the snail is interesting. In order to fulfill the given conditions the points 2.2′, 2″ – – – must lie ou an abridged pericycloid; in the given case, where 1.2 = 1.3 it is the form known as a homoeentric pericycloid.* The points of the re-entering angles lie on a similar curve. The circles rolling together to describe these curves are showu in the figure Ta rolling about 1, and Tb about 3; their radii are inversely as the angles of drop and advance. If the parts *b* and *a* move continuously, these circles roll on each other; for the actual movements which take place, the drops of the pawl occur as the successive ringed points coincide.

* See Reuleaux's Theoretical Kinematics, § 24.

In the preceding step ratchet (Fig. 687) the angle of drop and of release were given the ratio 1 : 2. In this case the points of the teeth were on cycloids, those on *a* being on a pericycloid, those on *b* on a hypocycloid. The contact point of the generating circle falls without the figure on 3.1 prolonged. Since the radii of the circles are as 1 : 2 with internal contact the hypocycloid becomes an ellipse. A portion of the curve is given in the figure; 3 *X*---, and 3 *Y* are the semi diameters. The simplest form for the line of the teeth will be obtained by making 1.2 = 1.3, since for this case the ellipse for one diameter of the base circle on *b* becomes the straight liue 3 *X*.

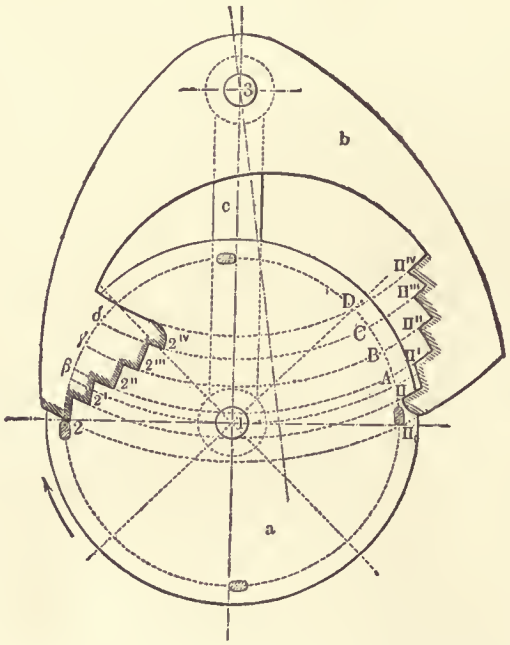


FIG. 689.

If it is desired to combine in the same piece two step pawls, Fig. 689, of which one set shall be iu tension and the other in compression, an anchor ratchet may be used. In this case a back and forth motion of the anchor permits an intermittent forward motion of the wheel. The anchor has ten steps and the wheel four teeth. This may be considered the general case of which Figs. 682 to 686 were special examples.

Numerous iuteresting problems may be solved by such devices, such as the conversion of continuous rotation of one piece into intermittent rotation of the second. Applicatious are found in clock aud watch-making.

The various modifications which may be made in the relative positions of the axes 2.1 and 2.3 permit a very great variety of step ratchets to be made.

§ 244.

STATIONARY RATCHETS.

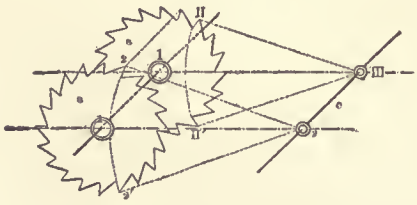


FIG. 690.

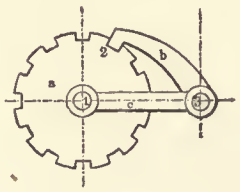


FIG. 691.

A stationary ratchet may be considered as a combination of a pair of running ratchets with the teeth facing in opposite directions. The scheme of such a combination is shown in Fig. 690. From the four possible positions of the parts 2.2′, II and II′ we may make the following double combinations:

2 with II,	2′ with II′,
2 with II′,	2′ with II.

The first two combinations are practically identical with the stationary ratchet, Fig. 691. The flauks of the two wheels give a notch for the space, while the teeth assume a dove-tail shape, and this form of stationary ratchet may be called a notched ratchet. The wheel will be firmly held by the so-called "dead" tooth, or when $(90° - \sigma) < \phi$, § 237. Mauy forms of this kind are used in practice.

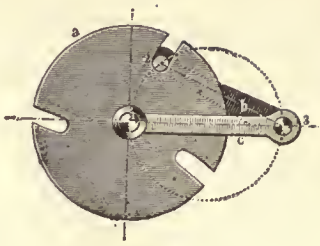


FIG. 692.

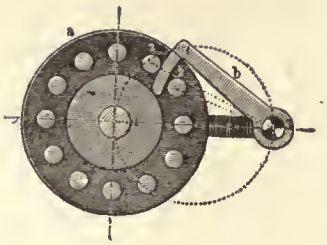


FIG. 693.

Figs. 692 and 693 show two modifications of the notched ratchet. The distinction between tension and compression pawls disappears, since the pawl is the same for either action. If the distance between the axes 1 and 3 is made infinitely great, the pawl becomes a sliding bolt. Such a form is shown in Fig. 694, which is for non-intersecting axes. The wheel is a crown wheel, and the pawl may have more than one notch.*

Another form of notched ratchet with axes 1 and 3 infinitely separated is shown iu Fig. 695, and is intended to hold a shaft from longitudinal motion, being used in connection with the disengaging gear of hoisting machinery, lathes and other similar machines.

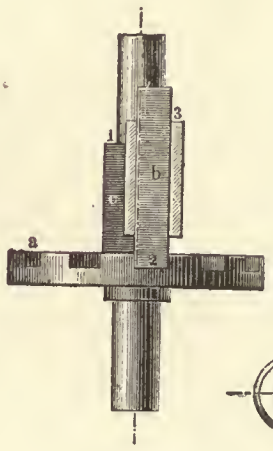


FIG. 694.

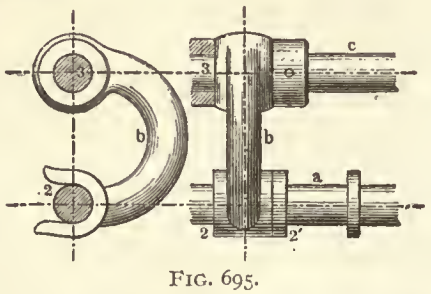


FIG. 695.

In this case the radius *a* is iufinitely great; the wheel *a* becomes a shaft.

The combination 2 with II′ and 2′ with II of Fig. 690, if we make 3 . 2 = III . II, gives a statiouary ratchet of the form shown in Fig. 696.

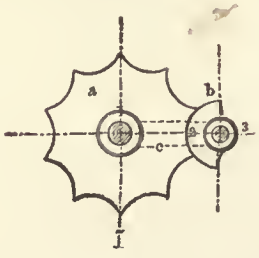


FIG. 696.

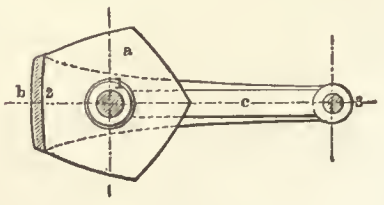
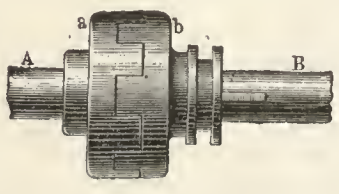


FIG. 697.

The pawl becomes a segment of a cylinder and works always in compression, or in the modification given in Fig. 697, always in teusion. This form may be called a cylinder ratchet. The form of Fig. 696 has many applications, as, for example, the Thomas' Calculating Machiue and similar work.

* This form of ratchet will be recognized as similar to the common jaw coupling. The shaft *A* carries the crown wheel *a*, the bolt corresponds to



the other half of the coupling *b*. The shaft *B* carries the part *b*, the latter sliding upon a feather.

The cylinder *b* may be entirely cut through as in Fig. 698, so that the segment shall fall entirely within the surroundiug circle. When it is placed opposite the teeth the wheel may be revolved in either direction as far as desired. If this movement is to be limited, as, for example, to a given pitch, it can be accomplished by cutting a correspondiug space in the cylinder, such as is shown in Fig. 699 *a*.

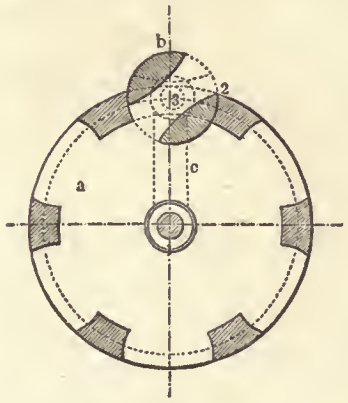


FIG. 698.

It is not necessary that the spaces in the wheel *a* should conform to the circular profile of the cylinder *b* (see § 237); the thrust is at two points on the right and left of 1 . 3, and it may be formed as at *b*, or pin teeth used as at *c*. This last figure shows the modification made in the notch of Fig. 698 to reduce the backlash of the wheel *a*. In Fig. 699 *a* the pitch circle of the pin gear *a* passes through the axis 3, and the gap in the cylinder is increased proportionally. When the wheel is impelled in the direction of the arrow, the pin 2 slips into the space in the

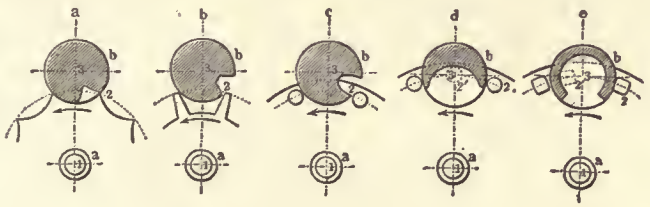


FIG. 699.

cylinder as soon as the opening is turned towards it far enough, but cannot pass out until the cylinder has turned back the same distance in the opposite direction, thus forming an intermitteut pitch movement.

This idea is more fully carried out in Fig. 699 *e*. In this case the inner profile of the space is concentric with the outside of the cylinder, as was also the case with the form shown in Fig. 697. In this case the tension and compression pawls are practically combined in one. When the opening moves into the proper position, the pin 2 moves to the point 2′, and completes the remainder of the pitch movement when the cylinder moves to the left again. This form may be made free from backlash by making the outside of the cylinder fill the space between two teeth, as in Fig. 700. If it is required that the intermitteut movement should divide the pitch into two equal parts, the arc of the pitch circle of *a*, which is the measure of the thickness of the teeth, must be equal to the arc cut off by the space in the cylinder. If backlash is permissible, the thickness of tooth may be reduced.*

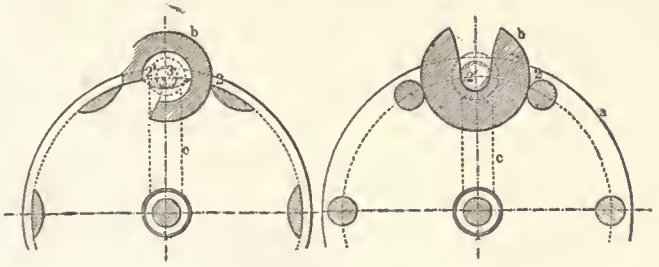


FIG. 700.

If we compare the various forms of cylinder ratchets with the notched ratchets, as, for example, in Fig. 692, it will be seen how the one may be derived from the other. If the pawl of Fig. 692 is giveu a row of teeth similar to the tooth 2, placed in a circle about a centre 3, and a space cut in *a* of the circular profile indicated, we obtain the same general and important form as is shown in Fig. 698.

In a similar manner the notched ratchet can be derived from the cylinder ratchet, and also inverted by transposing the parts, and all the modified forms obtained. The interchangeability of the two parts gives the midway form shown in Fig. 701, in which both pieces are the same, each being wheel and pawl for the other.*

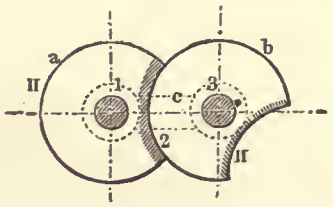


FIG. 701.

For the varied positions which may be given to the axes, a wide variety of cylinder ratchets can be made, many of these possessing useful applications. If the axes are at right angles, the cylinder may become a disc, as in Fig. 702; this form being used in Thomas' Calculating Machine, in which case the wheel *a* is made with but a single tooth.

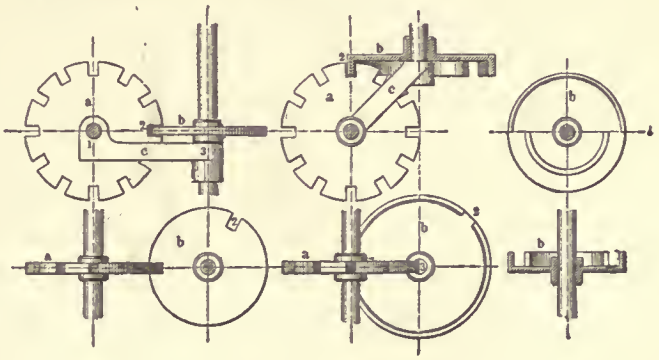


FIG. 702. FIG. 703. FIG. 704.

The form shown in Fig. 703 is derived from the globoid gearing of Class III, § 224, the ratchet being a cylindrical notched ring. Fig. 704 shows how a pitch ratchet can be made on this principle.

An examination of the preceding forms of stationary ratchets, in which the pawl consists of a revolviug member with a gap cut in it, will show one common property in all of them. This is the fact that an intermittent motion produced by successive release and engagement may be made either by a continuous rotation of the cylinder or by an oscillating movement. If, therefore, we have a continuously revolving shaft to deal with, or a vibrating member, the desired release or intermittent action of the part to be acted upon may in either case be obtained. Both forms are found successfully applied in actual practice.

§ 245.

RATCHETS OF PRECISION.

If we imagine the running ratchet of Fig. 682 so modified that upon the release of the pawl 2 that at 2′ shall enter at a point nearer the tooth than the middle of the pitch, as there shown, the principle will not be changed. If this modification is made to such an extent that the angle δ in both cases becomes zero, *i. e.*, the pawls so made that one enters into engagement at the instant of release of the other, we have the form shown in Fig. 705.

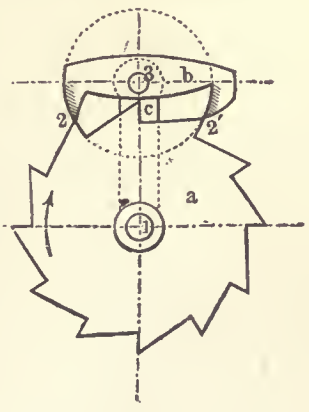


FIG. 705.

In this case the wheel *a*, being impelled in the direction of the arrow, can pass the points of both pawls at once. The slightest movement of the member *b* in either direction, however, will bring either 2 or 2′ into engagement and hold the wheel. This form is called a Ratchet of Precisiou, the especial one given being a running ratchet.

The principle is capable of various applications, and is also suitable for stationary ratchets, two forms of which are shown

* If the preceding forms are compared with Fig. 682, a similarity will be noticed. The "dead" ratchet with pawls of circular profile, of Fig. 682, are here, in Fig. 699, replaced by a gap of small angle; the compression pawl is at 2, the tension pawl at 2′, the arc 2 — 2′ is made very small, and the relative diameters very different.

* This form is similar to the running ratchet of Fig. 671.

in Figs. 706 and 707. In the latter case the pawl assumes the form of a bolt, shown in the illustration with several notches.

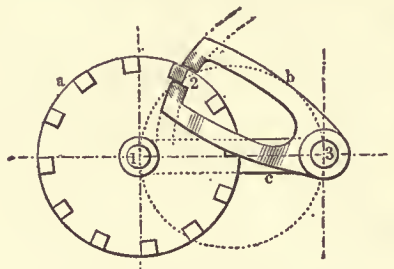


FIG. 706.

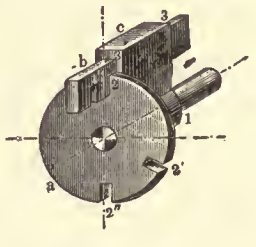


FIG. 707.

The practical applications of ratchets of precision are numerous, and examples will be given hereafter.

§ 246.

GENERAL FORM OF TOOTHED RATCHETS.

We have already seen that several forms of ratchet mechanism which have been described possess numerous points of similarity, and may be reversed and derived from each other, and hence it is not unreasonable to expect that some general form may exist from which the various special modifications can be derived, and in which the distinction between ratchet wheel and pawl, or checked and checking member, shall not exist, but each shall appear in both.

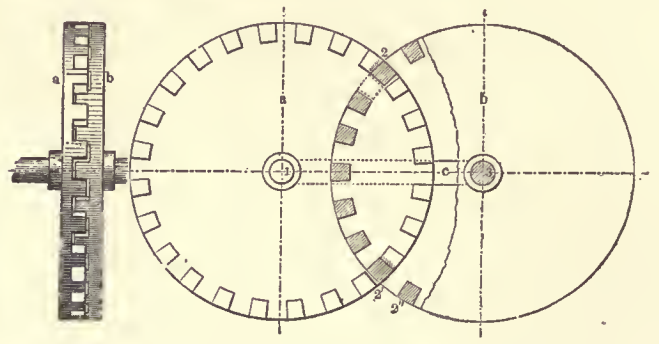


FIG. 708.

This general form is actually found in the combination of two disc face wheels (§ 211), with their centres carried on the same bar, Fig, 708, in such a manner that the teeth of both shall engage and be engaged by the other. In the illustration is shown such an arrangement made for a stationary notched ratchet. The wheel *b* engages as a pawl with the wheel *a* at 2 and 2′, and if it revolves a space of one-half a pitch, *a* is released. If *a*, however, revolves any given odd number of half-pitch angles only, *b* will be checked, and *a* become the pawl. In both cases we have a ratchet of precision of the same type as in Fig. 706.

The pitch ratchet with anchor pawl may also be thus derived; it is true the anchor form cannot so readily be shown as a pair of similar wheels, but it is clearly only another form of the same problem. The zig-zag ratchet, notched ratchet, step ratchet, or their combinations are all reducible to this general form, the only condition being that the direction of the force in the position of engagement of the checking member shall be such that the checked member cannot revolve. The intermediate forms show the "pawl lifting" action, § 237. It is evident that in some cases the checked member may have a forward movement, and in others a reverse movement. Since here, as in § 235, we may consider the link *c* as a checked member when the wheel is held fast, we may, from the combination of these parts, obtain four kinds of ratchets, viz.:

1. *c*,	stationary;	*a*,	checked;	*b*,	checking.
2. *c*,	"	*b*,	"	*a*,	"
3. *a*,	"	*c*,	"	*b*,	"
4. *b*,	"	*c*,	"	*a*,	"

As a general statement of the fundamental principle we have: *A toothed ratchet consists of a combination of a pair of gear wheels, or of portions of gear wheels, in which the teeth are so made that for certain positions of the wheels the resultant of the pressures on the teeth of one of the wheels either passes through its axis, or differs from such direction by less than the angle of friction.*

§ 247.

DIMENSIONS OF PARTS OF RATCHET GEARING.

The great variety of ratchet gears in use makes it almost impracticable to prepare any compact rules for the determination of the dimensions of the various parts. The general proportions can be obtained for the various forms by comparison with similar preceding devices. For spur ratchet wheels similar proportions may be used as for spur gears with thumb-shaped teeth. § 212. The action of the pawl tends to produce shocks and this must not be overlooked in determining the thickness of the teeth. It is generally most convenient to give the pawl a curved profile, in which case the discussion of combined resistance, § 18, is to be considered. Pawls which are subject to frequent vibration are best made of steel, as are also those in which the superficial pressure is high.

§ 248.

RUNNING FRICTION RATCHETS.

The mechanical devices which are constructed to modify the relations between two moving bodies by means of friction, may be called by the general term of friction clutches.* Such a device, when so arranged that one member opposes a positive frictional resistance or check to the motion of the other in one direction under the action of an impelling force, constitutes a friction ratchet. Such devices may be divided, as before, into running and stationary ratchets, § 235, and the first form will now be considered.

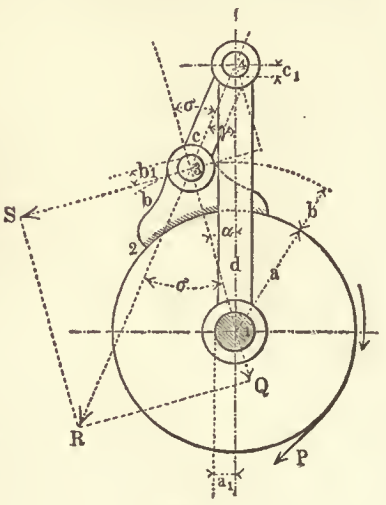


FIG. 709.

In Fig. 709 is shown a friction ratchet for parallel axes. In this case the friction block *b* is carried by the friction with the wheel *a*, when the latter begins to revolve in the direction of the arrow, that the pawl link *c* is crowded against the axis 4. The radial component *Q*, in the direction 4 . 3, exerts a pressure upon the brake block *b*. We also have the tangential component *S*, which we may consider as composed of two forces S_1 and S_2, acting in the same direction, which hold the friction at 1 and 2 in equilibrium. At 3 we have two opposite forces S_3 and S_4 which are capable of resisting the friction at 3 and 4 respectively.

The moment *M*, of the four friction forces is: $M=(S_1+S_2-S_3-S_4)(a+b)$. If we give the angles the symbols shown in the illustration, and make $1\,.\,2=a$, $2\,.\,3=b$, $3\,.\,4=c$, $4\,.\,1=d$, and call the radii of the several journals a_1, b_1, and c_1, we have:

$$S_1=\frac{Qfa_1}{a+b},\quad S_2=\frac{Qfa}{a+b},$$

$$S_4=\frac{Rfc_1}{c\cos\sigma}=\frac{\frac{Qf}{\cos\sigma}c_1}{c\cos\sigma},\text{ and }S_3=\frac{S_4=\frac{b_1}{c_1}(d\alpha+d\gamma)}{d\gamma}.$$

But we also have $(a+b)\sin\alpha=c\sin\gamma$.

From this we get

$$S_3=\frac{\frac{Rfb_1}{c\cos\sigma}d}{(a+b)\cos\alpha}=\frac{\frac{Qf}{\cos\sigma}\cdot\frac{b_1}{c\cos\sigma}d}{(a+b)\cos\alpha}$$

This gives for *M*:

$$M=Qf\left(\frac{a+a_1}{a+b}-\frac{1}{\cos^2\sigma}\,\frac{b_1d}{c(a+b)\cos\alpha}+\frac{c_1}{c}\right)(a+b)$$

The force *P* which acts at 2, to revolve the wheel in the direction of the arrow, may be considered as a couple. We then have for $M=Pa$:

$$\frac{Pa}{a+b}=Qf\left[\frac{a+a_1}{a+b}-\frac{1}{\cos^2\sigma}\left(\frac{b_1d}{c(a+b)\cos\alpha}+\frac{c_1}{c}\right)\right]$$

* This term only partially expresses the general scope of the German word "Bremswerke," for which there is no exact equivalent in English.—Trans.

But Q is a functiou of P, and in fact we have $\frac{Pa}{a+b}=Q\tan\sigma$.*

This gives:

$$\sin\sigma\cos\sigma - f\sin\sigma\,\frac{a+a_1}{a+b} =$$

$$= f\left[\frac{a+a_1}{a+b} - \left(\frac{b_1 d}{c(a+b)\cos\alpha} + \frac{c_1}{c}\right)\right]$$

aud since the augles σ and α are small, and become smaller under the action of the pressure, a sufficiently close approximation will be obtained by putting:

$$\text{siu}\ \sigma \leqq f\left[\frac{a+a_1}{a+b} - \left(\frac{b_1 d}{c(a+b)} + \frac{c_1}{c}\right)\right] \quad . \ . \ (233)$$

The followiug conditions must be noted. If an independent force outside of Q exerts a normal pressure N upon the circumference of the wheel, the friction Nf will diminish the force actiug to turu the wheel backward. If this is to enter iuto the resistance which is produced by Q, the magnitude of σ as given by equation (233) must be modified. If N becomes sufficiently great, Q may become zero; in such a case we obtaiu a stationary iustead of a running ratchet.

The pressure R on the pawl may become very great. We have $R=\frac{Q}{\cos\sigma}$ which may be made approximately:

$$R = \frac{Pa}{(a+b)\sin\sigma} \quad . \ . \ . \ . \ . \ . \ . \ . \ (234)$$

Example.—Let $a = 14.2''$, $a_1 = 1.6''$, $b = 2''$, $b_1 = 0.6''$, $c = 11.8''$, $c_1 = 0.6''$, and f at all four points $= 0.10$,† we have $d = a + b + c = 28''$ approximately, and

$$\sin\delta \leqq 0.10\left(\frac{15.8}{16.2} - \frac{0.6+28}{11.8+15.8} - \frac{0.6}{11.8}\right).$$

whence $\sin\sigma \leqq 0.0834$, which gives $\sigma = 4^\circ 47'$. To be on the safe side we will make $\sigma = 5\frac{1}{2}^\circ$, or $\sin\sigma = 0.0787$, and then get $R = P\frac{14.2}{16.2+0.0787} = 11.17P$. The exact length of d will be very slightly less than $a+b+c$.

As will be seen the ratio comes out unfavorably. The method of remedying this will be discussed hereafter.

The pawl c may also extend within the circle of the wheel, as in Fig. 710, iu which σ is au obtuse angle. The axis of the pawl

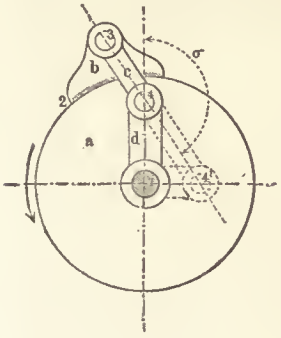


FIG. 710.

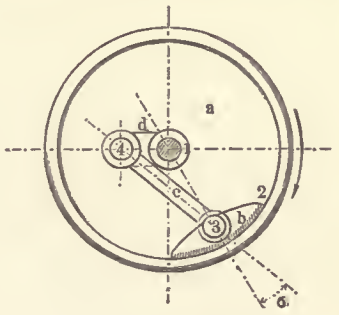


FIG. 711.

may be either at 4 or 4′, on 3 . 4 prolonged; the pawl is in this case a tension pawl. If a is made an internal wheel, we have the arrangemeut shown in Fig. 711, the pawl being uuder compression.

Especially noteworthy are those cases in which one or more of the axes are infinitely distant. In Fig. 712 is shown the case

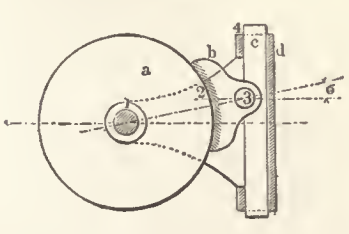


FIG. 712.

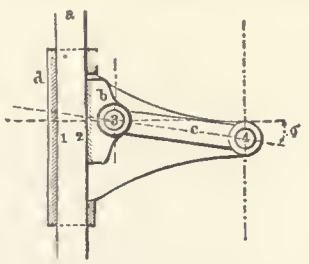


FIG. 713.

in which the length of pawl and also d and c_1 are of infinite length. We have for the augle of thrust, from (233)

$$\sin\sigma \leqq f\left[\frac{(a+a_1)}{(a+b)} - \frac{b_1}{(a+a_1)} - 1\right]$$

When a_1 is very small, release is difficult, and the arrangement does not appear to be very practical. If the arm a is made infinitely long, so must also a_1, and we get the case of Fig. 713. The wheel becomes a sliding bar. The relations

$$\sigma \leqq f\left[2 - \tfrac{1}{2}\left(\frac{b_1}{c}\right) - \left(\frac{c_1}{c}\right)\right]$$

give excellent action.

If with a and a_1 we make c infiuitely long, we obtain the constructiou of Fig. 714.

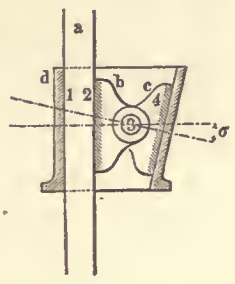


FIG. 714.

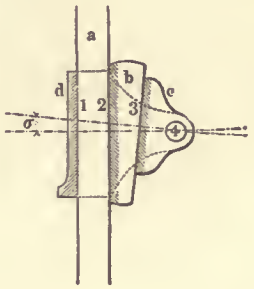


FIG. 715.

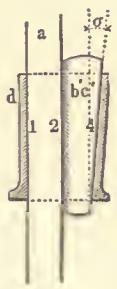


FIG. 716.

The conditions give: $\sin\sigma \leqq f(2-1)$. The joint at 3 insures full bearing for the surfaces at 1, 2 and 4. This is also the case with the joint at 4 in Fig. 715, iu which a, a_1, b_1, c and d are infinitely great, while b is the difference between two iufinitely long but opposite distauces, and hence is finite. We have the relation $\sigma \leqq f(2-1)$. By omitting the joint we obtaiu the simple construction shown iu Fig. 716. The friction block is in the form of a key or wedge, as in Fig. 715, aud the number of parts reduced to three (see also the following section).

The results as determined by calculation are not always practicable for the desired ratchet constructiou, which shows that the selection of the relation between the parts must be made with judgment, and care taken that those pieces which are subjected to heavy pressure shall not readily be deformed. As the preceding example indicates, the small size of the angle σ renders it an important point for consideration. In this case the actual length of d is only about $\frac{1}{16}''$ greater than $a+b+c$. The pressures Q and R act to lengthen d and shorten a, b and c, and if $P = 100$ pounds the differeuce may readily $= \frac{1}{32}''$, so that with only $\frac{1}{32}''$ additional wear, σ becomes zero, the parts b, c pass the centre, and the ratchet action ceases. This will indicate the couditions under which the ratchet becomes a useful device.

The uumerical maguitude of the parts a_1, c and d can be choseu so as to render the unavoidable wear least hurtful. This may be done with the sliding ratchet, Fig. 713, by making the length c sufficiently great. It is also important to devise means to prevent the block and pawl from being forced past the centre. A method of accomplishiug this is to substitute for the pin joint between the block and pawl, a curve or cam bearing, as in Fig. 717.

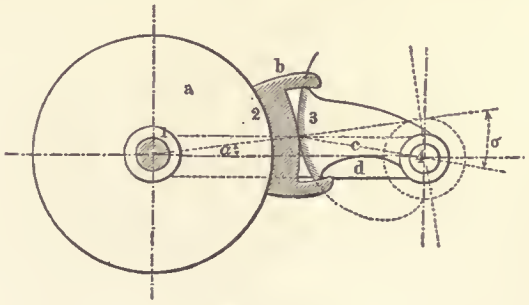


FIG. 717.

If the block is given a circular profile from the centre 1, and the pawl an evolute outline developed from a circle about 4, with radius d sin α, we shall have σ nearly coustant, notwithstauding the elastic yielding of the parts and the unavoidable wear. If the yieldiug between 1 . 4 is great, the radius of the circle ou which the evolute is generated may be made somewhat greater than d siu α. This construction appears to be new. A uuuiber of similar modificatious may be made, for which see the following section.

* The moment of the frictions produced at 2 and 1 by the force P is $= P(a+a_1) - Pa_1 = Pa$.

† If various coefficients of friction are to be used we have for S_1, S_2, S_3 and S_4, corresponding values f_1, f_2, f_3 and f_4.

It is desirable to examine the value of the coefficient of friction f, in order to increase it at the point 2. This cannot well be done by choice of material, since wood can scarcely be used in many cases, and lubrication of the rubbing surfaces is essential. The application of wedge profiles to wheel and friction block enables greater friction to be obtained, Fig. 718, as in the case of wedge friction wheels, § 196. Instead of the coefficient f, we have the value $\frac{f}{\sin \frac{1}{2}\theta}$. If the wedge angle $\theta = 60°$ this gives $2f$; for $\theta = 30°$, nearly $4f$. By combining this principle with the preceding forms, some very useful devices may be made.

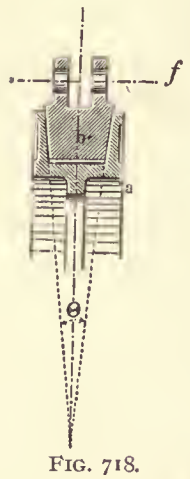


FIG. 718.

It is desirable to arrange the application of the force R so as to exert as small a distorting action upon the parts as possible. This may sometimes be done by arranging two or more friction pawls of similar kind to act upon one wheel. Some examples of such devices will be found in the following section. It must not be forgotten that the conditions for δ are not changed by the repetition of parts, since the numerical value of P does not enter into its determination.

There is yet another form of friction ratchet which is capable of being made very useful. By an examination of formula (223) it will be seen that the influence of the dimension a_1 is almost as great as that of a itself. If we increase a_1 to nearly the same magnitude as a, Fig. 719, we may approach

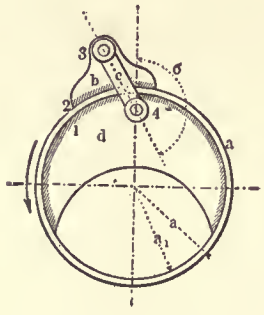


FIG. 719.

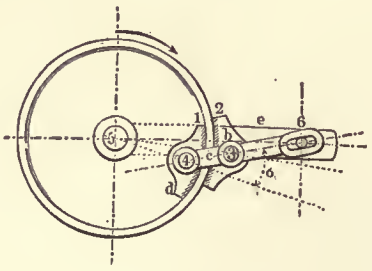


FIG. 720.

closely the minimum value of σ. This carries with it the disadvantage that the frictional resistance to the backward and forward movement at 1, is greatly increased, but this effect may be avoided by making a special bearing for the friction block axis and rearranging the parts somewhat as shown in Fig. 720. The attempt of a to move backward causes the pieces b and d to press upon the rim of a from without and within and grasp it firmly. The angle σ may now be made twice as great as in the previous forms without danger, all other things remaining the same. A practical form of this device is shown in Fig. 721, as applied to saw mill feed motion. Here the screw motion $F\,G$ is intended to permit of a suitable degree of play for the lever c.* If we make $a_1 > a$, we have the form shown in Fig. 722, which seems quite practical, and when applied to a friction rack we obtain the form in Fig. 724. We shall return to the consideration of these double friction ratchets hereafter.

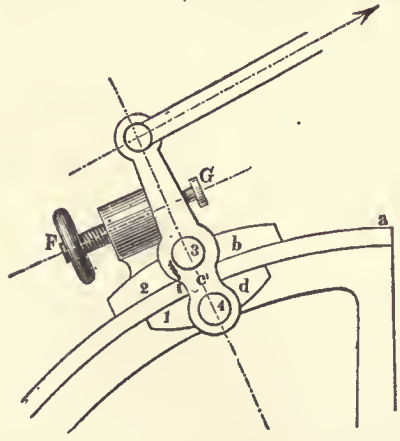


FIG. 721.

It must be remembered that these forms of friction ratchets are also applicable to other positions of axes and some resulting devices are in practical use.

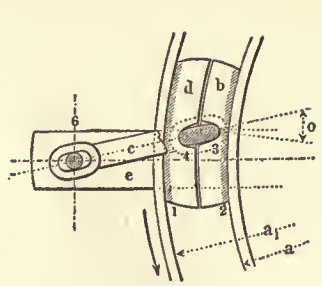


FIG. 722.

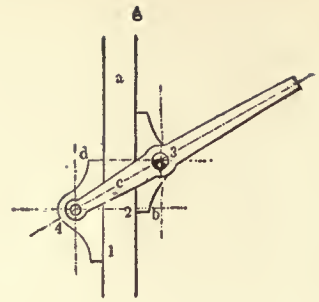


FIG. 723.

§ 249.

RUNNING FRICTION RATCHETS.

If the force to be transmitted is not very great, the intermediate friction block may be dispensed with and the curved contact surface be made directly upon the pawl. This reduces the mechanism to three parts: the wheel a, pawl b, and arm or connecting bar c, Fig. 724.

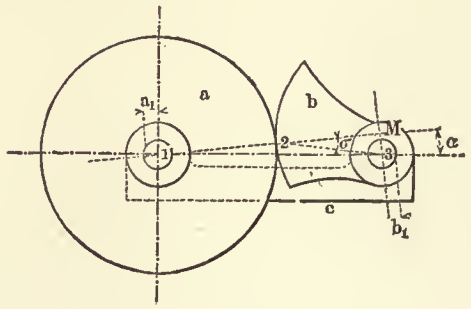


FIG. 724.

This form may be called a clamp ratchet, or since the pawl resembles the thumb-shaped teeth already described, the term "thumb-ratchet" may be used. The determination of the angle σ may readily be determined by what has preceded, and the following relations established:

$$\sin \sigma \leqq f\left(\frac{a + a_1}{a} - \frac{b_1}{b}\right) \quad \ldots\ldots\ldots \quad (235)$$

A suitable profile for the thumb pawl may be obtained as in Fig. 717, by using the evolute upon a base circle of radius $c \sin \alpha$, about 3 as a centre. This may be approximated by a circular arc struck from M, in which 3 M and 1 M are at right angles to each other.

If a and c are made infinitely great we have a form similar to Fig. 713, the straight profile 2 being an evolute of infinitely long radius, and the profile 3 a portion of the circumference of an infinitely great cylinder.

If the wheel be made a wedge friction wheel we have the form shown in Fig. 725. The wheel may be made with several grooves, by which means the pressure on each surface can be reduced (see § 196).

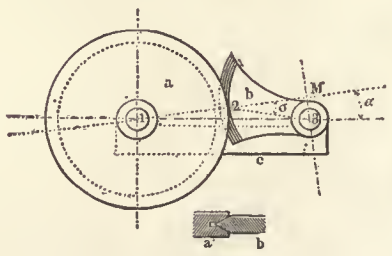


FIG. 725.

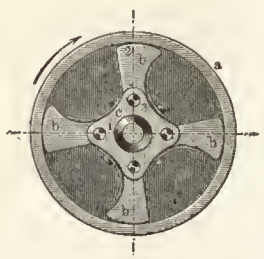


FIG. 726.

A variety of modifications can be made in the arrangement of the pawls. A clamp ratchet in which a repetition of pawls is used to distribute the pressure, is Dobo's ratchet, Fig. 726, which is very effectively used by A. Clair in his indicator.†

If we adapt the idea of Fig. 717 to a revolving journal using the "thumb" pawl at 3, we obtain a very useful modification of the clamp ratchet. The curve which is applied between 2 and 3 may be variously arranged. A very simple form is obtained

* See Goodeve. Elements of Mechanism, London, 1860, p. 49.

† See Morin, Notions géometriques sur les mouvements, Paris, 1861, p. 200.

by making the curves at 2 and 3 portions of the same circle, and the corresponding curve at 3 so found as to produce the required clamping action. The clamping piece *b* becomes a cylinder, Fig. 727.

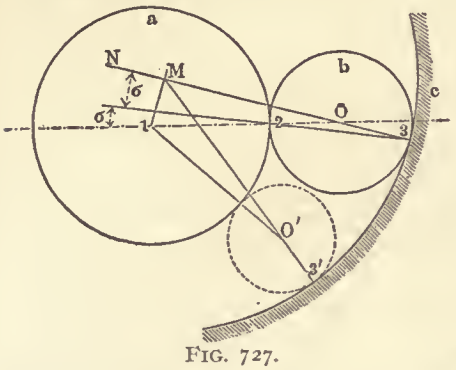


FIG. 727.

If we make the angle $O\ 2\ .\ 3 = \delta$, prolong the radius $3\ O$ to N, then will $3\ O\ N$ be the normal to the curve at the point of contact with *b* at 3, since the angle $3\ .\ 3\ O = \delta$. The curve for *c* is an arc of a circle struck from a centre *M*, on $3\ O\ N$, found by making $1\ M$ perpendicular to $3\ O\ N$. This curve is practically correct for a smaller clamping cylinder as at $O'\ 3'$, since the angle of thrust is very nearly the same as at $O\ 1\ .\ 2$, or in other words the effectiveness of the clamping action is not impaired as the cylinder is reduced by wear.

The pressures at 2 and 3, Fig. 728, are $T = \frac{Q}{\cos\sigma}$, $R = \frac{T}{\cos\sigma} = \frac{Q}{\cos^2\sigma}$, whence $O = P\left(\frac{a}{(a+a_1)f}\right)$.

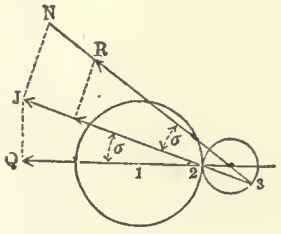


FIG. 728.

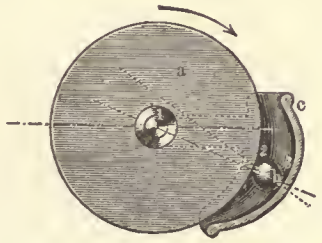


FIG. 729.

A practical application of the preceding form is shown in the checking device for sewing machines, Fig. 729. In this case a ball of rubber is substituted for the cylinder. Another similar device is the ratchet check used on the old Langen Gas Engine, Fig. 730. In this case a number of roller checks are used in order to distribute around the wheel *a*.

The whole forms a sort of continuous ratchet gearing in which the backward and forward movement of *c* imparts a continuous forward movement to the wheel *a*. When *c* moves in the direction of the arrow II, *a* is clamped and driven, while the parts are released when the motion is reversed in the direction I. The action of the centrifugal force tends to keep the checking cylinders in contact with the outer ring, and so insure prompt action upon the reversal of motion. The pressure upon these roller checks in the Langen Gas Engine was very great; wrought iron rollers wore out rapidly and phosphor bronze was substituted, although even these gradually altered their form under the pressure.

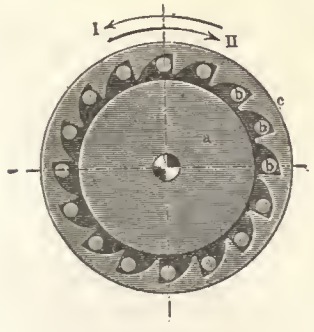


FIG. 730.

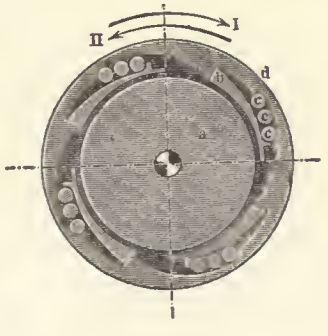


FIG. 731.

Another ratchet check used by Langen for the same purpose, is shown in Fig. 731. Here again we have a repetition of the parts, and also a return to the friction block, the rollers occupying the place of pawls. Comparing this with Fig. 709, the curved bearing surfaces correspond to the journals 3 and 4, and the action is similar to Fig. 727. The block *b* is arranged so that full clamping is obtained in a quarter turn. Friction ratchets with double clamps are also used as in Fig. 721 and the same principle appears in Fig. 732, which shows Saladin's "friction pawl."*

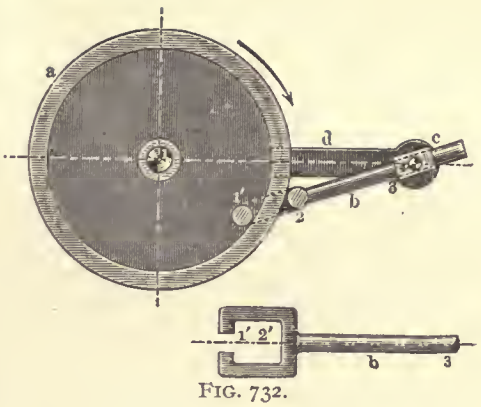


FIG. 732.

A similar device is shown in Fig. 733, as applied to a rod movement, and upon inspection the resemblance to the action of the "thumb" pawl will be seen.†

As long ago as 1798 Hornblower applied this idea to a rotary engine as shown in Fig. 734.‡

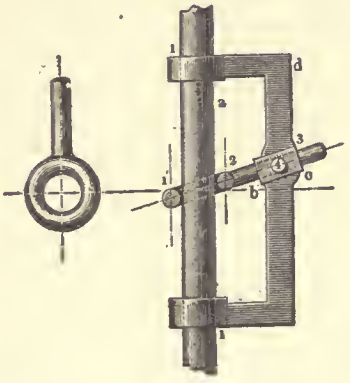


FIG. 733.

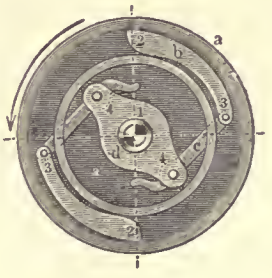


FIG. 734.

§ 250.

THE RELEASE OF FRICTION PAWLS.

The release of a friction pawl under pressure requires a certain degree of force, since there is always a friction between the rubbing surfaces which is at least equal to *P*, which must be overcome if the pawl is to be released under pressure. The release is to be effected under quite different conditions from those which obtain with toothed ratchets in which, for example, with "dead" engagement, only the "*f*th" part of *P* is exerted at the pawl point. The force required for release may be somewhat reduced by combining the action of two sets of friction surfaces of opposite direction of engagement, Figs. 735 and 736. The motion in the direction of the arrow tends to draw the pawl 2 into closer engagement, and at the same time to release that at 2'. By altering the relations of the distances 4–3 and 4–3', etc., any proportion of the moment may be used to hold the parts in gear. These forms appear to be new, and may be called "throttle ratchets." §

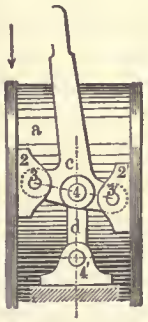


FIG. 735.

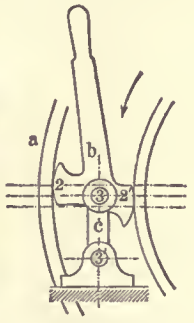


FIG. 736.

§ 251.

STATIONARY FRICTION RATCHETS.

A stationary friction ratchet may be defined as one in which the clamping action is not dependent upon the direction of

* See Bulletin von Mülhausen, XII. 1838, p. 296, also Salzenburg's Maschine details.

† A similar arrangement will be found to exist in the ring spinning frame. Here the pawl *b*, Fig. 732, is made of wire and held at 4 by the thread passing through an eye. Since the angle σ is made greater than we have taken it above, a tension or brake is necessary.

‡ A similar device is used by Carter, both being found in Farey's Steam Engine, Pl. XV, Figs. 8 and 9, also in Severin's Abhandlung, p. 141.

§ It may be noted that friction pawl actions are found in nature. Some fishes have such connections to certain bones or spines which they can thus elevate or depress. See O. Thiele, Die Sperrgelenke einiger Welse, Dorpat, 1879.

rotation of the wheel. Such a ratchet is shown in Fig. 737. 1 and 4 are parallel axes, the block acts with a radial pressure Q, such that for the circumferential force $\pm P$ the following conditions may exist:

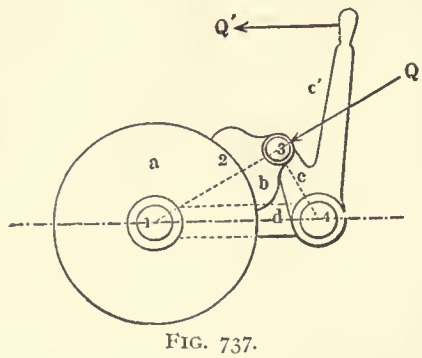


FIG. 737.

$$Q f (a + a_1) \geqq Pa, \text{ or:}$$

$$Q \geqq \frac{P}{f}\left(\frac{a}{a+a_1}\right) \quad (236)$$

If Q is less than the right hand expression, P will only be partially opposed, there will be motion from a toward d, with slipping at 2, or in other words, we have a brake, see § 248. This construction is frequently applied, although it requires a relatively large force at Q', acting through the lever $c\,c'$, giving increased pressure on the axle and much wear on the block. Various forms of lever connection are used to modify the ratio $Q' : Q$. By clearing the angle which the axes 1 and 4 make with each other, various convenient modifications may be made. The general scheme of such constructions is indicated in Fig. 738, in which the toggle connection gives a high ratio of Q' to Q; the block being guided in slides. By making a an internal wheel, a very practical arrangement is obtained as shown in Fossey's coupling, Fig. 450. Koechlin's coupling, Fig. 449, is also another form of friction ratchet gearing, the pressure in this case being applied by the medium of a right and left hand screw. The same is true of other forms of friction coupling, and the various methods of applying the pressure and reducing the wear, given in § 248, may also be applied in the design of mechanism for the purpose.

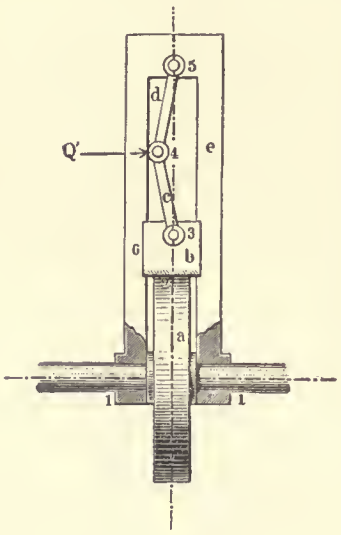


FIG. 738.

§ 252.

RELEASING RATCHETS.

Following the classification given in § 235, we have first discussed the various forms of ratchets for the general meaning of the term, and the five special classes remain to be considered, the next being the so-called Releasing Ratchets. Such ratchets must be considered primarily with regard to the question of release. When the release is to be effected by hand, various forms of handles or other connections to the pawls are readily devised. In most cases, however, the release is automatically effected, in which event, some mechanical tripping device is required.

The resisting force in such gearing is practically the same as the force required for release. It is applied usually by weights, springs, steam or air pressure, etc., and is variously intended to cause the released member to act with a predetermined velocity, either slow or rapid, as may be required. Many millions of releasing ratchets have been made for gun locks, and the various forms of releasing valve gears for steam engines, introduced by Corliss, but first invented by Sickles,* are of this class. In designing releasing valve gears, it is important that the valves should be closed quickly yet without sudden shock, and hence some form of buffer is essential. It is in the various devices for applying the force, for releasing, and for cushioning the released force that the many gears differ from each other. The original form of Corliss valve gear, and the modified form of Spencer & Inglis, are but little used on the continent, but these are well known, and hence examples will be given of some of the numerous modified trip valve gears which have been put into practical use.

Example 1.—Valve gear by Cail & Co., Paris, Fig. 739. a is the driven piece, a sector with one tooth, fast to the valve stem; b is the pawl; c the arm, loose on the hub of a; b' is the pawl spring; d the releasing cam. The trigger is that part of the pawl upon which journal 5 of the releasing cam is carried. The tooth profile at 2 should be "dead," but this is not the case, as the curve is struck from the centre 3. The bearing points on pawl and sector are of steel, separately inserted. The force which closes the valve is

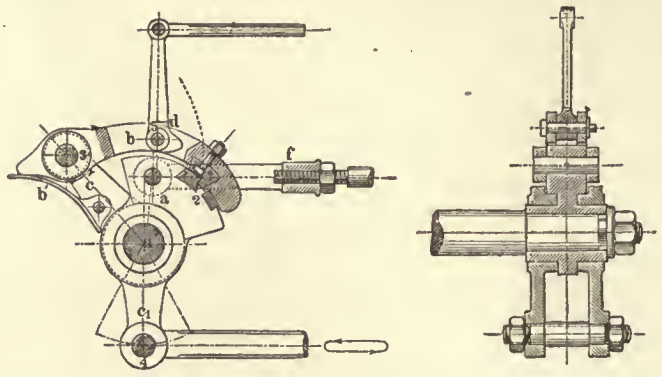
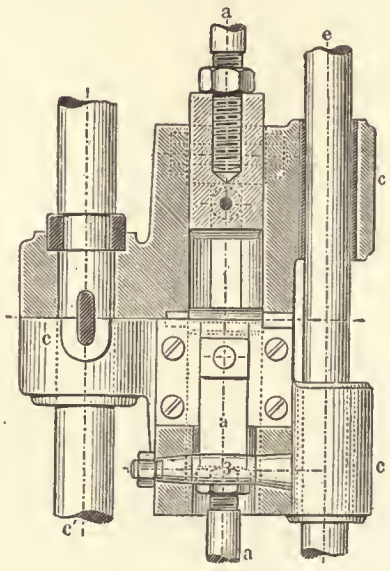
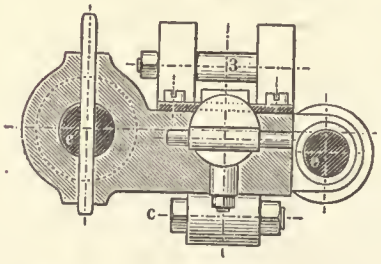


FIG. 739.

exerted by a spiral spring acting on the rod f, and the valve is opened by the rod connecting the arm c_1 with the engine motion. The cushion is effected by an air dash pot, also acting through the rod f, and the instant of release is determined by the governor.

Example 2.—Valve Gear by Wannich, of Brünn. In this case there are two





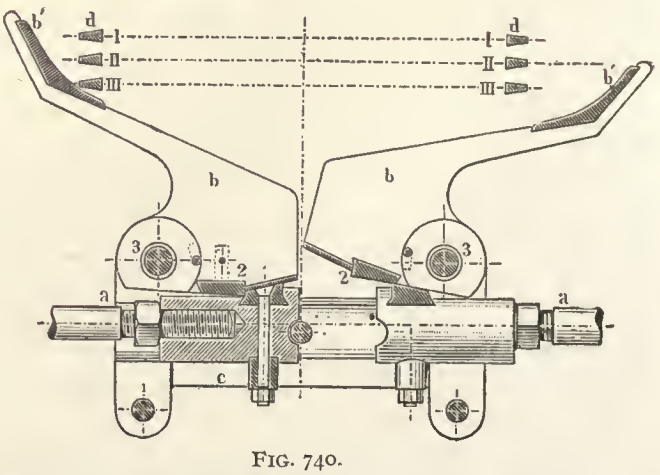


FIG. 740.

flat slide valves to be operated by the reciprocating movement of the piece c. It will be seen that this is a form of ratchet rack gearing. The valves are

* F. E. Sickles, of Providence, R. I., took out his first patent for a "trip cut-off" valve gear in 1842.

closed by steam pressure acting upon small auxiliary steam cylinders on the rods *a*, the cushion being provided by air buffers as in the preceding example. There are double pawls *b*, *b*, with "dead" tooth profiles, faced with steel; *c* is the pawl carrier, moved back and forth by the rod *c'*; *b' b'* are the triggers, and *d d* the releasing stops, the latter shown in three successive positions; *e* is a guide rod. The rod *c'* receives motion from an eccentric on the engine shaft.

Example 3.—Valve Gear by Powel, of Rouen, Fig. 741.* This is a form of rod ratchet gearing with bolt pawl. Here *b* is the driven piece in which the

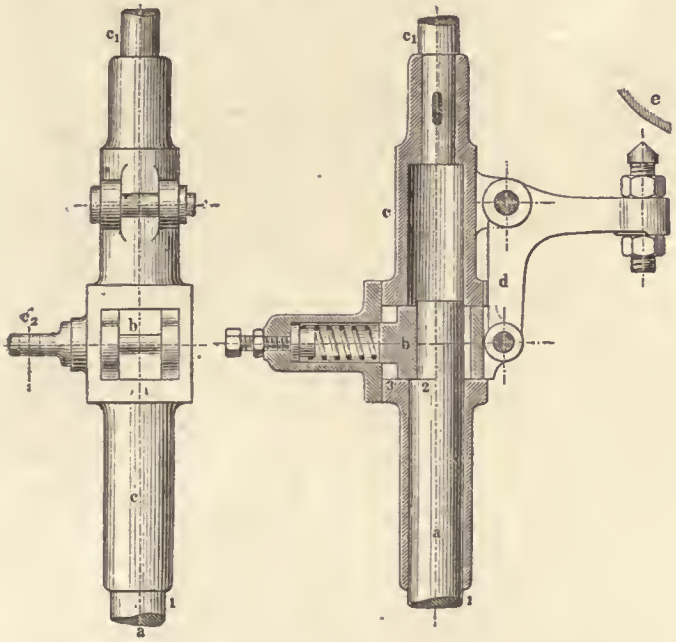


FIG. 741.

bolt *b* and its spring are carried. The rod *a* is moved up and down by an eccentric. The piece *c* is guided at c_2. The trigger *d* acts sooner or later, as the governor changes the position of the trip *e*. The force to close the valve is steam pressure acting on the upper part of the rod c_1, which also carries an air buffer.

The use of releasing ratchets in valve gear of steam engines is very old, being found in the old Newcomen pumping engines, and in the Cornish engine a similar gear is used to-day, while in recent times trip valve gearing of various designs have come into extended use, and some of the forms are shown in Figs. 670 and 671, not only for closing the valves, but also for opening them. These latter valve gears are intended to be operated by direct connection with the piston movement, while those in the preceding examples are operated from revolving crank shafts.

Releasing gears which are to be operated by reciprocating members, are sometimes constructed on quite a different principle, viz.: that of a weighted lever in nearly unstable equilibrium, so that it can be caused to fall to the right or left by means of a slight thrust, and so operate a releasing member. A form which was formerly much used, in which the lever is carried on a horizontal axis, is shown in Fig. 742.

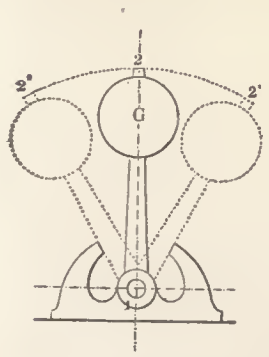


FIG. 742.

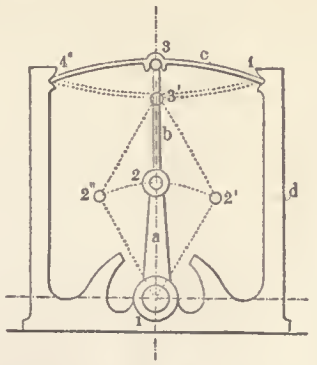


FIG. 743.

When the weight *G* is in the vertical position 1 . 2, the pressure acts directly downward upon the axis, the journal friction acting as a ratchet. The form is sometimes used on planing machines, screw-cutting machines, etc.

Another form is shown in Fig. 743. Here the pressure is due to a spring, acting through a link 3 – 2 upon 2 . 1.

* Further details of this and the preceding gear will be found in the Austrian Report on the Exposition of 1878. Section on Steam Engines by A. Riedler, Vienna, 1879.

A third form is that used in Shanks' planing machine, Fig. 744. In this case the lever, with its axis *a*, is at right angles to *b*, and the latter is provided with a roller. The limit of measurement of *a* is between 2′ and 2″.

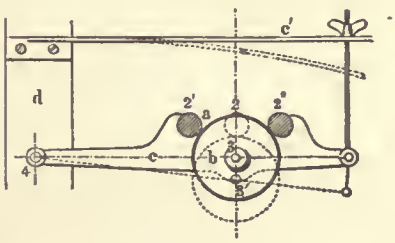


FIG. 744.

The forms of tumbling ratchets described in § 239, may be adapted as releasing gears, but it must not be forgotten that in such mechanisms provision must be made for the middle position of the ratchet.

A fourth form of tumbling gear, of which, indeed, there are many varieties, is the so-called "loop" of Hofmann's valve gear, Fig. 745. The loop *a* is made in the arc of a circle from a

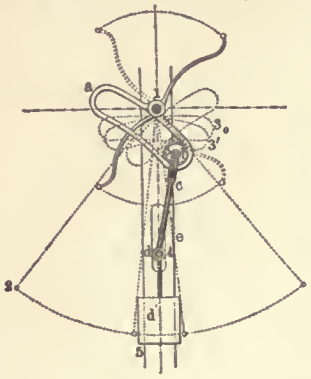


FIG. 745.

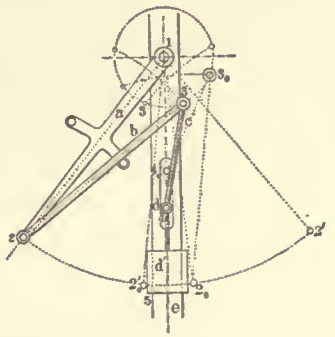


FIG. 746.

centre at 2, *b* is a heavy roller, with additional weight suspended at *d'*. When the loop or curved link is in either of the positions, 3_0 or 3′, the weight acts to continue the motion in the direction in which it started until the limit of travel is reached.

A swinging arm *b* may be substituted for the slot and roller, Fig. 746, and it will be seen that during the movement from the position $2_0\ 3_0$ to 2′ 3′ the tumbling action will take place and the arm *a* be carried over. The similarity to the previous tumbling gear will be apparent. If 2 . 3 be made infinitely long, the loop will become straight and the two forms will coincide. Hofmann has made the analogy to a ratchet train more complete by placing a ratchet so as to engage with the point 3 in the positions 3_0 and 3′, the release being made at the proper time by means of a cataract.*

In some cases it is desirable to make a gearing which shall be released by the action of a very small force. For this purpose a second releasing gear may be introduced, itself being readily released, and by its action permitting a blow to fall upon the trigger of the main gear. Such a device forms a releasing gear of the second order. Such an example is shown in the hair-trigger of a rifle.† Releasing ratchet gearings of higher orders are also found in textile machinery, as in the Jacquard loom, also in the striking gear of tower clocks and of repeating watches. Another example is found in the relay of the Morse telegraph, besides many other applications which will be considered hereafter.

§ 253.

CHECKING RATCHETS.

Checking ratchets are used in a great variety of machines, but their principal applications are found in machinery for hoisting and lowering heavy loads, as in mine lifts, elevators, and the like, to guard against accidents in case of the breakage of the 2 ropes. In the opinion of the writer these devices have not been as yet regarded as they should be, merely special cases of ratchet construction, and as such capable of utilizing all the various principles heretofore considered. When examined in this light their study will be greatly facilitated.

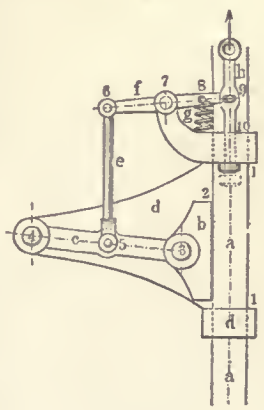


FIG. 747.

As a scheme of a general system for checking ratchets a rod friction ratchet may serve, Fig. 747, in which the rod *a* is held stationary, the loaded member *d* carries the ratchet, and the pawl *c* and friction block *b* are held out of

* See Zeitschrift des Vereins deutscher Ingenieure, 1860, Vol. IV., p. 209.

† Such hair-triggers were ingeniously applied in former times upon cross-bows.

engagement by the releasing lever *f*, and rod *e* as long as the hoisting connections *g* and *h*, are under stress. If the tension is released, the ratchet is thrown into gear and the parts clamped. If a toothed ratchet is used instead of a friction device, the block *b* is omitted. According to the manner in which the various constructive details from *a* to *h* are arranged, we obtain the various systems of checking ratchets which have found practical application.

A collection of such devices was exhibited by the Industrial Association (Verein für Gewerbfleiss) in 1879.* More than 80 designs were shown, of which only a few can be described. Many of the devices were rather designs for improved construction as regards strength and rigidity, rather than examples of mechanical ingenuity.

In most cases the clamping action takes place upon the upright timbers of the shaft; sometimes guide ropes are used. The greater number of designs shown used friction clamps, those of the type of Fig. 724 being shown, the thumb pawl being roughened, however, or finely toothed. The one which showed the most evidence of careful constructive design in accordance with the principles previously laid down in § 248, was that of Hoppe, shown, as attached to each side of the hoisting car, in Fig. 748.

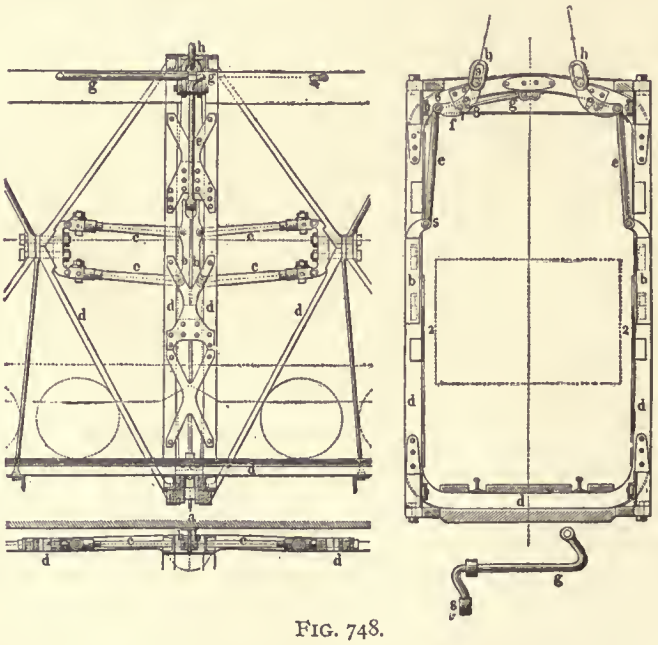


FIG. 748.

The form of friction pawl used is similar to that shown in Fig. 713, there being four pawls on each side of the car, or eight in all. The clamping action takes place upon the guide bars *a*, made of T iron, as shown. At 1 are the guide rods between *b* and *a*; at 2, the double clamp blocks of hardened steel, which are connected at 5 to the coupling rods *e*, *e*. The actuating spring *g* is a torsion spring (see Fig. VII, p. 19; also Fig. VIII, p. 19), secured to the roof of the car at *g'*, *g'*, and operated by the releasing gear *f* at 8, and transmitting action from 6 to 5 by the rods *e*, *e*, the connection being made by the links 9 to the double chain in such a manner that the arm *f* cannot be drawn too far out of position. The proper adjustment of the pawl arms is obtained by the keys on the rods *c*, *c*. Hoppe has taken into consideration the fact that the angle σ, see (233), must not pass beyond certain limits, or too great pressure would be exerted on the frame *d*, *d*, and hence has provided stops in the frame for the travel of the pawls *c*, *c*. The parts are so proportioned that a load of double that ever placed upon the car would be supported by the friction clamps before there would be an appreciable elastic yielding of the frame. The adjustments of the rods *c*, *c* provide for the change of relations due to wear. This apparatus does not bring the lowering car to a sudden standstill in case of breakage of the hoisting gear, but the shock is avoided by the gradual action of the friction brakes.

By using the author's device, shown in Fig. 717, at 3, the value of σ might be maintained constant, or by proper construction of the guides the wedge friction pawls, similar to Fig. 718, may be used; the blocks acting on both sides of the guide. This would reduce the stress upon the frame very materially.

The system of brakes used upon railway trains are really forms of friction checking ratchets. The shocks due to sudden stoppage are also to be avoided, and if the wheels are braked too firmly the sliding action is simply transferred to the rails.

§ 254.

CONTINUOUS RUNNING RATCHETS.

Continuous ratchets (§ 235, No. 4) consist of such combinations of pawl mechanism as act to drive a member in a given direction with practically a continuous determinate motion. This may be effected by combining two single running ratchets in such a manner that they both act upon the same wheel, one pawl attached to the arm *c*, which is stationary, the other swinging about the axis 1, Fig. 749, this being a very common form.

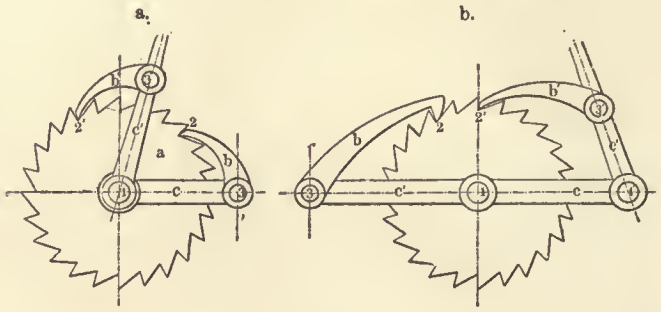


FIG. 749.

In this case 3 . 2 is the checking pawl, and 3′ . 2′ the driving pawl. A movement of the driving pawl, if a little more than one tooth space, moves the wheel one tooth; a little more movement than two spaces moves it two teeth; and a regular back and forth motion gives a forward movement at intervals of a single pitch space.

If this device is made with step ratchets, as in § 243, the pitch may be subdivided into 2, 3, 4 or more parts, and for some purposes, such as saw-mill feed motions, this is very desirable.

If the arm which carries the feed pawl swings about an axis 4, removed from 1, Fig. 749 *b*, there will be a movement between the pawl and the point of application 2 on the wheel; while in the arrangement shown at *a* the motion of the two is identical, and hence no wear occurs.

The two pawls may be connected so that both of them become drivers. If they are arranged so that their movement is alternate, as in Fig. 750 *a*, the wheel will be moved forward for

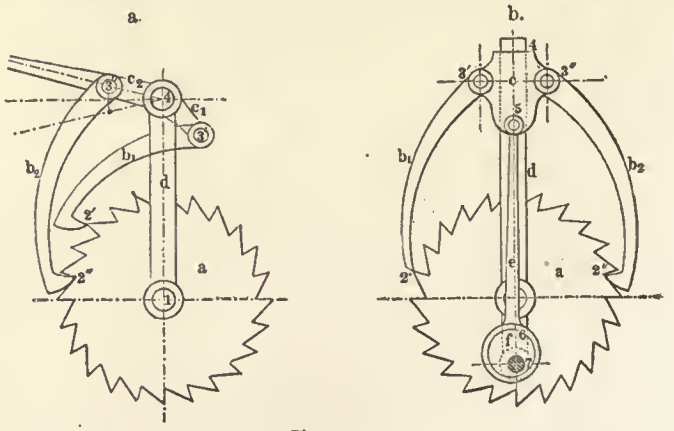


FIG. 750.

the movement of the lever in each direction, giving a double-acting ratchet motion, the so-called Lagarousse Ratchet.* This may be also accomplished in various ways, as in Fig. 750 *b*. For any movement of the arm which is less than 1 and more than ½ the pitch, the wheel will be moved 1 pitch for each vibration, and hence for a half vibration a feed of a half tooth may be obtained. Step pawls may also be used with these devices to obtain further subdivisions.

If, in Fig. 750 *a*, we hold the lever c_1 c_2 rigidly, and instead permit the arm *d* to vibrate with the same angle about the axis 4, the wheel moving with it, we obtain the same relative feed motion.† This has been used by Thomson in a telegraph apparatus.

* Berliner Verhandlungen, 1879, p. 345. Prize essay by Dr. F. Nitzsch, on Safety Checking Devices for Mining Apparatus. See also Mairs, Berg und Huttenmänn, Z., 1879, p. 361.

* Named from the inventor, M. de la Garousse, and used in 1737. Bélidor, Arch. Hydraulique.

† This is the ordinary kinematic inversion.

A contiuuous ratchet gearing may be so arranged that backward movement of the wheel is utilized to compel a uniform division of motiou.

This is the case with the feed motion used by Gebrüder Mauser, of Oberndorf, in their revolvers, Fig. 751. In this case

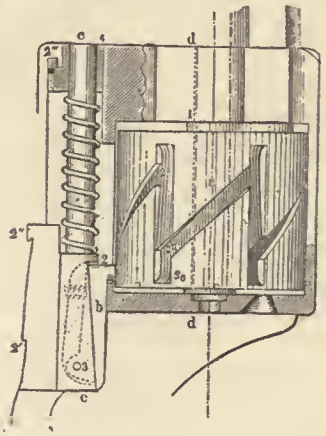


FIG. 751.

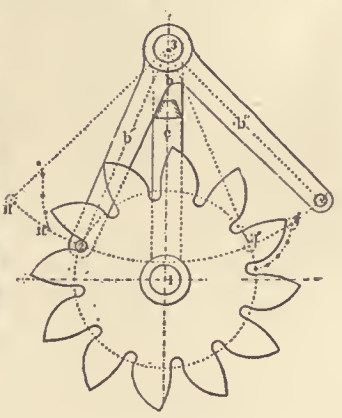


FIG. 752.

a crown wheel is used (see Figs. 677 and 678). The wheel is at *a*; *b* is the feed pawl, jointed at 3 to the slide *c*, the whole being carried in the frame *d*. The zig-zag profile is formed in the rim of the crown wheel, one portion being parallel to the axis, the other spirally inclined, so that the angle of thrust is $\sigma < 90° - \phi$ and $> \phi$ (§ 237, cases 4 and 5). The movement of the pawl produces a backward movement of the wheel. It should be noted that at 2′ and 2″ steps are made in ends of the tooth profiles in order to guide the pawl into the proper path and keep it from reversing.

The anchor ratchet of Fig. 682 may be used for a feed motion, as in Fig. 752, iu which there is also the reverse action of the wheel, in accordance with the notation of § 237. Here the wheel is at *a* and the anchor at *b′ b″*. When the latter is moved into the position shown by the dotted lines, the wheel is moved backward ½ pitch, and the return vibration completes the pitch movement. In order that the anchor shall enter the teeth properly, the movement should be quick, especially at the eutrauce of the pawl iuto the space. This is well obtained by electro-magnetic action.

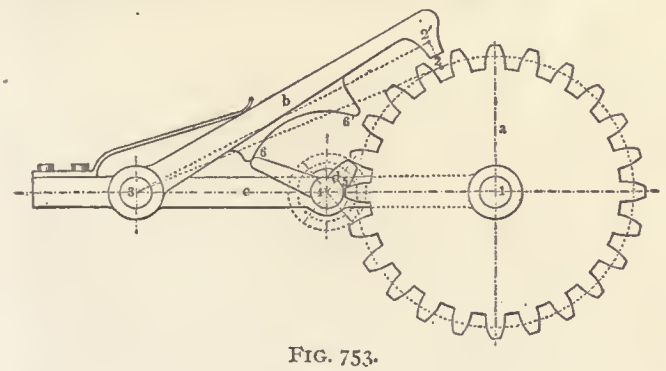


FIG. 753.

§ 255.

CONTINUOUS RATCHETS WITH LOCKING TEETH.

If it is desired to use ratchets according to the method given in Fig. 749, additional parts must be devised to move the pawl in and out of gear. A simple method of accomplishing this result is to use a single tooth wheel for the driver, and operate the pawl in the same manner as in Fig. 753.

Before the single tooth 5 begins to drive the wheel *a*, the arm 6 lifts the pawl *b* and lowers it into the next space just as the tooth ceases to drive. In this case the usual gear tooth profiles may be used. Still better is the "dead" tooth profile of Fig. 754, iu which the entrance and withdrawal of the pin tooth both lock the wheel while the pawl is being lowered.

This form may also be used for rack feed movement, Fig. 755. In this case the profile of the pin tooth is formed in several arcs; 2′ 2‴ being struck from 3, and 2″ 2‴ and 2′ 2ᴵᵛ being the paths of the corners of the space (see § 203).

By using the cylinder ratchet, as shown in Fig. 696, the number of parts can be reduced, since the driving gear and checking pawl may be combined in the same member. The resulting forms, Figs. 756 to 758, are variously called: Maltese Cross;

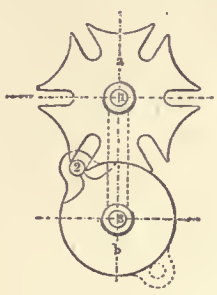


FIG. 756.

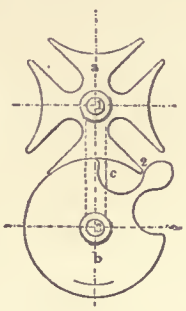


FIG. 757.

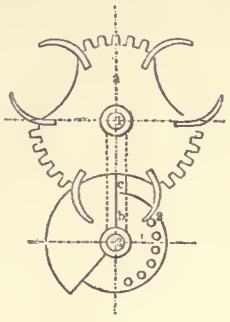


FIG. 758.

Geneva Stop, used in Swiss watches, in which case one of the tooth sections is filled out; or after Redtenbacher we may call them single tooth gears, although this is hardly correct, for the general form of Fig. 758 may have several teeth, and a second tooth is dotted in Fig. 756.

A great number of variations may be made of these cylinder ratchet motions. An interesting form is the intermittent gearing of Brauer, Fig. 759.*

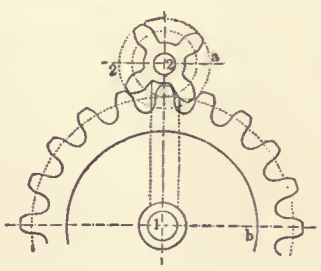


FIG. 759.

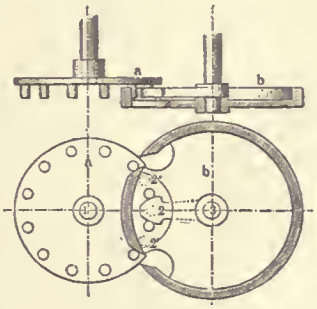


FIG. 760.

The pinion *a* is the driver, and the wheel *b* is driven, and between the passage of each tooth of the pinion the driven gear remains stationary for a short space, about ⅛ of the pitch. The points of the teeth of the driven wheel here act as ratchet teeth, in a similar manner to the arc of repose of the single ratchet gearing of Fig. 756.

The cylinder ratchet gearing of Fig. 760 is similar to that shown in Fig. 700, aud is used in the counting mechanism of English gas meters. In Fig. 761 is a modified spiral ratchet of

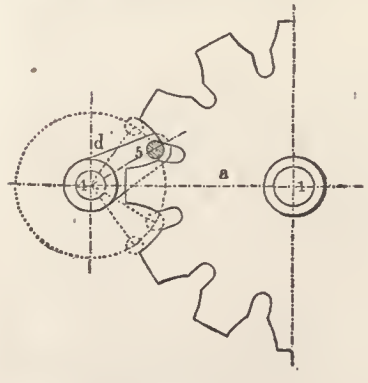


FIG. 754.

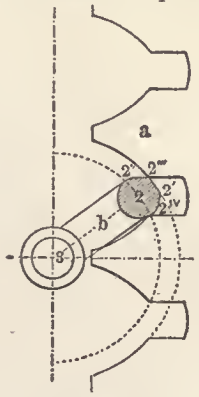


FIG. 755.

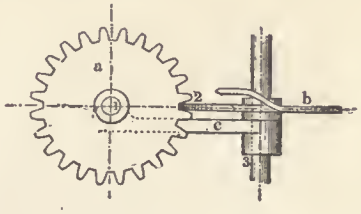


FIG. 761.

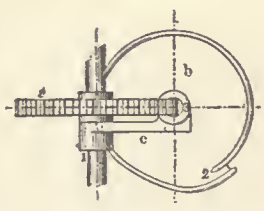


FIG. 762.

the same general type as Fig. 702, with only a portion of the path of *b* in a spiral, and a similar variation of Fig. 704 is shown in Fig. 762.

* Royal German Patent, No. 5583, 1878.

§ 256.

LOCKING RATCHETS.

Locking ratchets include all the numerous devices by which the parts of a mechanism are firmly held against the action of external forces, and yet readily and definitely released when desired (see § 235, No. 5); thus the various clutch couplings are included, also car-couplers and similar devices.

Locking ratchets occur frequently in the mechanism of firearms, especially to prevent the danger of premature discharge, etc. The great refinements which have been introduced in such weapons during the last ten years include especially the application of various forms of ratchets. The following single instance will serve to illustrate:

The mechanism of the well-known Mauser revolver may be divided into two series; one to effect the discharge and the other to unload or remove the empty shell from the chamber. The first may be called the discharging mechanism, the second the unloading mechanism. We then have the following details:

A. Discharging Mechanism.

This includes the revolving chamber, barrel, hammer, spring and accompanying smaller parts, giving as combinations:

1. Hammer, spring-rod and trigger = ratchet rack, as Fig. 659.
2. Spring-rod and trigger, acting as locking ratchet for the above, as Fig. 664.
3. Spring-rod, pawl and revolving chamber = continuous ratchet with crown wheel and bolt pawl, as Fig. 751.
4. Securing pawl and revolving chamber = locking ratchet, as Fig. 677.
5. Revolving chamber and pawl, forming a ratchet gearing with limited travel.
6. Tumbling ratchet and securing pawl = ratchet gearing for three positions, Fig. 669.
7. Catch on the axis of hammer = locking ratchet, as Fig. 695.
8. Trigger guard and pin = locking ratchet and stationary pawl.
9. Checking-plug and trigger = locking ratchet with stationary pawl.
10. Rifled barrel and bullet = screw and nut.

B. Unloading Mechanism.

This includes an axial slide which catches under the rim of the empty cartridge shell to withdraw it, actuated by a toothed sector and revolving clamp and axis called the ring clamp. These include the following combinations:

11. Unloading slide and sector = slide with rack and pinion, Fig. 381.
12. Axis of revolving chamber, with pawl to prevent endlong motion, = locking ratchet gear, as Fig 695.
13. Ring clamp, barrel and chamber bearing = locking ratchet gear with stationary pawl, as Fig. 654.
14. Ring clamp axis and axis of securing pawl = locking ratchet, as Fig. 701, forming with (13) a locking ratchet gear of the second order.
15. Ring clamp axis upon the reverse motion of the ring clamp forms, with the axis of the securing pawl, a locking ratchet gear, which combines with (4) to form a similar gear of the second order.
16. Securing pawl acts as a catch for the axis of the ring clamp in the axial direction to form a locking ratchet gear, as Fig. 695, forming also with (4) a similar gear of the second order.
17. Ring clamp hub and axis of securing pawl = locking ratchet, as Fig. 695, and with (4) gives one of the second order.

This analysis shows that in the Mauser revolver there are 17 mechanical combinations; these are composed of 26 pieces. Classified, these are as follows: 1 releasing ratchet, 1 continuous ratchet, 2 driving ratchets, 11 locking ratchets, of which four are of the second order, 1 screw motion and 1 slide motion.

A very important application of locking ratchet mechanism is found in the signal apparatus of Saxby & Farmer for use on railways, and made in Germany by Henning, Büsing and others. This includes many ratchets of higher orders, reaching to the tenth, twelfth, or even higher. When this is used in combination with the electric systems of Siemens & Halske, as in the block system, we have the further combination of two systems of the higher order with each other.

A branch of locking ratchets which exhibits a great variety of applications is found in the different kinds of locks, such as are used for securing doors, gates, chests, etc. These extend from the most primitive forms, made of wood, to the most refined productions of exact mechanism, and their study possesses an historic and ethnographic interest in addition to their mechanical value.

A door forms itself a ratchet combination; the door being the part *b*, the strike the part *c*, and the bolt or other piece which keeps it from being opened is the part *a*; doors with latch bolts being running ratchets, and doors with dead bolts being stationary ratchets. A simple lift latch and door, as the furnace door shown in Fig. 763, is really a section of a crown ratchet wheel with running ratchet gearing.

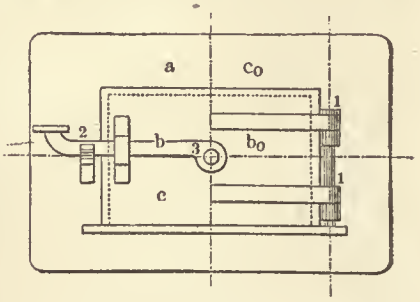


FIG. 763.

A door with sliding dead bolt, as used on common room doors, is a similar section of ratchet gear with stationary ratchet.

In key locks, the key is the releasing member of the ratchet train, and also serves to actuate the bolt after it is released. The key and ratchet mechanism are arranged in most ingenious manners, so that numerous permutations can be made to effect the release.

Some of the most important systems of lock construction are given as examples:

Example 1.—The common so-called French lock, Fig. 764, is similar to the ratchet of Fig. 753. The bolt is a sliding rack, the "tumbler" *b* being often,

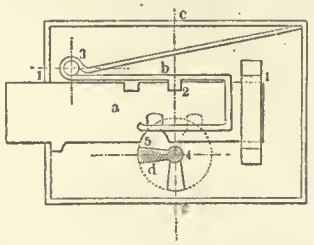


FIG. 764.

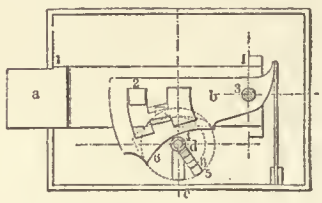


FIG. 765.

as in this case, made in one piece with its spring. The case of the lock corresponds to the frame for the ratchet mechanism, and the key acts as the releasing and actuating member.

Example 2.—The Chubb lock, Fig. 765, which is always made with a dead bolt, forms with the door and door frame a ratchet gearing similar to Fig. 691. The bolt is secured by means of several ratchets of precision, as in Fig. 706, and is moved by a ratchet as Fig. 755. The key, the axis 4, and the various bittings of the key form a system of pawls. The whole is a ratchet system of the second order with precision gear.

Example 3.—The Bramah lock, Fig. 766 *a* and Fig. 766 *b*, is differently constructed. In this case the dead bolt is actuated through the medium of a cylindrical driving ratchet gear, which does not contain the mechanism of security, the latter being in a distinct portion of the lock, Fig. 766 *b*. This consists of a number of sliding precision pawls, as Fig. 707, the number being 6 to 8 (in the illustration 5). The member *a* of Fig. 707 is here made in the form of a ring with internal teeth, secured to the escutcheon *a* by screws. The key is a prismatic adjuster of the slides, and the whole is a locking mechanism of the third order with ratchets of precision. The spiral spring around the pin restores the slides to their extreme position when the key is withdrawn.

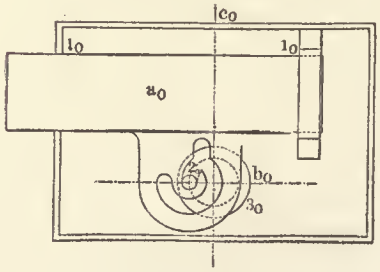


FIG. 766 *a*.

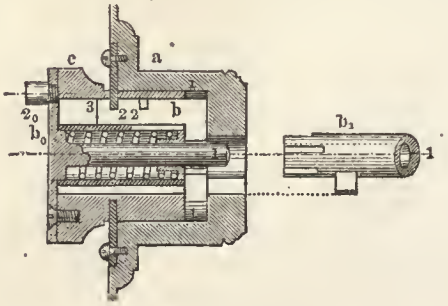


FIG. 766 *b*.

Example 4.—The Yale lock, Fig. 767 *a* and *b*, is also a system in which the mechanism of security is separated from the bolt mechanism. This is again a system of the third order, with ratchets of precision. The key is a flat prism (corrugated in recent locks) and serves to place precision bolts, or

pin tumblers in proper line, and also operate the bolt. The figure shows the method of connecting the cam b_2 to the plug a.

The so called combination locks are locking ratchets with precision pawls, operated without a key by being placed successively in the positions for release in accordance with a previously selected series of numbers and dial marks.

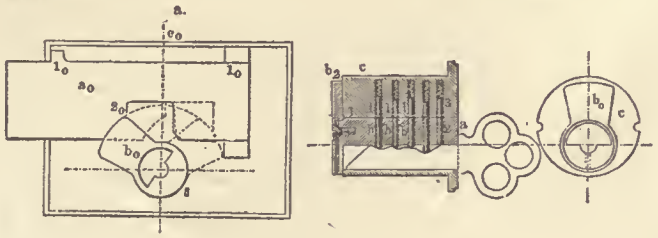


FIG. 767.

The numereus systems of Arnheim, Ade, Wertheim, Kleinert, Polysius, Kromer, and others are mostly locking ratchet systems of the fourth order, or combinations thereof. The American manufactûrers, especially the Yale and Towne Manufacturing Company of Stamford, Counecticut, have shown great ingenuity in this industry.*

§ 257.

ESCAPEMENTS—THEIR VARIETIES.

Escapements may fairly be considered as among the most important mechauical devices, since it is by their means that the elementary forces are used to regulate mechanical work. For this purpose they are used in the greatest variety, all forming ratchet devices in which the driven member is alternately released and checked. The arc, angle or path through which the driven member passes between the interval of release and check is called the "range" of the escapement. During the passage over this range there elapses a definite amouut of time, which may be called the "period" of movement of the escapement. This is followed by an amount of time when the driven member is stationary, called the period of rest. The sum of the two forms the "time of oscillation." The range and the period of oscillation may be (*a*) constant, (*b*) periodically variable, or (*c*) variable at will.

We therefore have

a, Uniform escapements,
b, Periodical "
c, Variable "

and these will be briefly considered.

§ 258.

UNIFORM ESCAPEMENTS.

If, in ordiuary running ratchet, Fig. 768, we have the wheel *a*,

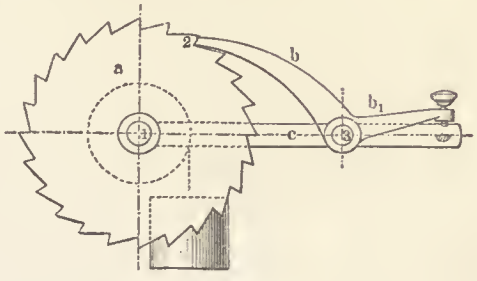


FIG. 768.

impelled by a weight or other force, and suppose the pawl *b*, lifted and dropped quickly, as by the arm b_1, the wheel will move one space, and au escapement will have occurred. In this case the range will be one pitch. If, after a definite time, this operatiou is again and again repeated, we shall have a uniform escapemeut. In mechanism the releasing and checking action is produced mechanically and not by hand, the impulse being obtained from the movement of the wheel.

* The ancient and modern Egyptian locks, also those of ancient Greece, Rome, India and Chiua, contain the principle of running ratchets with flat pawls, actuated by a key pushed directly into the lock. The Egyptian lock, with pin precision pawls, is quite similar to the Yale lock in priuciple, although very different in construction. Ancient Roman locks, found in Pompeii, are similar in principle. Wooden locks are still in use in China, Persia, Bulgaria, Russia and Southern Italy, also in the Farõe Islands and Iceland. At the suggestion of the author, Professor Wagner, of Tokio, succeeded in inducing some Japanese lockmakers to make a very complete and intelligible collection of native locks for the kinematic cabinet of the Royal Technical High School at Berlin.

The most general examples of uniform escapement are found in watches. In these impulses are isochronous, and obtained from the inertia of a vibrating body. The wheel *a* is called the escape wheel. The vibrating member, or balance wheel, makes its oscillations in nearly equal times for great or small vibrations. If, therefore, in a watch escapement, the time of the fall of the pawl is less thau the time of oscillation, the most important requirement is fulfilled, namely, that for uniform periods of time the same number of teeth of the escape wheel shall pass, and the corresponding angle may then be used as a measure of time. A given amount of work may also be abstracted from the motive power and used to produce the impulse. These important points have been fulfilled in the design of escapements, and it has been made possible to measure time with a great degree of accuracy. When the highest accuracy is demanded the greatest care must be given to the construction and execution, and to the reduction of friction and compensation of the balance.

In the case of watches the duty of the impelling force is simply that of overcoming the resistance of the mechanism, the function of the escapement being to provide against any acceleration of the rate motion, and the impulse which is required to operate the escapement may be cousidered as a portion of the resistance of the mechanism.

A systematic discrimination between the various kinds of watch escapements will show that they vary as to the checking ratchet device, the impelling device, the release and the accelerating device. We may have Simple or Compound escapements of the lower or higher orders. Some examples are here given.

A. Simple Escapements.

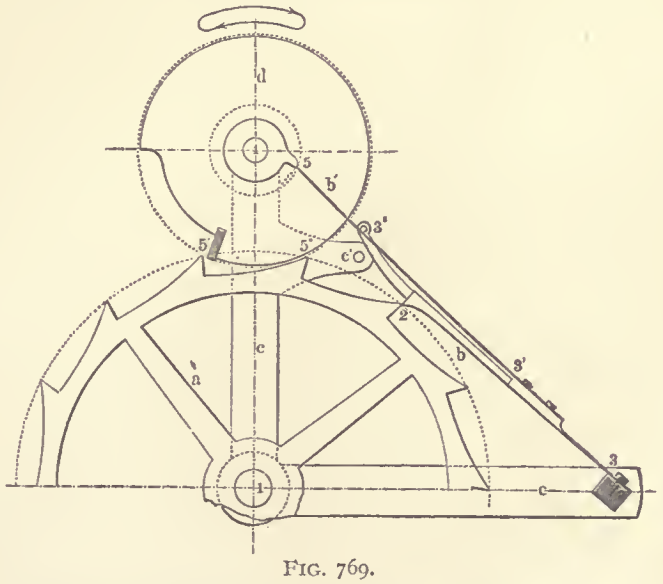


FIG. 769.

Example 1.—The Free Chronometer Escapement (Jullien le Roy, Earnshaw, Arnold, Jürgensen), Fig. 769. The runuing ratchet gearing *a*, *b*, *c*, is similar to Fig. 768. The pawl *b* is provided with a flat spring 3. The impelling device is the balance wheel *d*, which acts as a pendulum. The releasing device is at 4 . 5, and is attached to *d*, and when it swings to the left, impelled by the movement of the watch, it releases the pawl by means of a second running ratchet at 5. At *c'* is a stop for the pawl *b*. At 5′ is the accelerator which, for each tooth of the escape wheel *a*, swings from 5′ to 5″. As it returns, the pawl *b* engages with the tooth which has just left the point 5″. The spring *b'* permits the releasing tooth 5 to pass back during the return oscillatiou. The balance wheel can swing freely beyond 5″ and back without eugaging with the escape wheel, hence the name "free" escapement.*

Example 2.—The Duplex escapement, Fig. 770, is derived from the ratchet of Fig. 699. The escape wheel is upon the same axis as the checking pawl

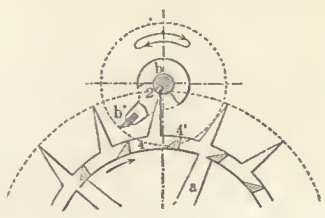


FIG. 770.

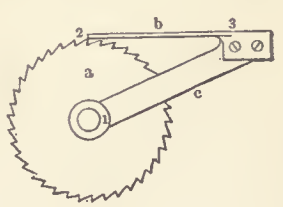


FIG. 771.

* This beautiful movement is apparently the first form which was applied as a peudulum escapement, having been used by Galileo in 1641.

b; the accelerator is at 4, acting upon the impelling pawl at every vibration between 4 . 4'.

The so-called "verge" escapement is similar in construction, except that the arm b' is longer and curved. The simplicity of this form as compared with the preceding is due to the fact that the impelling and checking pawls are made in one member. It will be noticed that the entrance of the tooth of the escape wheel into the space, causes a slight reverse movement at a, due to the fact that b is really a tumbling ratchet gear. This escapement has been called duplex by its English inventor, although some contend that it is properly a double wheel escapement, although the two wheels are combined in one.

Example 3.—Another method by which the checking and impelling pawls may be combined is shown in the Hipp escapement, Fig. 771. This consists of a simple running ratchet a, b, c. The pawl b is a plate spring, which is lifted and dropped by the passage of the teeth. The acceleration is given by the deflection of the spring. If the impelling force upon the wheel a is great, two teeth will pass, but this can be detected by the note emitted by the spring, which will then be one octave higher than before.

B. Compound Escapements.

Example 4.—Lamb's escapement. Those escapements which have two escape wheels are properly classed as compound, and to this class belongs Lamb's escapement. This consists of a running ratchet gear, similar to Example 1, and the same form of impelling device, but between these is an

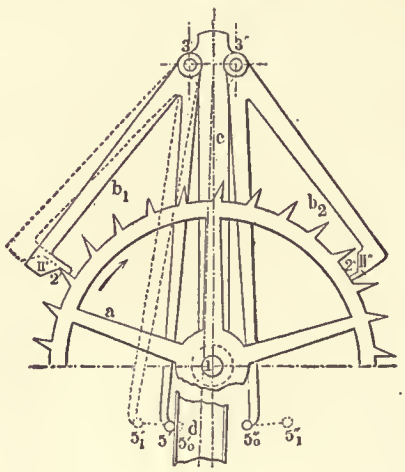


FIG. 772.

internal wheel with pitch ratchet gearing, similar to Fig. 686, which is impelled with each direction of vibration. Another double-wheel escapement is Enderlein's, based on Fig. 702, also one devised by the author, like Fig. 686.

Example 5.—Mudge's Escapement (also invented by Tiede), Fig. 772. This is a double ratchet gear system, with one pawl in compression and one in

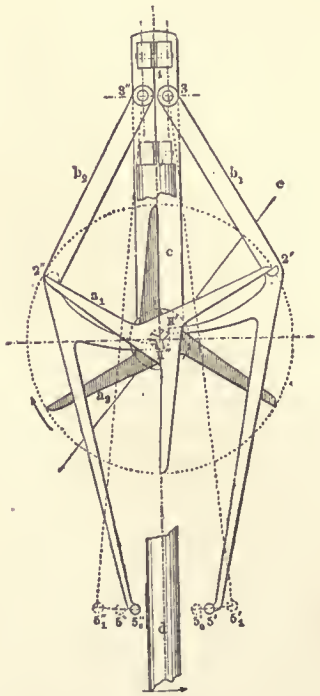


FIG. 773.

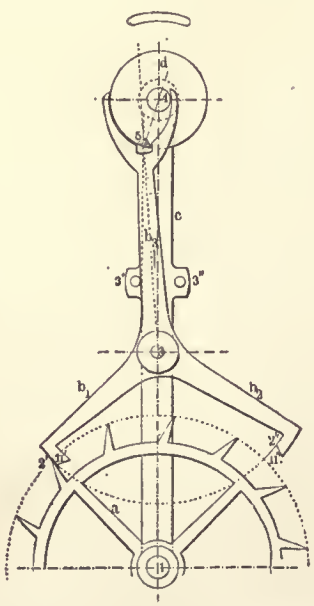


FIG. 774.

tension, b_1 and b_2. At 2' and 2'' is a "dead" pawl action for checking, and at II' and II'' a running pawl action for impelling. (See Cases 5 and 7, § 237). The pawls are lifted by the pendulum d. The releasing arms 3' . 5' and 3'' . 5''

are moved alternately by the pendulum; for example, the arm b_1, being moved into the dotted position, lifts the pawl out of gear, and the weight of the pawl and arm (sometimes assisted by a spring), gives an impulse to the return vibration of the pendulum, the acceleration being provided by the escape wheel acting on the portion II'. A similar action takes place on the other side.

Example 6.—Bloxam's or Dennison's so called "gravity" escapement, Fig. 773. The escapement is controlled by a pendulum suspended by a spring at 4. The escape wheel is made in two parts, as Fig. 686. The accelerating surfaces II' and II'' are much better arranged than in the preceding example, the friction being reduced. A fan is used also, as shown at e, for the purpose of preventing great acceleration of the escape wheel, which might otherwise occur in the large angle (60°) of escape. The fan is not fast to the axis of the escape wheel, but connected by running ratchet so that its momentum is not checked as the escape wheel is stopped.

Example 7.—Free Anchor Escapement, Fig. 774. The two pawls are combined into one anchor, as in Fig. 682, and the action is much the same as Fig. 772. The escape is controlled by a balance wheel at d. The pawls 2' and 2'' are operated through the arm b_3, and at the same time the impulses are given by the action of the escape wheel upon the inclined surfaces II'

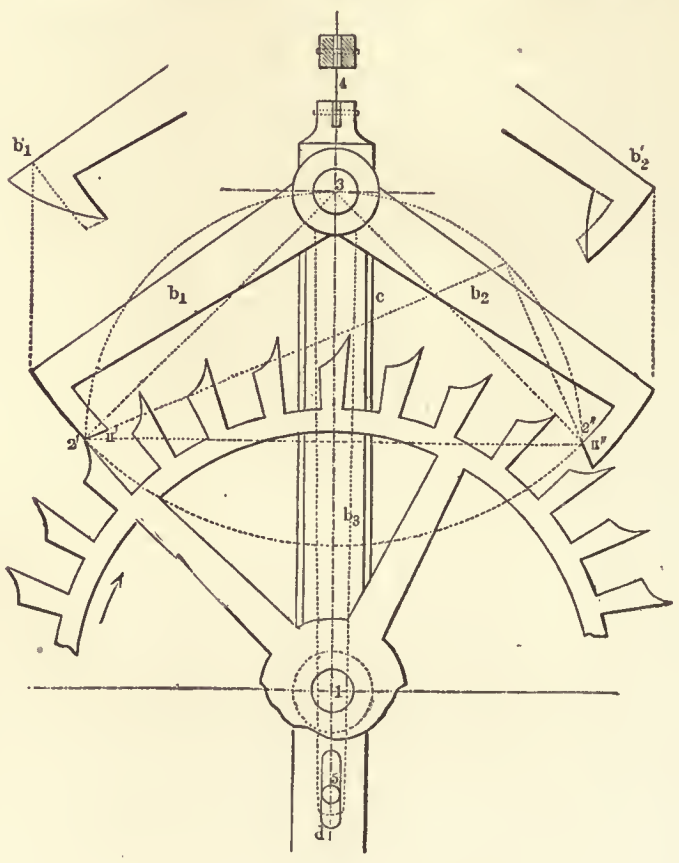


FIG. 775.

and II''. The pawls are technically known as pallets. The tooth action at 5 is a continuous ratchet gear similar to Fig. 754. The arm b_3 is limited in travel by pins at 3' and 3'', or in some forms by a fork at 4. Since there is a ratchet at 5 and also at 2, this forms a system of the second order.*

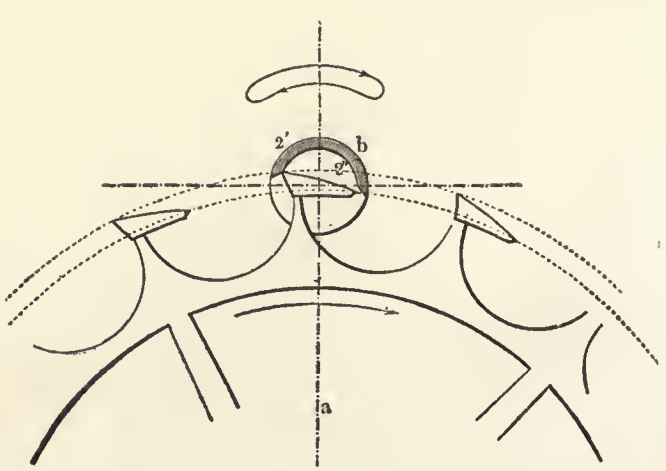


FIG. 776.

* A watch escapement of the third order has recently been designed by **A.** E. Müller, of Passau. This is made with a cylinder ratchet, as Fig. 699 *b*, between the arm and the escape wheel.

Example 8.—Graham's Escapement, Fig. 775. The construction is very similar to the preceding. The connection 5 between the anchor-arm b_3 and pendulum d, is different, and the arm b_3 does not come to rest, but both it and the pallets 2′ and 2″ slide upon the teeth while the escape wheel is stopped. An earlier form of pallets for this escapement is shown at b'_1 and b'_2 (called Clement's Anchor, from Clement, 1680; but described by Dr. Hooke in 1666). This form produces a brief reverse movement to the escape wheel at each oscillation.

Example 9.—The form of ratchet of Fig. 684 is used in Lepaute's escapement, which was really invented by the watchmaker Caron, afterwards Marquis Beaumarchais.

Example 10.—Cylinder Escapement, Fig. 776. This is made from the cylinder ratchet of Fig. 700, the impelling surfaces being divided between the anchor and the teeth of the escape wheel. The cylinder b is attached to the axis of the balance wheel, and the wide spacing of the teeth of the escape wheel permits a correspondingly wide amplitude of oscillation. If we imagine the pallets of Graham's anchor to be formed between two concentric circles (as, indeed, most watchmakers construct them), the "cylinder" will be seen to be a similar anchor.

Example 11.—Crown Wheel Escapement, Fig. 777. Escapements constructed with crown ratchet wheels (§ 241) are the oldest forms used in

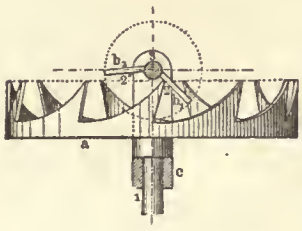


FIG. 777.

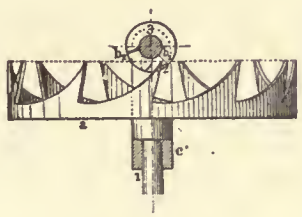


FIG. 778.

ratchets.* The form of the pallets causes a reverse movement, and in the old watches using a balance with its centre of gravity in the axis of oscillation, without any assisting spring action, this reverse movement was a necessity, which accounts for the long and extended use of this form of escapement. Toward the end of the fifteenth century the hair spring was introduced by Hele, in the form of a hog's bristle, and in 1665 Hayghens made the steel hair spring, which made the construction of the modern chronometer possible. The crown escapement is easily modified so as to remove the reverse action, as was done by the author in 1864. We then have a "dead" tooth action, as Fig. 699. The modified escapement is shown in Fig. 778; the pawls are practically hyperboloidal in form.†

C. *Power Escapements.*

In the case of watch escapements the impelling force is only used to overcome the resistance of the watch mechanism. Escapements can also be used to regulate greater forces, such as are intended to perform useful work, and these may be

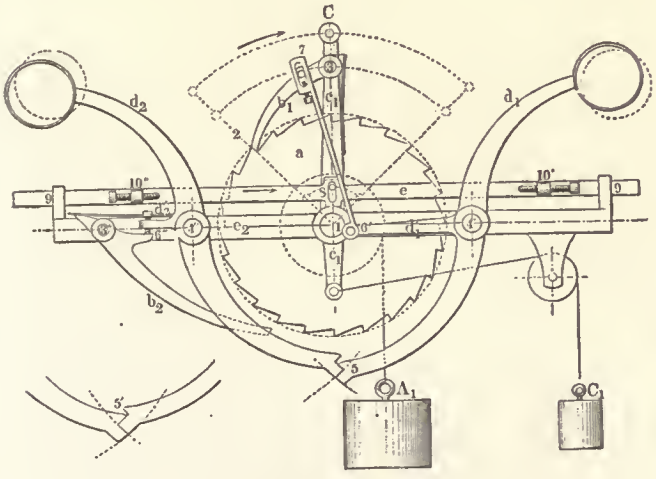


FIG. 779.

called power escapemeuts. Alarm and striking clocks are of this class, and there are numerous other forms. The following example will serve to illustrate:

Example 12.—Power Escapement for a Reciprocating Movement, Fig. 779. At $a\ b_1\ c_1$ and $a\ b_2\ c_2$ are ordinary running ratchets, the pawls b_1 and b_2 of which can be released and engaged by suitable auxiliary mechanism. This mechanism is either a substitute for or identical with the regulating device (balance wheel, pendulun, etc) of a watch escapement. The escapement is intended to control the motion of the swinging arm C by means of the lever c_1 and the descending arm A_1. This is accomplished by a double acting ratchet system $d_1\ d_2\ 5$ (as Fig. 671), by means of the slide e, driven from 8 by the arm c_1.

The action is as follows: When the parts are in the position shown in the figure, the motion of the wheel a to the right moves the arm c_1 by means of the pawl b_1 until the trigger 10″ trips the pawl d_2 and shifts the engagement at 5 into the position 5′ (in the small figure to the left). This action, by means of the trigger at 6″, throws in the pawl b_2 and stops the wheel a. At the same time b_1 is thrown out of gear by the connections d_1, 6′ and 7′, and the counterweight C_1 returns the arm c_1 to its original position. This brings the trigger 10′ against the lever d_1, and again shifts the engagement at 5. The pawl b_1 falls into gear, and the pawl b_2 is disengaged, leaving the wheel a free for another forward movement.

The preceding escapement can be readily converted into a double acting one by introducing a second ratchet wheel toothed in the opposite direction, with proper pawl on c_1 and trigger connections to d_2; the other portions would remain the same. This escapement appears to be new, and many important applications will suggest themselves.

§ 258.

Periodical Escapements.

A great variety of periodical escapements are to be found in the striking mechanism of clocks and repeating watches. The entire period is the revolution of the hour hand, and if the half hours are struck the order will be

1, 1, 1, 2, 1, 3, 1, 4, – – – – 1, 12,

making in all 90 strokes in the twelve hours. A fan regulator is used to cause the strokes to follow each other uniformly. There are two systems of escapement in use for this purpose, the German and the English, the latter also used for repeaters. An essential piece of the latter, the so-called "snail," has been shown in Fig. 688; its function is to control the number of strokes. Further subdivisions cannot be here discussed, but it must be remembered that the striking arm is itself a ratchet mechanism.*

Important applications of periodical escapements are found in the self-acting spinning mule, and both these and the clock striking mechanism are examples of power escapements.

The mechanism in Platt's mule is here briefly shown. Fig. 780, a and b. The shaft 1 is required to make rapid turns

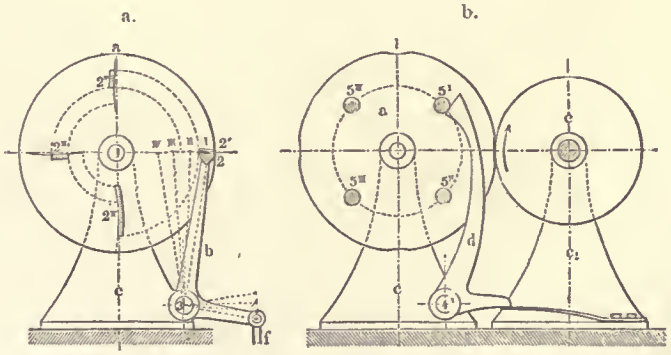


FIG. 780.

through 90° at intervals of different lengths of time. The wheel a is an escape wheel with teeth in four concentric rings, I, II, III, IV (compare Fig. 686), each ring having one tooth. The other side of the wheel a is shown in Fig. b, where is the ratchet chain $a\ d\ e$. When a is released, the pressure of d at 5′ moves it slightly and brings the running friction wheel e into contact, thus driving a through a quarter revolution, toward the close of which the pawl d again enters into eugagement.

* This has been used since the tenth century, having been invented by Bishop Gerbert, afterwards Pope Sylvester II, about 990; also by Heinrich von Wyck about 1370, and applied to a pendulum by Huyghens. The oldest tower clock in Nuremberg, built about 1400, has such an escapement.

† In the Kinematic cabinet of the Royal Technical High School there is a schematic series of models of clock and watch escapements.

* See Ruhlmann Rèdtenbacher, Denison.

The recesses in *a* permit the friction wheel to run free when *a* is at rest. This is evidently a form of ratchet gearing in itself. The order of escapements at 2 is as follows:

I II, II III, III IV, IV I.

This is controlled by a second escapement, shown in Fig. 781.

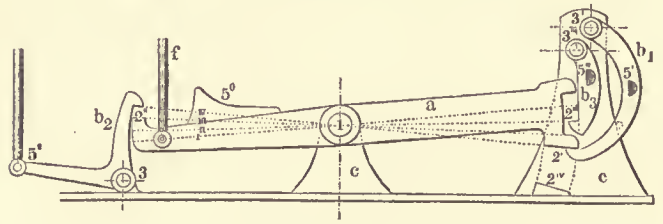


FIG. 781.

The pawl *b* of Fig. 780 is connected by the rod *f* to the beam *a*, as shown. This mechanism is a step ratchet of four steps. The steps are the pawls b_1, b_2, b_3, and the stop on the frame *c*; giving the positions 2^{I}, 2^{II}, 2^{III}, 2^{IV}. The action takes place in the four following periods:

1. Drawing and spinning—*a* checked at 2^{I}
2. Stretching and twisting " " 2^{II}
3. Holding and spun thread " " 2^{III}
4. Winding and returning " " 2^{IV}

The succession of movements is as follows: At the termination of the first period a projection on the carriage strikes the pawl b_1 at 5′. The step lever, which is heavier on the right end than on the left, moves from position I to position II, in which it is held by the pawl b_2; this, by means of the rod *f*, places the pawl *b* of Fig. 780 in the position 3 II, thus starting the second period.

At the close of the second period the pawl b_2 is released, the lever falls to the position III, shifting the pawl *b* to 3 III, and is held by the pawl b_3 at $2'''$.

The third period, which is very brief, is terminated by the winder striking $5'''$, releasing the pawl b_3, and the lever assumes the position IV, and the rod *f* moves the pawl *b* into the position 3 IV, and the fourth period begins.

During this period the carriage returns, and just before the close of its motion a roller acts upon the portion 5°, bringing the lever back into the first position. This returns the pawl *b* to its original position 3 I, and the succession is repeated.

The entire mechanism forms a periodical escapement of the second order, or, when the connections are included, the third order, and when taken together with the ratchet gearing, of the fifth order; while a sixth ratchet mechanism is used for the primary control.

§ 259.

ADJUSTABLE ESCAPEMENTS.

An escapement can be so arranged that the checked member, after the release, will again be checked by the impulse of its fresh start, thus forming what may be called a self-acting escapement. In a mechanism of this kind, the amplitude of the escapement is dependent upon the amount of displacement which is permitted to the releasing member. This may be made greater or less, and hence such devices may be called adjustable escapements. These devices are likely to play an important part in modern machine design.

A simple form of adjustable escapement is shown in Fig. 782. This apparatus, designed by the author, is based upon that of Fig. 674. The ratchet wheel *a* is stationary, being fastened to the frame *a′*; the pawl is at *b*, and the link is in the form of a disc *c*, driven by a force, *C*, and checked by the escapement. At 3 . 5 is the guide for the pawl. This can be adjusted by the wheel *d*, by turning the latter more or less in the direction in which *c* is impelled. If *d* is turned so far that the pawl *b* is lifted out of gear, the force at *C* will set the disc *c* in motion. This latter carries with it the axis 3 of the pawl, which, by the action of the guide 5, draws the pawl into engagement again, entering the space 2 and checking the disc. In order to avoid an uncertain or irregular action, a brake may be used as at *a″*. If the wheel *d* be moved forward regularly through two, three, or four arcs, the disc *c* will be released and checked successively in similar manner.

It will be evident from the foregoing that the ratchet gearings which form the foundation of the various kinds of adjustable escapements are so varied that the different constructions which may be used are very numerous. Among them may be mentioned those in which friction ratchets are used, these possessing the advantage that the arc of motion of the escapement may be varied from the smallest to the greatest without being dependent upon any especial pitch.

We have already intimated that the various forms of couplings may be considered as varieties of ratchet gearing. The

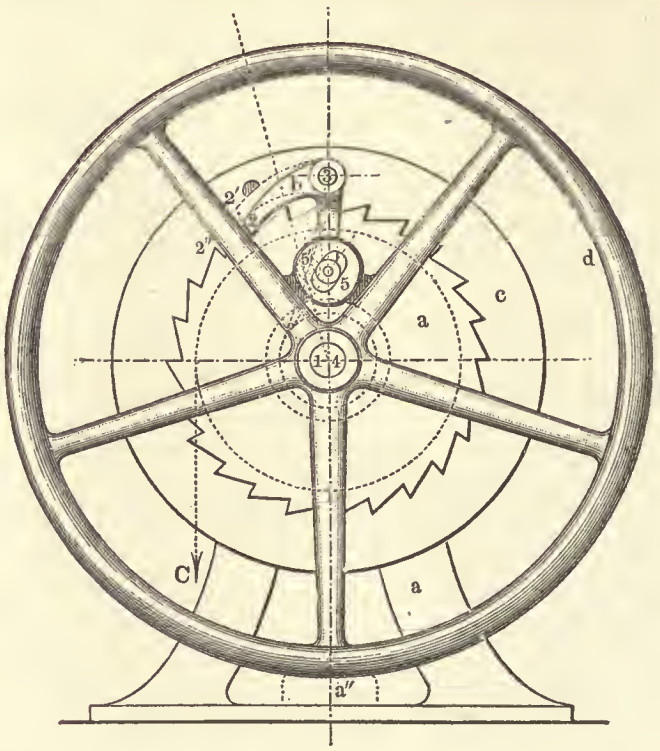


FIG. 782.

same is true of the present subject. If it is desired to use this adjustable escapement as a disconnecting coupling, the following arrangement may be adopted:

The disc *c* can be attached to the shaft which is to be set in motion, and the wheel *a* to the driving shaft, which is supposed to be in continuous revolution and is to be coupled to *c*. The teeth are then to be so arranged that by the revolution of *a* the pawl *b*, disc *c* and wheel *d* will be carried around together. When the disconnection is to be made, it is only necessary to hold the wheel *d* from revolving. The pawl-axis 3 will then move on and cause disengagement of the pawl at 2, and the disc *c* will come to rest. If the wheel *d* is then turned a short distance in the direction of rotation the pawl will again be thrown into gear and the parts once more connected. A coupling thus formed from an adjustable escapement may be called an adjustable coupling.

The suitability of the application of toothed ratchet gearing for this purpose is open to question, and indeed toothed gearing is only to be recommended for the lightest service of this kind. In most cases, if indeed not all, friction couplings are much better. An adjustable friction coupling is to be seen by reference to Fig. 448, in which *A* is the friction wheel, *B* is the pawl, disguised in the form of a cone, and *b* is the adjusting member.

If a combination is made of an adjustable friction coupling with some form of transmission to a machine, such as a rope or belt gearing, so that it is thrown into action when any reverse motion is attempted, we have what may be termed an automatic friction brake.*

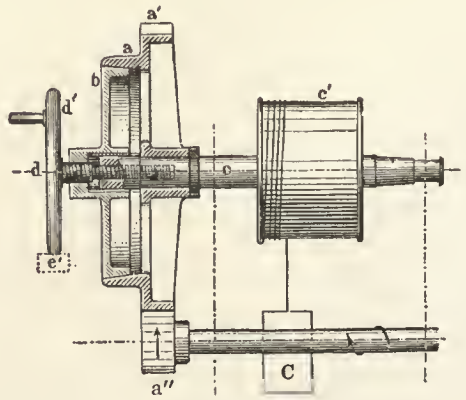


FIG. 783.

*See German Patent, E. Langen, No. 21,922.

Example.—Fig. 783 shows such an automatic brake device as applied to the pontoon bridge at Cologne. At *a* is a friction cone combined with a spur gear *a'*, driven by the shaft and pinion *a''* in the direction to wind up the cord on the drum *c'*. The drum is fast to the chaft *c*, but the cone *a* is loose on the shaft. The wheel *a* is connected firmly to the shaft *c*, when the cone *b*, which slides on a feather, is forced into engagement with it, and this engagement is effected by the differential screw *d* and hand wheel *d'*. The use of the differential screw enables the equisite pressure to be obtained, and also causes the motion of *d'* to be in the same direction as *c'* when lifting. The friction of the cones binds the parts firmly together, so that *a* is practically secured to the shaft until *d'* is revolved backwards, when *c'* follows by the action of the weight *C*, the cones slipping upon each other and the pressure being automatically regulated, and the motion at once checked when *d'* is stopped.

Other and most important applications of adjustable escapements will be given hereafter. It may, however, be here noted that by means of such mechanism the most powerful combinations may be controlled with the exercise of a minimum effort.

§ 260.

GENERAL REMARKS UPON RATCHET MECHANISM.

Ratchet mechanism, as already discussed, is applicable to a most extensive range of uses; in this respect far excelling every other form of mechanism. This is plainly due to the fact that ratchets are suited either to produce the effect of relative motion and relative rest. Considered in this light the six preceding classes may be grouped as follows: Common ratchets, checking ratchets, and locking ratchets are those which act to hinder motion, while releasing and continuous ratchets, as well as escapements, act to produce definite motion. The motion produced by ratchets is intermittent while that produced by the forms of mechanism previously considered, such as cranks, friction, or toothed gearing, etc., is continuous. Mechanism for continuous motion may be called "running gearing,"* and practically merges into ratchet gearing. The general province of ratchet gearing has only been partially covered in the preceding pages, where such forms as may strictly be considered machine elements have been included. An exception might be made as to the allied forms of springs, some of which, indeed, were referred to. There is, however, a large number of machine elements of a different kind, which usually involve the continuous action of the operative forces in one direction; these include tension organs, such as ropes, belts, chains, etc., compression organs, fluid connections, and many others, all of which are considered in the following chapters. It will be seen that these may all be so arranged as to be fairly considered ratchet devices also; as belts or chains may become frictiou or toothed ratchet gears, and even the valves of fluid connections are really pawls.†

The pawl mechanism must also be extended to include these classes of machine elements, and their limits thus greatly widened, especially in the case of pressure organs. Examples of this will be found in the pistons and valves of pumps, both for liquids and gases, which may act as checking or locking ratchets, or in hydraulic motors and steam engines as escapements, and in gas engines, as escapements and continuous ratchets combined. Similar comparisons may be made of the ratchet principle in the use of accumulators for hydraulic cranes, presses, riveting machines, and the like, and in the cataract for single acting steam engines we find a complete analogy to the ratchet. In these cases we have ratchet systems of the higher orders. The history of the development of these machines is really that of their pawl members.

A very interesting example is that of Fig. 779, in which, if we substitute a flow of steam for the ratchet wheel, we have the arrangement of the single acting high pressure steam engine with Farey's valve gear. The numerous modifications of escapement gear, which are included in the steam engine, have occupied the activity of designers down to the present time. A number of the more recent valve gears have been shown in § 252, and similar devices are used on engines for steam steering gear, called by the French "moteurs asservis," and such gear also plays an important part in the mechanism of some of the so-called "fish" torpedoes.

In this manner the applications of pawl ratchets may be extended before our eyes and yet the limitations are not reached, and the further researches are carried the broader and more general does the scope of this division of mechanism become. Not only does it include fluid pressure organs, both liquid and gaseous in a strictly mechanical sense, as in the case of pumps, etc., but also when these are considered in a physical sense with regard to their internal stresses. This gives a branch which may be called "physical" ratchet trains, of which the steam boiler is the most important example. In this, when taken in connection with a pipe full of steam, and suitable valves for opening and closing, forming what has been termed a steam column,* we have undoubtedly a physical ratchet train in which the particles of vapor are considered as a physical aggregate, which from the higher temperature, are under higher stress. Another example of a physical ratchet train is the apparatus for operation by liquid carbonic acid which has been recently used.

Electrical accumulators are also instances of physical ratchet trains, as well as some applications of galvanic batteries, the action taking place by make and break of electrical contact. The dynamo-electric machine also becomes a physical running ratchet and the electric motor a physical escapement, the whole forming a physical running gear train.

Again we may consider a "chemical" ratchet train, such as coal or any fuel, which, during combustion, releases the energy which is stored in it. This may be utilized in numerous ways, but for our present considerations, mainly in the production of motion. Chemical action is also included in hot-air engines, and in the operation of telegraph apparatus in a similar sense.

We may consider the principal factors in a steam motor plant as portions of a ratchet chain, somewhat as follows:

Chemical ratchet	= combustion of fuel,
Physical "	= steam generator, etc.,
Mechanical escapement	= steam cylinder and attachments,
Mechanical running gear	= crank shaft and wheel,

these four uniting to convert the released energy into mechanical motion. If we consider a locomotive engine, we have added to this another running gear in the shape of the driving wheels and rails, while the train and wheels and journal bearings unite to form a combination of the sixth order.

Another chemical train may be formed by the use of explosives, which are released either mechanically, as by percussion or friction, or chemically, by combustion of some auxiliary material. Again, we may have releasing gear of the first, second, or higher orders.

In the case of most firearms the release is of the second order, since the mechanism of the lock acts upon a fulminate by percussion, and the heat of the latter releases the powder.

If we examine and classify all mechanism of transmission in the above manner, it will be apparent that all forms are included in one or the other of the following classes, viz.: mechanical, physical, or chemical; these also entering into combinations of the higher orders with each other.

The steam engine itself, as we have already seen, consists of a driving train of the fourth order. Trains of still higher orders are of frequent occurrence.

In the recording telegraph, with relay, we have a physical ratchet train of the second order, releasing a mechanical running train and operating a recording train, both physical trains actuated by chemical trains, the whole forming a combination of the fifth order. The ordinary signal mechanism of a railway station, when mechanically operated, is a system of the fourth order.

The Westinghouse air brake, not considering the boiler, is a train of the fifth order, consisting of an escapement (steam cylinder), driving ratchet (air cylinder), intermittent ratchet (air vessel), escapement (piston and valve connections), friction checking ratchet (brake gear). If we include furnace and boiler, this becomes a train of the seventh order, and may be still further extended.

A still more noteworthy example is found in the application of compressed air for the purpose of operating pumping machinery at the bottom of deep mine shafts. In this case we have:

1. Furnace	=	chemical ratchet train.
2. Boiler	=	physical " "
3. Steam engine	=	mechanical escapement train.
4. Shafting and transmission to		" running "
5. Air compressor,		" driving ratchet.
6. Air chamber,		" intermittent "
7. Air cylinder in mine,		" escapement train.
†8. Water cylinder in mine,		" driv'g ratchet "

The preceding discussion and illustrations of the relationship existing between mechanical, physical and chemical trains shows the necessity of combining mechanical and technical research, and a complete mechanical training therefore includes these three branches, and also the later science of electro-mechanics. Modern methods of invention require research into all of these lines of science, and the constantly widening field of mechanical engineering is thus extending its work, while at the same time gathering into systematic form the many branches of applied mechanical science.

* See the author's Theoretical Kinematics, p. 486, in which this classification was originally made.

† See Theoretical Kinematics, p. 458 *et seq.*

* See Theoretical Kinematics, p. 493

† The system of clocks operated by pneumatic pressure from a central station, designed by Mayrhofer, at Vienna, forms a combination of 33 distinct systems.

CHAPTER XIX.

TENSION ORGANS CONSIDERED AS MACHINE ELEMENTS.

§ 261

Various Kinds of Tension Organs.

The various forms of machine elements which have already been discussed, have been those which offered resistance to forces acting in any given direction, forming more or less rigid constructions. We now have a series of elements which are only adapted to resist tension, and which are very yielding under the action of bending, twisting or thrusting forces. These include a great variety of rope, belt wire, chain belt and similar transmission devices, all of which may be included under the general term of Tension Organs. Their usefulness is limited by reason of the fact that they have only the single method of resisting force, but at the same time the element of flexibility permits the use of one and the same organ to transmit power in changing directions, and hence gives rise to many useful combinations. An especially valuable feature of tension organs in practice lies in the fact that many materials are excellently adapted for such use, and can be more economically applied.

Fig. 262.

Methods of Application.

A distinction is to be made between "standing and running" tension organs. The first are those used to suspend weights, support bridges, also in the construction of many machine details. Examples of such use are found in suspension bridges, pontoon bridges, hawsers, guy ropes, standing tackle, etc. Running tension organs are used in machine design in connection with other machine elements principally for the transmission of motion.

Running tension organs may again be divided into three classes according to their action in connection with other machine elements.

According as they are used:

1. For guiding.
2. For winding (hoisting or lowering).
3. For driving, this also being possible by winding and unwinding.

Combinations of these applications may be made, either with or without the use of standing tension organs. In order to understand the various applications it is desirable to consider some of the most important combinations, hence these will be briefly examined.

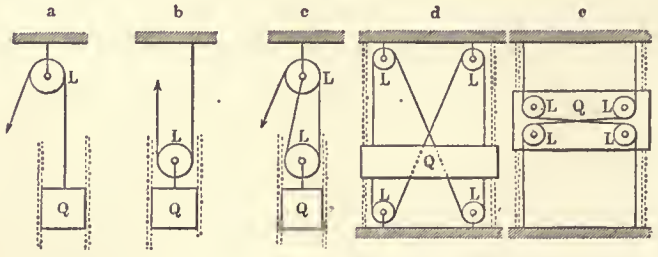


Fig. 784.

1. *Guiding.*—Fig. 784 shows several combinations, adapted solely for guiding. At *a* is the so-called stationary pulley, in which a cord, led off at any angle, is used to raise and lower a load *Q*. The dotted lines show the position of guides, or in the absence of these the direction of motion is governed by the action of gravity. At *b* we have the so-called movable pulley, the pulley being combined with the moving piece; the weight *Q* is here supported on two parts of rope. Form *c* is a combination of *a* and *b*, and is the well known tackle block. Form *d* consists of four sets of form *a*, and the action of the cords compels the piece *Q* to maintain a parallel motion. This is practically applied in Bergner's drawing board.

In like manner four pulleys of form *b* may be combined as in form *e*. This is the old parallel motion for spinning mules, also used as a squaring device for traveling cranes.*

The use of pulleys and bearings is to reduce friction at the point of bending, and roller bearings, as Fig. 566, are also used, but when the bending surface is well rounded the pulleys may be dispensed with. Fig. 785, at *a*, *b*, *c*, shows such arrangements, the action being the same as before, but with greater friction. The arrangement at *d* is a six-fold cord, aod in sail making eyelets are often used in similar manner, as at *e*. The friction is great in all such devices, because the cord presses hard upon the point of curvature; its magnitude increases rapidly with the arc of contact. This action, which here opposes the motion of the cord, is in other instances made of great utility. Cord-

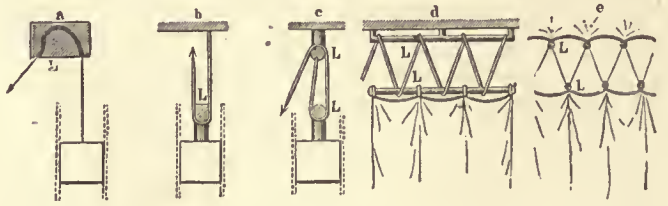


Fig. 785.

friction, which is to be considered as a particular case of sliding friction, plays a very important part in constructions, involving tension organs, and will be more fully considered hereafter.

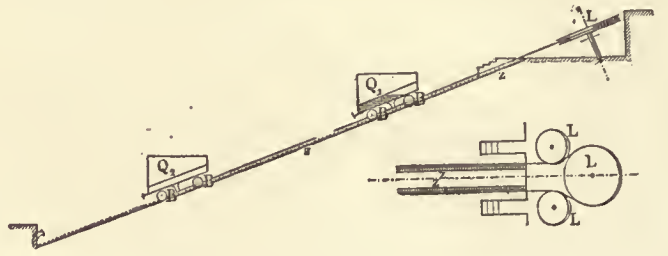


Fig. 786.

In Fig. 786 is shown Riggenbach's rope haulage system for use on inclined trackways, or so-called "ramps." In this arrangement, the descending car is loaded at the top of the ramp with sufficient water to enable it to draw up the ascending car by the power of its descent. The speed can be controlled by the descending weight, and also a weight acting upon wheels gearing into a rack *z*.†

2. *Winding.*—The most important forms of winding gear are

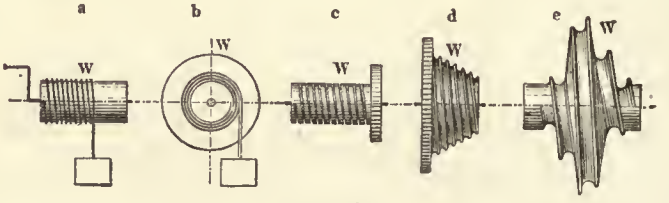


Fig. 787.

shown in Fig. 787. At *a* is the common windlass, also known as a winding barrel or drum, extensively used in many forms of hoisting machinery; *b* is a drum for spiral winding of a flat belt, the belt being wound upon itself, and side discs being provided as guides for the belt; *c* is a spirally grooved drum for winding chain; *d* is a conical drum, with spiral groove, used in clocks (there called a fusee), also for hoisting machinery with heavy rope; and *e* is a rope "snail" used on the self-acting mule, to produce the varied speed of the carriage. Many combinations of winding and guiding devices are made, also of winding devices with each other.

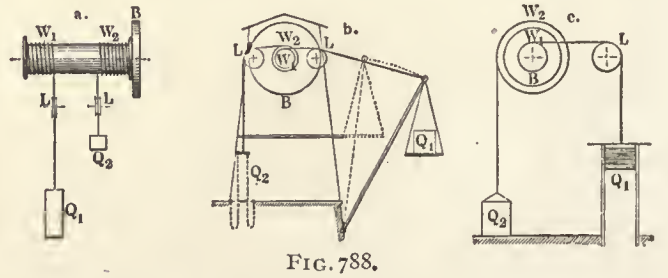


Fig. 788.

In Fig. 788 are shown several lowering devices. At *a* is a lowering drum for warehouse use; the unwinding coil at W_1 lowers the load *Q*, while the cord of the upward moving counterweight Q_2 is wound on the drum at W_2; a brake can be applied at *B*, and when necessary, guide pulleys used as at *L L*. Form *b* is a lowering apparatus for coal trucks, consisting of a combination of two winding coils, with a brake at *B*. The

* Form *d* is a kinematic inversion of the older form *e*.

† Numerous illustrations are in use in Switzerland and elsewhere, with inclines varying from 25 to 57 per cent.

counterweight Q_2 is in the form of Poncelet's chain, the action being to vary the rate of descent of the load W_2. This apparatus, which is called a "Drop," is much used in the coal mining districts in England. Form *c* is Althan's furnace hoist, and consists of two drums with steel bands. The load of water at Q_1, by its descent, raises the charge Q_2 to the top of the furnace, after which the water is drawn off, and the empty car descends and the water vessel is raised to the top again. The speed is controlled by a brake at *B*.

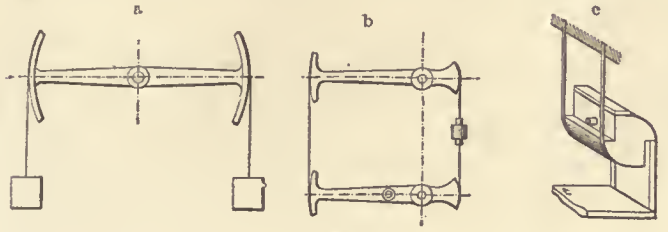


FIG. 789.

Wrapping connections have been used from early times in connection with beams and levers, as shown in Fig. 789 *a*, and the form *b* is especially applicable to scroll-sawing machines. Form *c* is a combination made with very fine steel bands, and used in the Emery weighing machine.

Combination windlasses are frequently used for lifting weights, some forms being shown in Fig. 790, and other combinations also in complete machines for hoisting, as in Fig. 791.

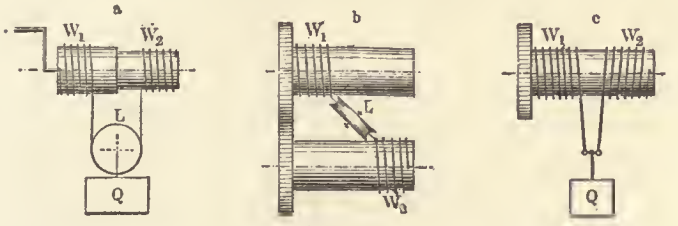


FIG. 790.

In Fig. 790, *a* is the so-called Chinese, or Differential Windlass, consisting of two windlasses and one sustaining combination; *b* is another differential combination used in a traveling crane designed by Brown, of Winterthur, the arrangement being intended to obviate the lateral motion of the load. Another arrangement for the same purpose is shown at *c* (devised by the author in 1862); it consists of two drums united in one. The signal arms and automatic safety gates, now so much used on railways, are operated by a combination of winding and guiding members, chains being used on the winding barrels and wire connections on the straight lines.

Winding and guiding members are much used in cranes and hoisting machinery, several combinations being given in Fig.

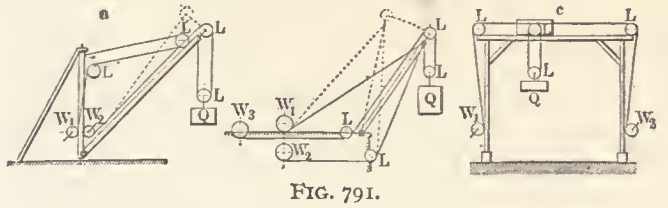


FIG. 791.

791. A crane with boom of variable radius is shown at *a*; *b* is a pair of shears operated by three windlasses, W_1 and W_2 for moving and holding the shear legs, W_3 for hoisting and lower-

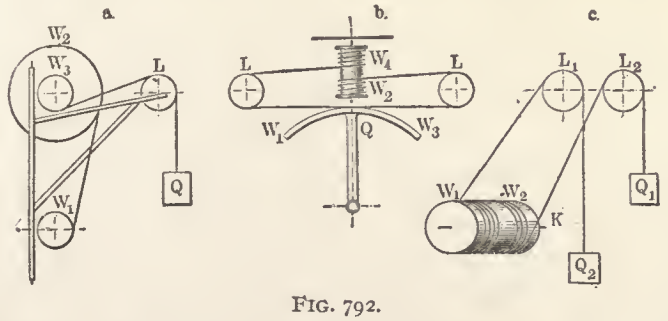


FIG. 792.

ing the load; *c* is a form of bridge crane, using a trolley in combination with two winches. If both winches are operated in parallel direction and uniform speed, trolley travel is effected, hoisting or lowering by unequal wind motion.

In Fig. 792 *a*, three drums and one guide sheave are used; *b* is made with four drums and two guide sheaves, a combination used in steering machinery for operating the tiller; and *c* consists of two drums and two guide sheaves so arranged that one load is raised as the other is lowered, this being used in mine hoists. This is also used for inclines or "ramps." When the load is always to be lowered, the descending load does away with the necessity of any motive power, and the speed is controlled by a brake. Examples of this form are found in some mines and stone quarries, and in apparatus for loading vessels, etc. (See Chap. XXII.) Power-driven cable railways for passenger service on inclines are sometimes made with two cables, one for driving, and a second for guiding and as an additional security, an example being the old road up the Kahlenberg at Vienna. When round ropes are used it is desirable to have the drums made with spiral grooves, in order to reduce the wear on the

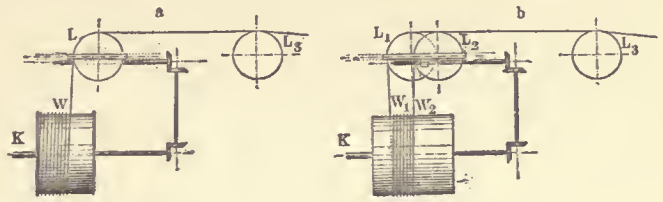


FIG. 793.

rope. The travel on the drum causes the angle of the rope between *W* and *L* to vary, and to prevent this the device shown in Fig. 793 has been used by Riggenbach on the cable incline at Lucerne; two forms being given. The guide sheaves are traversed by screw motion, the rope being led off in a plane parallel to the axis of the drum, and in the second form two guide sheaves are used for a double cable.

3. *Driving.*—This application of tension organs is most extensive. The principal forms are given in Fig. 794. The cap-

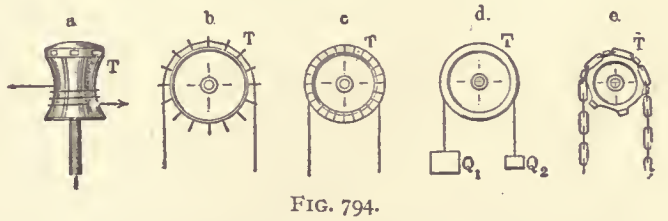


FIG. 794.

stan *a* consists of a hollowed drum, the surface of which is composed of numerous ribs and the rope is given several turns about it. The axial travel produced by the spiral path causes the rope to climb upon the larger diameter, from which it is easily forced back to the middle from time to time by hand. At *b* is a sprocket wheel with Y-shaped sprockets, much used in many modifications; *c* is Fowler's drum, a form of grip drum which grasps the rope automatically, and which is discussed more fully hereafter. At *d* is a simple rope pulley, partly encircled by a tension organ under such load as will produce sufficient friction to prevent slippage; *e* is a chain wheel with teeth to prevent the slipping of the links. In all five cases the wheel may drive or be driven by the tension organ.

By combination of driving and guiding devices many useful transmissions are made.

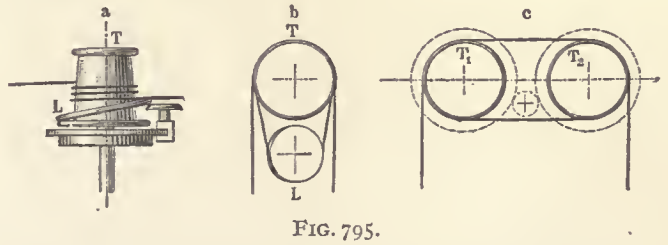


FIG. 795.

Several forms are given in Fig. 795: *a* is David's Capstan, with conical windlass, with a ring-shaped guide roller which constantly leads the rope from its travel toward the base of the cone. At *b* is a counter-sheave device, the main sheave *T* being made with two grooves and the counter-sheave set at a corresponding angle. This gives increased rope contact, which may be multiplied still more by increasing the number of grooves. The counter-sheave may also form the second pulley of the combination, as at *c*; this is used in rope transmission devices.

Driving tension devices are often capable of being used to

greater advantage than winding devices, since the direction of motion need not be changed and is not limited. For these reasons driving combinations are frequently used instead of drums, as in hoisting machinery. Chain sheaves with pockets to receive the ordinary oval link chain are here applied (see § 275), or with flat link chain the sheave engages with the pins of the chain.

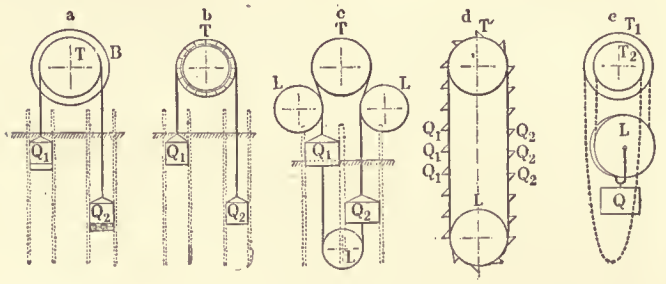


FIG. 796.

Other driving systems are shown in Fig. 796. At *a* is a double lift with water counter-weight. *T* is a pulley for round or flat belt; the weights Q_1 and Q_2 are nearly equal, so that a semicircle of contact is sufficient to prevent slipping at *T*, and the friction of contact is sufficient.

A reference to the Riggenbach cable road gear, Fig. 786, will show a similarity to this device, but in Fig. 786 a braking device is provided at Q_1 and Q_2 to protect from accident in case of breakage of the cable. A similar device, using strains at *T*, has been applied by Green for operating the sluices of the Great Western Canal. At *b* is shown the grip-wheel, which has also been used for cable driving. In this form the loads may be quite unequal without apprehension of a deep groove cutting in the drum. Koppen's system is shown at *c*; this uses a round or flat belt with tightening pulleys *L*, *L*, so that sufficient friction can be obtained for any given difference of loads; this avoids the unequal action upon the heavily-loaded side of the belt, by producing tension upon the otherwise slack side, and might be applied with advantage to the driving system of Fig. 795 *c*, requiring but a single tightening pulley, and subjecting the rope to only one kind of bending.

At *d* is shown a bucket gear, which combines driving and guiding, and is much used for conveying in mills, grain elevators, etc. If the difference in weight between the sides is slight, the tension organ may be a leather belt, but for heavy service a chain is used. This device has been in use from a very early period for well buckets, and in modern times in mud dredging machines. At *e* is the Weston differential pulley block, a modification of the Chinese windlass, Fig. 790 *a*. T_1 and T_2 are chain sheaves fast to each other, producing a differential action due to their difference in diameter, the whole forming a substitute for the older tackle block gear, Fig. 784 *c*.

The form shown at Fig. 796 *d* demands further consideration, as it can be given a series of most important applications.

If the tension organ is made a band and placed in a horizontal or nearly horizontal position, it can be used to convey finely divided material simply poured upon its upper surface. Examples of this are found in the transportation of grain, also in the movement of paper pulp, and many other such purposes; also for conveying straw upon chain lattice conveyors, etc. In all of these cases the material is kept on the conveyor simply by gravity. This condition may be avoided and the capacity extended by using a pair of belts, the material to be conveyed being carried between them. A very important application of this principle is found in power printing presses, the delivery of the sheets being effected by systems of tapes and bands with great speed and accuracy. Band conveyors are also used in needle machinery and in match making machines, and many similar situations.

An important application of driving gear is found in the construction of inclined haulage systems for mine ramps.

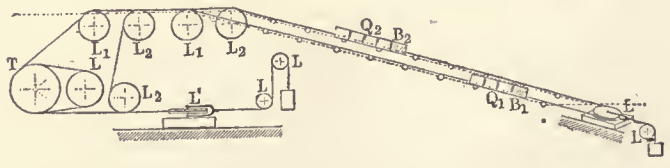


FIG. 797.

In Fig. 797 is shown the inclined cable system of the Rhenish Railway. The driving wind *T L*, operated by a steam engine, works the descending cable on one track and the ascending cable on the other. At *L'* is a tension pulley to take up the slack cable and maintain a proper tension. The trains Q_1 and Q_2, are connected to the brake cars B_1 and B_2, which are extra heavy and control the rate of descent by proper brakes.

In the anthracite coal region of Pennsylvania haulage systems are in extensive use for the transportation of coal, some being constructed with iron bands, but most of them using ropes. The arrangement will be understood from the diagrams in Fig. 798 and 799, which, with the accompanying data, have been obtained by the author from their engineer and constructor, the late Mr. W. Lorenz.

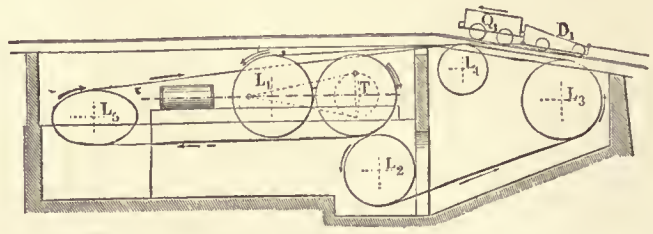


FIG. 798

The car in which the coal is hauled is not attached directly to the cable, but is driven by a dummy *D*, which is permanently connected to the cable. This dummy runs on a narrow gauge track, and at the foot of the incline the narrow track continues on, so that the dummy *D* can go below the main track, as shown in Fig. 799, and on the ascent it can thus be drawn up behind

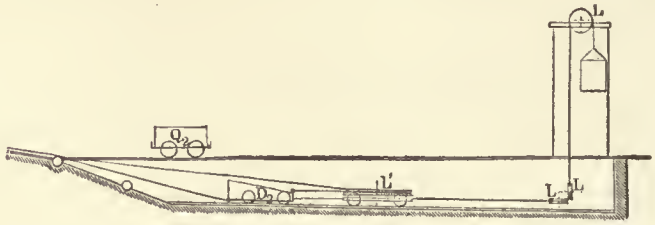


FIG. 799.

the cars which have been placed by the shifting locomotive The steam engine and drawing gear is placed at the head of the incline, as shown in Fig. 798, and the cable is led, as shown by the arrows, that it passes twice over the driving wheel *T*, each time covering about ¾ of its circumference. The dummy cars D_1 and D_2 are connected by a secondary cable passing over the tension sheave *L'*; this secondary cable maintains the proper tension on the main cable, whether the load is at the head or foot of the incline, or on the horizontal. The tension car is given a play of 75 feet to provide for the necessary variation.

A different form of cable haulage is found in the system in use between Lüttich and Ans, and sketched in Fig. 800.*

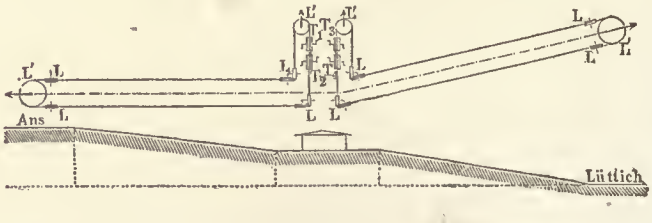


FIG. 800.

In this case the incline is divided into two sections, which make an angle with each other as shown on the plan, and between which is a short level space. On this space is placed the steam engine and driving wheels T_1, T_2, T_3, T_4, each wheel having its own engine, two engines always driving and two being at rest; *L'* are the tension sheaves.

In this, as in the preceding case, it will be noticed that the cable runs continuously in the same direction, differing in this respect from the previously described winding and reversing system. The cable is brought to rest in transferring the cars from one plane to the other in order that this may be readily and conveniently done, but should this be avoided by running them over the connection, by momentum or otherwise, the advantage and usefulness of the system would be greatly increased.

This has been done in the cable tramways of Halliday and Eppelsheimer, first used in San Francisco, and shown in dia-

* See Weber's "Portfolio John Cockerill."

gram in Fig. 801. This is most effectively applied on the trolley streets of the city, for which it is admirably adapted.

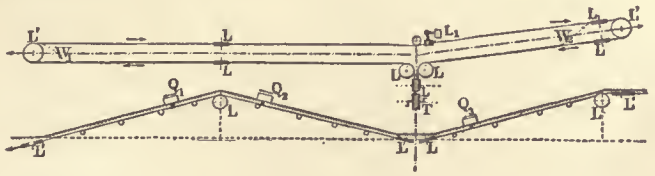


FIG. 801.

The endless cable runs in an iron way between and beneath the tracks, the power being at T and guide sheaves at L, L, with suitable driving and tension mechanism. The cars grasp the cable by a gripping device through a narrow slot in the trackway. The guide sheaves at the bases of the inclines and sides of the curves permit the grip to pass, and when the foot of the hill at the end of the road is reached, the grip is released and the car transferred to the other track as at W_1, and in similar manner shifted at the other end, W_2. The weight of the cars on the down grades counterbalances those on the up grades, and so the motive power has only to overcome the frictional resistance. The cable system of tramways has been extended to Chicago and many other American cities; also in London, and a cable system of canal towage has been projected by Schmick for the proposed Strasburg-Germersheim Canal.

When it is practicable to propel the cars by a suspended cable from overhead a different arrangement may be adopted.

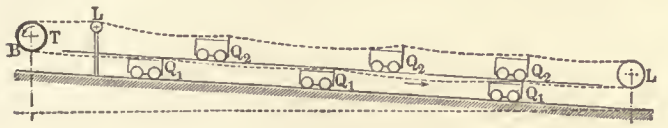


FIG. 802.

Fig. 802 is a diagram of a system operated by a suspended chain. The descending cars Q_1 are loaded and the ascending ones Q_2 are empty, and the speed is controlled by a brake at B. If the action is in the reverse direction, a driving engine must be applied at T. A similar arrangement is much used in coal mines which are entered by inclines. The chain is attached to a fork on the cars.

The system of overhead cable tramway, which has been brought to a high state of efficiency by Bleichert, is based on the same principle as the preceding, but for much lighter loads. The system consists of a cable tramway in which a stationary cable is substituted for the trackway. The running cable is commonly called the pulling rope, and runs underneath the stationary rope. The cars consist of a combination of grooved sheaves, from which the bucket or other receptacle is suspended by curved arms. The stationary cable is supported upon round poles, and the arrangement of the stations is shown in the diagrams of Figs. 803*a* and 803*b*.

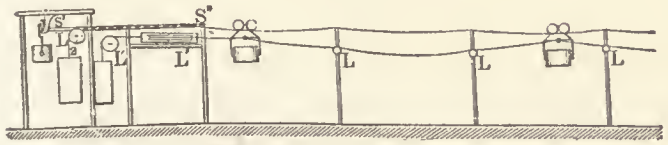


FIG. 803*a*.

The stationary cable connects with the suspended tramway at S^I S^{II} and S^{III} S^{IV}. At S_o is the anchor of the stationary

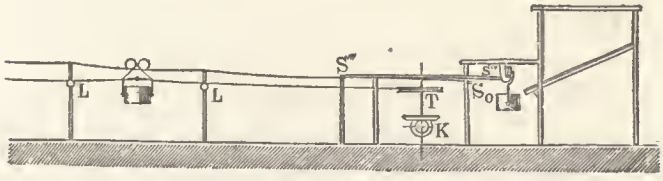


FIG. 803*b*

cable, with a tension weight at L_2. The driving sheave is at T, driven by connections to the engine at K, and at L' is the tension device for the pulling cable. If the service is heavy the cable is carried twice around the driving sheave T, using a counter-sheave, as in Fig. 795*b*, to obtain increased tractive power.

Fig. 804 shows a plan view of a double system.

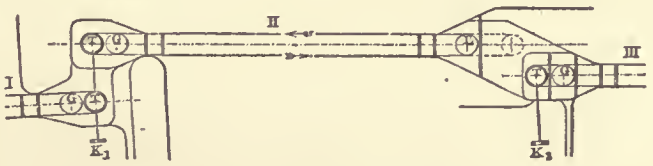


FIG. 804.

At K_1 is the motive power for systems I and II, and at K_2 the motor for system III. The driving sheaves are at T, the counter-sheaves at G, and the tension sheaves at L'.

The supporting columns for the stationary cable must be stiff, and often quite high.

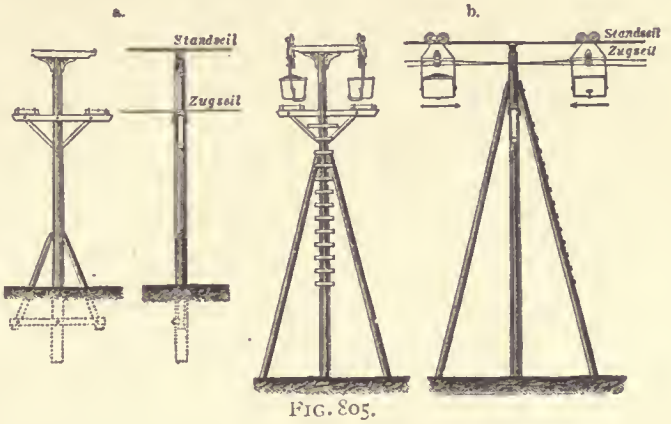


FIG. 805.

Fig. 805 shows the forms used by Bleichert, *a* being used up to 24 feet high, *b* for heights between 24 and 80 feet.*

In Fig. 806 is shown a combination of driving and guiding systems in which the guiding and driving sheaves are combined upon the car Q, and the tension organ is fastened at two points S_o S_o on the path of the car Q.

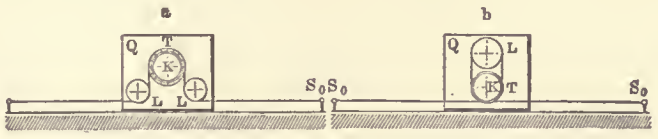


FIG. 806.

The motive power is on the car and operates the sheave T. In the form shown at *a*, a Fowler grip sheave is used at T, this form being suitable for a rope system, while the form shown at *b* is better adapted to be used with chain.

The system shown in Fig. 806 *b* is also adapted for hauling boats, and has been used by Harturch for operating the railway ferry across the Rhine at Rhinehausen. The ferry boat in this case is guided by a stationary cable securely anchored, as in Fig. 807, the anchorage being up the stream, and the force of

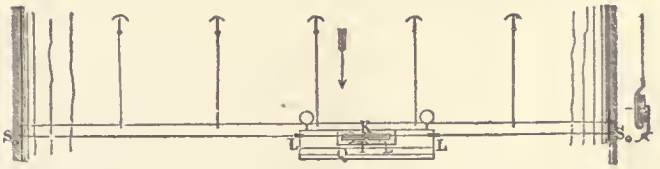


FIG. 807.

the current keeping the cables taut. The equilibrium of these forces enables this to act in the same manner as the stationary cable of the Bleichert system, the difference only being that the load, instead of being suspended from the cable, exerts a lateral stress. The driving cable is similar to Fig. 806 *b*, and is beneath the surface of the water.

If we imagine, in the combination of Fig. 806, that the traveling vehicle Q may be longer than the distance S_o S_o, which is the full length of the tension organ, the principle will not be altered, but the action will be modified, since the relations of the traveling vehicle and the tension organ are now inverted. The ends of the tension organ can now be joined

* On the tramway at Liker-Vashegy, poles of 140 feet high are used.

together, or in other words it can be made endless, and if heavy enough, its weight can be caused to produce enough friction on the bed of the stream to furnish the necessary resistance. This is the construction of Heuberger's chain propeller, Fig. 808, as improved by Zedé

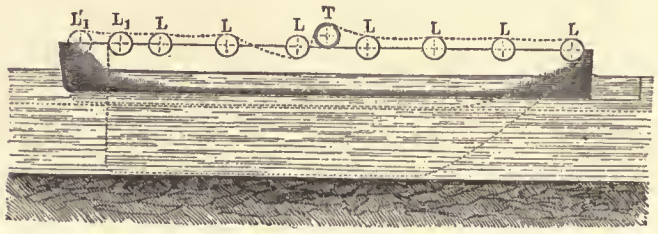


FIG. 808.

T is the driving sheave for the chain, L, L, L are guide sheaves, L_1 is a movable sheave to take up a portion of the slack chain when passing into shallow water. The system is made double, being placed on each side of the boat, and each side is driven independently, so that sharp curves can be turned.*

If, in the case of a tension organ driven by a revolving pulley, there is not sufficient tension given, the friction becomes insufficient to overcome the resistance of the load; if the necessary tension is externally supplied and removed periodically, a continuously revolving pulley can be caused to produce a lifting and dropping action of a given load. This plan has been adopted in some forms of drop-hammers, of which Fig. 809 is the arrangement. T is a pulley running continuously in the direction of the arrow, Q is the drop weight, H a handle by which the operator applies and releases the tension which causes the pulley to drive or slip.

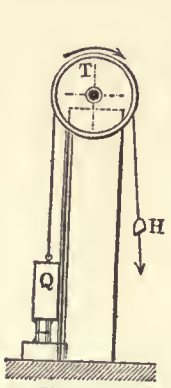


FIG. 809.

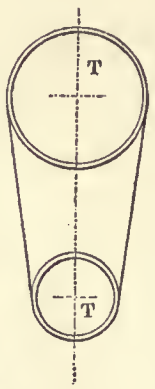


FIG. 810.

The applications of running tension organs which have been thus far considered, are those in which the device has been used either to lift weights or to transport the same from place to place. One of the most important applications, however, is that of transmitting rotative motion from pulley to pulley, an operation which can be almost indefinitely repeated. This combination includes all numerous forms of belt, rope and chain transmission, Fig. 810. The necessary tension for this purpose is sustained by the journals and bearings of the pulleys, also being modified by supporting or by tightening pulleys. The two portions of the tension organ are distinguished as the tight and slack sides respectively, and many modifications of this form of transmission are discussed more fully hereafter, (see Chap. XX to XXII).

There is one application, however, which is appropriately discussed in this place, namely, that in which rotative transmission between pulleys upon stationary axes is combined with pulleys upon a movable member, thus enabling motion to be transmitted from a stationary source to a moving body, Fig. 811.

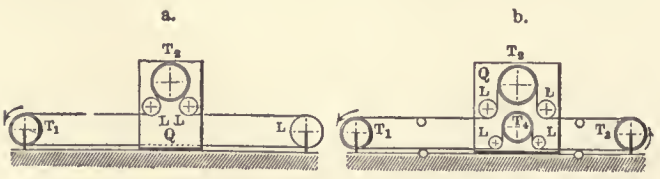


FIG. 811.

In case *a*, one of the driven pulleys is mounted upon a carriage, saddle, trolley, or the like, and may be shifted in position upon its ways or track; the tension is sustained by the three guide sheaves. Applications of this form, using belting, are used upon planing machines by Sellers, Ducommun & Dubied and others. With rope driving gear it is used to operate the spindles upon the carriage of the self-acting mule, also for operating traveling cranes by Ramsbottom, by Tangye, and by Towne; being combined by the latter with the squaring device as shown in Fig. 784 *e*, and effecting all the functions of the crane, including bridge and trolley travel, as well as the hoisting and lowering of the load.

The form of Fig. 811 *b* differs from *a* in that both sides of the belt or rope are used to transmit power. The stationary pulleys T_1 and T_3 here drive the movable pulleys T_2 and T_4. These driven axes can be utilized in various manners, as, for example, to operate a windlass device for the propulsion of the carriage Q; an example of which is found in Agudio's cable locomotive.† In this device the pulleys T_2 and T_4 drove a friction train which operated a drum connected with a stationary cable as in Fig. 806.

A more recent device is shown in a modification of Fig. 811 *a*, as shown in Fig. 812.‡

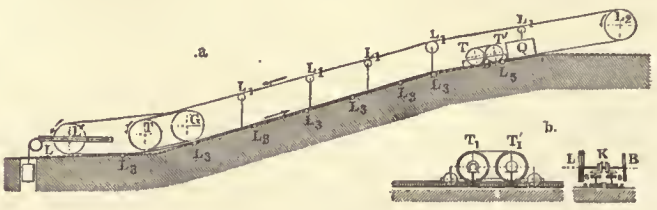


FIG. 812.

This construction, which is in use at the Soperga-Rampe at Turin, consists of a double rack, placed between the rails as shown at *b*, which also shows the gearing by which car is driven. The motive power is placed at the foot of the incline at T, G, the 500-horse power engine running continuously in one direction. The cable is carried upon the overhead guide sheaves L_1 and passes around the pulley L_2, and through the sheave system T T' of the locomotive, and is supported also on guide sheaves under the track, a tension pulley being placed at L'. The velocity of the driving cable is four times that of the cars, and the descent is effected by gravity alone under control of a brake. During the descent the bevel gears on the shaft of the driving pulley are released by friction clutches at K, thus rendering the car independent of the cable.

The foregoing condensed description is nevertheless fully sufficient to indicate the extreme service of which tension organs are capable in machine design. No less than seven systems have been shown for railway use, and four for boats. This is the more significant since it will be remembered that cable propulsion had been abandoned for railway use, but yet appears to now be revived with increasing success.

Our division into Guiding, Winding, and Driving systems enables different devices to be placed in corresponding classes. There yet remains to be considered the co-existing action of many of the devices, such as pulleys, windlasses, cranes, etc., in which a negative motion may be given to the tension organ by the descent of the load Q under the action of gravity.§ This action can be fully determined by reversing the previously considered movement for the backward motion. In the common belt transmission, Fig. 810, the action is reversible, as is also the case with the simple pulley, Fig. 794 *d*.

The case is different, however, with the rope tackle Fig. 784*c* and the differential block Fig. 796*e*, which are therefore here considered in the more general form of Fig. 813.

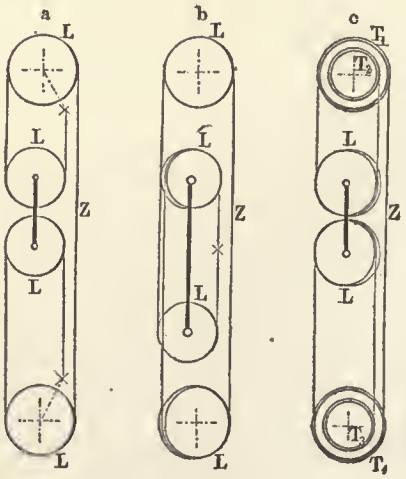


FIG. 813.

If in these forms the cord Z is pulled in either direction the lower sheave will be also moved up or down proportionally. At the present time systems using endless cords are under consideration, but frequently choice is to be made as to which portion is best used. It will be seen that the system of Fig. 806, which is made with both ends of the cable secured, can also be considered as a portion of an endless system similar to Fig. 808, and other endless systems are found in Fig. 784 *d* and *e*; also Fig. 813 *b*, which differs from *a* only in the running of the rope, the united ends being marked by a cross. If

* The following data of performance are given by Zedé: Capacity, 500 tons; length over all, 230 ft.; breadth, 21½ ft.; depth, 6½ ft.; midship draught, 31½ in. The chains were of cast iron, weighing 275 pounds per yard· two engines of 150 I. H. P. gave a speed of 3.72 miles (!) per hour.

† See Thomas Agudio. Memoire sur la Locomotive funiculaire, Turin, 1863.
‡ See Bulletin de la Soc. d'Encouragement, Vol. XVI., 1869, p. 48.
§ Kinematic force closure. First discussed in the Author's Theoretical Kinematics, p. 575.

If we bring the applications of Figs. 806 and 811 into a general form in which the path of travel shall return upon itself, we have Fig. 814 *a*. If the guide sheaves are removed and the

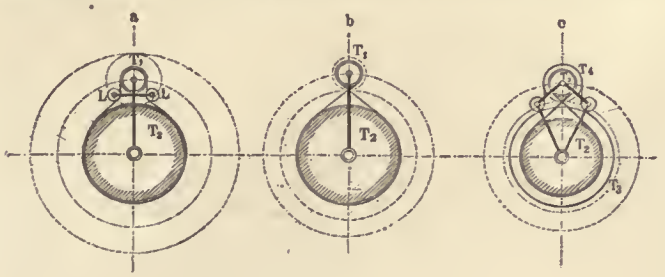


FIG. 814.

cord crossed, the simpler form of Fig. 814 *b* is obtained. The rotation of the pulley T_1 causes travel around the stationary pulley T_2. The old form of Agudio's cable locomotive may be represented by a similar diagram, Fig. 814 *c*. The shaded pulley T_2 is held stationary, while the conceutric pulley T_3 is assumed to revolve; this causes the system to revolve in a circular path, the whole forming a differential or epicyclic system.

Finally it may be remarked that in electric transmission systems a similar aualogy exists to the above combinations of teusiou organs of wire and cable in various forms.

§ 263.

TECHNOLOGICAL APPLICATIONS OF TENSION ORGANS.

In addition to the preceding applications of tension organs, they are also used iu numerous forms of machine tools, *i. e.*, as organs for the alteration of the form of bodies.

A straight blade of steel furuished with teeth forms the well-known frame or gang saw used in numerous wood-working machines. When made without teeth, aud used with sand and water, it becomes a stone-cuttiug saw, or in the form of a wire charged with oil aud emery or diamond dust, a saw for the hardest materials, in which case a high tension must be given to the wire to prevent lateral displacement. The saw blade may be given a vibrating motion in a device such as Fig. 789 *b* for use as a scroll saw. In all these cases a reciprocating motion is used. Teusion organs are also used as ruuning members for sawing, the form of Fig. 810 becoming the well-known band saw. Very fine band saws have been made, and also saws of wire, these having been used as long ago as 1877 by the writer, suggested by the saws used for precious stones.

An ingenious form of wire saw has been made by Zervas for cutting blocks of lava or stone from the original bed, as shown in the diagram Fig. 815.

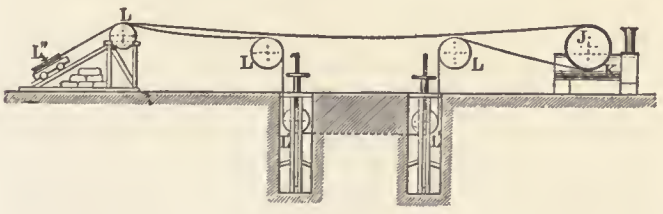


FIG. 815.

Two small shafts are sunk in the stone, and the guide pulleys inserted as shown, the endless wire being fed down by the screws. The cutting is effected by using water aud sand, and the cord is formed of three twisted wires, although more recently a single smooth wire, with twisted one wound above it, has been used, the outside diameters being $\frac{1}{4}''$ to $\frac{5}{16}''$.

A patent was takeu out in Germany by Paulin Gay in 1882 for an apparatus for cutting a block of stone into slabs by the use of a number of wire saws.

Polishing belts are another example of teusion orgaus used as tools, the flat side of the belt being used, impregnated with polishing material. Such belts, used in the nickel-plating establishment of Neumann, Schwartz & Weil, at Freiberg in Breisgau, are operated at a speed of over 6500 feet per minute.

Tension organs are of frequent use in many details of spinning machinery, acting both for guiding and winding; also in numerous other forms of textile machinery.

Chains are especially useful for dredging machinery, working in wet or dry material, also for handling coal.

In musical instruments we find tension organs of definite dimension and stress, as sound producing machines.

§ 264.

CORD FRICTION.

When a tensiou organ which is loaded at both ends is passed over a curved surface, there is produced between the tension orgau and the surface a very considerable sliding friction. Since this friction will first be mathematically considered in connection with the subject of cords, it will be given the general name of cord friction. The curved surface over which the cord is passed is the pulley, and the motion of the cord takes place in the plane of the pulley. If the tension T on the driving side of the cord is to overcome the cord friction F, as well as the tension t of the driven side, we have for the value of the friction, $F = T - t$. It is dependent upon the magnitude of the angle of contact a and upon the coefficient of friction f, but is iudependent of the radius R of the pulley; it is also dependent upon the iufluence of centrifugal force. For these conditions we have:

$$T = t\, e^{f a (1 - z)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (237)$$

$$F = t\,(e^{f a (1 - z)} - 1) \quad \ldots\ldots\ldots\ldots\ldots \quad (238)$$

In these e is the base of the natural system of logarithms = 2.71828, and $z = 12 \frac{\gamma v^2}{g S}$; v being the velocity of the tension organ in feet per second, S the stress in its cross section, γ the weight of a cubic inch of the material, and g the acceleration of gravity = 32.2.

Example.– In the capstan shown in Fig. 794 *a*, let $f = 0.21$, $a = 6\pi = 3$ convolutions, $z = 0$. We then have $f a = 0.21 \times 6 \times 3.14 = 3.958$, say 4, and $F = t\,(2.7184 - 1) = t\,(54.6 - 1) = 53.6\, t$. This shows the friction upon the capstan drum to be nearly 54 times the pull upon the free end of the cord.

The influence of centrifugal force becomes important at high speeds, and when the tension organ is under small stress. For hemp or cotton rope, or for leather belting, we may take $\gamma = 0.035$, and for wire rope about nine times so great.

The value of S in the formula $z = 12 \frac{\gamma v^2}{g S}$ is properly considered a function of a, and we may therefore assume a constant value for the arc a, and thus calculate the following table for the values of $1 - z$.

TABLE.

S.	Value of Coefficient $1 - z$ for Centrifugal Force.					S.
	Velocity of Rope in Feet per Second.					
Hempen Rope.	20	40	60	80	100	Wire Rope.
400 lbs.	0.987	0.948	0.882	0.791	0.674	3,600 lbs.
600 "	0.991	0.965	0.922	0.861	0.783	5,400 "
800 "	0.993	0.974	0.941	0.896	0.837	7,200 "
1000 "	0.995	0.980	0.953	0.916	0.870	9,000 "
1200 "	0.996	0.982	0.961	0.930	0.892	10,800 "
1400 "	0.996	0.985	0.966	0.940	0.907	12,600 "

This table serves both for hemp and for wire rope by taking the ninefold value of S in the right hand column for wire rope. It should be observed that the velocities are in feet per second. It will be seen that for high speeds a high stress in the tension organ is necessary in order to oppose the action of the centrifugal force.

In order to simplify practical calculations we may substitute for the exponent $f a\,(1 - z)$ in each case the form $f' a$; that is, instead of using the actual coefficient of friction f, taking another one f', which is equal to $(1 - z) f$. If it is a transmission system, as Fig. 810, which is under consideration, the friction of the cord, belt, chain, etc., must at least equal the transmitted force P, hence also must the stress be that of a cord friction $\geqq P$, which gives for a minimum value of T:

$$\frac{T}{P} = \tau = \frac{e^{f a}}{e^{f a} - 1} = \frac{\rho}{\rho - 1} \quad \ldots\ldots\ldots\ldots \quad (239)$$

wheuce

$$\frac{T}{t} = \rho = e^{f' a} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (240)$$

Both of these values are absolute numbers. The ratio $\frac{T}{P}$ indicates the amount of stress which must be given to the ten-

sion organ, and hence may be called the stress modulus, and is designated as τ. The ratio $\frac{T}{t}$, we may, in like manner, call the modulus of cord friction, aud indicate as ρ. A series of values for both are given in the followiug table.

Moduli for Cord Friction and Stress.

$f' a$	$\rho = \frac{T}{t}$	$\tau = \frac{T}{P}$	$f' a$	$\rho = \frac{T}{t}$	$\tau = \frac{T}{P}$
0.1	1.11	10.41	1.6	4.95	1.25
0.2	1.22	5.52	1.7	5.47	1.22
0.3	1.35	3.86	1.8	6.05	1.20
0.4	1.49	3.03	1.9	6.69	1.18
0.5	1.65	2.54	2.0	7.39	1.16
0.6	1.82	2.22	2.2	9.03	1.13
0.7	2.01	1.99	2.4	11.02	1.10
0.8	2.23	1.86	2.6	13.46	1.08
0.9	2.46	1.69	2.8	16.44	1.07
1.0	2.72	1.58	3.0	20.09	1.05
1.1	3.00	1.50	3.2	24.53	1.04
1.2	3.32	1.43	3.4	29.96	1.03
1.3	3.67	1.37	3.6	36.60	1.03
1.4	4.06	1.33	3.8	44.70	1.02
1.5	4.48	1.29	4.0	54.60	1.02

Example.—Arc of contact $= \pi$; coefficient of friction $f = 0.16$, velocity $v = 80$ feet. The tension organ is a leather belt under stress of 400 lbs. per square inch. We have from the first table $1 - z = 0.791$, hence $f' a = 0.791 \times 0.16\,\pi = 3.976$, or nearly 0.4. From the second table this gives $\rho = 1.49$ and $\tau = 3.03$, that is, over three times the above stress on the belt would be required to overcome the frictional resistance. If $v = 20$ ft., the value $1 - z = 0.987$, and $f' a = 0.496$ or about 0.5, and the modulus of stress $\tau = 2.54$.

In order to make these relations more apparent, they are shown graphically in the diagram, Fig. 816, in which the scale upon the upper horizontal line gives the values for both moduli, while the vertical scale ou the left gives correspouding values of the product $f' a$.

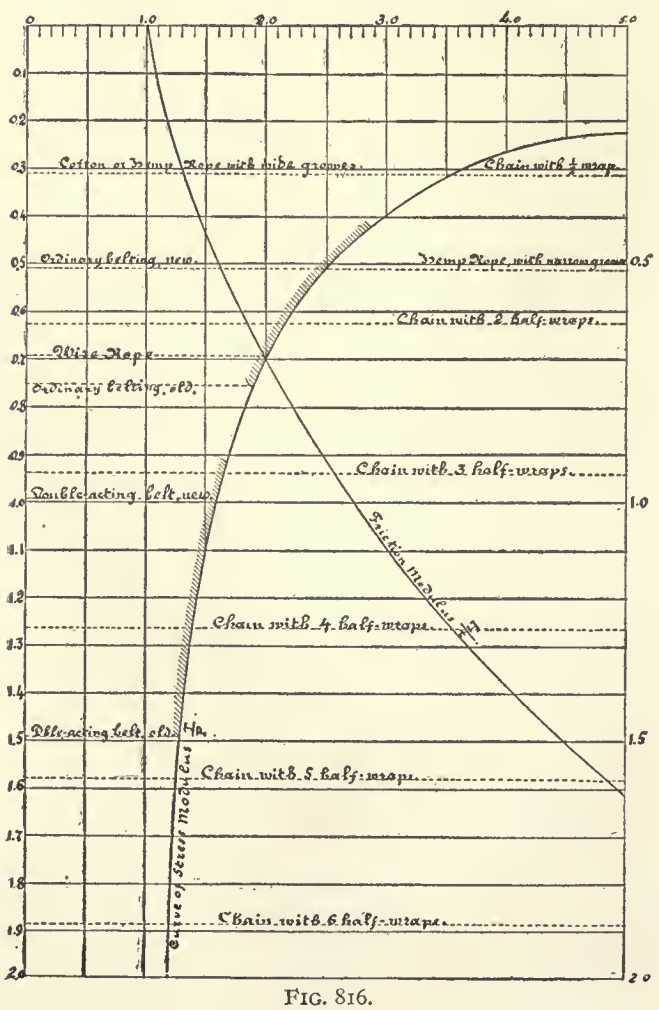


FIG. 816.

The superficial pressure p of the tension organ upon the circumfereuce of the pulley increases as the belt or cord passes from the slack to the tight side. It is equal to $\frac{Q\,d\,a}{b'\,R\,d\,a}$, in which b' is the breadth of the surface of contact of the belt. Now for any cross section q, the force $Q = q\,S$. Hence we have:

$$\frac{p}{S} = \frac{q}{b'\,R} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (241)$$

from which it will be seen that the pressure p can easily be kept with moderate limits.

Special applications of this formula, and of the diagram, Fig. 816, will be given hereafter.

§ 265.

ROPES OF ORGANIC FIBRES.

Hemp Rope.—The form in most general use is a round hemp rope twisted of three strands. This is twisted "loose" or "tight," according as it is desired to be more or less flexible. The cross section of a three-strand round rope, in which δ is the diameter of a single strand, is $3\,\frac{\pi}{4}\,\delta^2$, whence δ bears the following relation to the diameter d of the circumscribing circle: $d = \delta\left(1 + \frac{1}{\cos 30}\right) = 2.15\,\delta$, Fig. 817. This gives for the cross section $q = \frac{\pi}{6.16}\,d^2$. On account of the spiral twisting of the strands, and their compression upon each other, this may be taken $q = \frac{\pi}{5}\,d^2$, that is about 0.8 times the value of the full cross section. Good hemp rope, when loosely twisted, will bear a stress of 1700 pouuds, and when tightly twisted, about 1½ times as much. For convenience of calculation we may assume the cross section to that of the full circle d, if, instead of the full stress, we take ouly $\frac{4}{5}$ as much, or 1400 lbs., and 2100 lbs. We then have for the force P, for:

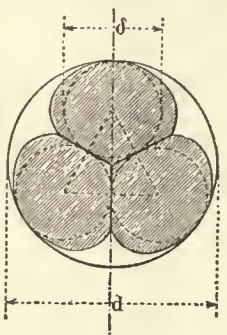


FIG. 817.

$$\left.\begin{array}{l} \text{loosely twisted rope } d = 0.03\sqrt{P}; \text{ and } P = 1111\,d^2 \\ \text{tightly } \quad " \quad " \quad d = 0.024\sqrt{P}; \quad " \quad P = 1677\,d^2 \end{array}\right\} \quad (242)$$

The radius R of the pulley should never be less than 3 to 4 d for loosely twisted rope, aud not less than 6 to 8 d for tightly twisted rope, the diameter being measured to the centre of the rope. For heavy service, as for hoisting machines, R should be not less than 25 d.

Flat hemp ropes are made by sewing 4 to 6 round ropes together, each rope being then proportioned to bear $\frac{1}{4}$ or $\frac{1}{6}$ the whole load.

The runniug weight G_o per foot is as follows:

$$\left.\begin{array}{l} \text{For loosely twisted rope, } G_o = 0.325\,d^2 \\ \text{For tightly } \quad " \quad " \quad G_o = 0.467\,d^2 \\ \text{and approximately for both } P = 3400\,G_o \end{array}\right\} \quad \ldots\ldots \quad (243)$$

The latter assumption is based on the same number of fibres in both cases. The followiug table gives valnes for three-straud hemp rope.

Dia.	Loose Twist.		Hard Twist.	
d.	P.	G_o.	P.	G_o.
½	276	0.081	397	0.116
¾	621	0.183	893	0.263
⅞	967	0.284	1,389	0.408
1	1105	0.325	1,588	0.467
1¼	1726	0.508	2,481	0.729
1½	2485	0.731	3,572	1.050
2	4420	1.300	6,351	1.868
2½	6906	2.031	9,923	2.919
3	9945	2.925	14,290	4.203

According to (243) a rope L feet in length, hauging vertically, is loaded $\frac{1}{3400}\,L$ of its working strength already by its own weight. If $L = 3400$, the entire practical load would already

be applied, and this may be considered practical working length of the rope. We have for the available practical working load: $P' + \frac{1}{3400} L P = P$ or $P' = P\left(1 - \frac{1}{3400} L\right)$.

A vertically suspended rope will break by its own weight when its length reaches about 2000 feet, since the modulus of rupture is about 8500 lbs. for loosely twisted rope, and about 14,000 lbs. for tightly twisted rope. The above length (2000 ft.) may be called the length of rupture. For a cord suspended in the water, as for deep sea sounding, the length of rupture is about twice as great. For very heavy stresses three simple strands are insufficient, and the strands themselves are each made of smaller strands, as in cable construction. Very heavy cables are also made of more than three strands.

Cotton Rope.—Cotton rope has been used of late for purposes of transmission, and is usually made with three strands, very loosely twisted. It opposes a resistance to rupture of about 7500 pounds, reckoning the full sectional area, and is operated under stresses ranging from 1000 to 2000 pounds. It is used for driving spindles in spinning frames and mules, and in the snail drum movement, as in Fig. 787,* and is also used for operating traveling cranes on the Ramsbottom system.

Driving ropes are usually operated over grooved pulleys, the radius of the semicircular groove being slightly greater than that of the rope. In machine construction the sheaves are usually of cast iron, and in ship's tackle they are made of lignum vitæ.

The sheaves revolve on cylindrical journals, and recently roller bearings are being used, Fig. 818.†

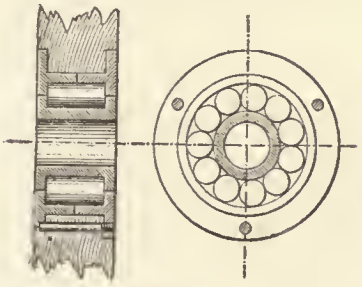


FIG. 818.

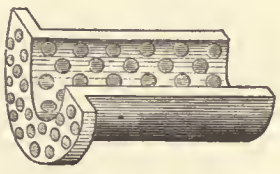


FIG. 819.

When the pressure is moderate, the rollers may be made of hard bronze, but for high pressures the rollers, ring and journal should all be made of hardened steel. In case of extremely high pressures bronze bearings with metaline may be used, the metaline being a solid lubricant imbedded in recesses in the box, Fig. 819.‡ Such bearings were used most successfully in the construction of the East River Bridge at New York, operating for an entire year without requiring lubrication.

§ 266.

WIRE ROPE.

Wire rope is usually round, and made with 36 wires, since six strands are used, each containing six wires. Each strand contains a small hemp core, and the strands are twisted about a central core of hemp. These hempen portions are of greatest importance in the construction of wire rope for transmission (see § 268), and should be made of the best material. For stationary ropes the hempen strands may be replaced by wire, giving 42 or 49 wires, and proportionally increasing the strength of the rope. The strands of six wires may be combined to make ropes of 48, 54, 60, 66, 72 wires, etc., and other combinations are also used.

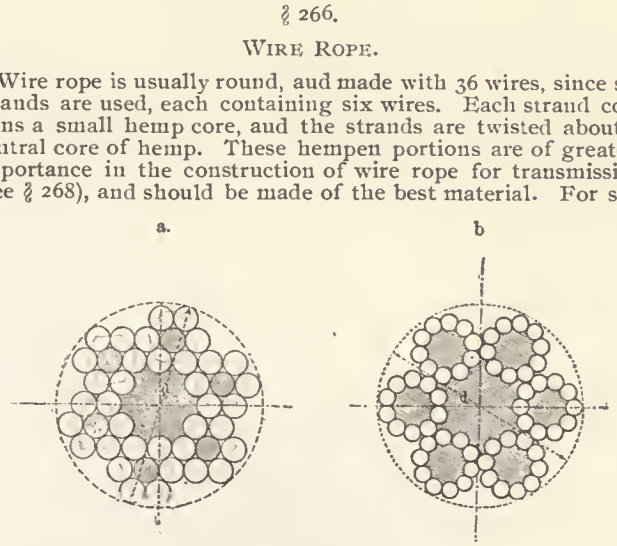


FIG. 820.

In Fig. 820 is shown at *a* a section of a rope of 36 wires, and at *b* a different form of 60 wires, both being made with cores of hemp for the strands as well as for the ropes. For the external diameter of the wire ropes of the preceding form, when the wires lie in close contact, we have:

$$\left.\begin{array}{lcccccc} i = & 36 & 48 & 54 & 60 & 66 & 72 \\ \frac{d}{\delta} = & 8.00 & 10.25 & 11.33 & 12.80 & 13.28 & 14.20 \end{array}\right\} \quad \ldots \quad (244)$$

in which i is the number of wires, δ the diameter of a single wire, and d the diameter of the rope.

In some later kinds of rope they do not lie in contact with each other, but are separated slightly by the hemp, in which case the diameter will exceed the previous figures by 10 to 15 per cent., but after a period of use the diameter becomes reduced to the sizes given above. When the strands are made without hemp cores they are arranged in the following manner: §

3 7 10 14 16 19

while with hemp cores the numbers are

5 6 7 8 9 10.

The number of strands runs from 3 up to 4, 5, 6, which latter is most used, and on up to 7, 8, 12, 14, 16, 19. For cables which are required to resist heavy stresses and also to possess great flexibility, the same construction is employed as for hempen cables, the strands themselves being composed of twisted ropes; the number of strands is 3, 4, 5 or 6. Flat cables are also made of a number of parallel ropes. The number of ropes is 4, 6 or 8; the number of strands in each rope 4 to 6.

Example.—A heavy cable of steel wire is made of 6 ropes, each rope of 19 strands, each strand containing 7 wires. The total number of wires $= 6 \times 19 \times 7 = 798$. Diameter of wire $\delta = 0.055''$.

Well made rope is so wound that the load produces a uniform stress upon all the wires, so that, when i = the number of wires, P the load, S the stress on the wire, we have

$$P = S\, i \frac{\pi}{4} \delta^2 \quad \ldots \quad (245)$$

The diameter of wire varies from 0.04″ to 0.14″. If the rope is required to be very flexible the wires should not be more than 0.1″ in diameter.

In the passage of the rope over a sheave or pulley, of a radius R, the individual wires are subjected to bending, which, under the action of tension and compression (see § 8), produces a stress of a magnitude $s = \frac{E\,\delta}{2\,R}$, in which E is the modulus of elasticity of the material. For steel or iron wire E may be taken at = 28,440,000.

$$\text{This gives } s = 14{,}220{,}000 \frac{\delta}{R} \quad \ldots \quad (246)$$

The stress s, which is produced on the tension side by bending, must be considered in connection with the stress S produced by the load P, in order to arrive at the total stress. In order to avoid a permanent set, it is necessary that the sum $S + s$ should not exceed the modulus of elasticity. The actual magnitude of R becomes a minimum when $s = 2S$; that is, the stress due to bending becomes double that due to the working tension.

Whatever may be the relation between the pulling stress S, and the bending stress s, the total stress on the material will be the sum $S + s$.

If it is desired to consider the security against rupture as well as the possible overstepping of the elastic limit, the value of $S + s$ must be taken into account. The Prussian Government rule places the modulus of rupture K if steel wire at 163000 pounds, or with a factor of safety of 6, the stress $S = \frac{163000}{6}$ = 27,166. If we take the case of a rope of 42 wires, its diameter $d = 10\delta$, and making the pulley diameter = 75 d, we get $R = 37.5\, d$. This gives the bending stress, according to (246),

$$S = \frac{14{,}220{,}000\,\delta}{37.5 \times 10\delta} = 37{,}920.$$

The sum $S + s = 27{,}166 + 37{,}920 = 65{,}086$. This gives an actual factor of safety of $\frac{163000}{65086} = 2.5$.

* In a spinning mule of 844 spindles, by J. J. Rieter & Co., of Winterthure, a rope 22 mm. (0.866 in.) operates under a stress of 1.6 kg. (2275 lb.), taking the full cross section.

† Martini's design, used in the Italian navy.

‡ John Wallace & Co., London; Selig in Berlin.

§ In American mining machinery, six strand ropes of 19 wires, with hemp cores in the middle, are much used.

The relation of the stresses in the various parts of the rope are shown in Fig. 821.

On the right, the tension side, there is the tension stress $(+S)$ and the bending stress $(+s)$, giving a total of $S+s$. On the left the tension stress $(+S)$ is diminished by the reverse bending stress $(-s)$. The neutral axis is therefore shifted from the middle at N, to a point toward the concave side of the bent rope at N^1.

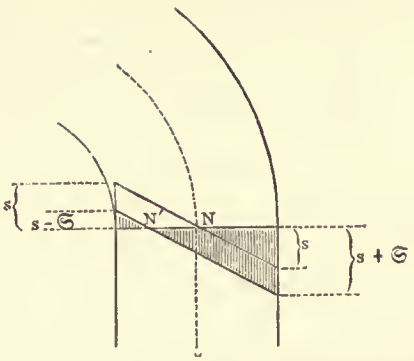


FIG. 821.

Wire rope may be made either of iron or steel wire, and its fabrication has greatly advanced within recent years. The following data are applicable to the various grades :*

Material.	Elastic Limit.	Modulus of Rupture
Annealed Iron Wire . .	42,000	56,000
Bright Iron Wire . . .	56,000	80,000
Steel Wire	64,000	85,000
Steel Wire	78,000	142,000
Steel Wire	99,000	170,000
Steel Wire	113,000	213,000
Steel Wire	142,000	256,000

It will be evident that no general rule can be given as to material, but that definite figures should be obtained for the material to be used in each case. For high speed rope the wire should be both smooth and strong, with a modulus of rupture of about 170,000 lbs. If we then take a working stress $S = 28{,}000$ lbs., and a bending stress $s = 28{,}000$ lbs., we have $S + s = 56{,}000$ lbs., which gives about threefold security.†

For $s = 28{,}000$ we have $R = \frac{14{,}220{,}000}{28{,}000}\,\delta = 500\,\delta$. If R is made less, the security will be reduced; if greater, it increases.‡ The durability of the rope for mining service is increased by galvanizing the wire.

For standing rigging of vessels galvanized annealed iron wire, with a value $K = 56{,}000$ is used, while for running rigging steel wire rope ($K = 170{,}000$) is being more extensively used; this also being galvanized. The latter rope is also suitable for cables. Hawsers are frequently made from iron wire, with a modulus of rupture $K = 56{,}000$ to 70,000. The cables for steam plowing machinery should be made of the strongest steel wire, $K = 256{,}000$.

Wire Cables for power transmission are discussed in Chapter XXI.

The cables for suspension bridges are not made from twisted strands, but the wires are laid parallel and held in position by bands of wire every two or three feet. §

* See the researches of J. W. Cloud on steel wire in connection with the Emery Testing Machine at the Watertown Arsenal. Trans. Am. Soc. Mech. Eng'rs, Vol. V.

† The Prussian rule requires $S = \frac{1}{6}K$, which gives about 28,000, and $R = 375\,\delta$, which gives $s = 38{,}000$, hence the security is only about 2½, or less than given above.

Prschibram has used with best results, $S = 23{,}000$, $s = 27{,}000$: also $S = 22{,}750$, $s = 36{,}000$, but finds that a value s 27,000 to 28,000 lbs. is better for the preservation of the rope. (See § 268.) In considering the question of pulley diameter, the ratio to the diameter δ of the wire should be taken, not that to the diameter d of the rope.

‡ If $\frac{R}{\delta}$ is made so small that $S + s$ is greater than the elastic limit, the rope will receive a permanent set. This, however, is not always dangerous.

In Fig. 822 the curvature 1 . 1 may produce a stress upon the concave side of the wires which, when added to S, may not exceed the elastic limit. If, however, a reverse curvature be given, as at 3 . 3, there may result a set, as 3′ . 3′, and too frequent repetition of this reversal may become dangerous. This is shown in the case of hoisting drums, such as Fig. 792 *c*, in which the rope $W_2\,L_2$, which is subjected to reverse bending, has been found to last only about ¾ as long as the rope $W_1\,L_1$.

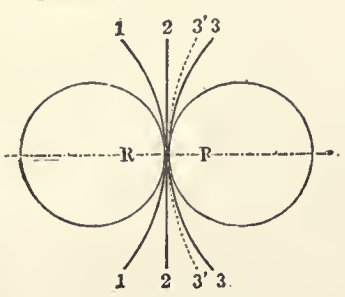


FIG. 822.

§ Among important suspension bridges are those built by Roebling in America, notably the Niagara, and the East River bridges.

§ 267.

WEIGHT OF WIRE ROPE AND ITS INFLUENCE.

A rope of parallel iron or steel wires, exclusive of any bands, will weigh, per foot, $0.28\left(12\frac{\pi}{4}\,i\,\delta^2\right)$, in which i is the number of wires and δ the diameter of each wire. For twisted rope, the twist and the hemp core increases this value from 1⅛ to 1¼ as much, or an average of 1⅙ times. This gives for the running weight per foot

$$G_o = 3.92\,\frac{\pi}{4}\,i\,\delta^2 = 3.07\,i\,\delta^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

This is also true for flat ropes, the value of the coefficient for cable ropes being increased as above from 1⅛ to 1¼, usually about 1⅙ times. For deep mine hoists the weight G_o exercises a marked influence upon the section of the rope. If $L\,k$ is the length in feet of the vertical hanging rope carrying a load P at its end we have: $P + L\,G_o = S\,\frac{\pi}{4}\,i\,\delta^2$, whence for ordinary round wire rope:

$$P = S\,\frac{\pi}{4}\,i\,\delta^2\left(1 - 3.92\,\frac{L}{S}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (248)$$

Example 1.—Let the depth of shaft $L = 1640$ ft. Wire rope of steel, $K = 170{,}000$, $S = 28{,}000$, $P = 4400$ lbs., and $i = 36$.

$$\delta^2 = \frac{\frac{4}{\pi} \times 4400}{28{,}000 \times 36\ (1 - 0.229)} = 0.0056$$

which gives $\delta = 0.075$. If $L = 0$ we get $\delta^2 = 0.0034$, and $\delta = 0.058$.

The above discussion enables us to determine the length L_t of rope which would produce by its own weight the stress S in the uppermost cross section:

$$L_t = 0.25\,S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (249)$$

This may be called the load-length for the stress S. Should the shaft reach a depth equal to the load-length, no weight could be suspended to the rope without exceeding the permissible stress S. If S is equal to the modulus of rupture, the rope would be broken by its own weight. This rupture-length may be designated by L_z, and is

$$L_z = 0.25\,K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (250$$

Example 2.—For round wire rope of uniform cross section the rupture-length L_z is as below for the corresponding strength:

K	56,000	80,000	85,000	142,000
L_z	14,000	20,000	21,250	35,500
K	170,000	213,000	256,000	
L_z	42,500	23,250	64,000	

For very deep shafts it has been found advantageous to make the rope a body of uniform resistance, which would make both load-length and rupture-length unlimited. The formulæ for this purpose have been already given in § 4. The taper to the rope may be given in two different ways. Either a constant diameter δ of wire, and varying number i, may be used; or a constant number i, and variable diameter δ. If the smaller diameter of wire $= \delta_o$, or the minimum number of wires $= i_o$, we have for any depth x:

$$\log\frac{i}{i_o} \text{ or } \log\frac{\delta^2}{\delta_o{}^2} = 0.4342945\,\gamma'\,\frac{x}{S}.$$

In this γ is the coefficient of weight which, for round rope, we have found to be $= 3.92$. Substituting this value we get:

$$\log\frac{i}{i_o} \text{ or } \log\frac{\delta^2}{\delta_o{}^2} = 1.68\,\frac{x}{S} \quad . \quad . \quad . \quad . \quad . \quad (251)$$

Example 3.—If the value of S be taken as 28,000, we have for the following depths:

$x =$	1000	1500	2000	2500	3000	3600
$\frac{x}{S}$	0.036	0.054	0.0714	0.089	0.107	0.121
$\frac{i}{i_o}$	1.115	1.123	1.318	1.411	1.512	1.597
$\frac{\delta}{\delta_o}$	1.072	1.110	1.148	1.187	1.230	1.263

These values will serve to approximate the intermediate cases ‖

‖ In the Prschibram mines taper ropes are in practical use. The rope in the Adalbert shaft is as follows: $P = 3850$ lbs., of which 2200 is useful load, $R = 74.8''$, and the rope is made in 7 sections of six part strands and eight hemp strands.

The great weight of the twisting rope has led to the use of a double lift, each half of the rope assisting to counterbalance the other half, or another plan is to use a conical drum, to equalize the power.* The spiral winding of flat ropes also serves to equalize the leverage of the drum, and by a judicious selection of drum diameter, this may be very successfully done. Flat ropes are little used in France, but are common in Belgium, and their use is increasing in England and America.† Ropes of copper wire are used for lightning conductors, and these are also made of iron wire rope with a core of copper.

§ 268.

STIFFNESS OF ROPES.

The resistance of stiffness of ropes must be considered both in hoisting and in driving ropes. The measure of this resistance is the force required to move a rope hanging over a very easy running pulley, both ends of the rope bearing the given load Q. It will be observed that the winding-up side of the rope does not hang as closely to the pulley as does the other side, and that the lever arm of the two sides is constantly changing. Eytelwein's formula gives for the stiffness S of a hemp rope of diameter d:

$$S = \delta \frac{d^2}{R} Q \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (252)$$

in which, when R and d are given in inches, $\delta = 0.463$. Coulomb gives the very inconvenient formula $S = \frac{C d^{1.7}}{R + C_2 Q}$.

Weisbach gives, from very limited data, for wire rope:

$$S = 1.078 + 0.093 \frac{Q}{R} \quad \ldots \ldots \ldots \ldots \quad (253)$$

Example 1.—Given a hemp rope 1" diameter, with a load of 880 lbs., bent over a pulley 4" radius, from Eytelwein's formula we have:

$$S = 0.463 \frac{880}{4} = 101.8 \text{ lbs.}$$

which seems very high. Coulomb's formula gives 66 lbs.

Example 2.—A wire rope, composed of 36 wires, each 0.039" diameter, with a load of 550 lbs., is bent over a pulley 44 inches diameter. From Weisbach's formula we get:

$$S = 1.078 + 0.093 \frac{550}{22} = 3.403 \text{ lbs.}$$

The utility of these formulas is doubtful, and a fuller investigation of the subject is much to be desired. It will be seen from formula (253) that for wire rope the value of R should be taken still greater than already considered for bending stresses (formula 246); this subject is also discussed in Chapter XXI.

The above rules are deficient in that they do not take into account the kind of mechanical work absorbed by the stiffness of ropes. The angle embraced by the rope is, in the investigations of Amontons, Navier, Poncelet and Morris, assumed to be constant, while in practice it is constantly changing, and exerts a very material influence upon the result.

The author's consideration of the subject is here given:

Referring to Fig. 823, it will be seen that the fibres or wires on the concave side of the rope which passes over a pulley R, are compressed, producing a reduction in the form of the convex side, the compression originating with the load Q, being transmitted along the whole length of the twisted strands. The bent position of the rope can no longer retain its original section, of diameter d, but its volume must be the same as that of a corresponding length of the straight portion. The alteration in cross-section

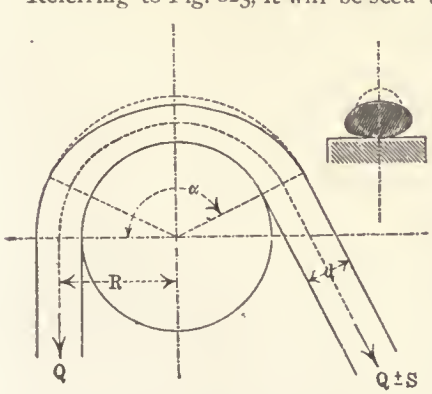


FIG. 823.

may be of two kinds; first: uniform compression; second, when this has reached its limit, a flattening of cross section.

Both deformations are observed in practice. Ropes which are very flexible are loosely twisted, and therefore readily compressed as they pass over pulleys. The general compression due to the tension of the load in the straight portion causes the twisted strands to press firmly together towards the axis, so that a heavily loaded rope is very hard. The compression is generally permanent, and not elastic, as may be deduced from the permanent reduction in diameter of ropes after use, and is generally due, in the case of wire ropes, to the compression of the hempen core; as is shown by the observations of Leloutre and Zuber ‡

The preceding remarks have not considered those wire ropes with metallic cores, used for running transmissions. Such ropes are always very stiff, and permit little or no compression. (According to Ziegler's experiments, only 0.22 to 1.2 per cent.)

It is really almost as important, so far as flexibility is concerned, that a rope should have a suitable soft core as that it should be made of the best and most elastic and flexible material. This is shown by the fact that even with ropes made entirely of hemp or of cotton, and used for transmission over pulleys, the inner fibres, which are never in contact with the pulleys, show great wear. This wear is evidently due to the friction of the fibres against each other, due to the flattening and changes of cross section. For this reason the desirability, or rather necessity, of lubricating the wires or fibres is evident, and this reduces the friction of the inner-lying portions of the rope. Rieten & Co. state that in the case of cotton ropes, "the rope always wears out by the internal friction of the strands upon each other, and that a load-twisted rope becomes useless in a shorter time than a soft, loosely twisted one, although the actual strength of the latter is the smaller."

In view of all these conditions the insufficiency of the existing rules for stiffness will be evident. It is apparent that the angle of contact must have a strong influence, and an entrance is found in cable roads where, when the cable is deflected through a small angle, small guide rollers are satisfactory, while much larger ones are necessary for greater angles. At a certain angle a, the deformation of the rope begins, and at another angle a maximum is reached, beyond which the resistance of stiffness is no longer dependent upon a. These points are of greatest importance with wire rope. It must be expected that the value of S will depend upon two functions of a, one for compression, and one for flattening. The first may be unimportant with old and compressed ropes, the latter will be much dependent upon the lubrication and upon the coefficient of friction.

§ 269.

ROPE CONNECTIONS AND BUFFERS.

The connection of one rope with another, when a smooth junction is required, must be effected by splicing. This may be accomplished by the short or German splice; or by the long, or

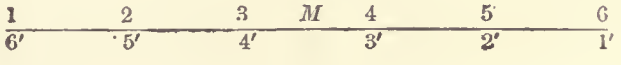


FIG. 824.

Spanish splice. The latter is the form to be used for wire rope. From the middle point M of the splice, Fig. 824, if, for example, a six strand rope is in hand, strands 1, 2 and 3 on the left are unwound, and strands 6′, 5′, 4′, of the other rope wound in, and the ends cut and worked in. The same is done on the other side, the whole length 1 — 6 being 30 to 50 feet.

To connect the end of a rope to another portion of the construction, the so-called hangers are used; three forms being shown in Fig. 825. At a is the so-called "swan neck," which is secured to the rope by through rivets; b is made with a conical socket, the wires being doubled up, and soft metal melted and run in; c is Kortum's hanger, the rope being held by two toothed wedges driven in, and secured by pins. Numerous tests have shown this fastening to be as strong as the rope itself.

In Fig. 826 a is shown a buffer coupling used in the Zentrum mine at Eschweiler, designed by the superintendent, Oster-

The details are as follows:

Depth x.	Dia. of wire δ.	Weight G_0.	S.	s.	$S + s$.
3936	0.1043	1.52	23,210	27,260	50,470
3280	0.0984	1.36	22,910	25,710	48,620
2624	0.0925	1.19	22,820	24,170	46,990
1968	0.0866	1.06	22,860	22,640	45,500
1312	0.0807	0.912	23,130	21,090	44,220
656	0.0748	0.785	23,690	19,550	43,240

The twisting of the rope was commenced at the small end, and the diameter of wires increased every 5 meters (16.4 ft.) after the first 200 metres (656 ft.). These ropes are very satisfactory, and last 3 to 4 years.

* Conical drums are used in the American anthracite coal mines.

† See Dwelshauvers-Dery in Cuyper's Revue des Mines, 1874; also F. Kraue in Zeitschrift der Berg u. Hüttenwesen, 1864.

‡ See Leloutre, Transmissions et courroies, cordes et cables. Paris, Tignol, 1884. Ziegler, Erfahrungs-resultant über Betrieb and Instandhaltung der Drahtseiltriebe, Winterthur, 1871.

kamp. The wrought iron thimble in the bight of the rope is fitted with a wooden block. Fig. 826 *b* shows the so-called

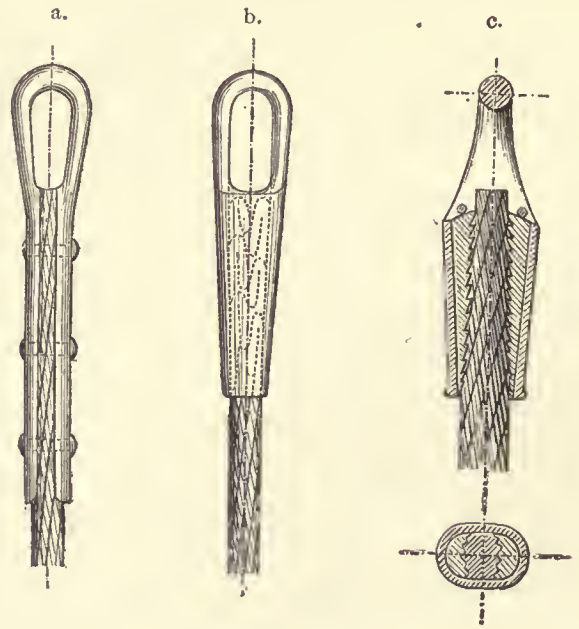


FIG. 825.

"friction hanger," both this and the previous form being arranged to be built into the upper part of the hoist cage.

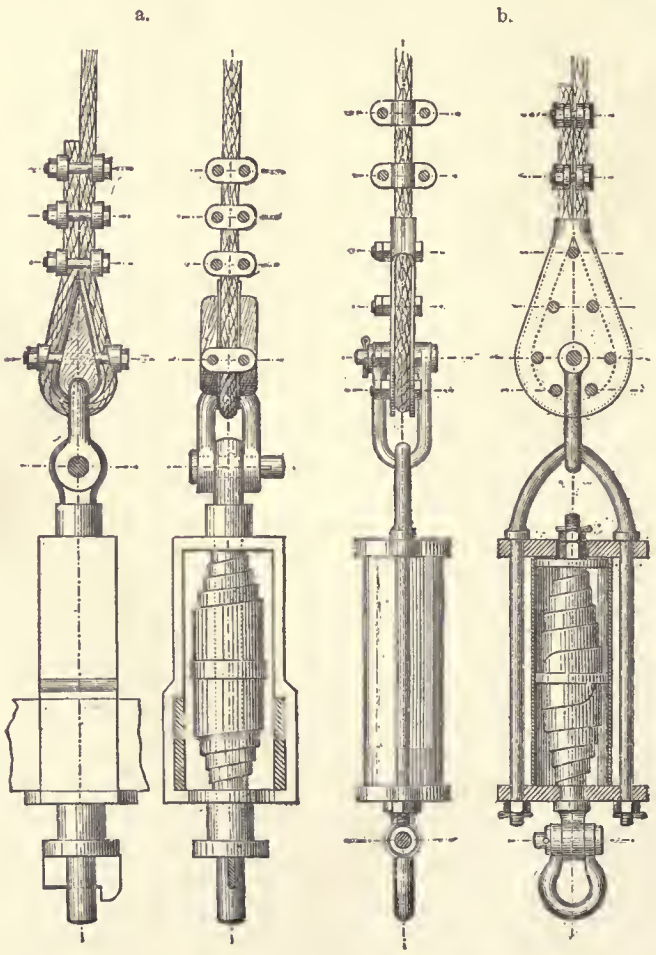


FIG. 826.

In Osterkamp's design the spring cage is built into the yoke of the frame, thus economizing room.

§ 270.

STATIONARY CHAINS.

Chains may be considered as jointed rods. Running chains are composed of very short members, in order that they may the easier pass over sheaves, while stationary chains, which are used in bridge, and other numerous constructions, are made with quite long links.

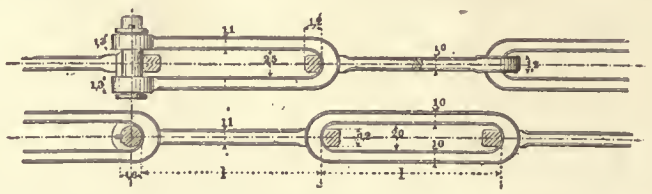


FIG. 827.

Fig. 827 shows the Admiralty form of stationary chain. The links are made ½ fathom long, not including the thickness of metal, and are divided into 10 fathom lengths, each length consisting of 20 links. The lengths are joined by a pin connection, shown on the left, and the pin is made of steel, galvanized.

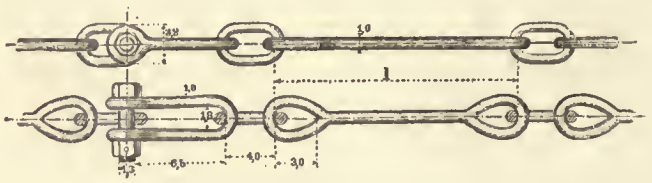


FIG. 828.

Another form, known as the Gemörsch chain, is shown in Fig. 828, and is well known in Germany.

Each long link is made 1.5 metres long, and these are connected by short oval links. The coupling link is secured by a common, but heavy screw bolt. The proportions in the illustrations are given in terms of the diameter of the rod.

In order to enable such chains to hang freely, the so-called "swivel" is used. A heavy swivel, for chains such as Fig. 827, is chosen in Fig. 829. The swivel bolt has a ring attached which can be readily opened, and is large enough to receive two chain links, while the upper ring can receive three. The limit of dimensions is the thickness of metal of the chain of Fig. 827.

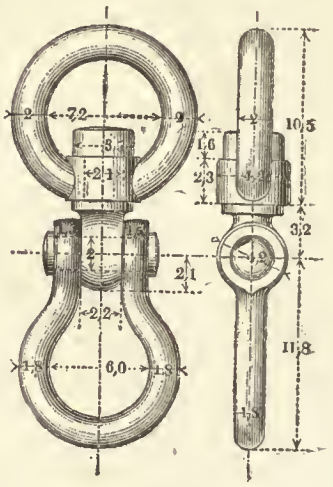


FIG. 829.

§ 271.

RUNNING CHAINS.

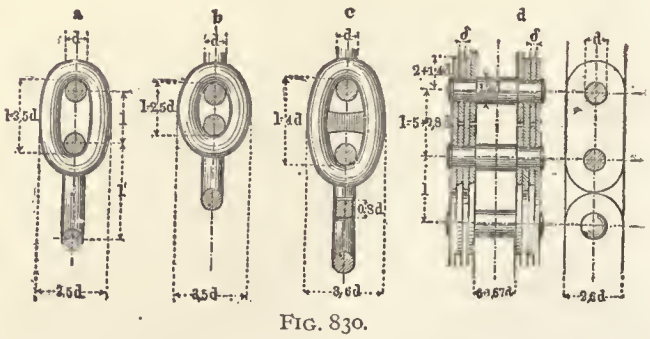


FIG. 830.

The most important forms of running chains used in machine construction are those shown in Fig. 830; *a* is an open link, and *b* is a close link chain; *c* is a stay link chain, and *d* a flat link chain. This latter is especially suitable for a pitch chain, on account of the parallel pins which are at uniform distance from each other. The other three forms are made with a higher order of linkage, viz.: the globoid form already discussed in Fig. 224.

In the wide open link chain *a* the globoid action can readily

be disarranged; less so in the close links of *b*, and hardly at all in the stay-link chain *c*, which latter closely resembles the globoid link of Fig. *a*, p. 142.

The proportional dimensions of chain links are not very closely determined. Those given in *b* and *c* are from the German Admiralty. The British Admiralty gives both for open and for stay-link chains, the pitch length 4 *d*, and width of link 3.6 *d*; in France, for open chains the length is made 3.25 *d*, and width 3.4 *d*, and for stay-link chains, 3.85 *d* and 3.75 *d* respectively.*

In crane and hoisting machine construction, a very important feature is the calibrating or adjusting of the links of chain.† This is also a matter of much importance in connection with the chain propulsion of boats used in France and Germany. The chain used on the Sweetwater canal at Suez was made with $d = \frac{11''}{16}$, and a pitch of 3 *d*, and breadth 3.2 *d*. The Magdeburg-Bodenbacher chain is very strong, *d* being given $\frac{15''}{16}$ to $1\frac{1}{8}''$, the links being proportioned as at *b*.

Flat link chains have been used by Neustadt, made of multiple plates (see § 94). The plates are made of the best quality and the pins made to project a little, and riveted over cold. Chains of this sort are also used for driving where heavy resistances are overcome, as in wire drawing, and in some spinning machinery.

§ 272.

CALCULATIONS FOR CHAINS.

The chains which are made at the best establishments are always thoroughly tested, every link being subjected to a stress closely within the limit of elasticity, or in some cases, slightly exceeding the elastic limit. A few links, usually three, are taken at frequent intervals every few weeks, and broken in the testing machine. The usual proof-load is such as to give the following stresses:

$S = 20{,}000$ lbs. per sq. in. for open link chain.
$S = 25{,}000$ " " " " stay "

The tests of chain for the German navy give for S:

17,000 lbs. test of elasticity,
19,000 " highest test, } for open links;
25,600 " proof load,
38,400 " breaking load for three links, } for stayed links.

For hoisting chain the elongation should be considered, and the metal should show an elongation before rupture of upwards of 20 per cent.‡

The permissible working stress per square inch section in Germany § is:

For open link chain = 9,000 lbs.
For stay link chain = 13,000 lbs.‖

From these we get for the proper total load P:

$$\left.\begin{aligned} &\text{For open links, } P = 14{,}000\, d^2 \\ &\text{For stay links, } P = 21{,}000\, d^2 \end{aligned}\right\} \quad \ldots\ldots (254)$$

Flat link chains are subjected to the heaviest stress at the portion which is in engagement with the toothed chain-wheel. (See Fig. 837.) For this reason there should be not less than five link pins in gear with the wheel at any time. If we assume that the tooth pressure is in arithmetic progression as 1 : 2 : 3 : 4 : 5 the pressure on the body of the last pin will be $\frac{1}{3}$ *P*, and on each journal also $\frac{1}{3}$ *P*, they being impelled forward by $\frac{1}{2}$ *P*. If we put as a maximum stress in the bolts of 17,400 pounds,* we have for the thickness of plates δ, pin diameter *d*, and number of plate *i*, for a given load *P*, the following values:

$$\left.\begin{aligned} \delta &= \frac{0.0107}{i+1}\sqrt{P} \\ d &= 0.0063\,\frac{i+2}{i+1}\sqrt{P} \\ \text{or} \quad \frac{d}{\delta} &= 0.58\,(i+2) \end{aligned}\right\} \quad \ldots\ldots (255)$$

The thickness δ is made the nearest convenient value, and *i* must be a whole even number. For the latter we may take the nearest whole number to the value given by the relation:

$$i = 0.26\,\sqrt[3]{P} \quad \ldots\ldots (256)$$

The following table has been calculated from these formulæ. The metal for the plates should be especially tough. Neustadt's chains had an ultimate resistance of four to five times the working load.

Example 1.—An open link chain of 1" iron, according to (254) should have a working load $P = 14{,}000$ lbs., while a stay link chain of the same iron would permit a working load $P = 21{,}000$ lbs.

Example 2.—Required to proportion a flat link chain to carry 22,000 pounds. We have from (256) $i = 0.26\,\sqrt[3]{P} = 0.26\,\sqrt[3]{22{,}000} = 7.32$, say 8. Then in (255) $\delta = \frac{0.0107\,\sqrt{22000}}{8+1} = 0.176''$ also $d = 0.0063\,\frac{8+2}{8+1}\,\sqrt{22000} = 1.04''$. The pitch length $l = 0.1875 + 2.8 \times 1.04 = 3.1''$; the width of plates $= 2.6 \times 1.04 = 2.7''$; the length of the body of the pins is $0.25 + 1.67 \times 1.04 = 1.98$, say 2", the diameter $= 1.2 \times 1.04 = 1.25''$ and the length of link beyond the pin centre is $0.08 + (0.9 \times 1.04) = 1.02$ say 1".

Table of Flat Link Chains.

Working Load *P.*	No. of Plates *i*	Thickness of Plates δ	Breadth of Plates *v.*	Dia. of Pin *d.*	Pitch *l.*
1,000	2	0.125	0.625	0.25	0.875
1,500	4	0.093	0.75	0.28	0.93
2,000	4	0.109	0.875	0.34	1.14
3,000	4	0.125	1.0625	0.40	1.375
4,000	4	0.140	1.1875	0.46	1.45
6,000	6	0.109	1.4375	0.56	1.625
8,000	6	0.140	1.6875	0.68	2.00
10,000	6	0.156	1.875	0.75	2.3125
12,000	6	0.171	2.00	0.93	2.375
16,000	8	0.156	2.375	0.93	2.50
20,000	8	0.171	2.625	1.00	2.8125

§ 273.

WEIGHT OF CHAIN.

The length *S* of rod required to make a chain of a given length *L* bears the same relation to *L* as the length *s* for a single link does to the pitch *l*. We have for the chains *a*, *b*, *c* of Fig. 830:

	Open Links.	Close Links.	Stay Links.	Stay Links, including stay.
$\frac{s}{d} =$	11.33	9.42	11.94	13.25
$\frac{s}{l} =$	2.52	2.69	2.39	2.65

From these relations the weight of iron rods required may be determined (see § 82). The greater the pitch of chain for a given weight of iron, the more economical is the form of construction.¶

The load length and rupture length for chains (see § 267) have been extended since that subject has been given practical consideration, this being especially the case with anchor chains (see next section). For this we may take the modulus of rupture *K* at 37,000 lbs. for open links and 38,000 lbs. for stay links, with a modulus of safety $T = 20{,}000$ and 24,000 lbs. respectively. We then have:

$$L_t = \frac{T}{6.29 \times \gamma \times \frac{s}{l}} \text{ and}$$

$$L_z = \frac{K}{6.29 \times \gamma \times \frac{s}{l}}, \ \gamma \text{ being the weight of a}$$

cubic inch of wrought iron = 0.27 lb., and hence:

* The length of flat links in Fig. 830 *a* is given as $5 + 2.8\,a$, and the projection of the ends as $2 + 1.4\,d$. These are in millimetres, and for inches the values $0.1875 + 2.8\,d$, and $0.08 + 1.4\,d$ should be used.

† Excellent pitch chain is made at the Guttehoffnungshütte at Oberhausen; also by Schlieper at Iserlohn, and by Dorémieux at St. Arnaud, and Plinchon Havez at Guérigny, and by Hawkes Crawshay at Gateshead on Tyne.

‡ At the Guérigny Works the required elongation is:

For rods 1½" to 1" . . . . . 18 per cent.
For rods 1" to ½" . . . . . 16 per cent.
For rods ⅜" . . . . . 14 per cent.
For rods ⅛" . . . . . 12 per cent.
For rods ¼" . . . . . 10 per cent.

§ At the Gutenhoffnunghütte.

‖ Henry R. Towne, Treatise on Cranes, Stamford, Conn., gives a permissible stress of 9,000 to 10,000 pounds.

¶ The pitch for stay link chain in the German navy was formerly 3 *d*, but has recently been made 4 *d*.

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	Open Links.	Close Links.	Stay Links.
L_t =	4672	4377	5458
L_z =	8672	8127	8567

§ 274.

Chain Couplings.

Chains which are used for transmission of motion (so called "endless" chains) require devices for coupliug, as do also those constructions with which chains are to be connected, and hence we have a variety of eyes, rings, coupling links, swivels, and the like.

Fig. 831.

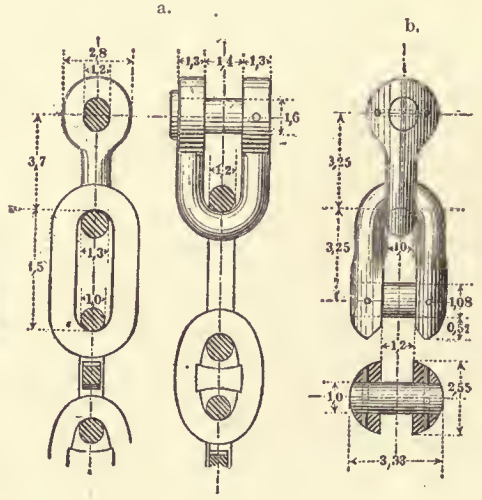

Fig. 832.

A piece which is sometimes used with anchor chains is the so-called "twin" link, Fig. 831. This may be made of cast steel, and because of limited space is formed with circular openings. The ordinary coupling link is shown in Fig. 832 *a*. The link is of wrought iron, the bolt and piu of steel, both galvanized. The pin is shorter than the diameter of the eye, and is secured on both sides by a plug of lead. The next link is made somewhat longer thau the other links of the chain, so that the coupling link may be more readily introduced. This form is used for joining pieces of chain to form greater lengths. The German Admiralty anchor chain is made with stay links, iu seven lengths of 25 metres (82 feet) each, joined with coupling links, two of which are swivels. A bow auchor chain is given two more lengths of chain and made of iron 3mm. (0.118″) thicker.*

The chains for the system of boat propulsion are fitted with a coupling liuk with rounded edges, and two are used together, as in Fig. 832 *b*, which shows the chain used on the Elbe. This coupling might also be suitable for power transmission chain.

The swivel is used to permit the chain to have a rotation about its axis of leugth without twistiug the links together.

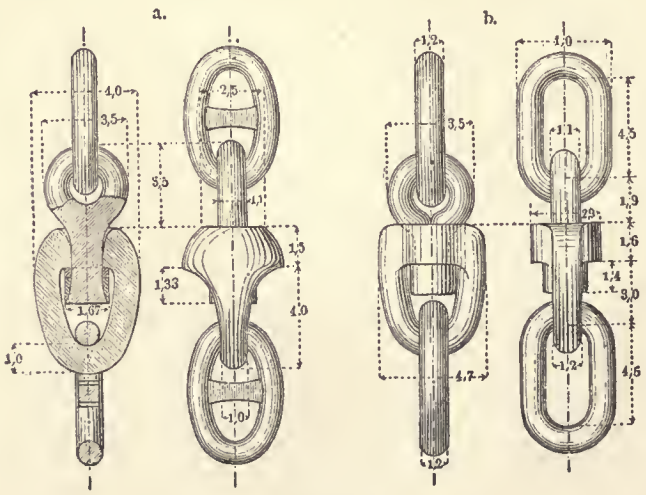

Fig. 833.

The form of swivel used in the German Navy is shown in Fig. 833 *a*, and at Fig. 833 *b* is showu the Euglish swivel.

* The lengths in the English Navy are 12½ fathoms.

Chains must also be provided with hooks for attachments to the load to be raised.

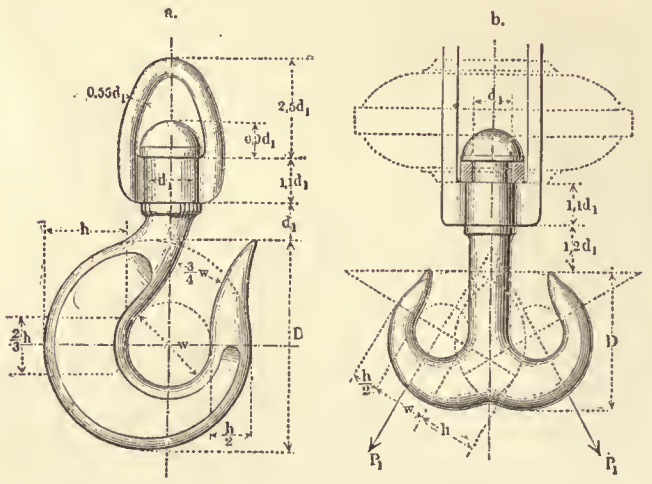

Fig. 834.

A single hook is given iu Fig. 834 *a*, and a double hook at Fig. 834 *b*. The construction of such hooks demands the greatest care, and according to Glynn, more lives have been lost aud damage incurred by the breakage of hooks thau by any other part of a crane. The case is one of combined resistauce and leads to unexpectedly great dimensions.

The diameter d_1 of the shank of the hook may be obtaiued from formula (72), so that we have for a load P:

$$d_1 = 0.02 \sqrt{P} \quad \ldots \ldots \ldots \ldots \quad (257)$$

This is based upon a stress of 3500 pounds, but an angular pull may increase this five-fold. Taking d_1 as the unit, we may obtain the proportions given in the illustratious iu the following manuer. Let w be the width of the opening of the hook, and h the width of the body of the hook, the thickness at the same point is made $\frac{2}{3}\,h$, and for a stress of 12,800 lbs. upou the metal of the hook we have:

$$\frac{h}{d_1} = 1.30 \sqrt{\frac{w}{h} + \frac{5}{4}} \quad \text{or} \quad \frac{h}{\sqrt{P}} = 0.026 \sqrt{\frac{w}{h} + \frac{5}{4}} \quad \ldots \quad (258)$$

The thickness at the point of the hook is made $\frac{h}{2}$, and hence the outside of the hook is a circle of diameter $D = w + 1.5\,h$. We then have for:

$\frac{w}{h}$ =	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\frac{h}{d_1}$ =	1.77	1.82	1.86	1.91	1.95	1.99	2.03	2.08	2.12	2.16
$\frac{h}{\sqrt{P}}$ =	0.035	0.036	0.037	0.038	0.039	0.040	0.041	0.042	0.042	0.043
$\frac{w}{d_1}$ =	1.06	1.27	1.49	1.72	1.95	2.19	2.44	2.70	2.97	3.24
$\frac{D}{d_1}$ =	3.72	4.00	4.28	4.59	4.88	5.18	5.48	5.82	6.15	6.48

The most useful ratio is $\frac{w}{h} = 1$. In wharf cranes a weight is often combined with the hook in order to facilitate the loweriug of the empty chaiu. This is shown iu the dotted lines in Fig. 834 *b*. In the case of a double hook each portion is calculated for its component P_1 of the entire load P. From this a special unit d_1' is obtained only for the dimensions w, h, aud D.

Example.—Let the load upon a hook be 4400 lbs. We have from (257) $d_1 = 0.02 \sqrt{P} = 0.02 \sqrt{4400} = 1.326''$. If we take $w = h$ we get from the above $h = 1.99 \times 1.326 = 2.638''$, and w is the same; while $D = 2.638 + 3.957 = 6.6''$.

In the case of a double hook the angle between the components is 60°; we then have $P_1 = \frac{0.5\,P}{\cos 30^\circ} = \frac{2200}{0.866} = 2540$, whence $d'_1 = 0.02 \sqrt{2540} = 1.008$ say 1″. If we make $\frac{w}{h} = 0.9$, $h = 1.91$ and $w = 1.72$. $D = 1.92 + 2.86 = 4.58$. For the upper portion we have as above $d_1 = 1.326''$.

§ 275.

CHAIN DRUMS AND SHEAVES.

Chain drums and sheaves are usually made of a radius $R =$ 10 to 12 d, measured to the middle of the chain. In some cases a rim is made on the chain sheave, as in Fig. 835 *a*.

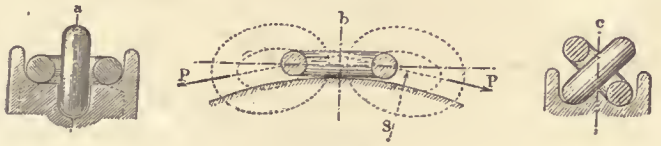

FIG. 835.

This form of sheave brings a bending action upon the links as shown in Fig. 835 *b*. Sometimes the flanges are omitted and the edges of the sheaves bevelled as in the dotted lines, and in other cases the links have a bearing as shown at Fig. 835 *c*, in which the bending action is somewhat reduced. The bending is entirely avoided, however, by the use of a pocketed sheave, as in Fig. 836.

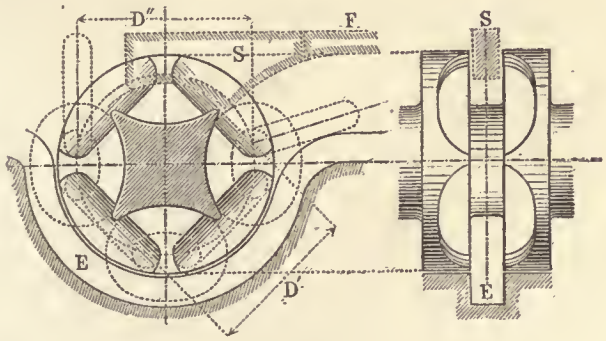

FIG. 836.

This form is useful both for chain transmission, and as a substitute for winding drums in hoisting machinery, as it enables a small pocketed sheave to serve instead of a large drum. When such a sheave is made with only four pockets, they form a square with a side $D' = l + d + 2(l-d)\sqrt{0.5} = 2.414\ l - 0.414\ d$; while the side of the square of the alternate links is $D'' = 1.414\ l + 0.414\ d$. The first gives the minimum, and the second the maximum, (double) lever arm with which the chain acts upon the sheave. If the pockets, instead of 4 and 4 are:

6 and 6, we have $D = 3.732\ l - 0.264\ d$
8 and 8, " " $D = 5.026\ l - 0.198\ d$.

Chain sheaves of this form require accurately made pitch chain.

When the load is heavy, the friction causes the chain to cling to the sheave, and a stripper *S*, Fig. 836, is required to lead the chain off in the proper direction *F*, while the entrance is properly effected by a guide channel *E*.

For flat link chain, a toothed chain wheel is used, Fig. 837.

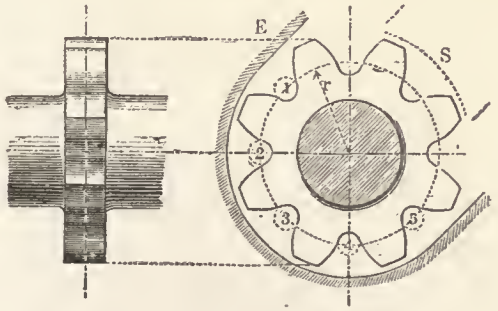

FIG. 837.

In this form a guide channel *E*, and stripper *S*, should also be used. The tooth profile is a circular arc with its centre at the link pin. If z, be the number of teeth, we have for the radius γ, of the pitch circle:

$$\gamma = \frac{l}{2 \sin \frac{180^\circ}{z}} \quad \ldots\ldots\ldots\ldots\ldots (259)$$

whence we get, for

$z =$	8	9	10	12	14	16	18	20
$\frac{\gamma}{l} =$	1.3066	1.4619	1.618	1.932	2.247	2.563	2.879	3.106

The minimum number of teeth is 8.

Neustadt uses the following:

$z =$ 8 for $P =$ 500 to 6,000 pounds.
$z =$ 9 for $P =$ 6000 to 50,000 pounds.
$z =$ 10 for $P =$ over 50,000 pounds.

Guide sheaves for either kind of chain are made with 16 to 30 teeth.

For chain propelliug cables ordinary smooth drums with parallel axes are used, with a groove for the chain.

In Fig. 838 *a* is shown a section

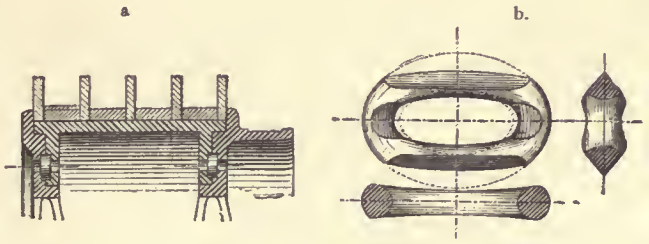

FIG. 838.

of the rim of the drum on the chain propelling gear on the river Elbe. This is made with steel flanges and channels on a wrought-iron rim. The last channel is made slightly larger in diameter in order to give a higher velocity to the driving side of the chain. The wear upon the chain is an important item. Fig. 838 *b*, shows a link of a chain as worn after long service. It must not be overlooked that the winding around the drum produces a twist in the chain, giving as many half twists in the chain as there are half convolutions about the drums. This twisting is not injurious if the chain is bent as frequently in one direction as in the opposite. In fact, however, the chain is usually bent into more concave than convex bends. This causes a twisting motion to the chain and as it drags upon the bottom and banks of the stream it produces much wear, and causes kinks to be produced at the shallow places. The chain must, therefore frequently be raised at such points and a link opened and the twist taken out.

This twisting may be prevented by using the drum arrangement shown in Fig. 839. This consists of simple drums all lying in one plane driven by gearing so that the proper relative motion is compelled.

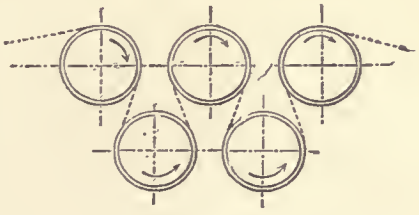

FIG. 839.

§ 276.

RATCHET TENSION ORGANS.

Tension organs may be combined with pawls, which in the case of cords are friction pawls, (§ 248, 249). and for chains are toothed pawls, acting upon the links in the same manner as upon ratchet wheels and ratchet racks.

The establishment of Felten & Guilleaume, at Mülheim a. Rhein, have devised a grip pawl for boat-cable driving, in which the rope is clamped to and released from a driving drum by an evolute shaped thumb clamp, the shock being reduced by a spring buffer.

Pawls for chains may be found used in connection with the heavy bow anchors of large vessels; Bernier, of Paris, has also used such devices upon chain hoisting machinery.

CHAPTER XX.

BELTING.

§ 276.

Self-Guiding Belting.

Belt pulleys are indirect acting friction wheels (§ 191) and the belt itself is a tension organ combining the functions of driving and guiding (§ 261). Those belts which act without requiring the use of special guiding devices may be called self-guiding belts. This action is attained by the use of cylindrical pulleys when the edge of the prismatic belt runs in a plane at right angles to the axis of the pulley; or in other words, when the middle line of the advancing side of the belt lies in the plane of the middle of its pulleys.

When a belt runs upon a conical pulley in a direction normal to its axis, its tendency will be to describe a conical spiral path upon the pulley, as will readily be seen upon the examination of the development of the surface of the cone, Fig. 840.

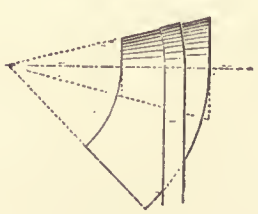

Fig. 840.

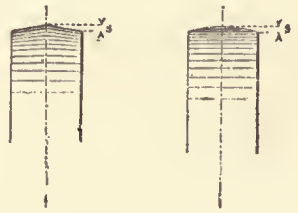

Fig. 841.

If the pulley is made with a double cone face or a rounded face, Fig. 841, the tendency will be for the belt to run at the middle of the face even when the direction of the belt is not exactly correct.

For leather belting, with a height of the crowning or curvature of the face $s = \frac{1}{20}$ of the width of face, the belt may deviate from the plane of the pulley by 2½° (tan = four per cent), while for cotton belting, on account of the lesser elasticity of the material, the crowning s should not exceed $\frac{1}{150}$ of the face, thus reducing very materially the permissible deviation. In ordinary circumstances at least one of a pair of pulleys should be made with rounded force.

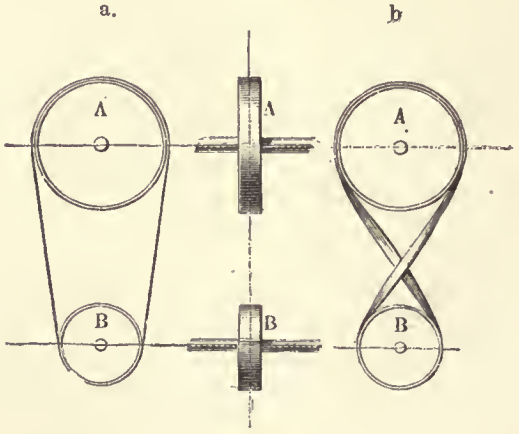

Fig. 842.

The simplest arrangement of self guiding belting is that for parallel axes, Fig. 842 *a* and *b*, *a* being for open belt and *b* for crossed belt, either arrangement being suitable to run in either direction.

For inclined and intersecting axes self-guiding belts are not suitable, except in the case of inclined axes in which the trace *S S*, Fig. 843, of the intersection of the planes of the two pulleys passes through the points at which the belt leaves the pulleys. The leading line then falls in the middle plane of each pulley, but the following side of the belt does not, hence such systems can only be run in one direction. The leaving points in the figures are at *a* and b_1. The arrangement gives an open belt when the angle β between the planes of the pulleys $= o°$, and a crossed belt when $\beta = 180°$. In the intermediate positions a partial crossing of the belt is produced. If $\beta = 90°$, the belt is half crossed (or as commonly called, quarter twist); if $\beta = 45°$, it is quarter crossed.*

* The above geometrical construction is only approximate; for an exact solution see a paper by Prof. J. B. Webb, Trans. Am. Soc. Mech. Eng'rs, Vol. IV., 1883, p. 165.

The leading off angle may be made as much as 25°, which occurs when the distance between the axes is equal to twice the

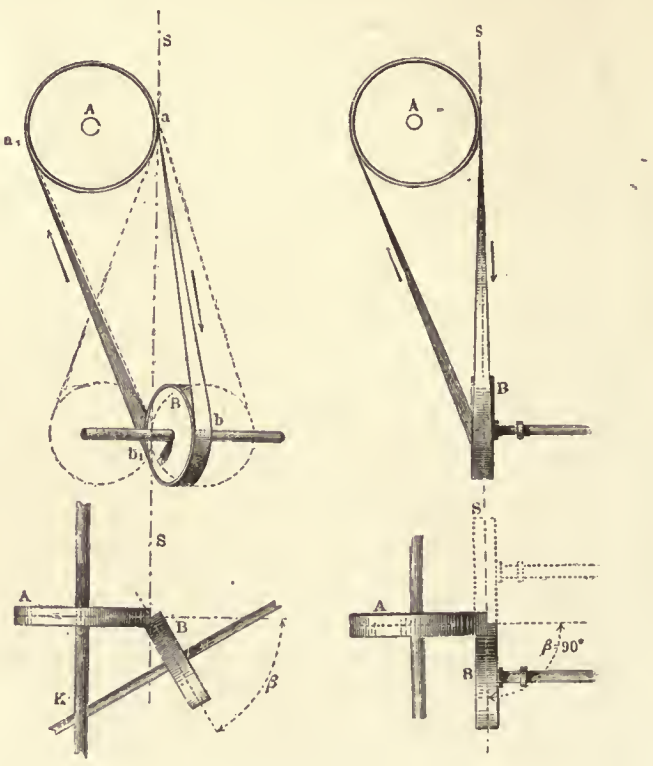

Fig. 843.

diameter of the largest pulley. Another rule for the minimum distance between shafts for quarter-twist belts is to make the distance never less than $\sqrt{b\,D}$.

§ 277.

Guide Pulleys for Belting.

When a belt transmission is arranged with guide pulleys, the proper guiding action is obtained when each guide pulley is placed at the point of departure of its plane with that of the next following pulley.†

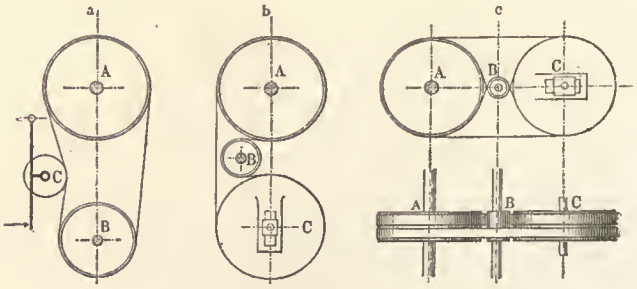

Fig. 844.

In Fig. 844 examples are given of guide pulleys for parallel axes, all three pulleys lying in the same plane.

At *a* is shown a belt transmission with tightening pulley, *b* is a device for transmitting motion when great difference of speed is desired. In this case the guide pulley *C* is as large as the driver *A*, and if desired may also be arranged to act as a tightener.‡ At *c* is Weaver's device for similar uses.§ In this case two belts are used, and the device has been used for driving circular saws. The pulleys should be fitted to run very smoothly in such devices.

The cases in Fig. 845-846 have parallel axes with two guide pulleys. In the first case the guide pulleys are placed in planes tangent to both operating pulleys, and hence driving may occur in either direction. Usually, however, it is required to provide

† See also the paper of Prof. Webb, referred to in the preceding note.
‡ Eckert's patent (German) for driving the drum of a threshing machine
§ See Cooper's Use of Belting. Phila., 1878, p. 171.

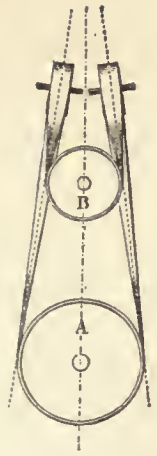

FIG. 845.

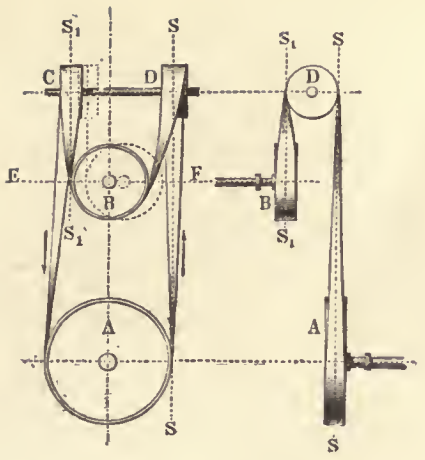

FIG. 846.

for motion in but one direction, in which case the second form is used as being simpler of installation. The pulley *B* may be used as one of the guide pulleys, in which case it may be placed loose upon the same shaft as *A*, and *C* or *D* be made drivers or driven.

By placing the guide pulleys between the axes of *A* and *B*, instead of beyond them, they will revolve in the same direction, and may be made fast upon one shaft, as in Fig. 847; this arrangement admitting of motion in only one direction.

In Fig. 848 is an arrangement for inclined axes, which is a modification of Fig. 846, as will be seen by the dotted lines. The guide pulleys run in opposite directions, but may conveniently be placed upon the same shaft.

In Fig. 849 is shown an arrangement of quarter-twist belts with guide pulleys. One side of the belt is placed in the intersection *S S* of the planes of the two pulleys. From any point *c*

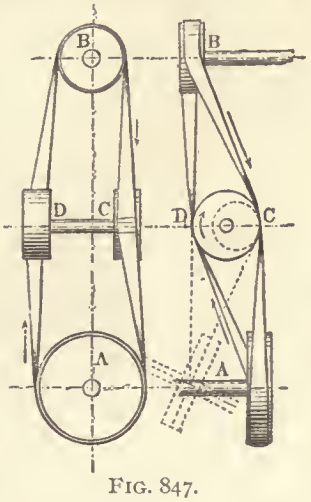

FIG. 847.

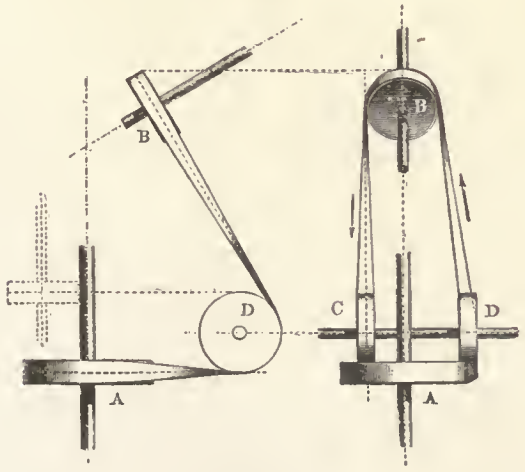

FIG. 848.

on *S S*, the tangents *c a* and *c b* are drawn, and in the plane of these the guide pulley *C* is placed. This arrangement permits of rotation in either direction.

Another arrangement for the same purpose is shown in Fig. 850. The side of the belt leading off from *A* is inclined towards *B*, the other side passing over the guide pulley *C*, which is in the same plane as *A* and *S S*. This arrangement is well adapted for driving a number of vertical spindles from one horizontal shaft.*

* An example is Jacob's grinding mill with 40 sets of stones; see Uhland's Praks. Masch. Konstr., 1868, p. 83, 1869, p. 242.

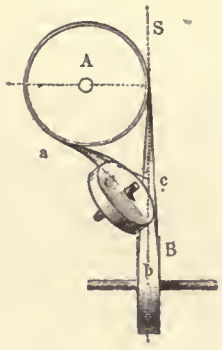

FIG. 840.

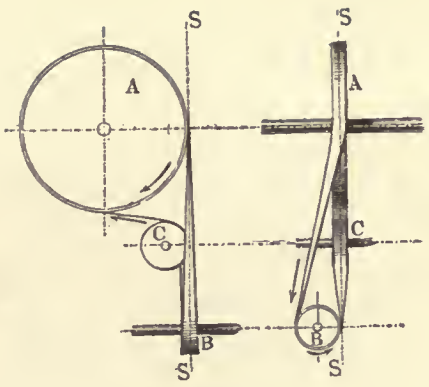

FIG. 850.

Fig. 851 shows the general case for inclined axes. Two points *c* and c_1 are chosen on the line of intersection *S S* of the planes of the two pulleys, and the tangents *c a*, *c b*, $c_1 a_1$, $c_1 b_1$

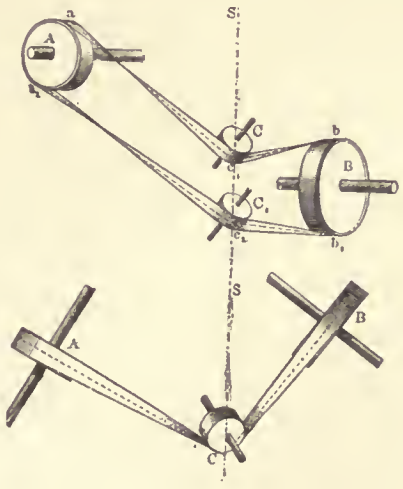

FIG. 851.

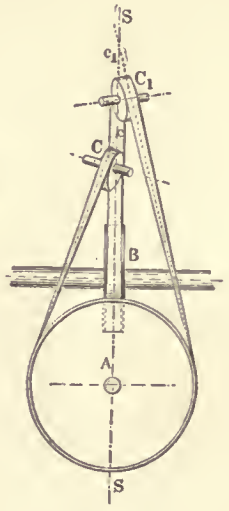

FIG. 852.

drawn, and in the planes of these tangents the guide pulleys *C* and C_1 are placed. Under these conditions the rotation may be in either direction. The arrangement shown in Fig. 852 occurs when the line *S S* passes through the middle of one of the pulleys.

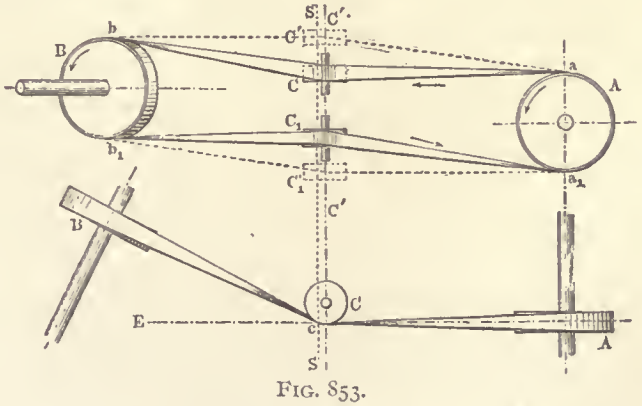

FIG. 853.

A simplification of the general case occurs when, as in Fig. 853, the guide pulleys fall upon one and the same geometrical axis which is parallel to the axes of both transmitting pulleys. In this case the only inclination of the belt is that given to it by the guide pulleys. The rotation can be in but one direction, viz.: that shown by the arrows; if the reverse is desired, the guide pulleys must be placed as shown in the dotted lines. If the inclination of the shafts is too great the belt will be liable to drop off when the pulleys come to rest. The use of guide pulleys involves special hangers, a practical form for which is shown in Fig. 854.†

† Patented in Germany by the Berlin-Anhaltischen Maschinenbau-Aktien-Gesellschaft.

The vertical axis is provided with an oil hole, and is fitted by a ball and socket bearing to the bracket *D*. The flange on the lower edge of the pulley keeps the belt from falliug off the

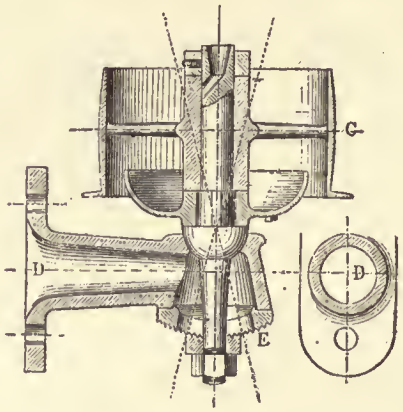

FIG. 854.

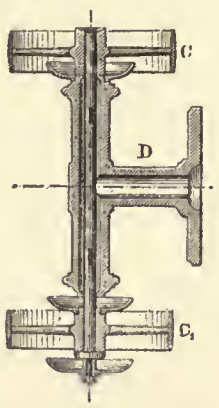

FIG. 855.

pulley when at rest. The form in Fig. 855 was desigued by the author for the arrangement of Fig. 848, both pulleys being loose upon the wrought iron shaft.

If the position of the shafts can be so chosen that the line *S S* touches at least one of the pulleys, the very practical arrangement showu in Fig. 856 can be applied. If the distance

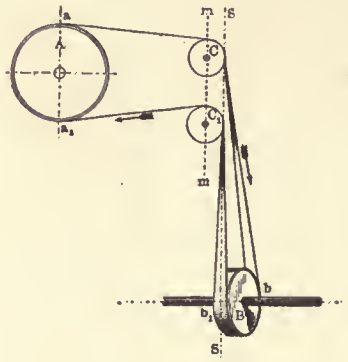

FIG. 856.

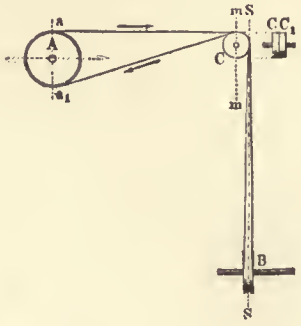

FIG. 857.

A C is great in comparison with the width of belt, the pulleys *C* aud *C*$_1$ cau be placed side by side instead of over each other, Fig. 857, iu which case round face pulleys should be used.

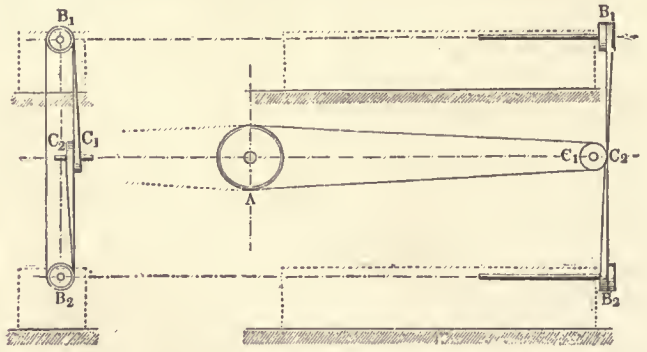

FIG. 858.

By the use of a fifth pulley the preceding arrangement may be so modified that two pulleys, B_1 and B_2, can be driveu from one driver, *A*. This is showu in Fig. 858 as applied in a spiuniug mill, in which the pulleys B_1 and B_2 are on different floors of the building, and are also provided with loose pulleys.*

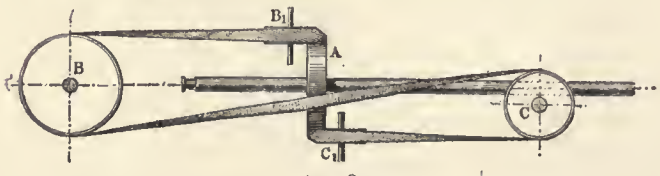

FIG. 859.

In the arrangement of Fig. 859 the pulley *A* drives two parallel shafts, one of which intersects its axis at right angles, the other passing beneath.

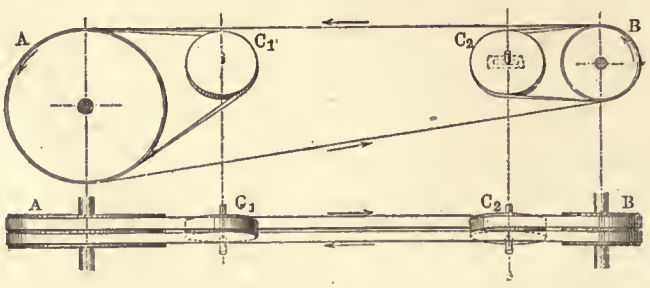

FIG. 860.

Another arrangement, devised by the author, is given in Fig. 860. In this case the followiug side of the belt is passed over an idler pulley, C_1 or C_2, and a second time arouud the driver (see also Fig. 795) by which the angle of contact *a* is doubled, aud the modulus of friction e^{fa} (§ 264) increased. This may be called a double-acting transmission. The cross section of belt may be made $\frac{6}{10}$ of a single acting transmission, so that in spite of the increase of length an economy of belting is obtained. One of the guide pulleys may also be used for a tighteuer. These devices will also be cousidered in connection with rope transmission (Chapter XXI.) to which they are especially applicable.

§ 278.

FAST AND LOOSE PULLEYS.

Fast and loose, or tight aud loose pulleys, as they are sometimes called, are generally used in counectiou with auother belt transmission in order to throw the latter in and out of action, the belt being guided by a belt shifter, which by the means of forks or finger-bars, enables the moving belt to be shifted. These shifting devices may properly be regarded as guide pulleys, aud are sometimes fitted with rollers, as showu dotted in Fig. 861, at *c* and c_0.†

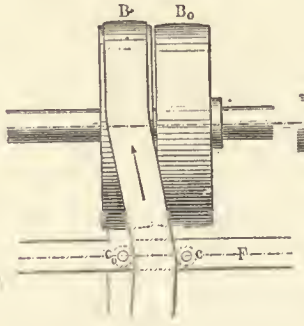

FIG. 861.

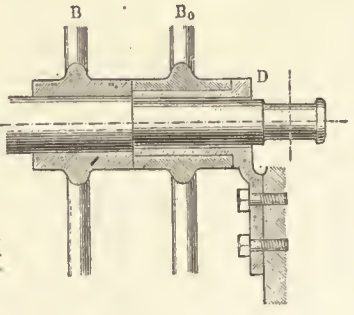

FIG. 862.

It is preferable to have the loose pulley upon the driven shaft, siuce the belt then can be shifted with a gradual spiral action by the shifter *F*, Fig. 861. It is best for the driving pulley to be made straight face, or if two fast pulleys are used side by side ou the driving shaft, these should have very slightly rounded faces, if the belt is to be shifted promptly and readily, and for the same object the shifter should be placed as close to the driven pulleys as possible. The loose pulley should be kept thoroughly lubricated, and for this purpose numerous oiling devices have been made. The frictiou betweeu the hub and shaft acts as a driving force upon the loose pulley, and this has been a source of numerous accideuts. This action is avoided iu the arrangement iu Fig. 862, in which the loose pulley is carried on a consecutive aud stationary sleeve *D*.‡

A variety of mechanical belt shifting devices have been made, § the desire being to prevent the action of the belt from moving the shifter. A useful form is Zimmermann's Shifter, Fig. 863.

* See Fairba ., Mills and Millwork, II., London, 1863, p. 103. For the theoretical discussion of these various arrangements, see § 301.

† Such rollers as especially necessary for shifting cotton belts, which are liable to catch on the shifter fingers, and even larger rollers are best in such cases.

‡ See Berliner Verhandlungen, 1869, p. 127. This has been used by the Society for Prevention of Accidents, of Mülhouse.

§ See Berliuer Verhandluugen, 1868, p. 171, Rittershaus, Belt Shifters.

The shifter bar F, to which the fork G can be clamped at any desired point, is operated by the lever H, which turns upon an

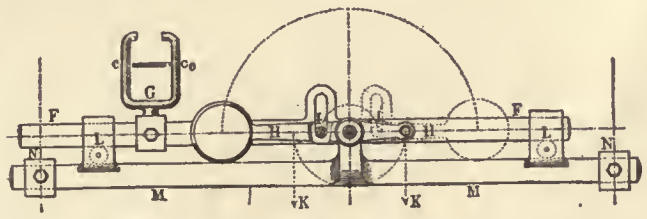

FIG. 863.

axis at I, forming a "dead" ratchet mechanism. The similarity to the ratchet devices of Figs. 754 and 755 will be observed. The movement of the bar is effected by connection at K or K_1.

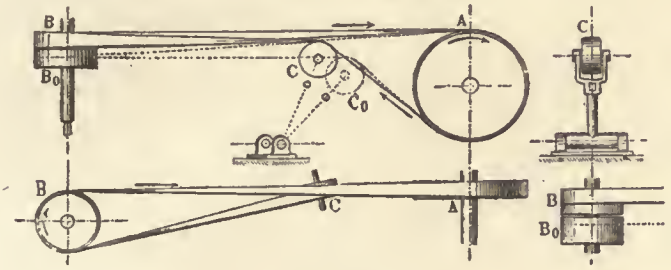

FIG. 864.

Fig. 864 shows a shifter for quarter-twist belt. In this form, devised by the author, the guide pulley, which is required to support the belt, also serves as a shifter to move the belt to and from the belt pulley B, and loose pulley B_0. If these pulleys are given greater width than that of the belt, as shown on the right, a vertical adjustment can be given to the upright shaft; a condition sometimes required in grinding mills and similar machines.

§ 279.

CONE PULLEYS.

When a number of pulleys are placed side by side in order to enable varied speeds to be obtained with belt transmission, and are united together in one member, we obtain what is called a cone pulley, such pulley being used in pairs. This construction involves the problem of determining the proper radii for the various pulleys, so that the same belt shall serve for all the changes, *i. e.*, so that the length of the belt shall be the same for each pair of pulleys in the set. The problem may be solved as follows:

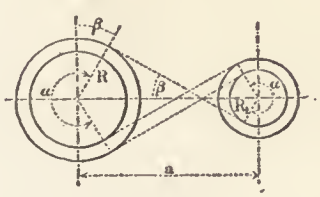

FIG. 865.

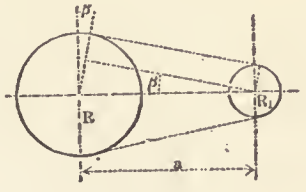

FIG. 866.

a. Crossed Belts, Fig. 865. The belt makes the angle β with the centre line of the pulleys R and R_1; and the half length of the belt, $l = R\left(\frac{\pi}{2}+\beta\right) + R_1\left(\frac{\pi}{2}+\beta\right) + a\cos\beta$, a being the distance from centre to centre of the pulley. We then have:

$$l = (R+R_1)\left(\frac{\pi}{2}+\beta\right) + a\sqrt{1-\frac{(R+R_1)^2}{a^2}} \quad . \quad . \quad (260)$$

This value is constant when $R + R_1$ is constant; that is, when the increase to the radius of one pulley is equal to the decrease in the radius of the other. Crossed belts are seldom used for this service, however, because of the injurious friction between the rubbing parts of the belt.

b. Open Belts, Fig. 866. In this case we have:

$$l = (R+R_1)\frac{\pi}{2} + (R-R_1)\beta + a\cos\beta,$$

and also $a\sin\beta = R - R_1$, which gives:

$$\left.\begin{aligned} R &= \frac{l}{\pi} \quad \cdot \frac{a}{\pi}(\beta\sin\beta+\cos\beta) + \frac{a}{2}\sin\beta \\ R_1 &= \frac{l}{\pi} - \frac{a}{\pi}(\beta\sin\beta+\cos\beta) - \frac{a}{2}\sin\beta \end{aligned}\right\} \quad . \quad . \quad (261)$$

This function is transcendental, but may be graphically represented in the following manner, Fig. 867. In the rectangle $A\,B\,B'\,A'$, with a radius $A\,B = a$, strike the quadrant $B\,M\,C$ about the centre A. Within this arc will fall all the values of

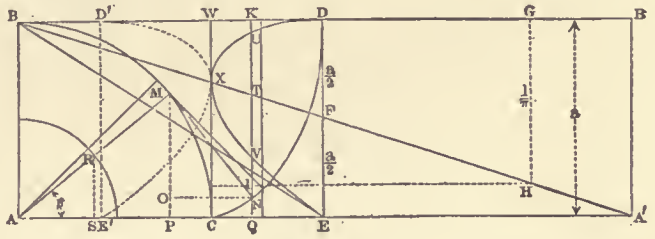

FIG. 867.

β which can occur. For any value of $\beta = C\,A\,M$, draw $M\,N$ perpendicular to $M\,A$ and make $M\,N =$ the arc $M\,C = a\,\beta$. Drop the perpendicular $M\,P$ to $A\,C$, and draw $N\,O$ perpendicular to $M\,P$. $N\,O$ will then $= a\,\beta\sin\beta$. Through N draw $Q\,N\,K$ parallel to $A\,B$, and we have $A\,Q = P\,Q + A\,P = a(\beta\sin\beta+\cos\beta)$. By taking successively all the values of β between 0° and 90° in this manner, we can determine the path of the point N, which will be the evolute of a circle, $C\,N\,D$ $B\,D$ being equal to the length of the arc $B\,M\,C = \frac{\pi}{2}a$. If we now draw $D\,E$ parallel to $B\,A$, and take its middle point F, we have $D\,F = E\,F = \frac{a}{2}$, and hence the proportion:

$$D\,F : D\,B = \frac{a}{2} : \frac{\pi}{2}a = a : \pi, \text{ and by similar triangles:}$$

$$T\,K = \frac{a}{\pi}\,Q\,A = \frac{a}{\pi}(\beta\sin\beta+\cos\beta).$$

This value is dependent upon $\frac{l}{\pi}$. If we prolong $B\,F$ until it intersects $A\,C$ prolonged, the resulting length $A\,A' = B\,B'$ will bear to $A'\,B'$ the ratio $\frac{\pi}{1}$. By then working $B\,G = l$, and drawing $G\,H$ parallel to $A'\,B'$, we have $G\,H = \frac{l}{\pi}$. This length being transferred to $I\,K$ gives $I\,T = \frac{l}{\pi} - \frac{a}{\pi}(\beta\sin\beta + \cos\beta)$. We then have only to use $\pm\frac{a}{2}\sin\beta$ to solve the problem.

Make $A\,R = \frac{a}{2}$, and we have the perpendicular $R\,S = \frac{a}{2}\sin\beta$. By laying this length off above and below T on $Q\,K$, we obtain the points U and V, and this finally gives $I\,U$ for the radius R of the larger cone pulley and $I\,V = R_1$, the radius of the corresponding smaller cone pulley.

By solutions for successive values of β, we obtain the curve $D\,U\,X\,V\,E$, which can be used for the determination of the radii of any desired pair of pulleys, each pair of ordinates measured from $H\,I$ belonging to corresponding pulley on each cone.

In practice it is usual to find one of the cone pulleys given and the dimensions of the other required. In this case $V\,U$ may be taken as the difference $R - R_1$, between the radii, were the steps uniform. By taking this difference $R - R_1$ in the dividers, and finding the equivalent ordinate $U\,V$ on the curve, and then adding $V\,I = R_1$, the axis $H\,I$ is found.

In order to use the curve conveniently, it may also be laid off left-handed, as shown in the dotted lines $D'\,X\,E'$.

The use of the diagram will be rendered still more convenient if we omit the unnecessary value l. This enables us to distort the curve in the direction of the abscissas to any desired extent.

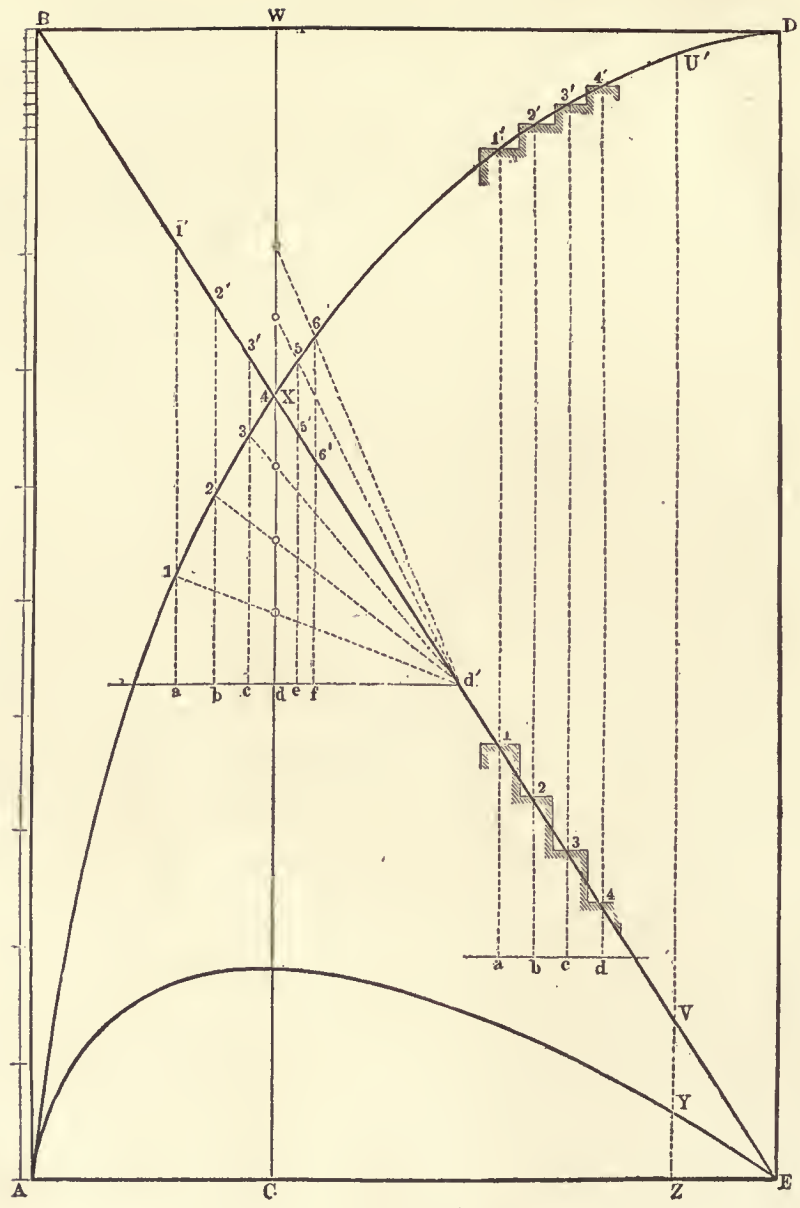

FIG. 868.

This has been done in the proportional diagram for cone pulleys, Fig. 868.

The method of using the diagram is as follows:

The sides $A\,B$ and $D\,E$ of the rectangle represent the distance a between the centres of the pulleys; all radii are given in proportional parts of a, for which reason $A\,B$ is sub-divided, the size of the diagram being selected so that $A\,B = 18$ to 20 inches. If, then, 1 a and 1′ a are two given radii for a pair of pulleys on a pair of cones, we take the vertical chord of the curve which $=$ 1′ a — 1 a, prolong the chord downward until its length $=$ 1 a, and draw the axis $a\,b\,c\,d$ parallel to $A\,E$. Then for the other pairs of pulleys on the cones, we have $b\,2$ and $b\,2'$, $c\,3$ and $c\,3'$, etc., which can be taken directly from the diagram with the dividers. If the given pair of radii to which the cones are to be made equal, the chord $R - R_1 = o$, and the axis will pass through X at right angles to $C\,X$.

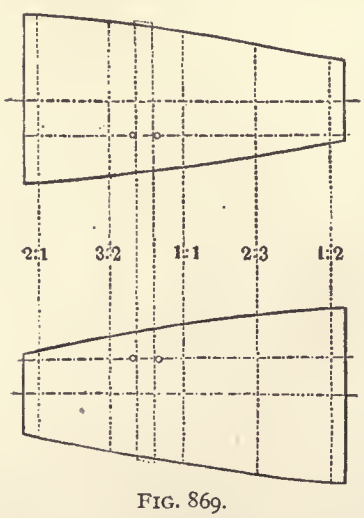

FIG. 869.

If it is desired to construct a pair of cone pulleys to any given speed ratio, this can readily be done. If, for example, the given ratio is 1 : 1, we lay off toward C, the corresponding radius $X\,d$ and prolong the axial line $d\,d'$ to its intersection d' with $B\,E$. Then lay off the given geometric ratio on $C\,X$, considering $X\,d$ as 1 (shown in the diagram by the small circles for the ratios $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{4}$), and draw rays from d' through the points of division, and these rays will intersect the curve at the correspouding points for the pulley radii R_1. We then have for the radii:

a 1	and	a 1′	for the ratio	1 : 4
b 2	"	b 2′	"	2 : 4
c 3	"	c 3′	"	3 : 4
$d\,X$	"	$d\,X'$	"	4 : 4
e 5	"	e 5′	"	5 : 4
e 6	"	e 6′	"	6 : 4

Cone pulleys may also be made continuous, thus becoming conoids upon which the belt can be shifted to any point by an adjustable guide or shifter. Such conoids are used for driving the rollers in spinning machinery. Such a pair of conoids are shown in Fig. 869, the proportions having been determined by the graphical scale. The angular velocity varies in an arithmetical ratio as shown.

The curve $E\,Y\,A$ in the scale shows the limit to which the axial line may approach $A\,E$; this distance must not be less than $R + R_1 = a$, from which $V\,Y = \frac{1}{2}(A\,B - V\,U)$.

§ 280.

CROSS SECTION AND CAPACITY OF BELTS.

A belt of rectangular cross section of width b, and thickness δ, will be subjected to a tension T on the tight side (see § 264), which it must be proportioned to sustain. If S is the permissible stress for the unit of cross sectiou, we have, therefore. $T = b\,\delta\,S$.

The minimum ratio which T bears to the transmitted force P is dependent upon the stress modulus τ, since $T = \tau\,P$ (§ 264). But $\tau = \frac{\rho}{\rho - 1}$, in which ρ represents the modulus of friction $e^{f\alpha}$. Hence, if N is the horse power transmitted for a belt speed of v feet per minute, we have: $N = \frac{P\,v}{33000} = \frac{b\,\delta\,S\,v}{33000\,\tau}$.

This enables us to determine the cross section of the belt, but iu practice the width of the belt is the variable factor, the thickness usually being determined by commercial considerations, and limited to few definite sizes.

If we let q represent the cross section of the belt in square inches, we have:

$$N = \frac{q\,v\,S}{33000\,\tau}$$

This formula is very useful, since it may be used to determine the capacity of a belt from its cross section and velocity. If we put $N_o = \frac{N}{q\,v}$ we have:

$$N_o = \frac{1}{33000} \cdot \frac{S}{\tau} \quad \ldots\ldots\ldots\ldots \quad (262)$$

The value depends upon the material and stress modulus, the latter including the arc of contact α, and upon f, which itself depends upon the material of both belt and pulley; it may also be considered as dependent upon α, independent of the material, in the same manner as was the subject of specific weight. The author has called this value N_o, the *specific capacity* of a belt. It will be seen that when this specific capacity is determined for any kind of belt, the proper cross section for the transmission of a given horse power N can readily be found, since the velocity v can be chosen, and we have at once

$$q = \frac{N}{N_o\,v} \quad \ldots\ldots\ldots\ldots \quad (263)$$

For the determination of the specific capacity of any kind of belt it is necessary to find the constants S and τ.

The materials used for belting are:

Tanned leather,
Cotton, woven and treated with oil,
Rubber, interlaid with linen or cotton webbing.

In practice the value of S to be used must depend much upon

judgment, the value being governed to a great extent by the quality of the material. Customary values are for:

Leather . . . . . $S = 4000$ to 6000 lbs.
Cotton . . . . . $S = 3000$ to 4000 lbs.
Rubber . . . . . $S = 3500$ to 5000 lbs.

The thickness δ for single leather belts varies from $\frac{3}{16}''$ to $\frac{7}{32}''$; double, triple, quadruple, and even quintuple thicknesses being sometimes used, the thicknesses being secured by cement, and sewed or rivetted together. Cotton belts are usually from $\frac{7}{32}''$ to $\frac{11}{16}''$ thick, while rubber belts are made of any desired thickness, a web of canvas being interlaid between the successive thicknesses of rubber.

The stress modulus τ depends upon a and f, and the latter coefficient varies with the age of the belt, being greater with belts which have been used some time than with quite new belts. It is advisable, however, to make all calculations as for new belts, in which case we have for smooth iron pulleys, for:

Leather and cotton, $f = 0.16$ to 0.25*, $\rho = 1.6$ to 2.1
Rubber, $f = 0.20$ to 0.25, $\rho = 1.8$ to 2.1

These give as approximate values for.

$$\text{Leather and cotton, } \frac{T}{P} \text{ or } \tau = 2.5 \text{ to } 1.9$$

$$\text{Rubber, } \frac{T}{P} \text{ or } \tau = 2.2 \text{ to } 1.9$$

By using these values together with those given for S, in (262) we get for the *specific capacity* for belting:

$$\left.\begin{array}{l} \text{Leather, } N_o = 0.0062 \text{ to } 0.0098 \\ \text{Cotton, } N_o = 0.0036 \text{ to } 0.0068 \\ \text{Rubber, } N_o = 0.0050 \text{ to } 0.0082 \end{array}\right\} \quad (265)$$

These are based upon low and moderate speeds; say up to 3000 feet per minute, and the variations between the limits given are those due to the differences in strength of various kinds of leather and canvas used.

The resistance to bending or stiffness of a belt must be taken into account, and the ratio of thickness δ to pulley radius R, must not be too great. Practical experience has shown that $\frac{\delta}{R} = \frac{1}{50}$ should not be exceeded to obtain best results.*

From the known stress and the thickness of the belt the superficial pressure p, between belt and pulley may be calculated. We have only to substitute in (241) for the width b' of the surface of contact, the width b of the belt itself, and since $q = b\,\delta$, we get the simple relation:

$$\frac{p}{S} = \frac{\delta}{R} \quad \ldots\ldots\ldots\ldots (264)$$

Example 1.—Required a leather belt to transmit 100 H. P. and the speeds of pulleys to be $n = 80$ $n_1 = 150$ revolutions. Taking the specific capacity at 0.007 and the lineal velocity of belt at 3000 feet, we have $q \frac{100}{3000 \times 0.007} = 4.8$ sq. in. cross section.

If we use a double belt 0.4" thick, the width should be $\frac{4.8}{0.4} = 12$ inches.

For the driving pulley we have: $\frac{2\,\pi\,R\,n}{12} = v$, and $R = \frac{3000 \times 12}{2\,\pi\,n} = 71.7$ say 72", or 154 inches. For the driven pulley we have $R_1 = \frac{80 \times Fz}{150} = 38.4$".

For the superficial pressure p, we have $P = \frac{33000 \times 100}{3000} = 1100$ lbs. Also $T = 2.5\ P = 2750$, hence $S_1 = \frac{2750}{0.4 \times 72} = 573$. We have also $t = 1.5\ P = 1650$, which gives $S_2 = 343$, or a mean of 458 lbs., which in (264) gives a mean value $p = \frac{458 \times 0.4}{72} = 2.5$ lbs. on the large pulley, and $p = \frac{458 \times 0.4}{38.4} = 4.98$, or nearly 5 pounds. This is verified since, if $f = 0.16$:

$$72 \times 3.14 \times 12 \times 2.5 \times 0.16 = 1100,$$

which is the value of P as above.

Example 2.—What horse power can be transmitted by a cotton belt 4 inches wide and 0.25" thick, at a velocity of 2000 feet per minute? Taking the specific capacity at 0.006, which has been found satisfactory in practice, we have from (262) $N = q\ v\ N_o = 4 \times 0.25 \times 2000 \times 0.006 = 12$ H. P.

Example 3.—A rubber belt is required to drive a centrifugal pump (rubber being especially adapted for damp locations). $N = 20$, the pump vane to make 300 revolutions, and the driving shaft 80 revolutions per minute, and the belt speed 2000 feet. Taking the specific capacity at 0.007, we have $20 = q \times 2000 \times 0.007$, hence $q = 1.43$ sq. in., and if we make $\delta = 0.2$ we have a width $b = 7.1$". For the driven pulley we have $R_1 = \frac{2000 \times 12}{2\,\pi\,300} = 12.71$, say $12\frac{3}{4}$", and for the driver $R = \frac{12.75 \times 300}{80} = 47.8$". A mean value of S is 425 lbs., whence $p = \frac{425 \times 0.2}{47.8} = 1.78$ on the large pulley and $\frac{425 \times 0.2}{12.71} = 6.7$ on the small pulley.

For extraordinary cases the fundamental formula should always be applied. For double-acting belts, as in Fig. 860, in which $a = 2\,\pi$ instead of π, the value $f\,a = 1$, and the modulus of stress is only 0.6 of the preceding value, hence q is reduced in the same proportion. If the belt velocity v is very high, it is no longer permissible to neglect the influence of centrifugal force. For a speed $v = 5000$ feet and a stress $S = 568$ pounds (see § 264) the exponent in the friction modulus becomes $0.84\,f\,a$ instead of $f\,a$, which for $f = 0.16$ and $a = \pi$, gives $f'\,a = 0.84 \times 0.16\,\pi = 0.42$. This gives $\tau = 2.91$ or about $\frac{7}{8}$ of the normal value, which requires one-sixth greater cross section q for the belt. The highest limit of belt speed in ordinary practice appears to be about 6000 feet per minute.†

§ 281.

Examples of Belt Transmission.

The table of existing examples of belt transmission on next page will serve to furnish data for comparison with calculated results.

The great variations in the values of S and N_o, in the following table are not surprising when the differences in the quality of material, and the various conditions are considered. Many leather belts are working under high stresses which are only practicable because of the excellence of the material. Some such belting can be operated under stresses as high as 2000 pounds, which enables much lighter sections to be used. Many belts which appear to have been excessively heavy have simply been calculated to work at a moderate stress.

The plausible but erroneous idea that the pressure of the atmosphere influences belt action cannot be admitted. It is contradicted not only by the fact that the same coefficient of friction exists for ropes as for belts, but also by the recent and careful experiment made in a vacuum by Leloutre which confirmed the theory of the modulus of friction.

§ 282.

Belt Connections.

The various methods of connecting the ends of belts generally give a greater stress at the point of connection than in the body of the belt. The attempts to reduce this weakness and also provide for the greatest facility in the making of the joint, has caused a great variety of methods to be proposed; some of the best of these are here given:

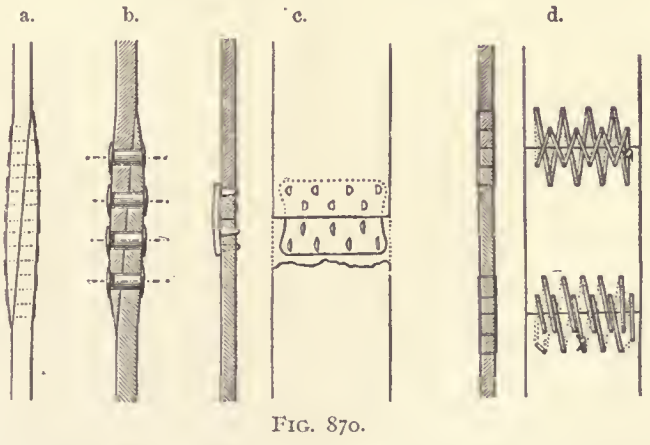

Fig. 870.

In Fig. 870, a is a lap joint sewed with hempen thread; b, a lap joint secured with screw rivets; c is a plate coupling, the plate and prongs being made in one malleable casting and the prongs bent over and clinched after insertion in the belt, several clamps being used for belts more than 4 inches in width. At d is shown belt lacings for use with single or double belts. The upper one has the defect of giving intersections which make the lacing cut itself, and the knot at the edge of the belt reduces the streugth of the joint.‡ These defects are both avoided in the lower form, which is an American belt lacing.§

* For cotton the thinner belts from 0.25" to 0.4" are preferable.

† In the construction of the Ariberg tunnel a hoisting machine was used in which the belt had a velocity of 4700 feet per minute, which worked well for fourteen months.

‡ Leloutre has used the upper form of lacing for a belt of 26" wide, 0.66" thick with excellent performance and durability.

§ See Cooper, Use of Belting, p. 189.

EXAMPLES OF BELT TRANSMISSION.

No.	Horse Power N	$\frac{n}{R}$	v	P	b	δ	S	N_o	REMARKS.
1	624	$\frac{40}{100}$ $\frac{271.8}{108}$	2887	7114	105	0.67	512	.0062	Leather, 2 belts side by side.
2	200	$\frac{52}{182}$ $\frac{137.8}{39.37}$	3749	1731	24	0.47	388	.0046	Leather.
3	190	$\frac{65}{223}$ $\frac{71.8}{22.6}$	2440	2528	21	0.24	1222	.0147	Leather.
4	175	$\frac{120}{228}$ $\frac{56.9}{30}$	3561	1573	29	0.35	388	.0046	Leather.
5	153	$\frac{120}{160}$ $\frac{63}{47.25}$	3955	1256	12.6	0.52	483	.0059	Leather.
6	130	$\frac{36}{94}$ $\frac{128}{45.3}$	2410	1544	10	0.40	981	.0121	Leather.
7	90	$\frac{65}{182}$ $\frac{83.8}{30}$	2833	1034	12	0.35	612	.0075	Leather.
8	81	$\frac{55}{137.5}$ $\frac{98.4}{39.37}$	2833	928	9.8	0.52	455	.0056	Leather.
9	60	$\frac{100}{100}$ $\frac{59}{59}$	1535	631	12.2	0.47	270	.0033	Leather.
10	54	$\frac{45}{125}$ $\frac{98.4}{35.4}$	2318	660	17.3	0.24	400	.0092	Leather.
11	42	$\frac{60}{90}$ $\frac{70.8}{47.25}$	2224	614	11.8	0.20	654	.0082	Leather.
12	40	$\frac{66}{102}$ $\frac{49.2}{38.9}$	2066	630	13.8	0.24	483	.0059	Leather.
13	530	$\frac{60}{262}$ $\frac{144}{27}$	5156	3337	38	0.72	313	.0036	Cotton.
14	497	$\frac{70}{144.4}$ $\frac{99}{48}$	3620	4457	30	0.72	526	.0065	Cotton.
15	470	$\frac{62.5}{114}$ $\frac{96}{49.5}$	3130	4877	32	0.72	540	.0065	Cotton.
16	413	$\frac{48}{120}$ $\frac{120}{48}$	3000	4453	30	0.72	512	.0105	Cotton.
17	325	$\frac{125}{172.4}$ $\frac{60}{43.5}$	3915	2583	22	0.72	412	.0049	Cotton.
18	134	$\frac{125}{133.3}$ $\frac{48}{45}$	3130	1390	10	0.72	412	.0049	Cotton.
19	60	$\frac{70}{175}$ $\frac{73.8}{29.6}$	2706	722	16.5	0.47	228	.0049	Cotton.
20	35	$\frac{81.3}{99.3}$ $\frac{38}{31}$	1633	704	5	0.72	498	.0062	Cotton.
21	66	$\frac{165}{243}$ $\frac{55}{37.5}$	4763	451	11.8	0.52	185	.0023	8-ply Rubber.

Fig. 871 *a*, shows Bötter's belt fastening. This is a form of belt hook which has been found very serviceable, reducing the strength of the belt but little, and permitting easy renewal. Another form is Moxon's belt fastening,* shown at *b*, is a pin

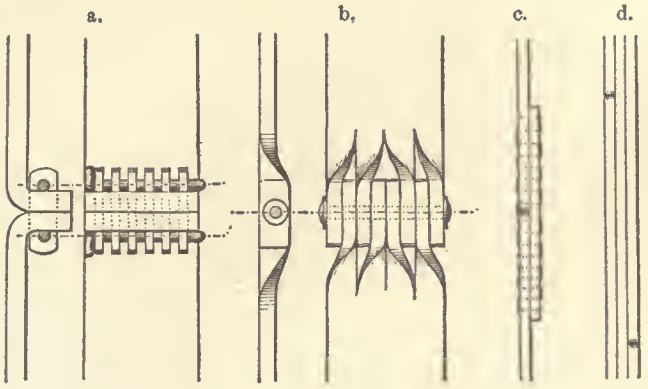

FIG. 871.

point, the ends of the pin being riveted over, and from its construction should be very strong. At *c* is a butt joint with a reinforcement piece especially suited for cotton belts. When a belt is made for special service it can be in several layers as at *d*; the joints overlapping, but thus giving no opportunity for change of length.

The stretching and joining of heavy belts is a matter requiring much care in order to secure the desired tension, $= \frac{1}{2}(T + t)$.† Belts which are subjected only to light tensions may be cemented by scarfing the ends and using a cement composed of common glue mixed with fish glue, or of rubber dissolved in bisulphide of carbon,

§ 283.

THE PROPORTIONS OF PULLEYS.

Pulleys are usually made of cast iron and of single width, *i.e.*, one set of arms. The arms, which formerly were made curved, in order to resist the stresses due to contraction, are now made straight, and for wide face pulleys two or even three parallel sets of arms are used.

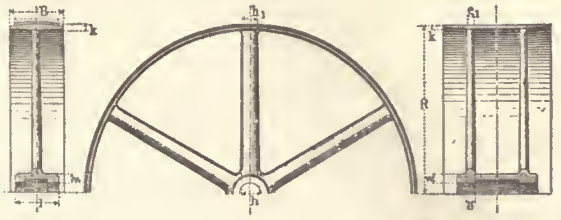

FIG. 872.

Fig. 872 shows both single and double arms. The dimensions of arms and rim have been determined by experience, based upon practical considerations. For the number A of arms for a single set, we get serviceable values from:

$$A = \tfrac{1}{2}\left(5 + \frac{R}{b}\right) \quad \ldots\ldots\ldots \quad (266)$$

which gives, for:

$\frac{R}{b} =$	1	2	3	4	5	6	7	8	9	10	11	12	13
$A =$	3		4		5		6		7		8		9.

The width h of the arm, if prolonged to the middle of the hub, may be obtained from:

$$h = 0.25'' + \frac{b}{4} + \frac{1}{10}\frac{R}{A} \quad \ldots\ldots\ldots\ldots \quad (267)$$

The width h_1 of the arm at the rim is equal to $0.8\,h$, and the corresponding thicknesses are $e = \frac{1}{2}\,h$, and $e_1 = \frac{1}{2}\,h_1$.

Pulleys with two or three sets of arms may be considered as two or three separate pulleys combined in one, except that the proportions of the arms should be 0.8 or 0.7 times that of single arm pulleys, or in the proportion of $\sqrt[3]{\frac{1}{2}}$ and $\sqrt[3]{\frac{1}{3}}$

The thickness of the rim may be made: $k = \frac{1}{5}$ to $\frac{1}{4}\,h$, this being frequently turned much thinner. The width of face should be from $\frac{2}{3}$ to $\frac{3}{4}$ the width of the belt.

The thickness of metal in the hub may be made $W = h$, to $\frac{3}{4}\,h$. The length of hub may $= b$, for single arm pulleys and $2\,b$ for double arm pulleys. Light pulleys are usually secured to the shaft by means of set screws, as in Fig. 875 and 877; heavier ones are keyed as in Fig. 191, either with or without set screws.*

For many purposes pulleys are made in two parts, such being commonly called "split pulleys. The forms of split pulleys are shown in Figs. 873 to 875.

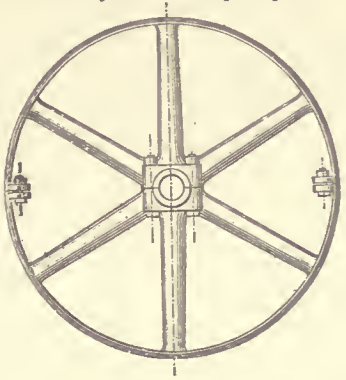

FIG. 873.

The arrangement of the two halves is clearly shown, that of Fig. 874 with hollow clamping section, being especially good.†

The form in Fig. 875 is the design of the Walker Mfg. Co. of Cleveland, Ohio, the clamps being made of malleable iron or steel. In all three cases there is no especial method of fastening to the shaft. In England and America pulleys are frequently made with wrought iron rims and cast iron hubs. This construction greatly simplifies the casting of the arms, and at the same time gives pulleys 25 to 60 per cent. lighter than those of cast iron, which in large transmissions greatly reduces the friction

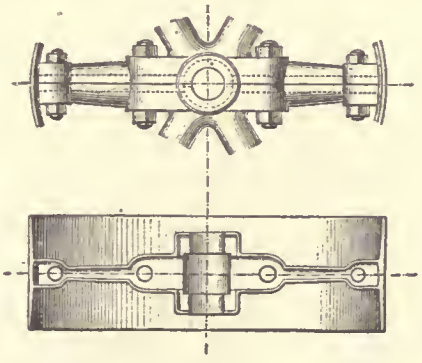

FIG. 874.

at the bearings of the shafting. Fig. 876 shows the Medart pulley. The rim is curved in bending rolls, and also given a rounding face, and is countersunk for the rivets at the attachment of the arms. The pads on the arms are truly finished, as is also the rim after it is riveted on, thus giving an accurate and useful pulley.‡

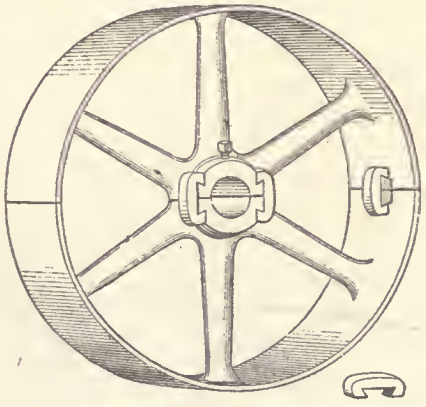

FIG. 875.

A metal pulley by the Hartford Engineering Company 60″ diameter and 16″ face weighed 320 pounds. A cast iron pulley of the same dimensions made by the Berlin-Anhalt Works, weighed 700 pounds, and one by Briegleb, Hansen & Co., a little narrower face weighed 528 pounds.

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*See Chronique industrielle, 1882, Vol. 5, p. 97; also Mechanical World, 1882, Vol. 12, p. 56.

†Leloutre has used a dynamometric belt-stretcher for tensions of $\frac{1}{2}(T + t) = 8800$ pounds.

*In order to determine the necessary friction to secure a pulley to the shaft, the force p on the belt will serve. In ordinary cases, assuming a coefficient of friction on the key of one-half that on the belt, there should be a pressure p' on the key of about 4000 times that on the belt, which, according to § 20 will not give more than 5000 to 7000 lbs. for p'.

†This is the construction of the Berlin-Anhalt Machine Works.

‡Made in England by George Richards & Co., Manchester.

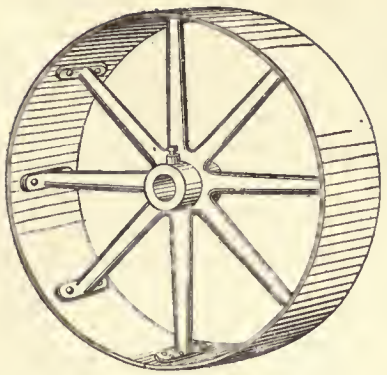

FIG. 876.

Fig. 877 shows Goodwin's split pulley, with wrought rim, the face of the rim being rounded by turning.

These constructions naturally led to the use of wrought iron arms also, although these are somewhat difficult to make; but for very large diameters (say 16 to 25 feet) they possess advantages.*

Pulleys made entirely of steel are used by J. B. Sturtevant of Boston, in connection with fan blowers, Fig. 878. The hub with web, is screwed on the steel shaft of the fan wheel, and the rim, which has a groove turned in it, is expanded by warming, and shrinks into place, the whole being finally turned in position, and carefully balanced. Sturtevant uses these pulleys up to 10 in. in diameter, and 7 in. face, the thickness of rim being from 0.08 to 0.16, and the velocity at the rim reaching 5000 feet per minute.

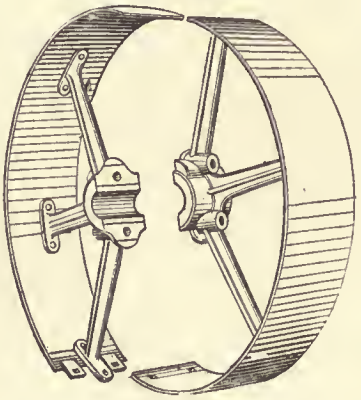

FIG. 877.

By covering the rim with leather the co-efficient of friction, f, and can be increased between the belt and pulley, and the modulus of stress τ reduced, and the specific capacity of the belt increased. This is sometimes useful because a reduced modulus of stress τ permits a smaller cross section of belt and lighter pulley. In large transmissions reduction of stress is important since it is accompanied with reduced journal friction and higher efficiency. The observation of the author leads him to believe the specific capacity of a belt is not greater with leather covered pulleys than with uncovered ones, and the cost of covering is an important item.

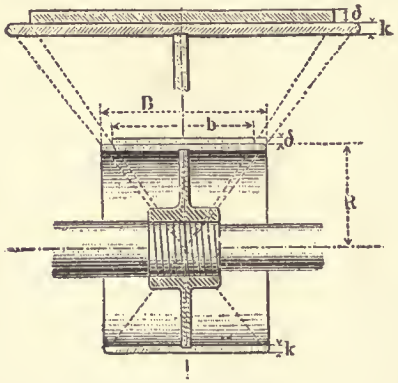

FIG. 878.

The greater the angular velocity of a pulley the more important it is that its geometric axis should be a so-called "free axis." This requires that the center of gravity of the pulley should be on the axis of rotation and also that the various portions of the mass should be so distributed that the axis of inertia should coincide with the axis of rotation and the centrifugal moment equal zero.† This can be done empirically by so-called balancing, the unequal distribution of material being equalized by attaching pieces of lead or other metal, or more accurately by balancing when revolving, for which purpose a beautiful apparatus has been made by the Defiance Machine Works, Defiance, Ohio. Careful balancing of pulleys is of great importance at high speeds, the rapidly increasing vibrations will soon limit the speed. This is to be considered in connection with the advantages to be gained by the use of high speed shaft as discussed in § 146.

NOTE.—The recent investigations upon paper rim pulleys‡ are instructive. This construction gives a very high modulus of friction, the modulus of stress τ being only 1.2. This gives $T = 1.2\,P$ as against 2.5 P, for iron pulleys. Hence follows a great increase in the specific capacity of the belt, and increased efficiency with smaller and lighter pulleys. This leads the way to further investigations which prove of material value in the science of belt transmission.

* Pulleys with wrought iron arms are made in Germany by Starck & Co., Mainz; in England by Hudswell, Clark & Co., Leeds, these latter with arms of round bar iron.

† See an article by the writer, "Ueber das Zentrifugal-Moment," in Berliner Verhandlung, 1876. p. 50.

‡ See Am. Machinist, May 23, 1885, p. 7.

§ 284.

EFFICIENCY OF BELTING.

Three causes of loss exist in belt transmissions, viz.: journal friction, belt stiffness, and belt creeping. For horizontal belting we have, according to formula (99) for the journal friction, expressed at the circumference of the pulley a loss E_z when $T = 2.5\,P$, $t = 1.5\,P$:

$$\frac{F'}{P} = E_z = \frac{T}{P} + \frac{t}{P}\,\frac{4}{\pi} f\left(\frac{d}{2R} + \frac{d_1}{2R_1}\right) = \frac{8}{\pi} f\left(\frac{d}{R} + \frac{d_1}{R_1}\right) \quad \ldots\ldots\ldots (268)$$

in which d and d_1 are the journal diameters, and f the coefficient of journal friction. This loss is doubtless the greatest of the three. For lack of better researches the loss of belt stiffness may be deduced from Eytelwein's formula for ropes. For the coefficient of stiffness s, for force S', which includes both pulleys;

$$\frac{S'}{P} = E^s = s\,\frac{T+t}{P}\left(\frac{\delta^2}{R} + \frac{\delta^2}{R_1}\right) = 4\,s\left(\frac{\delta^2}{R} + \frac{\delta^2}{R_1}\right) \quad \ldots\ldots\ldots\ldots (269)$$

in which $s = 0.009\,\frac{4}{\pi} = 0.012$.

The loss from creep is due to the fact that the greater stress on the driving pulley over that on the driven requires for a given volume of belt, a longer arc of contact; for the expenditure of force G' for creep on both pulleys, we have for a stress S_1 on the leading side of the belt:

$$\frac{G'}{P} = E_s = \frac{1 - \frac{t}{T}}{1 + \frac{E}{S_1}} = \frac{0.4\,S_1}{E + S_1} \quad \ldots\ldots (270)$$

In this E is the modulus of elasticity of the belt, which for leather is 20,000 to 30,000 pounds. The losses from stiffness and creep are small.

Example.—Let d and $d_1 = 4''$; $R = R = 20''$, $\delta = 0.2$, $f = 0.08$, $S = 0.012$, $E = 28{,}440$, $S_1 = 425$, we have $F^1 = P\,\frac{8 \times 0.08}{\pi} \times 0.4 = 0.08\,P$;

also $S^2 = P\,(0.048 \times 2)\,\frac{0.2}{20} = 0.0048\,P$,

and $G^1 = P\,\frac{0.4 \times 425}{28{,}440 + 425} = 0.0059\,P$.

The total loss is therefore: 0.08 + 0.0048 + 0.0059 = 9.1 per cent.

CHAPTER XXI.

ROPE TRANSMISSION.

§ 285.

VARIOUS KINDS OF ROPE TRANSMISSION.

If in the tension driving gear, shown in Fig. 810, the rope be used only for the transmission of power we have what is called a Rope Transmission. Since the details of construction must vary, according as fibrous or wire rope is used, we may distinguish between three kinds of rope transmission, viz.: those for Hemp, Cotton or Wire Rope, and these will be considered in this order. The oldest of all these is hemp rope transmission, but this was gradually being superseded by belting until Combes, of Belfast, revived it, about 1860, since which time it has been extensively used for heavy transmissions. The character of the material permits a wide variety of applications. The same is true of cotton rope, which is extensively used for driving spinning frames, travelling cranes and many other machines, the softness and flexibility of the material giving it advantages, but within limits. Wire rope transmissions, since its introduction by the brothers Hirn, at Logelbach, in 1850, have developed a high degree of efficiency and utility for long distance transmission. As will be seen hereafter, the applications of rope transmission appear to be capable of still further extension.

A. HEMP ROPE TRANSMISSION.

§ 286.

SPECIFIC CAPACITY. CROSS SECTION OF ROPE.

It is important first to determine the specific capacity for hemp rope (§ 280). This is obtained from the geueral statement according to (262):

$$N_o = \frac{4}{3}\frac{S_1}{\tau},$$

in which S_1 is the stress on the tight side of the rope, and τ the modulus of stress. The value for the co-efficient of friction f, depends upon the form of the groove or channel in the sheave over which the rope runs.

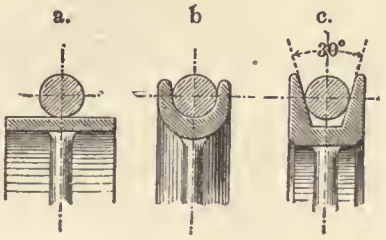

FIG. 879.

If the groove is semicircular, as at *b*, Fig. 870, the friction is but little greater than it is upon an ordinary cyliudrical pulley, as at *a*; if, however, the groove is made wedge-shaped, as at *c* (see wedge friction wheels § 196), the driving power is increased although the surface of contact is reduced. In determining the value of τ, from formula (239) the influeuce of the shape of the groove can be iucluded by using a corresponding co-efficient of friction f^1. According to the recent investigations of Leloutre and others, the value of f for cylindrical pulleys and new hemp rope is 0.075, for semicircular grooves, 0.088, and for wedge grooves with an augle of 60°, $f = 0.15$, which accords well with the action of the wedge, doubling the pressure, see (185). For $f^1 = 0.088$ and a contact of a half circumference, we have $f^1 a = 0.3$, and hence $\tau = 3.86$; with $f^1 = 0.15$, $f^1 a = 0.47$, and $\tau = 2.67$. The latter value, which is even reduced in actual practice, may be adopted, since wedge grooves in general use. The stress is usually taken while low, and may be put at $S =$ 350 lbs., which, taking $\tau = 2.67$, gives $N_o = \frac{1}{33000} \cdot \frac{350}{2.67} =$ 0.0039; see (262). Iu practice N_o is found even one-half this value, aud we may take as a practical rule in hemp rope transmission for the specific capacity, *i. e*, the horse power transmitted per square iuch of cross sectiou, for each foot of linear velocity per minute;

$$N_o = 0.004 \text{ to } 0.002 \ldots\ldots\ldots\ldots (271)$$

the cross section being taken as in § 265, as that due to the full outside diameter of the rope.

When great power is to be transmitted a number of ropes are used side by side, the pulleys being made with a corresponding number of grooves. For machine shop transmission such ropes are conveniently made about two inches in diameter, although they are used as small as 1¼, and as thick as 2¾ inches.

Example 1. A steam engine of 60 *H. P.* has its power transmitted through five ropes of 2 inches, the pulley being 11.28 feet diameter, making 45 revolutions per minute. This gives $v = 1592$ feet per minute. The cross section of the rope 3.14 sq. inches. Hence $N_o = \frac{1}{5} \frac{60}{1592 \times 3.14} = 0.0024$. This is taken from an existiug installation.*

Example 2. In the jute mills at Gera the fly wheel of the engine is grooved for 30 ropes, of 2.36″ diameter, each rope transmitting 25 *H. P.*; the velocity beiug 3000 feet per miuute. This gives a specific capacity of $N_o = \frac{25}{3000 \times 4.375} = 0.0019$.

Example 3. The Berlin-Anhalt Machine Works has design rope transmissions in which ropes of 1 18″, 1.57″, 1.97″ diameter transmit forces, respectively, of 92.4, 165 aud 264 pounds. The respective cross sections of the ropes are 1.09, 1.93 and 3.04 square inches. Siuce $\frac{N}{v} = \frac{P}{33000}$ we have $N_o = \frac{P}{33000\, q}$ which gives in each of the three cases $N_o = 0.0026$.

* See Zeitschrift d. Verein deutscher Ingenieure, Vol. XXVIII, 1884, p. 640.

The cross section of the rim of a pulley for five ropes is shown in Fig. 880. For large steam engines the grooves are sometimes made on the fly wheel, such constructions sometimes being very large and heavy.†

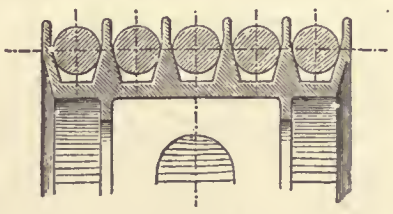

FIG. 880.

The application of rope transmission in manufacturing establishments simplifies the mechanism very materially, since it enables the jack shaft and gearing to be dispeused with.

Such an arrangement is shown in Fig. 881, iu which five

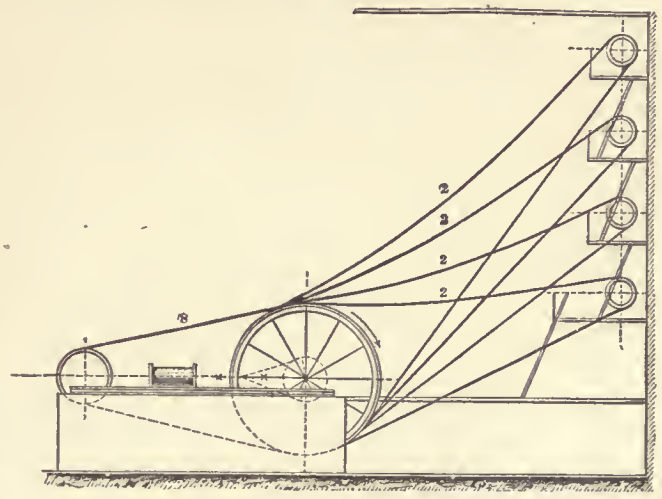

FIG. 881.

different lines of shafting are driven from one horizontal steam engine, sixteen hemp ropes being used in all.

§ 287.

SOURCES OF LOSS IN HEMP ROPE TRANSMISSION.

The use of hemp rope transmission reduces many losses which exist in other methods and which materially reduce the efficiency; the principal ones which need to be considered are the resistances due to journal friction, stiffness of ropes, and creep of ropes.

a. Journal Friction.—In rope transmissions from steam engines the journal friction is usually great, because the large fly wheel requires journals of large diameter. The usual calculations can only be given by indeterminate results, because the tension of the ropes sometimes acts with the weight of the other parts, and sometimes against it.

If we consider the rope tensions T and t by themselves, as acting horizontally, we have from formula (100) the friction $F = \frac{4}{\pi} f (T + t)$, which reduced to its corresponding resistance to the rope, taking $\tau = 2\frac{2}{3}$, gives a loss due to one shaft $= \frac{4}{\pi} f \left(2\frac{2}{3} + 1\frac{2}{3}\right)\left(\frac{d}{2R}\right)$. If we take $f = 0.09$‡ and double the result for both shafts, calling this combined loss E_z we have: $E_z = \frac{8}{\pi} \times 0.09 \times 4.33 \left(\frac{d}{2R}\right)$ which reduces to:

$$E_z = \frac{d}{2R} \ldots\ldots\ldots\ldots\ldots (272)$$

Example 1. In the first of the preceding examples we have also $d = 6.3$ inches, and $2R = 135\frac{1}{2}$ iuches, hence $\frac{d}{2R} =$ 0.046 or a little over 4 per cent.

† See Engineer, Jan., 1884, p. 38, for such a fly wheel 15 ft. face, 30 ft. dia., weighing 140 tons, to transmit 4000 H. P. by 60 ropes.

‡ See § 300.

b. Stiffness of Ropes.—If we apply Eytelwein's formula (252) we have $Q = \frac{1}{2}(T + t)$ taking both pulleys into cousideratiou, aud taking $\tau = 2\frac{2}{3}$ and introducing $T + t$, gives $Q = 4\frac{1}{3} P$.

It must be cousidered that the ropes are usually quite slack, and that the co-efficieut stiffuess S, may be takeu somewhat less than Eytelwein's value. If we take $\frac{2}{3}$ as a fair approximatiou, the ratio of loss is

$$\frac{S}{P} = \frac{2}{3} \times 0.463 \frac{d^2}{R} \times 4\frac{1}{3}$$

and calliug this loss E_s, we get:

$$E_s = 1.33 \frac{d^2}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (273)$$

iu which d is the diameter of the rope.

Example 2. In the case of the preceding example, $d = 2''$, $R = 67.75''$. This gives $E_s = 1.33 \frac{4}{67.75} = 0.078$ or 7.8 per cent.

c. Creep of Ropes.—The loss through creep is more important iu rope trausmissiou than with belting (see § 284) aud should not be neglected, although it cannot be so readily determiued, owing to the divisiou of the power among a number of ropes. It is practically impossible to insure a uniform teusion upon a number of adjaceut ropes, or to have them of exactly the same diameter, besides which the "working" diameters of the various grooves differ slightly, so that additional slippage must occur.* The resulting frictional loss is estimated by some at as much as 10 per cent., when the number of ropes is 20 to 30, aud it is at all times important enough to be given cousideratiou. The losses from stiffuess and creep should be investi gated whenever practicable, as the resulting information would be of much technical value.

Assuming the loss frou creep in the case previously considered to be 5 per cent., we have a total resistance of $4 + 7.8 + 5 = 16.8$ per cent.; which, since small values were takeu iu all cases, is not to be considered higher than the actual loss. This explains the numerous objections which have been raised (as in England) agaiust the use of hemp rope transmission for very large powers (see § 301).

§ 288.

PRESSURE AND WEAR ON HEMP ROPE.

As already seen, the surface of coutact of the rope and pulley may be one of three kiuds: upon a cylindrical pulley, iu a semicircular groove, or iu a wedge-shaped groove (Fig. 879), aud to these formula (241) cau be applied. In case *a*, we can approximate b' as equal to $\frac{1}{8}$ the circumference of the rope. This gives for the superficial pressure

$$= S \frac{\frac{\pi}{4} d^2}{\frac{\pi}{8} dR} \text{ whence:}$$

$$\frac{p}{S} = 2\frac{d}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (274)$$

For case *b*, we have $b^1 = \frac{\pi}{2} d$, whence

$$\frac{p}{S} = \frac{1}{2}\frac{d}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (275)$$

In case *c*, the radial pressure Q, of the rope is divided iuto two forces Q' actiug normal to the wedge surfaces aud equal to $\frac{\frac{1}{2}Q}{\sin \frac{1}{2}\theta}$ in which θ is the angle of the groove, and takiug the contact surface on each side as $\frac{1}{8}$ the circumference of the rope, we have

$$\frac{P}{S} = \frac{1}{\sin \frac{\theta}{2}} \frac{d}{R}$$

which, for $\theta = 30°$, gives approximately: (276)

$$\frac{P}{S} = 4\frac{d}{R}$$

*The variation in adjacent ropes may be shown by putting a little coloring matter on the ropes and watching its distribution.

Even under these unfavorable conditious the superficial pressure is not important, on accouut of the small value of S; which, as already seen, is about 350 pouuds.

Example.—If $S = 350$ pounds, and $\frac{d}{R} = \frac{1}{25}$ we have for a cylindrical pulley $p = 350 \times 2 \times \frac{1}{25} = 28$ lbs. for semicircular grooves, $p =$ lbs and for wedge grooves, when $\theta = 30°$, $p = 56$ lbs. per square inch.

These low pressures cause but little wear upon the rope, hence the great durability of hemp transmission ropes, sometimes extending to two or three years of use.

§ 289.

B. COTTON ROPE TRANSMISSION.

Cotton rope is not so extensively used for purposes of transmissiou as hemp rope, although it possesses the advautages of great strength and flexibility; the impediment to its use being its higher price. The application of cotton rope for driving spinning mule spiudles, referred to in § 265, is shown iu Fig. 882, iu which T_1 is the driving pulley aud T_2 the driveu pulley

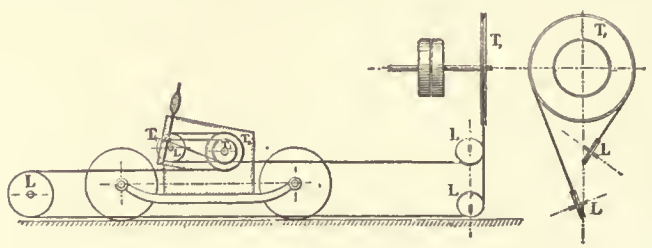

FIG. 882.

on the carriage. This latter pulley is on the axis of a drum T_3 from which light cords drive the spiudles T_4. At L, L, are guide pulleys. The usual diameter of rope for T_1 T_2 is 0.86″, and for large machines with many spindles two such ropes are used, the pulleys being made with double grooves, these always being of semicircular section.

On the ring spinuing frame cotton rope of 0.4″ diameter is used on cone pulleys of 12 steps, giving changes of speed from 3:1 to 2:3. The proportions of such pulley may be determiued as shown iu § 279, the grooves being semicircular.

As already shown in § 265 cotton ropes have been used by Ramsbottom for driving traveliug cranes. For this purpose ropes of $\frac{1}{2}$ to $\frac{5}{8}$ inch diameter are used, running at speeds of 2500 to 3000 feet per minute, a weighted idler pulley keeping the rope taut.

Iu view of the slow movement of the load, viz.: 20 to 40 feet per miuute, it is questiouable whether cottou rope transmission involving such a great trausforuiation of speed, is advantageous.†

C. WIRE ROPE TRANSMISSION.

§ 290.

SPECIFIC CAPACITY. CROSS SECTION OF ROPE.

Iu cousidering the transmissiou of power by means of wire rope the points to be deteruined are the cross sectiou of the rope, and the deflection of the two portions of rope due to its weight. The cross section will first be considered by determiuing the specific capacity (See § 280). This we get from (262)

$$N_o = \frac{1}{33000} \cdot \frac{S_1}{\tau}$$

iu which S_1 is the stress in driving half of the rope, considered either in connection with the driving or the driven pulley. The modulus of frictiou ρ is taken somewhat higher thau for belting, since the angle of contact a is greater, and also because the co-efficient of friction f, for pulleys fitted with diagonal leather strips (see below) is very high; early and recent tests giving $f = 0.22$ to 0.25 and higher. The first value gives $e^{fa} = 2$ (See Fig. 816), and also the stress modulus $\tau = \frac{2}{2-1} = 2$ (See 239). This gives, iu (262) if we neglect ccutrifugal force:

$$N_o = \frac{1}{33000} \cdot \frac{S_1}{2} = \frac{S_1}{66000} \quad \ldots\ldots\ldots (277)$$

†In some instances leather transmissiou ropes are used, formed of twisted thongs, these being used for light driving, as foot lathes, or light spindles.

This gives high numerical values, which is also borne out in practice, since large powers are successfully transmitted with wire ropes of small diameter. It is good practice to take S_1 for irou wire as high as 8500 pounds, and for steel wire up to 20,000 ponuds and even higher. This gives for the specific capacity, when:

$S_1 =$ 2000, 4000, 6000, 8000, 10,000, 12,000, 14,000, 16,000, 18,000, 20,000.

$N_0 =$ 0.03, 0.06, 0.09, 0.121, 0.151, 0.182, 0.212, 0 242, 0.273, 0.303 or approximately:

For Wrought Irou Wire $N_0 = 0.03$ to 0.121.

For Steel Wire . . . . $N_0 = 0.03$ to 0.303.

The cross sectiou q is readily obtained, since $N = q\,v\,N_0$ heuce:

$$q = 66{,}000\,\frac{N}{v\,S_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

We theu have, if i is the uumber of wires in the rope, a diameter of work: $\delta = i\,\frac{\tau}{4}\,\delta^2$. The speed v, of the rope may be as high as 6000 feet, but should uot exceed this velocity on accouut of the great stress upon the rim of the cast iron pulleys.

§ 291.

INFLUENCE OF PULLEY DIAMETER.

The bending of a rope about a pulley of a radius R produces a stress in each wire equal to

$$s = E\,\frac{\frac{\delta}{2}}{R}$$

which, if we take both for iron and steel wire $E = 28{,}400{,}000$, gives:

$$s = 14{,}200{,}000\,\frac{\delta}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (279)$$

The driving half of the rope is therefore subject to a tension stress, both at the poiut of advancing and departing contact equal to $S_1 + s$ in each wire. It is this sum which must be considered in determiniug the stress upon the material, aud it must uot be permitted to exceed the proper limits (See § 266). A practical upper limit for wrought iron wire is 25,000 pouuds, while for steel it may be takeu much higher; for hard drawn steel wire of good quality as high as 50,000 or even 60,000 pounds.

If we take as upper limits 25,000 lbs. for wrought iron and 50,000 lbs. for steel wire, we have for the given values of S, the following values of s and of $\frac{R}{\delta}$:

WROUGHT IRON WIRE.

S	s	$\frac{R}{\delta}$
711	24,885	571
1422	24,174	588
2844	22,752	625
4266	21,330	667
5688	19,908	714
7110	18,486	769
8532	**17,064**	**833**
9954	15,642	909
11,376	14,220	1000
12,798	12,798	1111
14,220	11,376	1250
15,642	9954	1429
17,064	8532	1667
18,486	7110	2000
19,908	5688	2500
21,330	4266	3333
22,752	2844	5000
24,174	1422	10,000

STEEL WIRE.

S	s	$\frac{R}{\delta}$
1422	49,770	286
2844	48.348	294
5688	45,504	313
8532	42,660	334
11,376	39,816	357
14,220	36.972	385
17,064	**34,128**	**417**
19,908	31,284	455
22,752	28,440	500
25,596	25,596	551
28,440	22,752	625
31,284	19,798	718
34,128	17,064	834
36,972	14,220	1000
39,816	11,376	1250
42,660	8532	1667
45,504	5688	2500
48,348	2844	5000

If a still greater value of $\frac{R}{\delta}$ is used for any given value of S_1 than in the above table, the durability of the cable will be increased. The minimum pulley radius for any giveu sum of stresses $S_1 + s$ is obtained when $\frac{s}{S_1} = 2$, which in the tables gives for $\frac{R}{\delta} = 833$ aud 417 respectively, as iudicated by the full-faced figures. Even in this advantageous proportiou the stress due to the bending of the wire around the pulley is double that due to the tension of the driving force.

Example 1.— Let $N = 60$ H. P. $v = 2952$. The material is iron wire, $S_1 = 8532$ and the number of wires $i = 36$. We then have for the cross section of rope:

$$q = 66{,}000\,\frac{60}{2952 \times 8532} = 0.16 \text{ sq. in}$$

$$\text{also } \delta = \sqrt{\frac{0.16}{36}\,\frac{4}{\pi}} = 0.076''$$

and the minimum pulley radius is $R = 833 \times 0.076 = 63.3''$ or appoximately 10 feet diameter. In order to obtain a velocity of 2952 feet per minute this requires about 93 revolutions.

If we take $S_1 = s = 12{,}798$ we find:

$$q = 66{,}000\,\frac{60}{2952 \times 12{,}798} = 0.108''$$

$$\text{whence } \delta = \sqrt{\frac{0.108}{26} \cdot \frac{4}{\pi}} = 0.06''$$

$R = 1111 \times 0.06 = 66.6''$ and $n = 72$.

The question here arises, to what extent should the effect of ceutrifugal force be takeu into account? If the velocity $v = 100$ feet per secoud, with a stress $S = 9000$ lbs. we have from the first table in § 264 the value $1 - z = 0.87$, so that iustead of fa we have $fa' = 0.87\,fa$. If $fa = 0.22$ we have $f'a = 0.87 \times 0.22 \times \pi = 0.70$, and if $f = 0.22$ we have $f'a = 0.87 \times 0.22 \times \pi = 0.60$. These give, by reference to the second table, in § 264, for the first value, the modulus of friction $\rho = \frac{T}{t}$ 2.01, aud for the second, $\rho = 1.82$ and a modulus of stress $\tau = 2.22$, which makes the specific capacity $= \frac{2}{2.22} = \frac{10}{11}$ as great as previously obtained. This may be compensated for by making the cross section of the rope 1.1 times that obtained by the previous calculation. For lesser velocities up to 2000 to 3000 feet per minute the effect of centrifugal force is much less and may safely be neglected, especially in the case of steel cables, in which much greater stresses are permissible.

Example 2.—How many horse power can be transmitted by a cable of 36 wires, each 0.078" diameter; the velocity beiug 6500 feet per minute. We have $N_0 = \frac{10}{11} \cdot \frac{8500}{66{,}000} = 0.117$; also $q = (0.078)\,\frac{\frac{2}{7}\pi}{4} \times 36 = 0.172$ sq. in.

We then have $N = q\ v\ N_0 = 0.172 \times 6500 \times 0.117 = 130$ H. P. For R we have from (279) $R = \frac{14{,}200{,}000 \times 0.078}{17{,}064} = 64.9''$ say $65''$.

Example 3.—What would the horse power be if steel wire were used? (See § 266). $S_1 = 17{,}000$ lbs. and N_0 will be twice $= 0.234$ whence $N = 274$ H. P. If we desire durability of the cable we may make s only 28,440 instead of 34,128, and thus obtain $R = \frac{14{,}200{,}000 \times 0.078}{28{,}440} = 38.9$ say only $40''$

When the resistance P is directly given, which is rarely the case, we have from the relation $q\ S_1 = \tau\ P$, taking $\tau = 2$.

$$q = 2\ \frac{P}{S_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (280)$$

The maximum statical moment which may have to be overcome upon the driven shaft is sometimes given, as in the case of pumping machinery, etc. Dividing the preceding equation by (279) we have

$$q = \frac{2\ s}{14{,}200{,}000\ \delta\ S_1}.\ P\ R$$

and since $q = i\ \frac{\pi}{4}\ \delta^2$, this reduces to:

$$\delta = 0.00564\ \sqrt[3]{\frac{1}{i}}\ \sqrt[3]{\frac{s}{S^1}}\ P\ R \quad \ldots\ldots\ldots\ldots (281)$$

and if we substitute for the moment $P\ R$ the quotient of effect from formula (135) $P\ R = 63{,}020\ \frac{N}{n}$ we get

$$\delta = 0.251\ \sqrt[3]{\frac{1}{i}}\ \sqrt[3]{\frac{s}{S_1}\ \frac{N}{n}} \quad \ldots\ldots\ldots\ldots (282)$$

Example 4.—A pressure pump operated by a crank on a shaft driven by wire rope, offers a resistance of 880 pounds, at a crank arm of 14.2 inches. This gives a maximum statical moment of $P\ R = 14.2 \times 880 = 12{,}496$ inch lbs. If we take $i = 36$ wires, and $S_1 = 8500$ pounds, we have from (281):

$$\delta = 0.00355\ \sqrt[3]{\frac{1}{36}}\ \sqrt[3]{\frac{17{,}000}{8500}}.\ 12{,}496 = 0.05''$$

This gives from the table $R = 833 \times 0.05 = 41.65$, say $42''$.

§ 292.

Deflection of Wire Ropes.

In order that the desired tensions T and t shall be attained in the two parts of a wire rope transmission, the deflections must be of predetermined values. The centre line of the rope will hang in a curve which lies between the catenary and the elastic line and which approximates closely to a parabola.*

For the parameter c, of this parabola, we have for a deflection h, in a horizontal rope, Fig. 883,

$$c = \frac{a^2}{8h} \quad \ldots\ldots\ldots\ldots\ldots\ldots (283).$$

in which a is the distance between two points of suspension; the deflection in the driving half being called h_1, in the driven half, h_2, and in the stationary rope h_0. This gives for the tangential force K at the point of suspension:

$$K = p\left(h + \frac{a^2}{8h}\right) \quad \ldots\ldots\ldots\ldots (284).$$

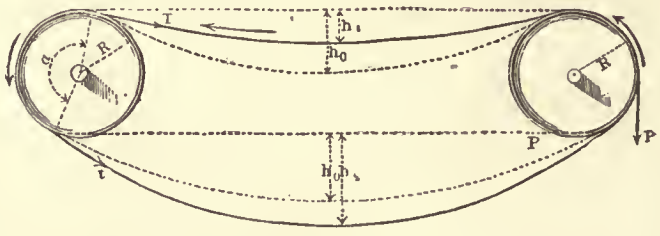

FIG. 883.

All dimensions are to be taken in inches. For any cross section q, we have $K = q\ S$, and $p = q\ \phi\ 12\ \gamma$, in which γ is the weight of the rope per cubic inch, and ϕ a coefficient, dependent upon the twist of the rope and upon the hemp core. The weight per cubic inch is $\gamma = 0.28$ pounds, ϕ is not always constant, but may be taken $= \frac{7}{8}$. These values give $p = \frac{7}{8} \times 0.28 \times q = 0.3266q$, and calling the coefficient $0.3266 = \psi$, we have:

$$S = \psi\left(h + \frac{a^2}{8h}\right) = 0.3266\left(h + \frac{a^2}{8}\right) \quad \ldots\ldots (287).$$

From this we get:

$$h = \frac{1}{2}\ \frac{S}{\psi} \pm \sqrt{\frac{1}{4}\ \frac{S^2}{\psi^2} - \frac{a^2}{8}} \quad \ldots\ldots\ldots (288).$$

Since $\frac{1}{\psi} = 3.061$ we have, taking the negative sign.

$$h = 1.53\ S - \sqrt{\left(1.53\ S\right)^2 - \frac{a^2}{8h}} \quad \ldots\ldots (289).$$

If we neglect the first member in the parenthesis in (287) we have for a close approximation:

$$h = 0.0408\ \frac{a^2}{S} \quad \ldots\ldots\ldots\ldots\ldots (290).$$

Example. Let a, which we may take as the distance from centre to centre of pulley, be 262 feet, or 3144 inches; also let S in the driving side of the rope be 8500 pounds, and on the driven side 4250 pounds. We have from (289)

$$h_1 = 1.53 \times 8500 - \sqrt{(1.53 \times 8500)^2 - \frac{3144^2}{8}} = 48''$$

$$h_2 = 1.53 \times 4250 - \sqrt{(1.53 \times 4250)^2 - \frac{3144^2}{8}} = 95''.$$

The approximate formula (290) gives:

$$h_1 = 0.0408\ \frac{3144^2}{8500} = 47.45'', \text{ and}$$

$$h_2 = 0.0408\ \frac{3144^2}{4250} = 94.89''.$$

The following method may be used to show the deflection h, graphically. The positive and negative signs before the radical sign in (288) indicate two values for h, as will be seen in Fig. 884.

The greater value is not of practical use, as it gives unstable (*labil*) equilibrium. Between the two lies a value $h = \frac{1}{2}.\ \frac{S}{\psi}$, which is obtained when the quantity under the radical sign $= O$, *i.e.* when $S = \frac{\psi\ a}{\sqrt{2}}$. This we will call the "mean" deflection and designate by h_m. This deflection is important because with it the absolute minimum stress exists in the rope (see note at the end of this section); and this stress, which occurs with the deflection h_m, will be designated S_m, and is:

$$S_m = \psi\ \frac{a}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots (291).$$

or for the preceding value $\psi = 0.3266$,

$$S_m = \frac{a}{4.33} \quad \ldots\ldots\ldots\ldots (292).$$

and since $h_m = \frac{1}{2}.\ \frac{S_m}{\psi}$, we have for the mean deflection

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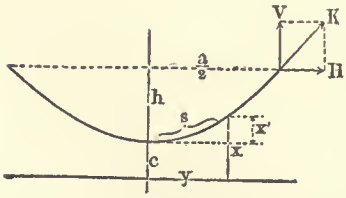
* The equation of the catenary is as follows:



$$x = \frac{c}{2}\left(e^{\frac{y}{c}} + e^{-\frac{y}{c}}\right) \quad \ldots (285)$$

in which the tangential, vertical, and horizontal forces at a point xy may be designated as px, ps and pc, p, being the weight for a unit of length, and $S = \sqrt{x^2 - c^2}$. For the point of suspension this gives:

$$K = p(h + c),\ \sqrt{} = p\sqrt{h^2 + 2hc},$$
$$H = pc \quad \ldots\ldots\ldots\ldots (286)$$

in which the parameter c is yet undetermined. In order to determine the latter let the equation of the curve be developed into the following series:

$$x = \frac{c}{2}\left(1 + \frac{y}{c} + \frac{y^2}{1.2\,c^2} + \frac{y^3}{1.2.3.c^3} \cdots + 1 - \frac{y}{c} + \frac{y^2}{1.2c^2} - \frac{y^3}{1.2.3.c^3} + \cdots\right)$$

Since the curve is always flat in rope transmission, the quotient $\frac{y}{c}$ is a proper fraction, and both series are converging. Stopping at the third member as giving sufficient accuracy we have:

$$x = \tfrac{1}{2}\,c\left(\frac{2 + y^2}{c^2}\right) = c + \frac{y^2}{2c} \text{ and}$$

$x^1 = x - c = \frac{y^2}{2c}$, which is the equation of a parabola.

$$h_m = \frac{a}{\sqrt{8}} \quad \ldots\ldots\ldots\ldots \quad (293).$$

Dividing (288) by (293) we have, after some reductions:

$$\frac{h}{h_m} = \frac{S - \sqrt{S^2 - S_m^2}}{S_m} \quad \ldots\ldots \quad (294).$$

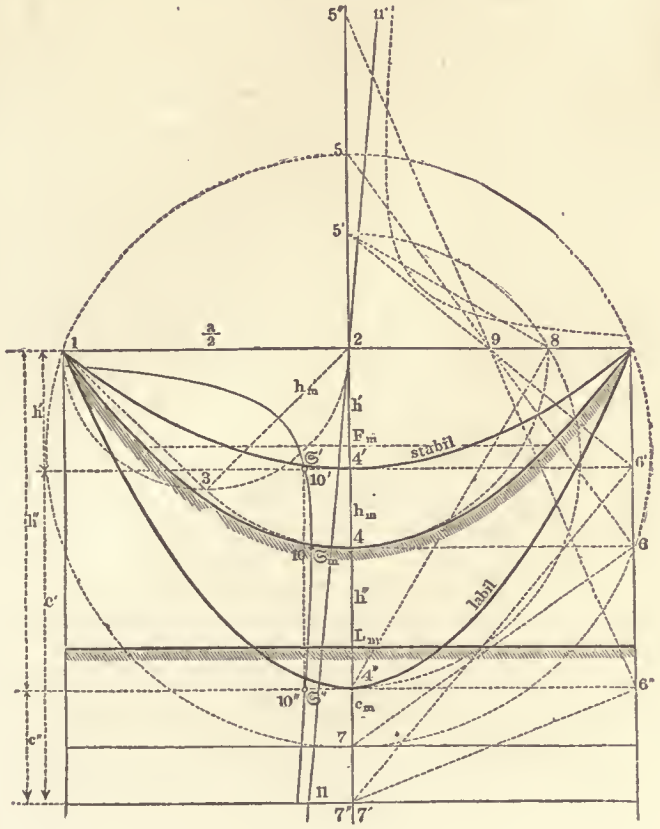


FIG. 884.

From this we obtain the following geometrical construction of Fig. 885. With a diameter $= \frac{1}{2}a$, describe the semi-circle 1 . 2 . 3, and join the point 3 of the quadrant 2 . 3 with 2, or 1; then $3 . 2 = 3 . 1 = \frac{\frac{a}{2}}{\sqrt{2}} = \frac{a}{\sqrt{8}}$ = the mean deflection h_m.

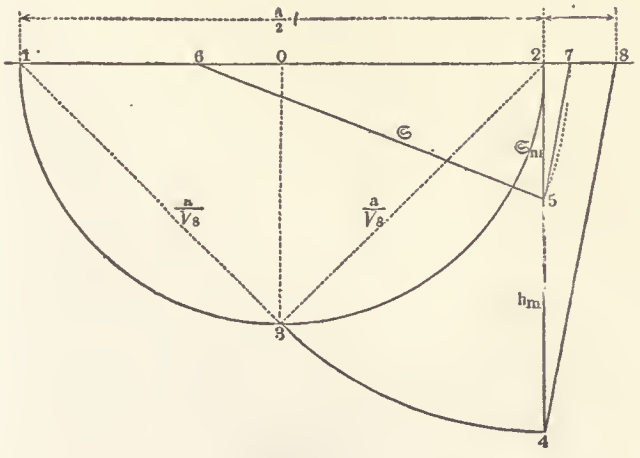


FIG. 885.

Lay off this distance perpendicular to 1 . 2 at 2 . 4, and on any scale (not too small) lay off from 2 to 5, the stress S_m, determined from (292). From 5 lay off, on the same scale, 5 . 6, equal to the given stress S, and from 6 draw the arc 5 . 7. This gives $2 . 7 = 6 . 5 - 6 . 2 = 6 . 5 - \sqrt{(6 . 5)^2 - (5 . 2)^2}$, which is $= S - \sqrt{S^2 - S_m^2}$. If we now draw 4 . 8 parallel to 5 . 7 we have $\frac{2 . 8}{2 . 4} = \frac{2 . 7}{2 . 5} = \frac{h}{h_m}$, and hence $2 . 8 = h$.

The value h_o of the stationary rope is that of a parabola of a length midway between those for h_1 and h_2 and is equal to:

$$h_o = \sqrt{\frac{h_1^2 + h_1^2}{2}} = 0.67h_2 + 0.28h_1 \quad \ldots\ldots \quad (295)$$

It may readily be constructed graphically from the first expression. It is not essential that the driving part of the rope should be the upper portion, as the lower part may drive, as in Fig. 886. The

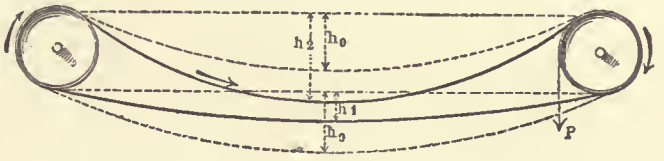


FIG. 886.

ropes will not touch, when stationary, if $h_2 - h_1 < 2R$. Owing to the fluctuations due to the action of wind, or of sudden changes of load, the minimum distance should not be too small, and is best kept greater than 20 to 24 inches.

NOTE.—We have from (287):

$dS = \psi \left[dh + \frac{1}{8}\left(o - \frac{a^2}{h^2} \right) dh \right]$ which gives for the minimum of S:

$o = \psi \left[1 - \left(\frac{1}{8} \frac{a^2}{h^2} \right) \right]$. But $\frac{a^2}{8h}$ is the parameter, or C_m, hence $o = 1 - \frac{C_m}{h_m}$ or $C_m = h_m = \frac{a}{\sqrt{8}}$ and hence from (287):

$$S_m = \psi \left(h_m + \frac{a^2}{8h_m} \right) = \psi \left[\frac{a}{\sqrt{8}} + \frac{a}{\sqrt{8}} \right] + \psi \frac{a}{\sqrt{2}}.$$

In Fig. 884 is showu graphically how for each value of h, the parameter c cau be found, by constructing the proportion $\frac{2h}{\frac{a}{2}} = \frac{\frac{a}{2}}{c}$. In the figure, $2 . 5' = 2 . 4' = h'$; also $4' . 6' = \frac{a}{2}$; and $6' . 7'$ perpendicular to $6' . 5'$ gives at $4' . 7'$ the parameter c'. In like manuer: $2 . 5'' = 2 . 4'' = h''$; also $4'' . 6'' = \frac{a}{2}$, and $6'' . 7''$ perpendicular to $6'' . 5''$ gives the parameter $4'' . 7'' = c''$.

To determiue the vertex $4''$ of the lower parabola we have:

$$h' + \frac{a^2}{8h'} = h'' + \frac{a^2}{8h''} \text{ whence } h' - h'' = \frac{a^2}{8}\left(\frac{1}{h''} - \frac{1}{h'} \right)$$

$$= \frac{a^2}{8} \frac{h' - h''}{h' h''}. \text{ This gives } h' h'' = \frac{a^2}{8} \text{ which as shown above } = h_m^2.$$

If, as before, we make $2 . 8 = h_m$, and $2 . 5' = h'$ and draw through 8 a normal to $8 . 5'$ the uormal will intersect $2 . 4''$ at $4''$ which is the desired apex. The lines 5 . 6, $5' . 6'$, $5'' . 6''$ intersect each other at the middle of the half-chord of the parabola at 9. This may be used in the construction by drawiug from 9, the line $9 . 6$, $9 . 6'$, $9 . 6''$, and the corresponding normals give the parameter points 7, $7'$, $7''$. The directrix of the parabola lies at a point distant $\frac{1}{2}\ c$ from the vertex. For the mean parabola the directrix is L_m, midway between 4 and 7, and the focus F_m is at the middle of h_m, and is also the centre of the circle $5 . 6 . 7 . 1$.

In the figure is also showu another curve which indicates the values of S. The proportional value of h from formula (287) taken from the line $2 . 11$, shows that h is in inverse proportion to the hyperbolic line $10' . 10 . 10''$. The ordinates of the hyperbola, taken from the axis of abscissas $2 . 7$ gives the values of S for the corresponding values of h. The ordinates $4' . 10'$ and $4'' . 10''$ give the equal stresses S' and S'', and $4 . 10$ the minimum stress S_m. The dotted hyperbola on the upper right, gives the corresponding thrusts in a parabolic arch, and the curve in an arch corresponding to the cateuary is the liue of thrust. In this also we find the mean height the most economical, the lower ones being stable, and the higher in an unstable equilibrium, dependent upon the thickness of arch ring and distribution of load for their stability.

§ 293.

TIGHTENED DRIVING ROPES.

The deflection of transmission cables often becomes inconveniently great. In many cases, however, it is possible to reduce its amount by increasing the tension to a greater extent than is necessary to prevent slippage. This requires the cable to be made correspondingly stronger in order to resist the increased tension. The modification in the preceding discussion of forces and dimensions is here given, the various terms being given the subscript s to distinguish them, (T_s, t_s, S_s, δ_s, instead of T, t, S, δ). The tension T, as shown in § 290, should not be less than $2P$, and if this is increased by a given factor m, we have $t_s = T_s - P$, and also:

$$\left.\begin{aligned} T_s &= m\,T = 2m\,P, \\ t_s &= (2m - 1)\,P, \\ \frac{t_s}{T_s} &= \frac{2m-1}{2m} \end{aligned}\right\} \quad \ldots\ldots\ldots (296).$$

In order that the stress S_1 in the driving part shall not be changed we have for the stress in the driving part, instead of $\frac{S_1}{2}$, the following:

$$S_{2s} = S_1 \frac{2m-1}{2m} \quad \ldots\ldots\ldots\ldots (297).$$

The diameter δ of the wire, if calculated from (280) is modified to

$$\delta_s = \delta\sqrt{m} \quad \ldots\ldots\ldots\ldots (298).$$

or if taken from (281) or (282), we take

$$S_s = \delta\sqrt[3]{m} \quad \ldots\ldots\ldots\ldots (299).$$

from which the following table has been calculated. Tightened cables are frequently applicable where moderate powers are to be transmitted.

TABLE FOR TIGHTENED CABLES.

$m = \frac{T_s}{T}$	$\frac{T_s}{P}$	$\frac{t_s}{t} = \frac{t_s}{P} = \frac{S_{2s}}{S_2}$	$\frac{S_{2s}}{S_1} = \frac{t_s}{T_s}$	$\frac{\delta_s}{\delta} = \sqrt{m}$	$\frac{\delta_s}{\delta} = \sqrt[3]{m}$
1.6	3.2	2.2	0.69	1.26	1.17
1.8	3.6	2.6	0.72	1.34	1.22
2.0	4.0	3.0	0.75	1.41	1.26
2.2	4.4	3.4	0.77	1.48	1.30
2.4	4.8	3.8	0.79	1.55	1.34
2.6	5.2	4.2	0.81	1.61	1.38
2.8	5.6	4.6	0.82	1.67	1.41
3.0	6.0	5.0	0.83	1.73	1.44
3.2	6.4	5.4	0.84	1.79	1.47
3.4	6.8	5.8	0.85	1.84	1.50
3.6	7.2	6.2	0.86	1.90	1.53
3.8	7.6	6.6	0.87	1.95	1.56
4.0	8.0	7.0	0.88	2.00	1.59
4.2	8.4	7.4	0.88	2.05	1.61
4.4	8.8	7.8	0.89	2.10	1.64
4.6	9.2	8.2	0.89	2.14	1.66
4.8	9.6	8.6	0.90	2.19	1.69
5.0	10.0	9.0	0.90	2.24	1.71

Example.—Given, $N = 5.5$, $n = 100$, $a = 590.4$ ft. $= 7086$ in. It is required to cover this distance with a single stretch of cable. If we take $S_1 = 14{,}220$ lbs., and $s = 11{,}376$ lbs., we have $\frac{s}{S_1} \times \frac{H}{n} = \frac{11{,}376}{14{,}220} \times \frac{5.5}{100} = 0.044$. If $i = 36$ we have from (282) $\delta = 0.251 \sqrt[3]{\frac{1}{36}} \sqrt[3]{0.044} = 0.024$ inches. We then get from (290): $h_1 = 0.0408 \frac{(7086)^2}{14{,}220} = 144'' = 12$ feet, $h_2 = 24$ feet, and $h_2 - h_1 =$ 12 feet. According to (279) $R = \frac{14{,}200{,}000 \times 0.026}{11{,}376} = 32.45$, say $32\frac{1}{2}$ inches, which gives $h_2 - h_1 > 2R$ and the driving part of the cable must be above. The above result shows that the centre of the pulleys must be more than $R + h_2$ or $24^{ft} + 2^{ft}\,8\frac{1}{2}''$ above the ground in order to clear. To reduce this height we must tighten the cable. Suppose we made the diameter of the wires $= 0.04''$ instead of $0.024''$. This gives $\frac{\delta_s}{\delta} = 1.67$, and from the table, columns 4 and 6, line 11, $S_{2s} = 0.89$, $S = 12{,}650$, and hence we have $h_{2s} = 0.0408 \frac{(7086)^2}{12{,}650} = 162''$, and $h_{2s} - h_1 = 162 - 144 = 18''$.

We also have $R = \frac{14{,}200{,}000}{11{,}376} \times 0.04 = 50''$. These values give $h_{2s} - h_1 < 2R$ and we may therefore place the driving side below without danger of interference. The greatest deflection occurs when the cable is at rest, and from (295) we have $h_{os} = 149$ inches, and the total height for the pulley centres is $149 + 50 = 199''$ or $16^{ft}\,7''$. This example is shown in Fig. 887, in which the dimensions, however, are in the metric system.

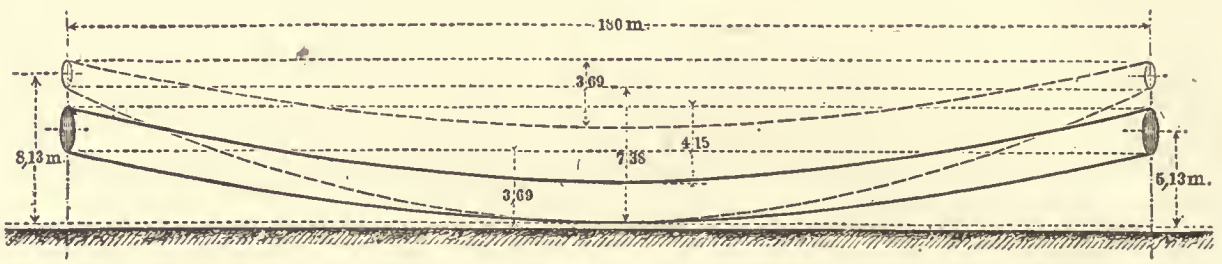


FIG. 887.

§ 294.

SHORT SPAN CABLE TRANSMISSIONS.

When the distance between pulleys is short the deflection must not be too small if good results are to be expected. To this end a small value should be taken for S_1, and hence the deflection is first to be chosen and the corresponding value determined from (287) which is readily done. For moderate powers wire rope transmission may be used in this way for short spans very successfully.

Example.—Let $N = 5$ horse power, to be transmitted over a span of 65.6 ft., or 787.4 inches; the number of revolutions to be 150, and the deflection 40 inches. We have from (287) $S_1 = 0.3266 \left(40 + \frac{(787.4)^2}{40 \times 8}\right) = 645$ lbs. Taking iron wire, and making $S + s = 25{,}600$ lbs. We have $s = 25{,}600 - 645 = 24{,}955$. If we make the number of wires $i = 36$ we have from (282)

$$\delta = 0.251 \sqrt[3]{\frac{1}{36}} \cdot \sqrt[3]{\frac{24{,}955}{645} \cdot \frac{5}{150}} = 0.083''.$$

We then have from (279)

$$R = \frac{14{,}2000{,}000 \times 0.0f3}{25{,}600} = 45.9 \text{ say } 46'',$$

$$\text{and } v = \frac{150 \times 2\pi \times 46}{12} = 3600 \text{ feet},$$

all of which values are quite practicable.

§ 295.

TRANSMISSION WITH INCLINED CABLE.

A transmission at which the pulleys are placed at different heights is called an inclined transmission, and the curve in such a case is unsymmetrical. For a given distance a, between the verticals through the ends of the curve, and for a difference in height H, we have for the deflections $h' = x_1$ and $h'' = x_2$, Fig. 888, and for the ordinates y_1 and y_2 of the two branches of the curve:

$$\left.\begin{aligned} x_1 = h' &= \frac{a^2}{8c} + \frac{c}{2}\frac{H^2}{a^2} - \frac{H}{2} \\ x_2 = h'' &= \frac{a^2}{8c} + \frac{c}{2}\frac{H^2}{a^2} + \frac{H}{2} \end{aligned}\right\} \quad \ldots\ldots (300)$$

and

$$\left.\begin{aligned} y_1 &= \frac{a}{2} - c\frac{H}{a} \\ y_2 &= \frac{a}{2} + c\frac{H}{a} \end{aligned}\right\} \quad \ldots\ldots (301)$$

in which the parameter c is yet unknown.*

* Deduced as follows: $y^2 = 2cx_1$, $y_2^2 = 2cx_2$, $y_1 + y_2 = a$. $x_2 - x_1 = H$, whence: $y_2^2 - y_1^2 = 2c(x_2 - x_1) = 2cH$, *i. e.* $(y_2 + y_1)(y_2 - y_1) = 2cH$ and hence $y_2 - y_1 = 2c\frac{H}{a}$, etc.

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For the parameter c, we have from (286) $K' = p(h+c)$ or $Sq = \psi q(h+c)$ and if we consider the lower pulley as bearing

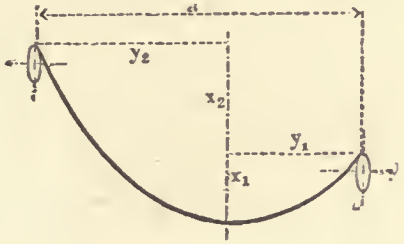


FIG. 888.

the lighter load we have: $S' = \psi(c + x_1)$ whence $c = \frac{S'}{\psi} - x_1$.

Substituting the value of x_1, from (300) we obtain after reduction,

$$c = \frac{\frac{S'}{\psi} + \frac{H}{2}}{2 + \frac{H^2}{a^2}} + \sqrt{\left(\frac{\frac{S'}{\psi} + \frac{H}{2}}{2 + \frac{H^2}{a^2}}\right)^2 - \frac{a^2}{8\left(1 + \frac{1}{2}\frac{H^2}{a^2}\right)}} \qquad (302)$$

The plus sign before the radical indicates that we have chosen the "stabil" parabola (see Fig. 884), and hence obtain the greater of the two values for the parameter. The parameter thus being determined, we have x_1 and y_1 from (300) and (301). For the upper branch of the curve the stress S'' is to be determined at the upper pulley. We then have $S'' = \psi(c + x_2)$. Subtracting from this $S' = \psi(c + x_1)$ we have

$$S'' = S' + \psi(x_2 - x_1) = S' + \psi H \ldots\ldots (303)$$

and if $\psi = 0.3266$ we get:

$$S'' = S' + 0.3266\, H \ldots\ldots\ldots (304)$$

Example 1.—Let $a = 328$ felt $= 3936''$, $S' = 8500$ lbs. If $H = O$, we have from (302) $c = \frac{\frac{8500}{0.3266} + o}{2} + \sqrt{\left(\frac{\frac{8500}{0.3266}}{2}\right)^2 - \frac{3936^2}{8}} = 25{,}951$ inches.

The deflection is:

$h_2' = \frac{3936^2}{8 \times 12{,}962} + \frac{12{,}962}{2} \times 0.0025 - 98.5 = 67.1''$, and $h_2'' = h_2' + 197 = 264.1$, whence

$$y_1 = 1968 - 0.05 \times 12{,}962 = 1319.9$$

The stress on the rope, instead of being exactly 8500 and 4250 pounds, will be, according to (304):

$8500 + 0.3266 \times 197 = 8564$ lbs., and
$4250 + 0.3266 \times 197 = 4314$ lbs. respectively.

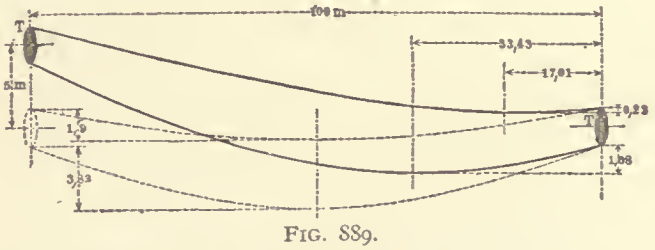


FIG. 889.

The arrangement is shown in Fig. 889. the vertical dimensions being three times the scale of the horizontal, and all dimensions being in metres.

Example 2.—Suppose the distance $a = 3936$ inches, and $S_1 = 8500$, and S_2 4250, as before, but the vertical distance $H = 1968''$, or $\frac{a}{2}$. We then have

(*a*) For the Driving Side:

$$c = \frac{\frac{8500}{0.3266} + 984}{2 + 0.5^2} + \sqrt{\left(\frac{\frac{8500}{0.3266} + 984}{2 + 0.5^2}\right)^2 - \frac{3936^2}{8(1 + 0.125)}} = 23361, \text{ whence}$$

$$h_1' = x_1 = \frac{3936^2}{8 \times 23361} + 0.25 \times \frac{23361}{2} - 984 = 2019 \text{ inches},$$

and $y_1 = 1968 - 23361 \times 0.5 = -9712$ inches,

the minus sign indicating that the apex of the parabola lies without the space between the pulleys.

(*b*) For the Driven Side:

$$c = \frac{\frac{4250}{0.3266} + 984}{2 + 0.5^2} + \sqrt{\left(\frac{\frac{4250}{0.3266} + 984}{2 + 0.5^2}\right)^2 + \frac{3936^2}{8(1 + 0.125)}} = 12271, \text{ whence}$$

$$h_2' = x_2 = \frac{3936^2}{8 \times 12271} + 0.25 \times \frac{12271}{2} - 984 = 708 \text{ inches},$$

and $y_1 = 1968 - 12271 \times 0.5 = 4167$ inches,

and the apex again lies outside.

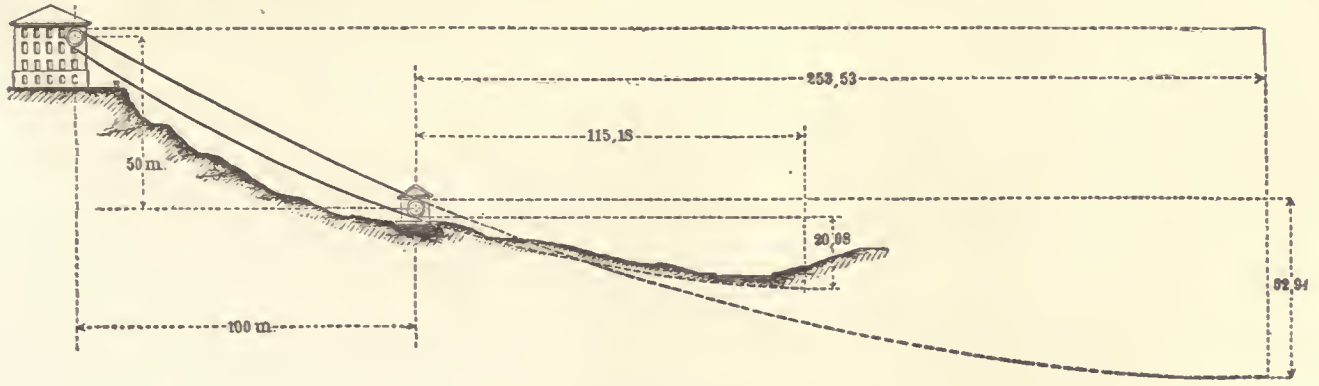


FIG. 890.

This value in (300) gives $x_1 = x_2 = h_1 = \frac{a^2}{8c} = \frac{3936^2}{8 \times 25{,}951} = 74.62''$.

For the slack half of the rope we have $S_2 = 4250$, and

$$c = \frac{\frac{4250}{0.3266}}{2} + \sqrt{\left(\frac{\frac{4250}{0.3266}}{2}\right)^2 - \frac{3936^2}{8}} = 12{,}862 \text{ whence } h_2 = \frac{3936^2}{8 \times 12{,}862} = 150.5''.$$

Suppose now that $H = 0.05\, a = 197''$. We then have:

(*a*) For the Driving Side:

$$c = \frac{\frac{8500}{0.3266} + 98.5}{2 + 0.05} + \sqrt{\left(\frac{\frac{8500}{0.3266} + 98.5}{2 + 0.05^2}\right)^2 - \frac{3936^2}{8(1 + 0.00125)}} = 26{,}018',$$

which is slightly greater than when $H = O$.

We have also, from (300) $h_1' = \frac{3936^2}{8 \times 26{,}018} + \frac{26{,}018}{2} \times 0.0025 - \frac{197}{2} = 8.45''$, and $h_1'' = h_1' + 197'' = 205.45''$.

The distance y_1 then becomes:

$$y_1 = \frac{3936}{2} - 0.05 \times 26{,}018 = 667.1.$$

(*b*) For the Driven Side:

$$c = \frac{\frac{4250}{0.3266} + 98.5}{2 + 0.05^2} + \sqrt{\left(\frac{\frac{4250}{0.3266} + 98.5}{2 + 0.05^2}\right)^2 - \frac{3936^2}{8(1 + 0.00125)}} = 12{,}962.$$

The general arrangement is shown in Fig. 890 all dimensions being given in the metric system, and the vertical and horizontal scales being the same. The increase in the stresses is more marked than in the previous example, on account of the increase in the value of H. We have $S_1' = 8500 + 0.3266 \times 1968 = 9142$ lbs., and $S_2' = 4250 + 0.3266 \times 1968 = 4892$ lbs.

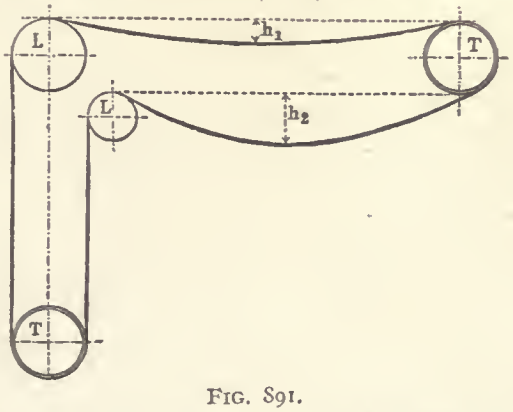


FIG. 891.

The upper limit for this form of rope transmission is that in

which the parts of the rope are vertical, in which case the parameter = ∞. In this arrangement the necessary tension must be obtained by the use of weights, spring, or the like. By using guide pulleys, a combination of horizontal and vertical transmissions may be made, as in Fig. 891, and the tension obtained by the deflection in the horizontal part.

§ 296.

CONSTRUCTION OF THE ROPE CURVE.

We have considered the curve as an ordinary parabola.

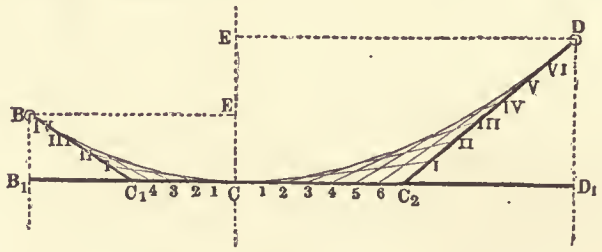


FIG. 892.

When the apex C, Fig. 892, has been determined, bisect the two parts $B_1\ C$ and $D_1\ C$ of the horizontal tangent $B_1\ D_1$, at C_1 and C_2, join $B\ C_1$ and $D\ C_2$, and these two lines will give the direction of tangents to the curve at the points of suspension B and D. Then divide $C\ C_1$ into equal parts C, 1, 2, 3 – – – – and $C_1\ B$ into the same number of equal parts C_1 I, II, III – – – –, and by joining these points we obtain a number of tangents which include the curve. The other portion $C\ C_2\ D$, of the curve is constructed in a similar manner. When the apex of the parabola falls beyond the lower pulley, only one portion of the curve is used.

§ 297.

ARRANGEMENT OF PULLEYS.

When the transmission pulleys are far apart, and not high above ground, supporting pulleys must be used for the rope. In some instances this is only necessary for the driven part of the rope, the driving part being left unsupported, as in Fig. 893.

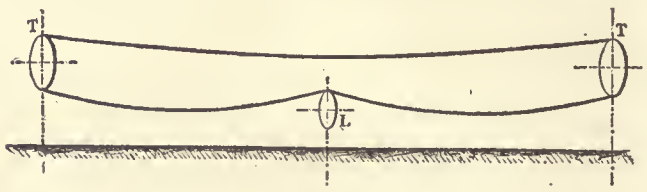


FIG. 893.

Each portion of rope between two pulleys may be called a "stretch" of rope, so that in the above instance we have the driving part in one stretch and the driven part in two stretches. If it is necessary to support both parts it is often practicable to use half as many supporting pulleys for the driving part of the rope as for the driven part as in Fig. 894.

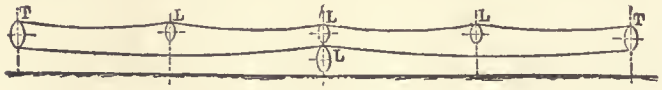


FIG. 894.

These pulleys are called guide pulleys to distinguish them from the main transmitting pulleys and their supporting structures are called supporting stations.

Another arrangement has been used by Ziegler, as shown in Fig. 895.

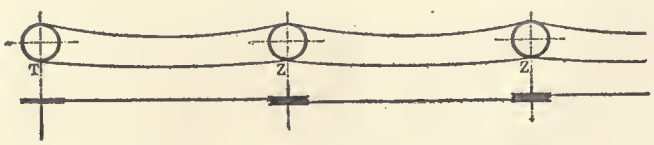


FIG. 895.

This consists of a number of shorter transmissions, using double grooved pulleys, or two single grooved pulleys at each station. In this arrangement it is advisable to make the stretches of equal length so that a single reserve cable will answer to replace any one which may give out.

It is always desirable to run a transmission in a straight line, and especial care must be taken to have the successive pulleys all in the same vertical plane. If it is impracticable to run the entire distance in a straight line it is necessary to introduce angle stations. These may be constructed as in Fig. 896 *a*,

FIG. 896.

using vertical and horizoutal guide pulleys, but this requires six pulleys, three for each part of the rope. A simpler arrangement is shown at Fig. 896 *b*, two pulleys and a pair of bevel gears being used.

In many cases it is desirable to take off a portion of the power at intermediate stations either by shafting or rope transmission, and this may readily be done by a variety of arrangements of gearing and shafting.

It is most important that the pulleys both for supporting and transmission should be amply large in diameter. Many rope transmissions have worn out rapidly, simply because the diameter of the pulleys has been too small. The intermediate pulleys for the driving side ought to be the same size as the main driving pulley in order that the total stress $S + s$ (see § 291) shall not be greater in the former case than in the latter. The supporting pulley for the driven side may be smaller because the stress S_2 is smaller generally, being ½ S_1, or for tightened transmissions (§ 289) being equal to $(2m - 1)\ 2m\ S_1$. The smallest permissible pulleys may be determined from formula (279) and the table of § 291.

Example 1.—In an ordinary wire rope transmission let $S_1 = 8500$, $S_2 = 4250$, and the wire of wrought iron, δ being $= 0.06''$. From the table in § 291 we have for the minimum radius of pulley, $R = 833 \times 0.06 = 50''$ or $8^{ft}\ 4''$ dia. and for the supporting pulleys: $R_2 = 667 \times 0.06 = 40$ or $6^{ft}\ 8''$ dia.

Example 2.—Let $\delta = 0.04$, $S_1 = 5688$, $S_2 = 2844$, and for iron wire we have $R = 28.56$ say $30''$, $R_2 = 25''$.

Example 3.—In a tightened transmission let $m = 3$, and $\delta = 0.06''$, $S_1 = 8500$. $S_2 = S_1 \frac{(2 \times 3) - 1}{2 \times 3} = \frac{5}{6}\ S_1 = 7080$. $R = 50''$ as before, and $R_2 = 769 \times 0.06 = 46''$, a difference which is hardly great enough to be of practical importance.

§ 298.

THE CONSTRUCTION OF ROPE PULLEYS.

The low value of the coefficient of friction of iron on iron makes it impracticable to run the wire cables directly upon the bare metal rim of the pulley, and hence various attempts were early made to fit the groove of the pulley with some soft material. After early experiments with wooden rims fitted with leather, or rubber, it was practically shown that turned iron rims fitted with leather filling placed edgewise in the bottom of the groove gave the best results.*

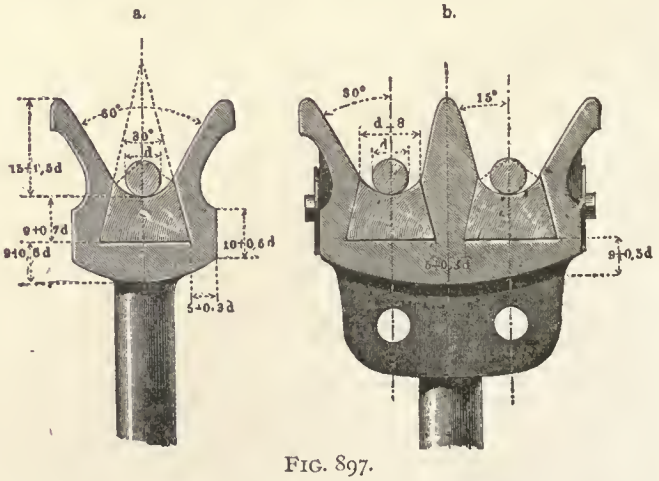


FIG. 897.

In Fig. 897, is shown at *a*, a rim for a single pulley and at *b*, for a double one, both being of cast iron. The proportions are given in terms of the diameter d, of the cable, and in the illustrations the constants in the various proportions are in millimeters. The sides of the grooves are made at an angle of 30° with the plane of the pulley in the case of the single groove

* See D. H. Ziegler, "Erfahrungs resultate über Betrieb und Instandhaltung des Drahtseiltriebs." Winterthur, 1871.

pulley, but this gives an excessively heavy middle rib for the double pulley, and hence the inner angles are made 15° as shown. The smallest diameter of rope for practical use is $d = 0.04''$. The superficial pressure p, may be calculated from (274). If, for example, $i = 36$, we have from (244) $\frac{d}{\delta} = 8$, and if $\frac{R}{\delta} = 1000$ and $S = 8500$, we have:

$$p = 2S\frac{8\delta}{1000d} = 136 \text{ lbs. per sq. in.}$$

a pressure readily borne by the leather filling.

The bottom grooves are made with a dovetail bevel in order to keep the filling from being thrown out by centrifugal force. The filling of leather may be made of pieces of old belting placed on edge and forced by driving into the dovetail groove; if new leather is used it should be softened by soaking in train oil. Rope sheaves for hoisting machinery, which are only used for guiding and supporting the rope, were formerly used without any filling, the rope resting on the bare metal. It is becoming more and more the practice to use a filling in the bottom of the grooves of such pulleys, vulcanized rubber giving good results.

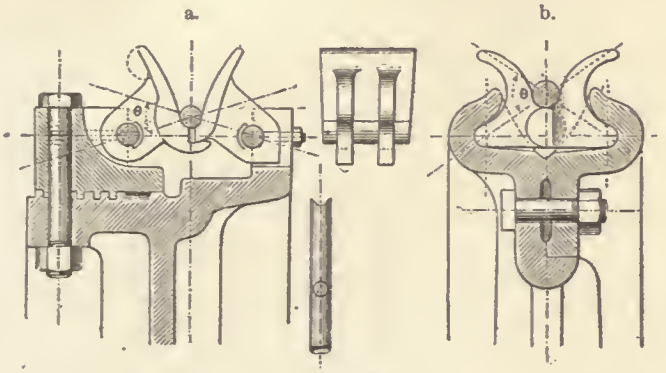


FIG. 898.

The construction of the rim of Fowler's "Clamp Pulley," referred to in Fig. 794 *c*, is shown in Fig. 898 *a*, the clamps being pivoted to blocks by means of bolts with anchor-shaped heads. The pressure upon the rope is the same as in the case of a wedge groove of equal angle, and the pulley as made by Fowler, has one clamp ring mounted upon a screw thread cut upon the pulley, thus enabling adjustment to be made for wear upon the clamps and for the reduction in the diameter of the rope. Fig. 898 *b* shows an American form of clamp pulley, somewhat simpler in construction than Fowler's. The clamps are pivoted on half-journals (see § 95) and the angle is not so small as in the preceding form.

The arms of rope pulleys are usually made of cast iron as well as the rim, although the intermediate supporting pulleys are sometimes made with wrought iron arms, as in Fig. 901. Large pulleys, when of cast iron, are usually made in halves, for convenience of transportation.

The number of arms A, may be obtained from:

$$A = 4 + \frac{1}{40}\frac{R}{d} \quad \ldots\ldots\ldots\ldots (305)$$

Cast iron arms may be either oval or cruciform in cross section, and the width of arm, h, in the plane of the pulley, if prolonged to the centre is:

$$h = 4d + \tfrac{1}{4}\frac{R}{A} \quad \ldots\ldots\ldots\ldots (306)$$

For arms of cruciform section, the thickness of the arms e may be made $\frac{1}{3}\,h$, and the rib thickness $e' = \frac{2}{3}\,e$. Arms of oval section may be made of the same proportions as for belt pulleys, the thickness being made one-half h at all points and the width at the rim being $\frac{2}{3}\,h$.

Arms of cruciform section are usually made straight as at *a*, Fig. 899, but arms of oval section are frequently made curved as at *b*.

To draw the curved arm make the circle $O\,A$ of a radius $= \frac{1}{2}\,R$ and divide it into spaces for the desired number of arms. Make $A\,E = \frac{2}{3}\,A\,B$, and draw $O\,C$ normal to $A\,O$ and C will be the centre for half the arm, and the other centre will be at D, the radius $D\,E$ being equal to $C\,E$.

When straight arms are used the hub should be divided as in Fig. 899 *a*, in order to avoid injurious stresses from shrinkage in casting. The spaces are afterwards filled in with fitted pieces of iron and a ring shrunk on each side to hold all together. The proportions of hubs are the same as in § 283.

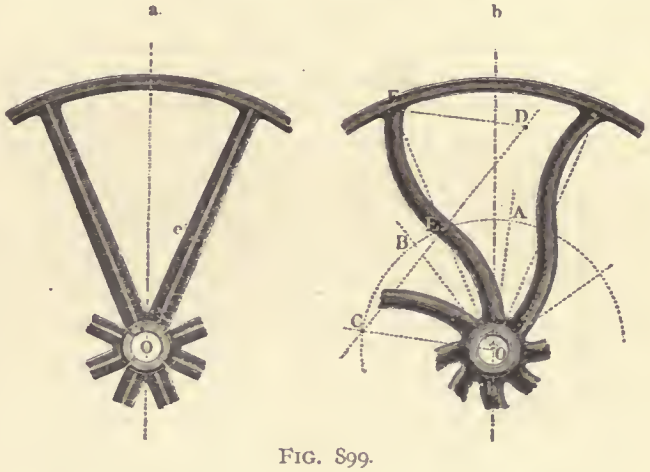


FIG. 899.

The distance between journals for the intermediate and supporting pulleys varies from $\frac{1}{5}\,R$ to $\frac{1}{2}\,R$. The load upon the bearings consists of the sum of the weight of the pulley and the vertical component of the various forces upon the rope, and this can best be determined graphically as shown in Fig. 900.

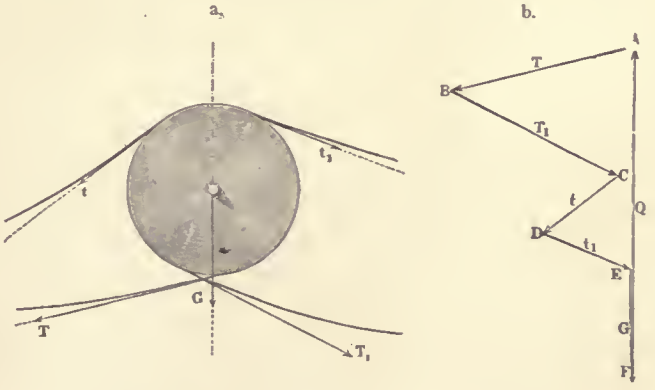


FIG. 900.

The weight G, of the pulley is so dependent upon slight variations in the thickness and section of rim and arms that a general formula of practical value cannot be given. The following examples from practice are given:

Example 1.—In an executed transmission by Rieter & Co., at Oberursel, near Frankfurt a. M., the pulleys are made with twelve straight arms of oval section and are 12 ft. 3.6 in. diameter. The main driving pulleys at the end of the transmission, with single groove, each weigh 2525 lbs. and the intermediate supporting pulleys, with double groove, each weigh 2780 lbs. The rope is made of 36 wires, each being 0.07" diameter.

Example 2.—The Berlin-Anhalt Machine Works Company makes a line of rope pulleys with wrought iron arms as in Fig. 901, the weights being as follows:

$R =$	20"	24"	28"	32"	36"	40"	50"	60"	70"
$G =$	176	214	248–308	281–343	316–387	316–506	528–570	748	968

In these instances the weight upon the bearings is not great. The journals for these pulleys should be made long, in order to reduce the superficial pressure, and swivel bearings with cast iron boxes (§ 116) can be used, which with self-oiling devices will give good service. In many cases the journals are made of hardened steel in order to combine the greatest security with the minimum size.

Example 3.—The intermediate pulleys in Example 1, give a total pressure, according to Fig. 900 *b*, upon the bearings, of 3036 pounds, or 1518 pounds on each journal. If we make $l = 1.5\,d$ according to the table in § 91 we get for d only 1¼ in. In the actual case, however, the journals are 3¼ in. diameter, giving a greatly reduced superficial pressure and thus insuring the most complete lubrication. In this case we have the actual length $l = 4.7$, whence $p = \frac{1518}{3.75 \times 4.7} = 86$ lbs. per sq. in. If, in order to use formula (89) we take $\frac{l}{d} = H$ and make $S = 8500$ as before we have:

$$d = \sqrt{\frac{16 \times 4}{8500\,\pi}}\sqrt{1518} = 1.9'' \text{ say } 2''$$

and $= 4d = 8''$. This gives:

$$p = \frac{1518}{2 \times 8} = 95 \text{ lbs. per sq. in.}$$

which is such a low value that even half boxes, similar to those in Figs. 324-325 could be used. By using hard steel bearings even this small frictional resistance could be reduced to ¼ the amount due to the above dimensions.

The pulleys for rope transmission should be most carefully balanced, as any vibration causes serious oscillation of the rope

§ 299.

CONSTRUCTION OF THE PULLEY STATIONS.

The extraordinarily high specific capacity of wire rope transmission has, as already said, caused it to be used especially for

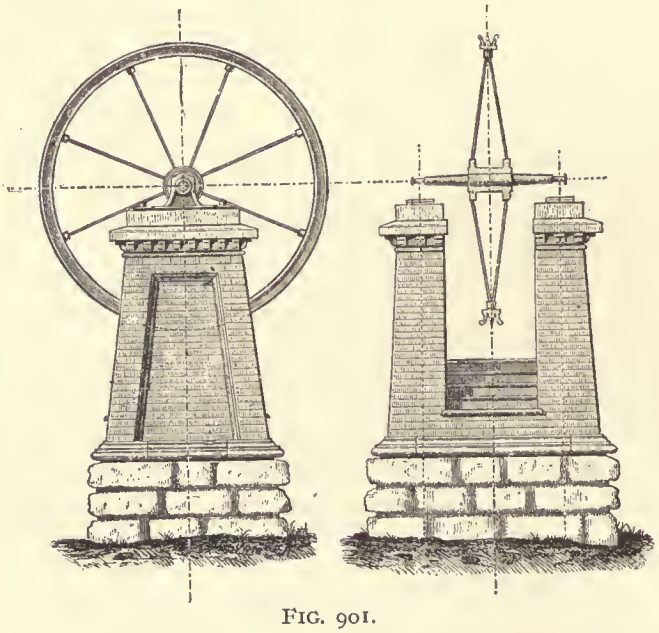


FIG. 901.

the long-distance transmission of power. It has been found particularly adapted for the transmission of the power of natural falls of water to places where it can be utilized and has thus

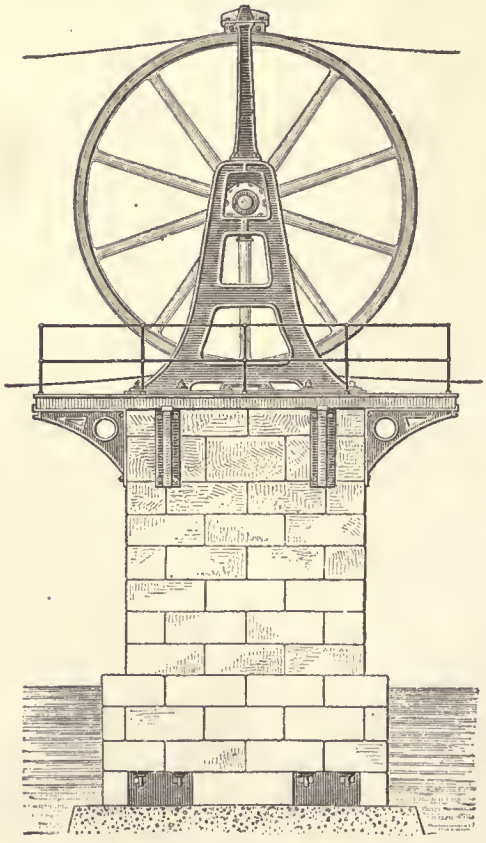


FIG. 902.

materially advanced the use of natural sources of power. In such transmissions one of the most important and difficult portions of the work consists in the construction of the stations upon which the pulleys are carried. * The following are examples of well designed and constructed stations.

Fig. 901 shows a design for an intermediate station of masonry. The foundation is of rough stone-work and the superstructure of brick-work.

Stations similar to this are used in the transmission at Oberursel, referred to in the preceding section, and erected in 1858. This installation is used to transmit 104 horse power over a dis-

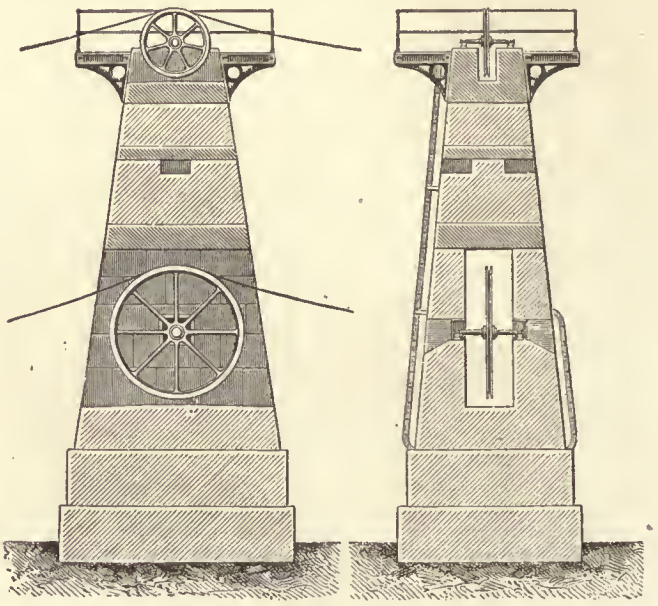
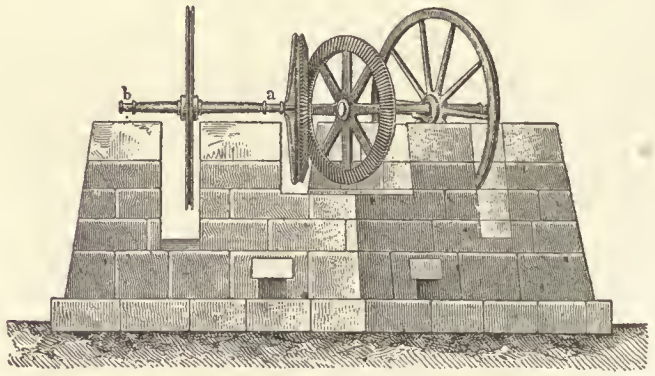


FIG. 903.

tance of 3168 feet (966 meters) divided into eight stretches, giving two terminal and seven intermediate stations. Each stretch $= \frac{3168}{8} = 396$ feet long; $K = 74$ inches, $n = 114.5$, $v =$



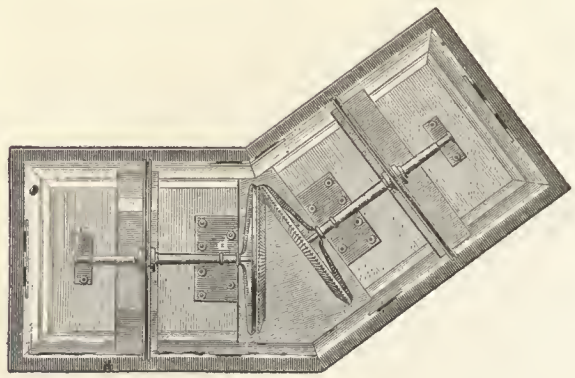


FIG. 904.

4400 ft, $\delta = 0.07''$, $i = 36$. The differeuce in level between the two terminal stations iu this case is 145 feet.

The transmission of the water power from Schaffhausen, cou-

* These have been fully discussed in a work by D. H. Ziegler treating of the installations made by Joh. Jak. Rieter, Winterthur, 1876, and printed privately.

structed by J. J. Rieter & Co. and in operation since 1866, is used to transmit a total of 760 horse power developed by the Falls of the Rhine. Of this 200 horse power is transmitted direct to the left bank by means of shafting; 560 horse power is carried across the Rhine in one stretch, the distance *a* being 385 feet, using two similar ropes carrying 530 horse power. ($n = 180$, $R = 88\frac{1}{2}''$ $v = 4636$ ft.) and a third single rope carrying 30 horse power ($n = 180$, $R = 35.4''$). Of this power there is about 480 horse power transmitted over three principal stretches of 378, 332, and 455 feet. The number of wires in the heavier cables is $i = 80$, the thickness of wire $\delta = 0.074''$, the rope being made in 8 strands of 10 wires each. One of the intermediate pulley stations is shown in Fig. 902, and this is an excellent example of good style in construction. In this case there are two pulleys, side by side. There is a guard shown over the pulleys, to prevent possible jumping of the cables out of the grooves in the pulleys, but this has been omitted in later instances as unnecessary.

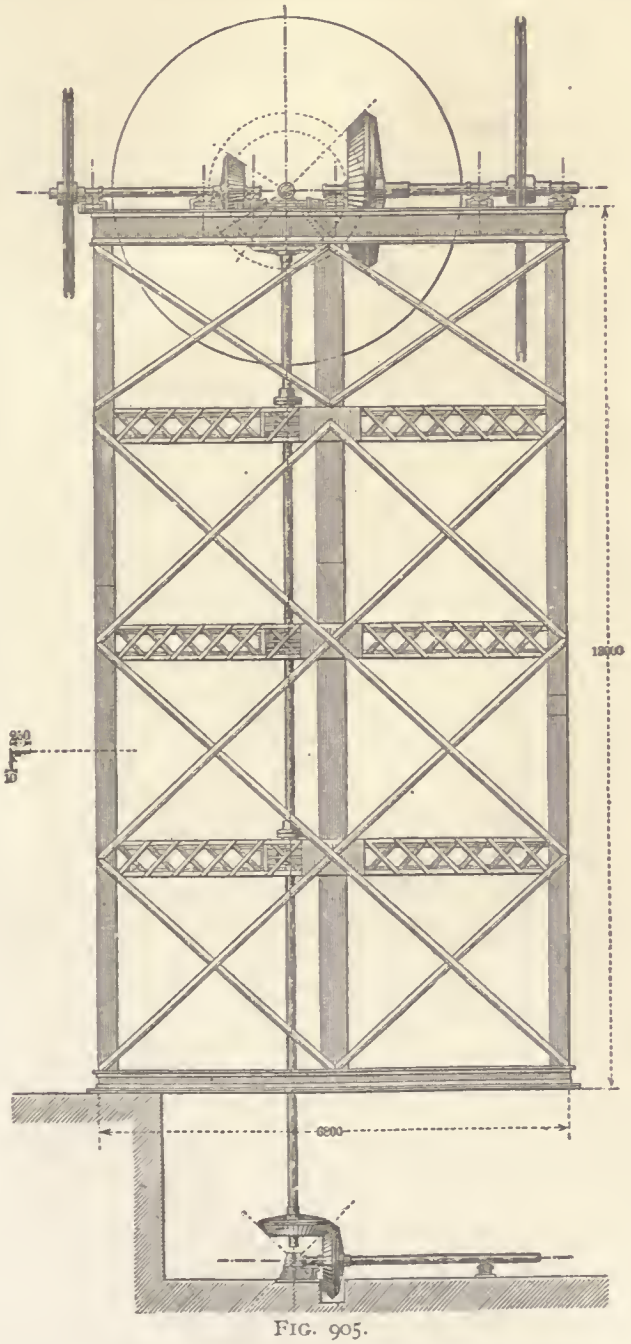


FIG. 905.

Messrs. Rieter & Co. have also installed a system of turbines and rope transmission at Freiburg, for the *Societè gènèrale suisse des eaux forèts*, of which 300 horse power is in a long-distance transmission. The power is carried in five stretches of 502 feet each, to a saw mill, the difference in level being 268 feet. One of the stations with two supporting pulleys is shown in Fig. 903, this one being quite high; a similar station, No. II, is placed in a tunnel, through which the rope passes. The number of wires $i = 90$, the diameter of wire $\delta = 0.072''$, the cable being made in 10 strands of 9 wires each, $R = 88.6''$, $n = 81$, $v = 3743$ ft. From this point the power is divided by an angle station and one part is delivered to the saw mill and balance transmitted to a number of minor establishments.

An angle station is shown in Fig. 904, and this form is also used when a portion of the power is to be taken off.

A fourth large installation of turbines and rope transmission has been executed by the firm of Rieter & Co., for the *Compagnie gènèrale de Bellegarde*, at the latter place, for the utilization of the well-known *Perte du Rhone*. The combined power of the Rhone and the Valserine is exerted upon five turbines of 630 horse power each, giving a total of 3150 horse power which is transmitted by cable to the Plateau of Bellegarde.*

At Zurich, the city has utilized the power of the Limmat by means of turbines and rope transmission built by the firm of Escher, Wyss & Co. In this case the stations, which for various reasons are quite high, are made of wrought iron, as shown in Fig. 905. The entire installation develops 1150 horse power, of which 750 horse power is used for the city water works.

At St. Petershurg a rope transmission in ten stretches is used to drive the Imperial Powder Works, the power being delivered into the buildings by shafting from each of the ten stations.

A modification of Herland's device for putting on belts, has been made by Ziegler for the purpose of putting the wire cables upon the pulleys. As shown in Fig. 906, it consists of a curved piece of angle iron, clamped temporarily to the arm of the pulley in such a manner as to lead the rope into the groove of the pulley. The short radius to which the rope is thus once bent does not appear to have an injurious effect.

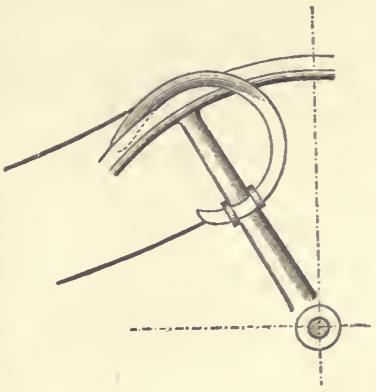


FIG. 906.

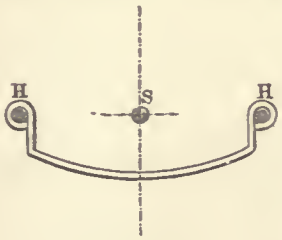


FIG. 907.

When a transmission rope is carried over a public or private road a guard should be used as a protection in case of breakage of the rope. A simple form used by Rieter & Co. is shown in Fig. 907, and consists of a sheet iron trough about 18 inches deep and ten feet wide, carried by two stationary suspension cables is *H H*.

§ 300.

EFFICIENCY OF ROPE TRANSMISSION.

The injurious resistances in wire rope transmission are mainly those due to journal friction and stiffness of the rope; the slip and the atmospheric resistance of the pulley arms being insignificant.†

a) Journal Friction.—We have from formula (100), $F = \frac{4}{\pi} f Q$,

in which Q is the load upon the journal. For a circumferential speed c, at the journal, we have a resistance in foot pounds:

$$F_c = \frac{4}{\pi} f Q \frac{\pi n d}{12}$$

or:

$$F_c = \frac{f n d Q}{3} \quad \ldots\ldots\ldots\ldots (307)$$

Example 1.—In the case of the transmission at Oberursel a number of experimental determinations were made. For a pair of journals $Q = 2948$ lbs., $d = 3.75''$ and $n = 114.6$. For a coefficient of friction $f = 0.09$ (experimentally determined) we have:

$$F_c = \frac{0.09 \times 114.6 \times 3.75 \times 2948}{3} = 37{,}658 \text{ foot lbs.}$$

or $\frac{37658}{33000} = 1.14$ horse power.

This gives for 8 stations a total loss of $8 \times 1.14 = 9.12$ horse power. The maximum power transmitted is 104 H. P. and the minimum 40.3 H. P., so that this gives a loss of about 9 per cent. of the maximum and 22 per cent. of the minimum. This shows the objection to the use of too large journals.

* See *Engineering*, Vol. 37, 1874.
† See Leloutre.

b) *Stiffness of Rope.*—Using Weisbach's formula (253) given in § 268:

$$S = 1.078 + 0.093\,\frac{Q}{R}$$

we have, calling T' the tension on the rope:

$$S_v = 0.093\ v\left(11.6 + \frac{T'}{R}\right) \quad \ldots\ldots (308)$$

for the resistance in foot pounds.

Example 2.—In the preceding case, $v = 4400$ ft., $R = 73.8''$, and $T' = \frac{1}{2}(T + t) = 0.5 \times 2028 = 1014$ lbs., whence:

$$S_v = 0.093 \times 4400\left(11.6 + \frac{1014}{73.8}\right) = 10368 \text{ ft. lbs.}$$

This resistance comes twice at each station, and for eight stations we have a total of $2 \times 8 \times 10{,}368 = 163{,}888$ foot lbs., or nearly 5 horse power. Adding to this the journal resistance we have a total of $9.12 + 5 = 14.12$ H. P. The direct measurements of Ziegler gave 13.341 H. P., which is a reasonably close verification of the calculations. The total loss of efficiency is therefore:

$$\frac{14.12}{104} = 13.6 \text{ per cent. of the maximum,}$$

and $\frac{14.12}{40.3} = 35$ per cent. of the minimum,

the lesser of these being a very excellent result.

§ 301.

REULEUX'S SYSTEM OF ROPE TRANSMISSION.

In the preceding sections the utility and importance of wire rope transmission has been shown. The various applications of the methods already discussed exhibit much ingenuity and ability on the part of the designers. At the same time there appears to be a possibility of improvement, especially in the case of the transmission of large powers over long distances involving a number of stretches.

The Ziegler system of intermediate pulleys has given excellent results, but the following points may be enumerated as objections:

a. The great height of the supports usually necessary because of the large size of the pulleys.

b. The large base required for the supports, not only for clearance for the lower part of the rope, but also to resist the tension of the rope.

c. The necessity of making the supports of great strength when gearing is to be carried.

These three points are all well shown in the Zurich station, Fig. 905.

d. The resistance due to stiffness of the rope. This has usually been considered unimportant, until the recent investigations have shown otherwise. (See the preceding section.)

e. The loss of power when the rope becomes slack.

f. The necessity of giving sufficient tension to the rope to insure satisfactory action in warm weather and consequent excessive tension in winter.

g. The unsightly soiling of the exterior of buildings caused by the grease from the cable defacing the wall upon which the receiving pulley is placed.

h. The necessity of making the intermediate pulleys strong enough to carry the heavy stress of the cable, thus increasing the weight and consequently the journal friction.

It therefore appears advisable to devise a system which should permit the supports to be made low and light, to use a light cable under moderate tension, also to reduce the number of splices, and to place the terminal pulleys inside of the building, the pulleys being made as light as practicable.

All these points have been attained to a great extent in the following system.

In the first place, the cable, whenever possible, is made in one endless length from the driving to the driven pulley, thus making the intermediate pulleys merely supports and permitting them to be constructed very light. It is also desirable to arrange the cable so that both parts shall be at the same height from the ground and that this height should be as moderate as possible.

In Fig. 908 is shown the arrangement of the power house, the first driving pulley T_1 being directly upon the motor shaft and lying in a horizontal plane. The driving part of the rope then passes around a stationary pulley L_1 and is carried off in the desired direction. The driven part of the rope passes around a pulley L' mounted on a carriage running on a track parallel to the direction of the line of transmission and by means of weights a pull somewhat greater than $2t$ is brought upon the carriage. This tightener pulley L' is placed so as to bring the driven part of the rope to the same height as the driving part. The whole arrangement may be protected under roof as shown and the rest of the building used for other purposes, but if necessary the track and carriage may extend out of doors.

The intermediate stations may all be supporting stations merely, unless power is to be taken off at an intermediate point. If the transmission is a normal one, not using the method of increased tension (see § 293) the same deflection will be obtained

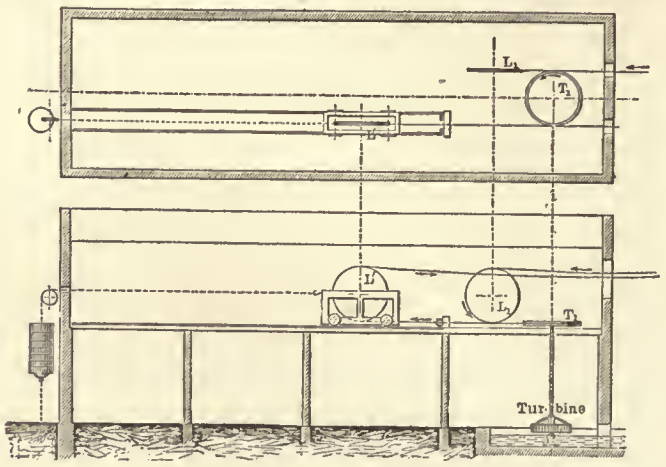


FIG. 908.

in both portions of the rope by making the stretches for the driven part half as long as those of the driving part, so that every other station may be provided with a double-grooved pulley, Fig. 909.

FIG. 909.

If no change in direction is necessary the cable is thus carried to the driven pulley, the two parts being separated by a distance equal to the diameter of the driving pulley T_1, and entering the building where the power is to be received the cable passes over guide pulleys L_6, L_7, and around the driven pulley T_2.

When the load is reduced by throwing off machinery in the manufactory, the tightener carriage is drawn toward the turbine (Fig. 908) by the driving part of the rope, since both parts give a pull of $\frac{1}{2}(T + t)$. A spring buffer is provided to check the motion of the carriage in that direction. A spring dynamometer may be connected with the bearing of the other pulley L_1 and the tension thus measured experimentally. When the transmission is set in motion from a state of rest the tightener pulley L moves slowly back until the tension in the driven part of the rope becomes equal to t. Should the rope have much stretch, the carriage must have sufficient travel provided, and when necessary the rope must be shortened. The stretch of the cable is less in this arrangement than with intermediate driving pulleys, because it is bent less frequently around the pulleys, and the wear of the rope is much reduced for the same reason.

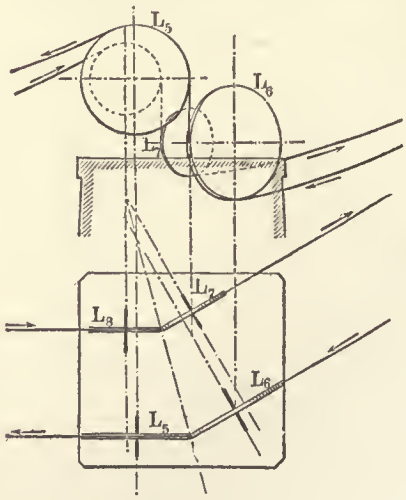


FIG. 910.

If angle stations are needed the arrangement of Fig. 910 is used; this requiring only two pulleys to each part of rope, instead of three, as formerly, and the use of gear wheels is avoided.

If the first driving pulley is in a vertical instead of a horizontal plane, the arrangement shown in Fig. 911 *a* is used, this requiring one more guide pulley than before. In this case the track for the tightener carriage is inclined so that its weight is used to produce the required tension. If it is desired to place the tightener pulley horizontal the arrangement shown in Fig. 911 *b* is used. In the cable of the Brooklyn bridge the tightener carriage is provided with a brake in order to check the suddenness of motion due to variations of load. A friction device similar

to σ Fig. 709 will serve for this purpose if the angle δ is made somewhat greater than is given by formula (233).

If it is desired to place the driven pulley T_2 in the same plane as one of the parts of the main line cable, the other part must

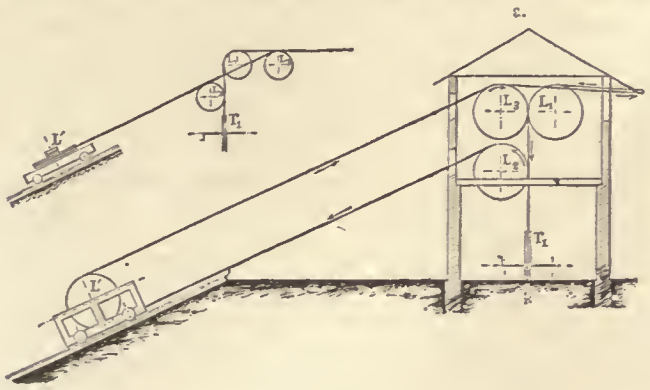


FIG. 911.

be led over another angle pulley. If power is to be taken off at intermediate stations these may be constructed as the angle stations of Fig. 910.

Various other forms of intermediate power stations may be used without involving the use of gearing, as shown in Fig. 912,

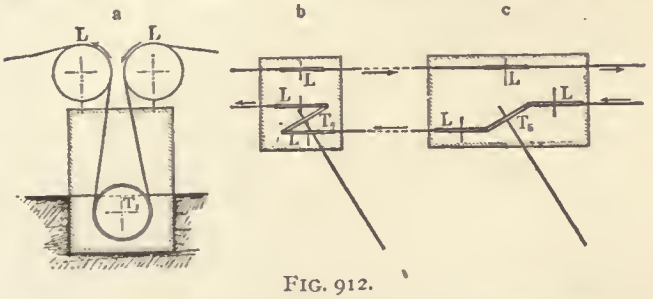
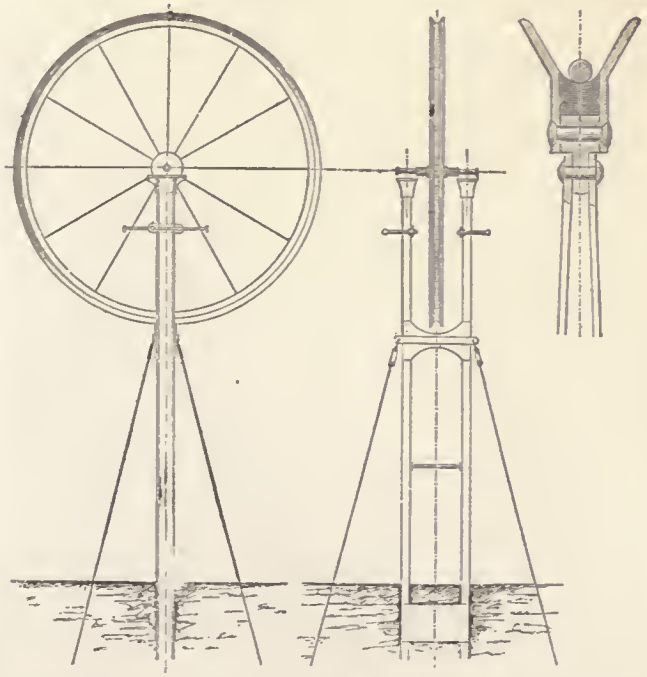


FIG. 912.

in which *a* is for a shaft at right angles to the cable, and *b* and *c* for inclined shafts for either direction of revolution.

The very moderate force which this system brings upon the supporting pulleys permits them to be made very light. This has been difficult of accomplishment with a cast iron rim. A light wheel can be made of wrought iron, using angle iron riveted to a special shaped centre piece, as shown in Fig. 914.

FIG. 913. FIG. 914.



These rims are bent by means of special rolls, and a tongue is formed in the sides of the groove to hold the leather filling in place. The arms are made of light flat iron and the hub of cast iron; the arms either being bolted fast or cast into the hub, the latter being made in halves. Pulleys made in this manner are very light.

The construction of the supports is also peculiar, as shown in Fig. 913. The two posts are made of channel iron secured to a block of stone in the ground by means of lead run in around the holes in the stone. The whole is steadied by guy-rods, and brackets are provided so that the bearings can be reached by a ladder. In many cases these supports of iron are cheaper than those built of stone.

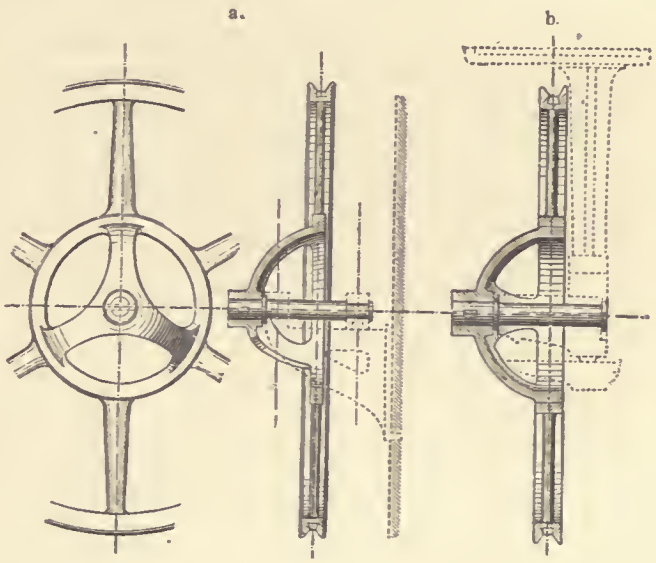


FIG. 915.

For the intermediate driving pulleys of cast iron, the form shown in Fig. 915 is used. The hub is outside of both bearings, but the plane of the pulley is midway between the journals. The connection between the arms and the hub is made by means of a hemispherical shaped device, somewhat resembling the frame of an umbrella, and hence these have been called "umbrella" pulleys. This construction enables the pulley to be firmly secured and readily removed without disturbing either bearing.

In Fig. 915 *b*, a modification of this form of pulley, the umbrella-shaped hub being made separately, and a straight arm pulley fitted upon it. This permits a single pattern to be used for the centres of a number of sizes of pulleys, or wrought iron pulleys may be used on cast iron hubs of this form. Instead of two journals a single longer one may be used, two forms of hangers being shown in dotted lines.

The use of the umbrella pulley enables a very simple form of support to be used, either for single or double stations.

Fig. 916 *a*, is a single station composed of a wooden post upon which a projecting bearing is bolted, and in which the journal of the pulley runs. At *b*, is a double station, the post being made of iron The dotted lines at *D* indicate a small roof to protect the bearings from the weather. A comparison of these forms with the older style, as for example, Fig. 903, will show that merely the use of the coutiuuous rope and the umbrella pulley will effect a great economy in construction. The umbrella pulley is also well adapted to be used for rope sheaves for hoisting machinery and for chain sheaves.*

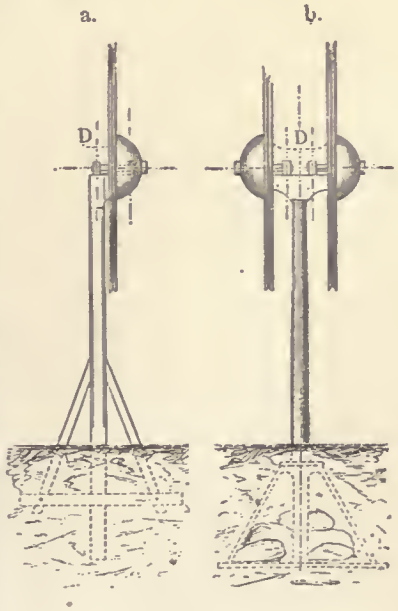


FIG. 916.

*Various applications of the umbrella pulley will be shown hereafter. The principle is also applicable to bell pulleys. At *a*, is a simple counter-

A comparative example with that in § 300 will be a practical illustration.

Example.—The transmission at Oberursel is made in eight equal stretches and seven stations with two pulleys each, one driving pulley and one driven. This gives 16 semi-circular wraps of the rope about the pulleys, causing a loss of 5.13 H. P. from stiffness. By the adoption of the new system there would be three semicircular wraps at the power house (see Fig. 908), one on the driven pulley and two quarter-wraps on the guide pulleys L_6, L_7 (see Fig. 909) There are also 11 short arcs of contact, about $\frac{1}{10}$ of a circle each, on the supporting pulleys, which latter would be very light and on supports constructed as already described. The combined arcs of contact make practically about 5 semi-circular wraps or $\frac{5}{16}$ of the resistance of the old arrangement, that is $\frac{5}{16}$. 5.13 or about 1.6 H. P. This is not too favorable an estimate, as we have not included the effect of the excessive tension which often occurs by the contraction of the cable in cold weather, and which is entirely avoided by the use of the tightener pulley and carriage.

The reduction of journal friction is also important, as the weight of the pulleys and the effect of the rope tension are both much less. The total weight of the pulleys will be only about $\frac{2}{3}$ that of the old system, although more pulleys would be used, and the journal diameter may be reduced to $\frac{1}{3}$ of the previous value. This gives a loss of $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ of the previous value of 9.36 H. P., which is 2.08 H. P. To this we must add a resistance of 0.40 H. P. for the guide pulleys which have been added in the new system, giving a total loss of $N_{08} = 1.60 + 2.08 + 0.40 = 4.08$ H. P. The loss in the first instance is with the new system 4 per cent. and in the second 10 per cent., as against 13 9 and 35.9 per cent. for the old system.

In this example there are no intermediate power stations, the entire amount of power less only the hurtful resistance. In considering the question of the stress in the driving part of the cable it is important to know whether the entire power is to be transmitted to the end of the line or if a portion is to be taken off at intermediate stations. If the initial forces at successive intermediate power stations be indicated by P_1, P_2, P_3, P_4, etc., the successive tensions in the cable will be reduced, and hence the deflection h should be determined for the stretches preceding and following each station, and the tension in the cable will vary according to the power taken off at intermediate points. The sum of all the forces P, will in every case be determined by taking the tension t, in the driven part at the first driven pulley, from the initial tension T, so that we have $T - t = \Sigma P$. From this equation we can deduce important results.

As an illustration we can assume the entire power transmitted to be divided up among a number of intermediate stations, all being operated by one continuous cable, as shown in diagram in Fig. 917.

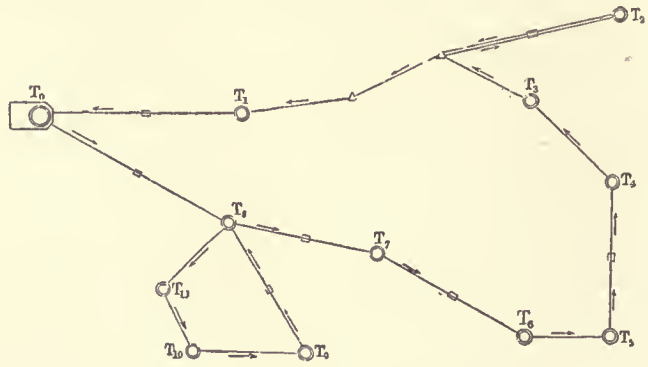


FIG. 917.

In this case the rope passes the entire round of stations T_1, T_2, T_3, T_4 to T_{11}, returning to the main power house. The rope returns to the power house at any angle with a tension t, giving $T = \Sigma P + t$. All stresses are regulated automatically for each stretch of the rope, as the forces vary at each station. If the work at any station is reduced or even becomes zero, the tightener carriage responds and alters the deflection so that $T - t = \Sigma . P$, in which t remains constant. A transmission of this kind, in which the cable makes a complete circuit of a number of stations, may be called a "ring" system. In Fig. 917, the supporting stations are indicated by small rectangles or triangles, according as the line is straight or makes an angle, and the power stations as shown are circles. At T_8 the rope passes off into an auxiliary circuit, which may be called a "ring" transmission of the second order (see § 260). The stations may all be constructed very simply. The supporting stations are made with one pulley when the line is straight, and with two at the angle stations; the power stations can generally be made with only two pulleys, providing the necessary arc of contact a, is obtained, or three pulleys used if necessary, see Fig. 918.

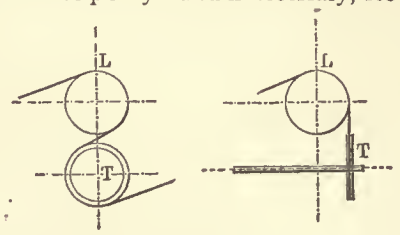


FIG. 918.

In many cases it is desirable to use the system for underground transmission, as in Fig. 919.*

FIG. 919.

In order to determine when an arc of contact α, of the proper magnitude has been obtained, we have, from (239), if P is the greatest force to be transmitted by the pulley with a tension T':

$$P = T' \frac{ef'\alpha - 1}{ef'\alpha} = T' \frac{\rho - 1}{\rho} = \frac{T'}{\tau}.$$

We will call the ratio $\frac{P}{T'}$, which is the reciprocal of the modulus of stress, the modulus of transmission, and let it be represented by θ, whence:

$$\theta = \frac{1}{\tau} = \frac{ef'\alpha - 1}{ef'\alpha} \quad \ldots\ldots\ldots\ldots\ldots (309)$$

Neglecting the influence of centrifugal force, we have, from § 290, for f' the values $f = 0.22$ and 0.25 to consider. Taking these we get the following values for various angles:

MODULUS OF TRANSMISSION θ.

α =	15°	30°	45°	60°	90°	120°	150°	180°	270°	360°	450°	540°
$f = 0.22$	0.06	0.11	0.16	0.21	0.29	0.38	0.44	0.50	0.65	0.75	0.86	0.88
$= 0.25$	0.07	0.12	0.18	0.24	0.33	0.41	0.48	0.54	0.69	0.79	0.81	0.87

These values are shown graphically in the following diagram, Fig. 920:

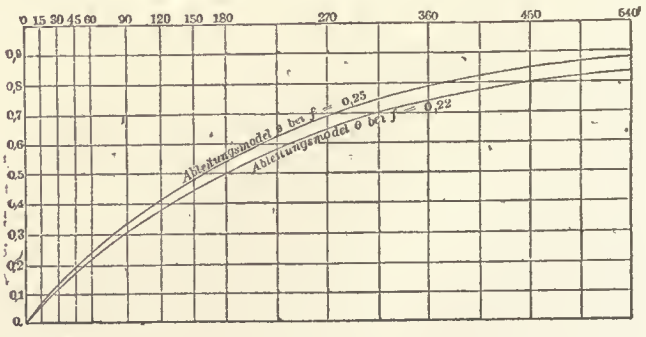
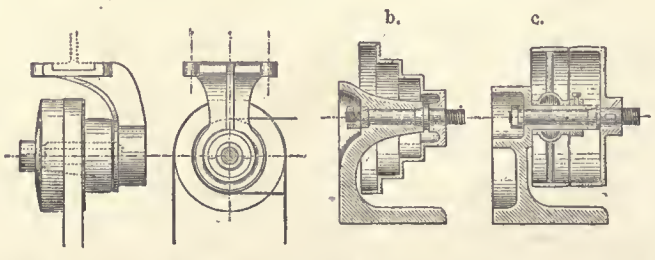


FIG. 920.

From this it will be seen that an arc of contact of 30° will permit the transmission of $\frac{1}{10}$ the power due to the tension T', and an arc of 90° gives about $\frac{1}{3}$.

A convenient application of this principle is found in the arrangement of a "ring" transmission when a large arc of contact is obtained upon the first or main driving pulley by redu-



shaft, at *b*, a simple headstock for a small lathe, and at *c*, is a head for a boring machine, the loose pulley running on a stationary sleeve, as already shown in Fig. 862.

* This has been done in San Francisco by Boone, using a conduit for the rope similar to a cable railway.

plication of the rope over a counter pulley, as in Fig. 795, and also shown in the case of the double-acting belt transmission in Fig. 860. By using a single-grooved counter pulley and double-grooved driver we get $\alpha \geq 360°$, so that θ is at least equal to 0.75. In this way the specific capacity of the rope can be materially increased, practically about 1½ times. If we give $\tau = \frac{1}{\theta}$ the value $\frac{4}{3}$ in the first equation of § 290, we have for the specific capacity of a cable transmission with a counter pulley:

$$N_0 = \frac{1}{33000} \cdot \frac{S_1}{\frac{4}{3}} = \frac{1}{24750} \cdot S_1,$$

$$\text{or say } N_0 = \frac{S_1}{25000} \quad \ldots\ldots\ldots\ldots\ldots\ldots (310)$$

The adaptation of the mechanism to receive the counter pulley is usually not difficult.

The adaptability of the "ring" system of transmission to use in distributing power in manufacturing establishments is apparent, and for this purpose hemp rope is very suitable. This will be shown by the following example:

Example 1.—The transmission shown in Fig. 881, § 286, contained 16 hemp ropes 2 inches in diameter, each having a specific capacity $N_0 = 0.0021$ and $v = 2360$ feet per minute. The cross section of each rope is 3.14 sq. ins. Hence $N = N_0 q v = 0.0021 \times 3.14 \times 2360 = 15.57$ H. P. for each rope, or 249 H. P. for the 16 ropes.

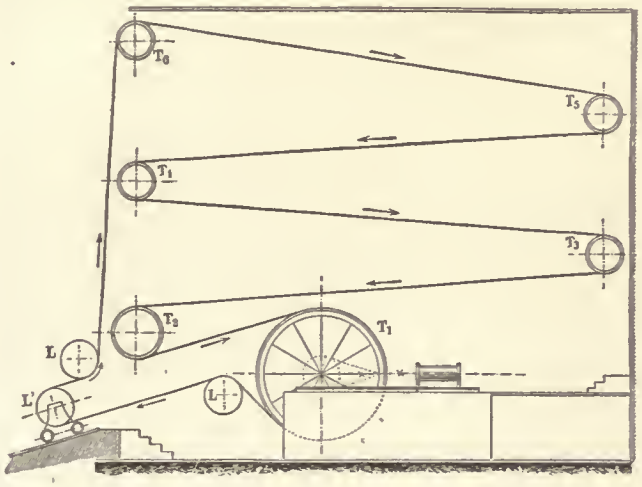


FIG. 921.

Substituting the arrangement shown in Fig. 921, we take a single wire cable composed of 60 steel wires, and use a stress of 17,000 pounds in the driving side of the cable and increase the speed to 3150 feet per minute. We then have from (278):

$$q = 66{,}000 \frac{249}{17{,}000 \times 3150} = 0.307 \text{ sq. in.}$$

and hence the area of each wire is

$$\frac{0.307}{60} = 0.0051 \text{ sq. in., and the diameter } \delta = 0.08''.$$

In the original hemp rope transmission the main driving pulley had a radius of 71 inches, and as we have increased the speed ⅓ times, the driving pulley must be proportionally increased, and hence the radius will be 95". This gives a stress due to bending,

$$s = 14{,}200{,}000 \frac{0.08}{95} = 12{,}000 \text{ lbs. nearly; see formula (279),}$$

this being not too great to give satisfactory results. We have, instead of a wide face pulley made with 16 grooves, a single groove pulley made with leather filling, as in Fig. 879 *a*, of 15 ft. 10" diameter. An important point to be considered is the stress due to the bending of the rope over the pulleys T_2, T_3, etc. These pulleys were 36" radius for the hemp rope, and hence ⅓ . 36 = 48" radius for the wire rope, or 8 feet diameter. We then have from (279)

$$s = 14{,}200{,}000 \frac{0.08}{46} = 23{,}666, \text{ which added to the working stress of}$$

17,000 lbs. gives a total of 40,666 pounds, which is not too high for steel wire, according to § 266. The idler pulley L, is made the same size as the driven pulleys T_3, T_4, etc., and the tightened pulley L' can be made a little larger. The loss of efficiency will be somewhat less than in the case of hemp rope, since for wire rope there is a smaller modulus of stress τ, (*i. e.* 2 instead of 2½, see § 287), and the initial force P, is smaller, because of the increase in velocity and the loss from stiffness will be less. The loss from stoppage and creep should also be considered as not unimportant (see § 287).

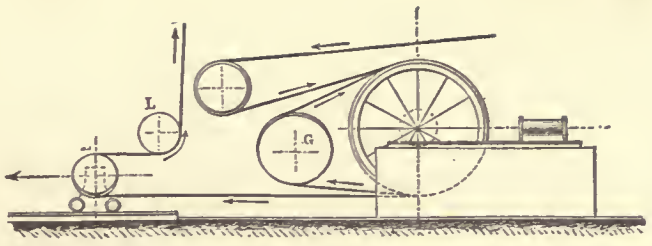


FIG. 922.

If it is desired to use a counter pulley with the above transmission, the arrangement in Fig. 922 may be adopted. In this case the counter pulley G, and tightened pulley L', are both inclined so that the rope shall be properly guided for the double grooves in the main driving pulley. The arc contact α is in this case greater than 360°, and the specific capacity will be 1½ times greater. This will enable the cross section of the rope to be reduced to ⅔ the previous value, or $q = \frac{2}{3} . 307 = 0.204$ sq in. If we use wires instead of 60, we have for the cross section of each wire

$$\frac{0.204}{36} = 0.0056 \text{ sq. in., and } \delta = 0.084 \text{ in.}$$

The diameter of the rope will be from 8 to 9 δ or $\frac{5}{8}''$ to $\frac{3}{4}''$, the latter when the rope is new.

The conditions of this example are hardly such as to demand the introduction of the counter pulley, but when large powers are to be transmitted its use is most advantageous. In some instances the counter pulley may be arranged, as in Fig. 911, so as to sustain a part of the weight of the fly wheel of the engine, and hence materially reduce the journal friction.

In many instances the power in factories may be arranged so as to use the "ring" system of transmission, and dispense with the use of the spur or bevel gearing, and some examples are here given.

In Fig. 923 *a* is shown the usual arrangement of the transmission of power in a weaving establishment.

FIG. 923 *a*.

In this instance the two shafts which extend each way from K, drive the line shafting by seven pairs of bevel gears, while in some factories as many as 12 to 18 pairs are used.

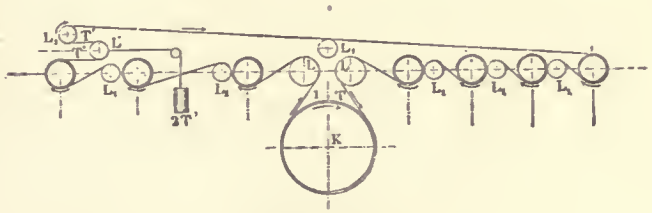


FIG. 923 *b*.

Fig. 923 *b* shows how a ring transmission can be used to drive the same shafting, there being seven guide pulleys and one tightener L', the guide pulleys being of the "umbrella" pattern, as in Fig. 915. The tension weight for the tightener is equal to 2 T'.

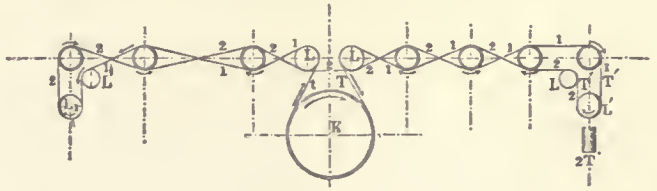


FIG. 923 *c*.

Another arrangement is shown in Fig. 923 *c*, this being used when the alternate shafts are to revolve in opposite directions. This permits the rope to be used double acting, as described in § 277 and shown in Fig. 860. Those portions of the rope marked 1 in Fig. 923 *c*, are in one plane, and those marked 2 in a second plane, giving clearance to the parts of the rope, and the rope is guided from one plane to the other by the guide pulley L_1 and tightener L'. Five of the seven driven pulleys are double acting, and hence are made double grooved.

Shafts which lie at right angles but in parallel planes, one above the other, are also readily driveu by use of a ring trausmission system.

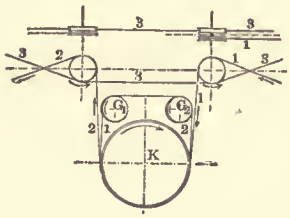


FIG. 924.

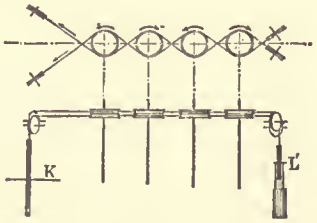


FIG. 925.

In the preceding cases it is desired to obtain a double wrap of the rope about the driving pulley K, the arrangement in Fig. 924 may be adopted. In this case two idler pulleys G_1 and G_2 are used to guide the rope from one plane to the other. The rest of the rope, when either of the planes showu in Fig. 923 *b* or *c* is used, is guided in a third plane by suitable pulleys. In Fig. 925 is shown au arrangement by means of which a series of parallel vertical shafts, revolving alternately in opposite directious, can be driveu from a siugle horizontal shaft K.

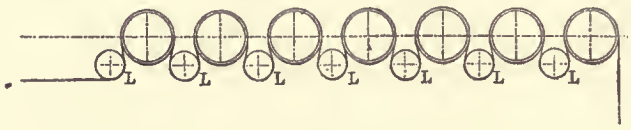


FIG. 926.

The ring system is well adapted for driving a number of mill stones, as arranged in Fig. 926, for example, in which all the mill spindles revolve in the same direction. The direction of the stones may be readily reversed by a corresponding change in the cutting of the furrows, and hence the double-acting arrangement as in Fig. 925 can be used if so desired.

The arrangement of the double-grooved pulley on the spindle in this case is shown in Fig. 927. This is a modified form of the umbrella pulley of Fig. 915, the hub being made in the form of a hollow sleeve carrying a cone or other suitable clutch K, K', by which any pair of stones can be stopped without interfering with the others. An adjustable step for the spindle is provided at H.

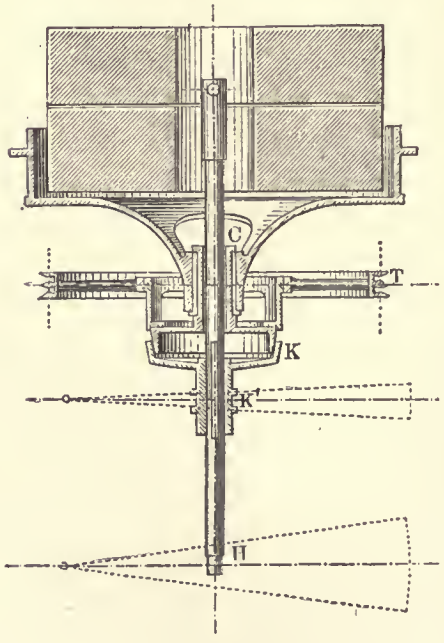


FIG. 927.

In machine shop transmissions it is frequently required to drive a series of parallel counter shafts, which revolve in one direction, sometimes in the opposite, and any of which may need to be stopped. Such an arrangement is showu in Fig. 928. The rope is carried over the various pulleys of one series arouud the tightener pulley L', and back over the other series At K_1 and K_2 are friction clutches which are throwu into engagement on one side or the other, according to the direction of revolution required, or which may be left disengaged. If two adjacent shafts are desired to revolve iu the same direction, an intermediate guide pulley is introduced, as shown at L_1. The subsequent belt transmissiou from these counter shafts can be

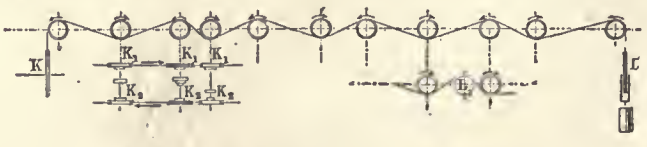


FIG. 928.

greatly simplified by using the above system. In all these arrangements the modulus of transmissiou is determined as already discussed in formula (309) and the proper arc of contact determined. For exauple, if iu an arrangement similar to Fig. 925, each part of the rope is in contact 30° ou the pulley, and the coefficient of friction f, is 0.22, we have from the preceding table for the modulus of transmission $\theta = 0.11$. If the tension on the respective sides of cable be T' and t', acting upon the pulleys, we have for the maximum force trausmitted by the rope $P' = 0.11\ (T' + t')$. In this case we have always $T' + t = T + t$, and hence $T = 2\ \Sigma\ P$, $t = \Sigma\ P$ (see § 264). Heuce we have $P' = 0.11 \times 3\ \Sigma\ P$, or about $\frac{1}{3}\ \Sigma\ P$. If there were but three driven pulleys, each offering the same resistance, the system would operate well, and still better with a greater number of driven pulleys. For mills of 20, 30 or more pairs of stoues, this arrangement is especially applicable, since it furnishes a far simpler transmission system than heretofore. This system, however, should not be carried beyond its proper limits, and for small, light running mills, such as are used for grinding paints, graphite, etc., belts are generally more advantageous, being easier thrown in and out; the rope system beiug better adapted for the transmission of greater powers.

In all the various classes of heavier mills, such as are used for grinding plaster, cement, and the like, also for paper mill machinery, the rope transmission is best adapted, replacing all the heavy shafting, gearing and belting otherwise necessary.

An example will illustrate the method of applying the foregoiug principles.

Example 2.—Let there be two sets of wood pulp mills each requiring 60 H. P. to be driven from a pair of turbines by a "ring transmission" system, the first moving shaft making 125 revolutions per minute. The main shaft is driven by the two turbines by means of spur gearing, and carries the driving pulley of the rope system, makiug also 125 revolutions. We have for the specific capacity of the rope, from (277) $N_0 = \frac{S_1}{66000}$, and if we use steel wire and take $S_1 = 21{,}300$ we get $N_0 = 0.323$. Also we make the velocity v of the rope = 3150 feet per minute, and we get for the cross section of the rope from (278)

$$q = \frac{\mathcal{N}}{v\,N_0} = \frac{120}{3150 \times 0.323} = 0.118 \text{ sq. in.}$$

If we make the cable of 36 wires we have for the cross section of one wire $\frac{0.118''}{36} = 0.0033$ and the diameter $\delta = 0.073''$.

From the number of revolutions and the chosen speed of rope we have for the pulley radius, $R = \frac{3150 \times 12}{2\,\pi \times 125} = 48''$, and using this in (279) we get for the bending stress, $s = 14{,}200{,}000\ \frac{\delta}{R} = 21{,}500$ lbs., which is satisfactory. The entire 120 H. P. is carried by the rope to the first set, where 60 H. P. is used, and the balance is transmitted to the second set, the necessary supporting pulleys being introduced between the two points, and the required tension t being given by the tightener carriage. Following the course of the rope, we have at the driving pulley the tensions T and t, respectively equal to $2\ \Sigma\ P$ and $\Sigma\ P$, whence $\Sigma\ P = \frac{23000 \times 12}{3150} = 1257$ lbs. $= t$ and $T = 1257 \times 2 =$ 2514 lbs. The tension at the first set is reduced by $P' = 628.5$ lbs., whence $T' = 2514 - 628.5 = 1885.5$ lbs. At the second point again is taken off $P' = 628.5$ lbs., and the tension becomes $1885.5 - 628.5 = 1257$ lbs., which is equal to the above value of t, and which is obtained by loading the tighteuer carriage with 2514 lbs. or a little more.

This system requires the use of clutches for starting and stopping the machines, and for this purpose Adyman's coupling, (§ 307), is suitable.

In some instances it is found practicable to drive two pulp mills with one pulley, the pulley being between the machines on an intermediate shaft with a fraction coupliug at each end.

Another case may be given where a number of machines with horizontal shafts each requiring the same amount of power, are arranged in a row and drawn by a ring transmission system, Fig. 929.

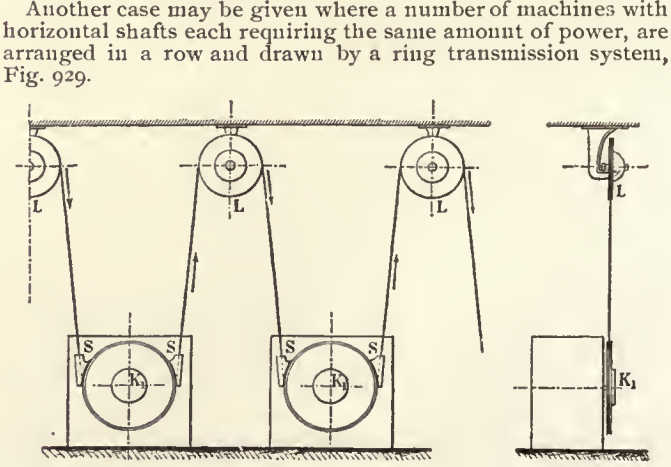


FIG. 929.

In this case frictiou clutches are placed at K_1 K_1 for stopping and starting the machines, while the intermediate pulleys L L, which may be of the umbrella pattern, are carried on haugers from the ceiling. The rope and the driving pulleys are covered by guards S S to protect the workmen. This arrangement is

especially convenient if there is a second series of machines on the floor above, when the pulleys *L L* become the driving pulleys of the upper set, and no guide pulleys are required at all. It is sometimes desirable to make the driving pulleys of umbrella form, supported on independent bearings, so that any machine can be repaired or entirely removed without interfering with the rest of the transmission.

It should not be forgotten that the riug system of rope transmission generally involves an entire rearrangement of the establishment, and that it can rarely be substituted for a shafting transmission to much advantage.

A comparisou of the last example with the older system in which a separate rope is used for each portion of the transmissiou will be of interest. In the previous method the pulleys could not be brought close together because the tension would require to be too great, and slight variations in temperature would produce excessive variations in tensiou. These difficulties are overcome in the ring system by the use of the tightener carriage, which may also be used to much advantage in those systems of belt transmission which lie in one plane, such as have been shown in Fig. 844. The construction is similar to that for rope transmission, and the umbrella hub may be used to advantage. In many cases the specific capacity may be much increased in this way.

The new system is also highly advantageous for long distance transmission, especially where power is to be taken off at several points, or it may be used in combination with the old system, retaining the latter aud usiug the new system for purpose of distribution.

The difficulties of construction are much less for long distance transmission than with the old system, and the cost of installation and supervision much smaller.

The application of the new system appears likely to increase very greatly, since it involves less first cost than electrical trausmission plant, and also a higher efficiency when the losses from transformation of electrical currents are considered.

This subject will be further considered in Chapter XXIV.

CHAPTER XXII.

CHAIN TRANSMISSION. STRAP BRAKES.

§ 302.

Specific Capacity of Driving Chains.

The use of chain for purposes of power trausmission is necessarily more restricted than the use of rope, but for single transmissions in special cases it is well adapted, and its applications are increasing. Chain is especially capable of resisting variations of temperature and exposure to the weather and to dust, and hence is well adapted for driving revolving drums in miuing machinery, washiug machiuery, the machines in bakeries, etc. In mining machinery chains are very extensively used, both above and below ground, not only for continuous tramway driving, as in Fig. 802, but also for the transmission of rotary motion over extended distances.

Chain sheaves are made either with smooth wedge shaped grooves, or with pockets for the chain links as already indicated in § 275. In the first case the driving is due to friction in the same manner as with belting and ropes, while in the second case the action is similar to that of toothed gearing.

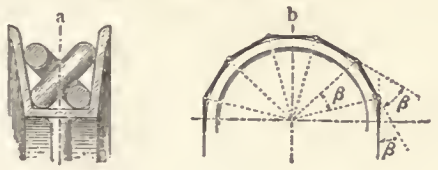


FIG. 930.

The method of friction driving can be used with ordinary link chain as at *a*, Fig. 930, and may also be used with the flat link chain of Fig. 830 *d*, if so desired. The circumferential friction $F = T - t$ may be determiued from the following relation (see § 264):

$$T = t = \left((1 + 2f \sin \frac{\beta}{2}\right)^{m} \quad \ldots\ldots \quad (311)$$

in which T and t are the tensious in the driving and driven sides of the chain respectively, and f is the coefficient of friction. The angle β is that subtended by the pitch length of a link of the chain at the centre of the chain sheave, and may be obtained from $r \sin \frac{1}{2}\beta = \frac{1}{2} l$; the exponent m is the number of link contacts, hence $m = \frac{\alpha}{\beta}$. A sufficiently close approximation may be obtained by taking $\beta = \frac{l}{r}$, which gives for the modulus of friction ρ:

$$\rho = \frac{T}{t} = \left(1 + f \frac{l}{r}\right)^{\alpha \frac{r}{l}} \quad \ldots\ldots \quad (312)$$

In chain transmission the modulus of friction is not independent of r, as with rope transmission, but varies somewhat with the ratio $\frac{r}{l}$. This latter ratio in practice seldom goes below 5. Taking this limit, and also putting $f = 0.1$, we have for practical values of ρ the following, in which u equals the number of half wraps of the chain around the sheave:

$$\rho = \frac{T}{t} = \left(1 + \frac{1}{50}\right)^{5\pi u}$$

whence:

$$\rho = \frac{T}{t} = 1.37^{u} \quad \ldots\ldots \quad (313)$$

The following table has been calculated for 1 to 8 half-wraps, and gives the modulus of friction $\rho = \frac{T}{t}$, the modulus of stress $\tau = \frac{T}{P}$ and the modulus of transmission θ (see (309)).

$u =$	1	2	3	4	5	6	7	8
$\rho =$	1.37	1.88	2.57	3.53	4.83	6.61	9.06	12.41
$\tau =$	3.69	2.13	1.64	1.39	1.26	1.18	1.12	1.09
$\theta =$	0.27	0.47	0.61	0.72	0.79	0.85	0.89	0.92

These values for ρ and τ are similar to those obtained for tension organs generally, as indicated in the diagram already given in Fig. 816. It will be noted that the transmitting capacity of chain even with a single half-wrap about a smooth sheave is good.

Since the specific capacity of a driving tension organ (see (262)) is equal to

$$N_o = \frac{1}{33000} \cdot \frac{S}{\tau} \text{ or } \frac{1}{33000} S\theta$$

we have for ordinary open link chains the following values for various stresses S:

S	$u =$	1	2	3	4	5	6	7	8
5000	$N_o =$	0.042	0.071	0.092	0.109	0.120	0.129	0.135	0.140
7000	$N_o =$	0.057	0.099	0.129	0.153	0.168	0.180	0.189	0.197
8500	$N_o =$	0.070	0.121	0.157	0.185	0.203	0.219	0.230	0.237

The specific capacity is in all cases high, and for the generally accepted stresses in the chain cross section it varies from 0.042 to 0.237. Various applications permit variations in the value of S, the value being taken lower when it is desired that the wear through friction shall be reduced. The cross section of chain is determined from the equation $N = 2\,q\,v\,N_o$ (see § 280) in which N is the horse power to be transmitted at a velocity v, and q is the sectional area of the iron of which the chain links are made. We have:

$$q = \frac{1}{2v} \cdot \frac{N}{N_o} \quad \ldots\ldots \quad (314)$$

The value of v is always low, and hence the influence of centrifugal force upon ρ may be neglected.

Example 1.—It is required to transmit 10 H. P., by means of a chain making a half wrap about a smooth sheave, the velocity v being 1180 feet per minute and $S = 8500$ lbs. We then have for the cross section q of metal:

$$q = \frac{1}{2 \times 1180} \cdot \frac{10}{0.070} = 0.0609 \text{ sq. in.}$$

which corresponds to a diameter of 0.3 in.

Example 2.—By using the counter sheave (Fig. 795) and thus obtaining three half-wraps the value of δ may be reduced to 5000 lbs., whence

$$q = \frac{1}{2 \times 1180} \cdot \frac{10}{0.092} = 0.046 \text{ sq. in.}$$

or a diameter of 0.27 in.

This gives a lighter chain and at the same time a more durable one, as the friction is materially reduced when entering and leaving the sheave (see § 303).

By using grooved and pocketed sheaves the specific capacity may be greatly increased, the chain being held so securely that

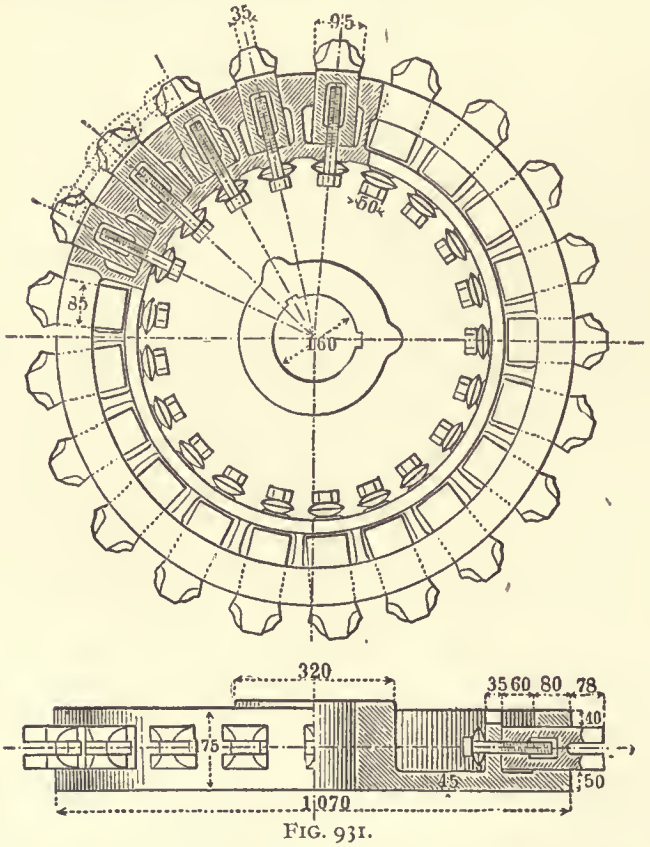


FIG. 931.

as many as eight half-wraps may be used. Two very practical arrangements for such sheaves are shown in illustrations, which are from executed examples in the chain tramway of the Decido iron mines in Spain, built by Brüll, of Paris. The dimensions are given in millimetres, and the chain is operated under a stress of about 5000 pounds per sq. in.

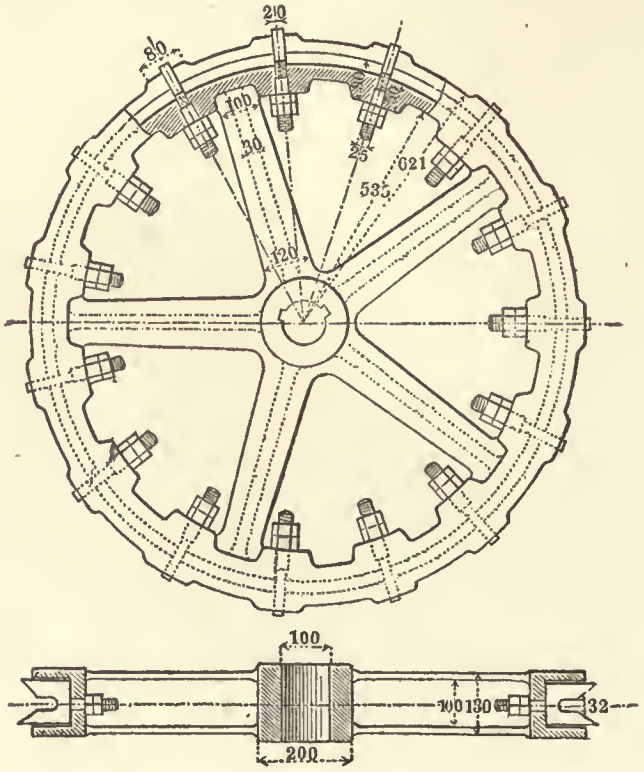


FIG. 932.

The sheave shown in Fig. 931 is for a 25 mm. (1″ nearly) chain, and is made with inserted teeth of steel, and the form of Fig. 932 is similar, and is for an 18 mm. (0.7 in.) chain. In both cases the teeth are radial, and formed to receive the chain links, being secured by jam nuts in the second case, and by nuts fitted with the Belleville elastic washers, which latter have worked well in practice.

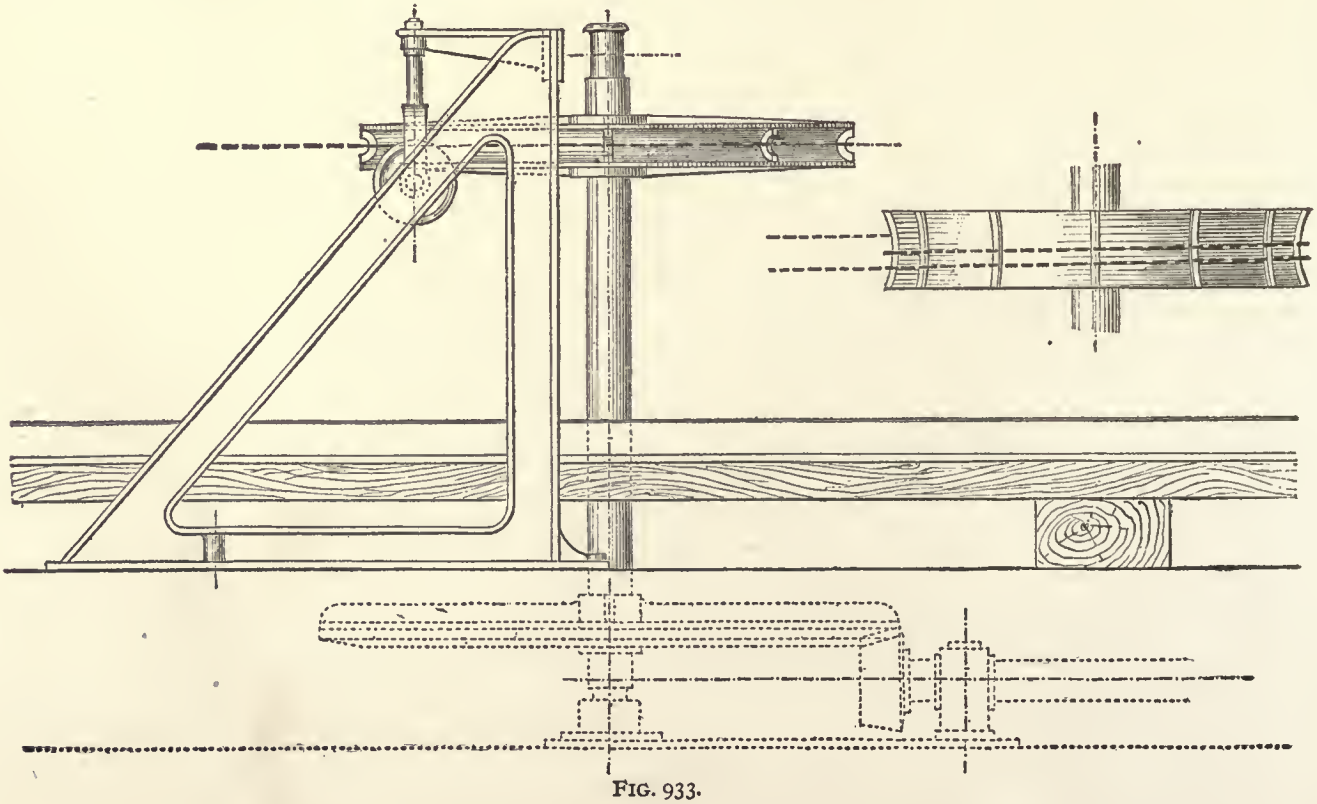


FIG. 933.

In Fig. 933 is given an arrangement of chain sheave gearing, including a solid massive form of bearing, as used in many English collieries.* Here the sheave is made with eight semi-circular ridges or ribs, similar to the old form of capstan shown already in Fig. 794 *a*; and both parts of the chain are carried on supporting pulleys. In many instances this arrangement is used, by widening the face of the sheave, to receive several wraps of chain, as shown in the upper right corner of Fig. 933. If we may safely assume that the ridges increase the coefficient of friction at least three times, in the preceding formulas (311) and (312), we have for the corresponding modulus of friction ρ':

$$\rho' = 2.5^{u} \quad \ldots\ldots\ldots\ldots \quad (315)$$

which gives for

u	½	1	2	3	4
$\rho' =$	1.58	2.50	6.25	15.63	39.06
$\tau' =$	2.72	1.67	1.19	1.07	1.03
$\theta' =$	0.37	0.60	0.84	0.94	0.97

from which the security against slippage and also the specific transmitting capacity may be determined for any given case.

Within moderate limits chain transmission may be used as a "ring" system, as for instance in driving the rollers of carding machines, also in wood pulp grinding mills a ring chain transmission is used for driving the feed rolls.

§ 303.

Efficiency of Chain Transmission.

The loss of efficiency in a chain transmission is due to journal friction, dependent upon the chain tensions T and t; and upon the friction of the links in entering and leaving the sheaves. The journal friction is determined as already shown in § 300, and for high values of θ, it is not great. The loss from chain friction is due to the rotation of each link about its adjoining link as an axis through an angle β. This gives, with a coefficient of friction f_1, a circumferential resisting force F_1, due to chain friction (see formula 100)

$$F_1 = f_1 \left(T + t\right) \left(\frac{4}{\pi}\right) \left(\frac{\frac{1}{2}\, d}{r}\right).$$

In this case the pitch length l of the links is taken $= 3.5\, d$, and making $r = 5\, l$, we get $F_1 = (T + t)\ 0.036 f_1$, and if we put for the loss at both sheaves:

$$\frac{2\, F_1}{P} = E_k$$

we get:

$$E_k = 0.072\, f_1 \frac{\rho + 1}{\rho - 1} \quad \ldots\ldots\ldots \quad (316)$$

Example 1.—Taking the coefficient of friction $f_2 = 0.15$ on account of the small bearing surface we have for a chain transmission on smooth sheaves with half-wrap; ρ being $= 1.37$, as in the preceding section:

$$E_k = 0.072 \times 0.15 \frac{2.37}{0.37} = 0.0692$$

or say 7 per cent.

Example 2.—If the sheave is made with ridges, as in Fig. 933, we have $\rho' = 2.5$, and hence

$$E_k = 0.072 \times 0.15 \left(\frac{3.5}{1.5}\right) = 0.025$$

or only 2½ per cent.

Example 3.—By using carefully made pocketed teeth and making $u = 8$, we have $\rho = 12.41$, whence

$$E_k = 0.072 \times 0.25 \left(\frac{13.41}{11.41}\right) = 0.0126$$

or only 1¾ per cent., this reduction being due to the reduction in the tension on the chain, showing the importance of considering the question of chain tension in this connection.

In the preceding examples the friction of the links upon each other has been considered, but not that of the links upon the sheave. This latter is a very variable quantity, being unimportant with a smooth sheave, as Fig. 930 *a*, and sometimes becoming excessive, as shown already in Fig. 838 *b*, § 275.

In every case all possible care should be taken to produce as little rubbing contact as possible.

§ 304.

Intermediate Stations for Chain Transmission.

The most important applications of chain transmission are in mining work, both above and below ground; and especially in coal mines. In this branch of work England takes the lead, followed by America, where, however, wire rope is more extensively applied, while in Germany the most applications are found in the Saarbruck district.

A very interesting application of endless long distance chain transmission is shown in Fig. 934, which gives two views of the

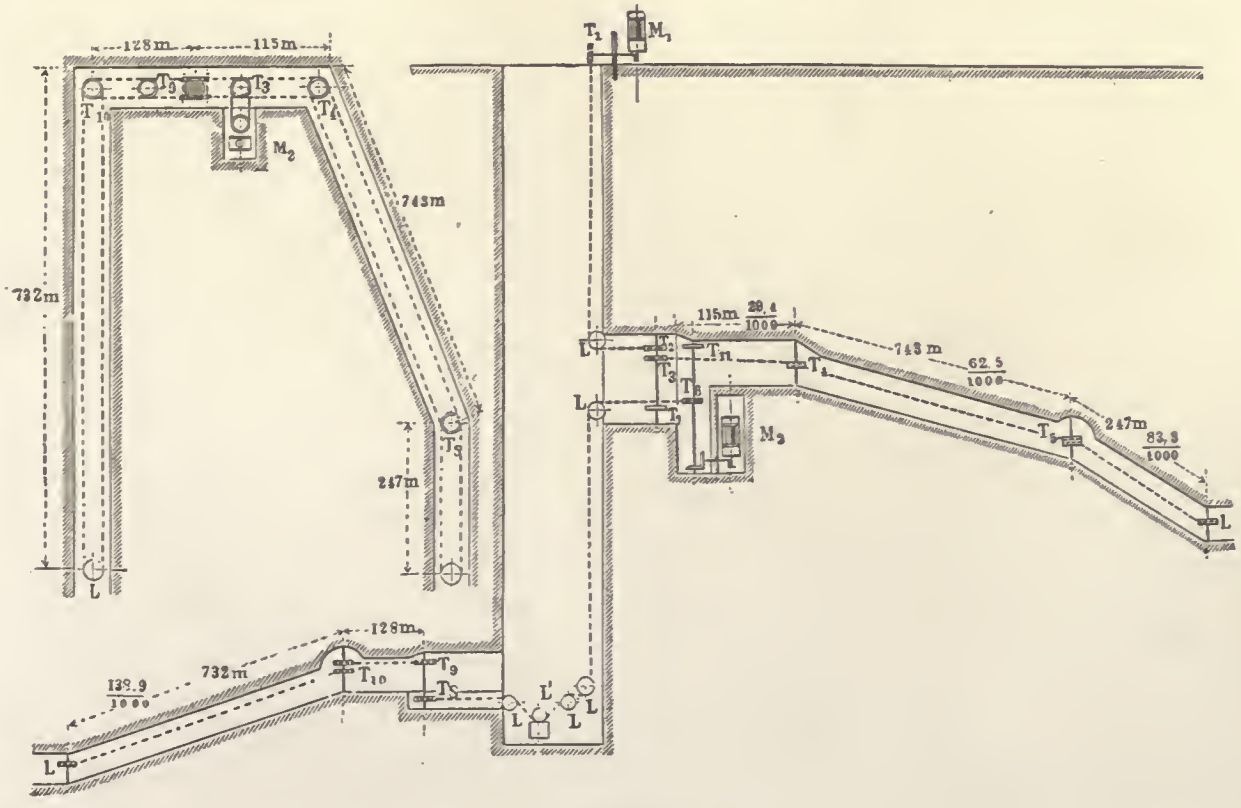


FIG. 934.
(Dimensions in Metres.)

* The illustration is from Newchurch colliery at Burnley.

Gannow mine at Burnley in Lancashire. The driving pulley is at T, and guide pulleys at L, while at L' is a tightener pulley hung between two idlers, a construction which is frequently used.

The rotation is modified in various ways in the English mines, stations similar to those of rope transmission systems being used.

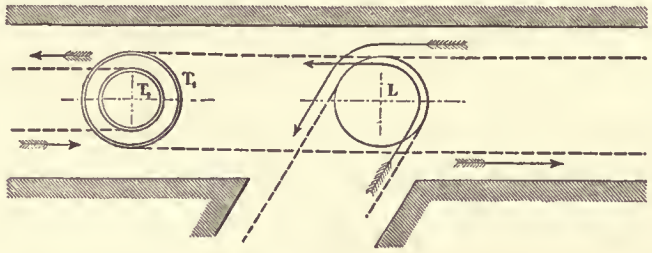


FIG. 935.

In Fig. 935 is shown an intermediate station at T_1 T_2, and also on angle station at L. In many instances combinations of bevel gearing and shafting are found in connection with chain transmission, but the examples here given are confined to the use of chain alone.

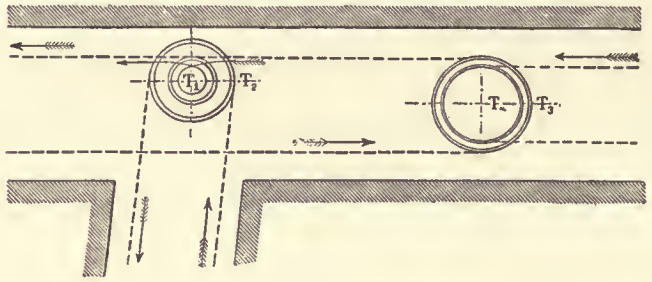


FIG. 936.

In Fig. 936 an intermediate station is shown at T_3 T_4, and a chauge station at T_1 T_2. At T_1, Fig. 936, the chain makes an entire wrap around the sheave, the latter being made with a wide groove, and interference of the two parts of the chain prevented by guide sheaves. The simple supporting stations are made with small horizontal guide sheaves, with wide grooves. The velocity of the chain varies from 200 to 500 feet per minute.

§ 305.

STRAP BRAKES.

If a driven pulley is embraced by a tension organ, either belt, rope, strap or chain, the ends of which are subjected to tensions T and t, and also held from moving, the pulley is hindered from moving toward t, so long as the force acting to rotate it does not exceed $P = T - t$. The tension organ then forms, with the pulley and stationary frame work, a friction ratchet system in which the tension organ forms the pawl. If the tension T be reduced until $T - t < P$, the pulley will slip in the strap, overcoming the frictional resistance due to $T - t$, and the motion can be made slower, if T and t be made great enough, so long as their difference is only slightly smaller than P. The mechanism then becomes a form of checking ratchet (§ 253) better known as a friction brake, or simply as a brake. Such brakes, when made with tension organs, are called strap brakes.

Strap brakes are made in various forms to suit the application.

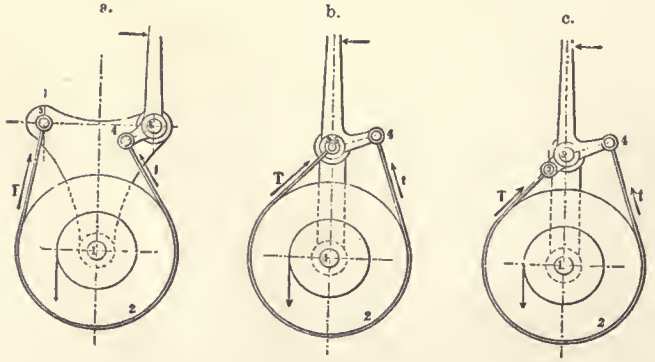


FIG. 937.

(*a*) *Clamping Brakes.*—When a strap brake is to be used to act as a complete clamping brake, to check motion entirely, the tensions T and must be determined. These are obtained from formulas (239) and (240) or from the graphical diagram of Fig. 816. Such strap brakes are frequently made with straps of iron or steel. It is generally desirable to so arrange the parts that the motion of the pulley acts to draw the strap into closer engagement, which may be done in various ways. Fig. 937 shows several such arrangements.

The various parts are indicated as follows: 1 is the axis of the pulley; 2, the point of application of the brake; 3, the attachment of the tight side of the strap; 4, the attachment for the slack side; 5, the axis for the brake lever. In Fig. 937 *a*, 3 and 5 are separate; in Fig. 937 *b* they are combined in one, and in Fig. 937 *c* both 3 and 5 are separate, but 3 and 5 are made movable, and 3 and 5 are so nearly in line with T that a very slight effect is produced on the lever by T.

In Fig. 938 *a* 3 and 4 are combined, and at the same time 3 and 5 are nearly in line with T. Fig. 938 *b* is the so-called

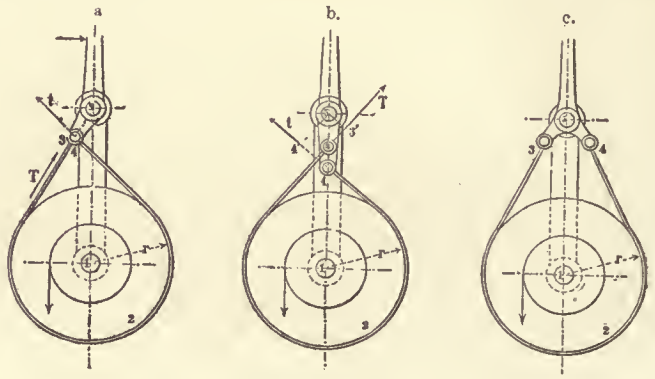


FIG. 938.

"differential" brake of Napier, in which 3 and 4 are so placed that perpendiculars to the directions of T and t are inversely proportional to those tensions, thus reducing the action of the strap upon the brake lever to a small amount. Fig. 938 *c* shows an arrangement adapted to permit the pulley to revolve in either direction. The angle 3 . 5 . 4 can be so chosen that the force upon the lever may be very small.

For heavy hoisting machinery, the braking power required

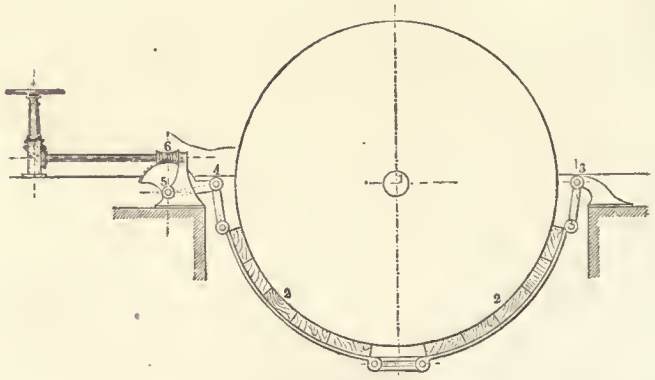


FIG. 939.

makes the arrangement shown in 939 suitable. In this case the strap is filled with blocks of wood in order to obtain a higher coefficient of friction and at 6 is shown an application of the globoid worm and worm wheel shown in Fig. 641.

Example.—Required a brake for a shaft driven by a force of 2200 pounds at a lever arm of 7.875 inches. The form chosen is that of Fig. 938 *a*. the arc of contact α of the strap being 0.7 of the circumference. The coefficient of friction $f = 0.1$, the strap being lubricated. We then have $f\alpha = 0.1 \times 1.4\,\pi$, $= 0.14\,\pi = 0.43$. We then have from the second table of § 264, the tension modulus $\tau = \frac{T}{P} = 2.88$ nearly, and the friction modulus $\rho = \frac{T}{t} = 1.5$ (see also the diagram, Fig. 816). This gives $\frac{t}{P} = \frac{2}{3} \cdot \frac{T}{P} = \frac{2}{3} \times 2.88 = 1.92$. If we make the brake pulley with a radius of 15.75 in., the braking force at the circumference of the pulley must be $\frac{7.875}{15.75} \cdot 2200 = 1100$ lbs., and $t = 1.92 \times 1100 = 2112$ pounds, and $T = 2.88 \times 1100 = 3168$ pounds. If the brake is to be operated by a hand lever with a force of 44 pounds, the ratio of the length of the hand lever to lever arm 4 . 5 must be $\frac{2112}{44} = 48$. The strap is under a tension of $T = 3168$ pounds. If we assume a permissible stress of $S = 14{,}220$ lbs. and a thickness of strap $\delta = 0.08''$ the width will be:

$$b = \frac{3168}{14{,}220 \times 0.08} = 2.78'',$$

which is quite practicable.

The question of the pressure between the braking surfaces is of interest. According to formula (241) $\frac{p}{S} = \frac{q}{b' R}$ we have for the tight end, where $S = 14{,}220$.

$$p = 14{,}220 \frac{0.08}{15.75} = 72 \text{ lbs.}$$

and at the slack end, since $\frac{t}{T} = \frac{2}{3}$, $p = \frac{2}{3} \cdot 72 = 48$ lbs., both of which such small values that the wear must be very slight.

This example shows how, in a properly arranged construction, a great ratio of force to resistance can be obtained. In large winding engines the brake pulley can readily be cast in one with the rim of the drum gear.

The method of securing the ends of the metal strap is shown in Fig. 940. The form at *a*, is secured by countersunk rivets, and that at *b*, by an anchor head and a single small rivet to prevent lateral slippage.

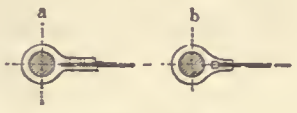


FIG. 940.

(*b*) *Sliding Brakes.*—In using clamp brakes operated by hand for lowering heavy loads in hoisting machinery, great care must be taken, since the throwing out of the checking pawls puts the entire resistance on the brake. With this arrangement there is always more or less insecurity, the safety depending upon the handling of the lever, and serious accidents have frequently occurred. This danger can be avoided by the use of automatic sliding brakes, the following form being designed by the author, and shown in two forms in Fig. 941. The brake pulley *a*, is loose on the shaft, but engages with it by means of a ratchet system *a' b' c'*. The brake is subjected to a tension equal to

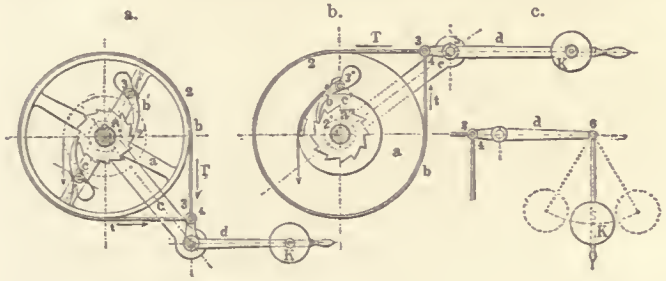


FIG. 941.

the greatest braking force desired; *i.e.* so that the weight *K* must be raised in order to permit the load to run down. If the lever is let go, for any reason, the descent is checked. In form *a*, the pawls are attached to the pulley, and the ratchet wheel *a'* keyed to the shaft; in *b*, the pawl is on a disk *c'*. When the load is raised the combination forms an ordinary ratchet train. A silent ratchet, Figs. 673, 674 may be used for this device. At *c*, is shown a pendulum counterweight, which can be adjusted so as to vary the braking power to suit various loads.

Another form of sliding brake, also designed by the author, is shown in Fig. 942. In this design the strap *b*, is given such

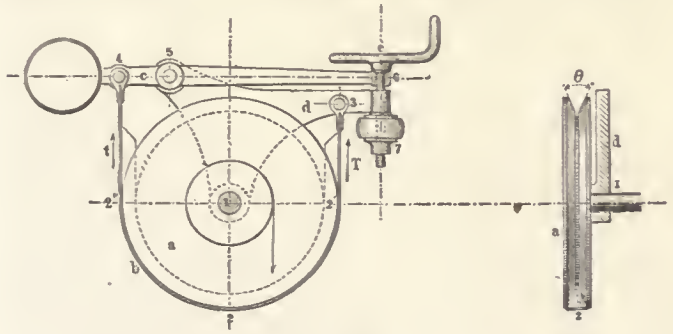


FIG. 942.

tension *t*, by means of the screw *e* 7, and lever *c*, as to hold the load from descending; a rubber spring being introduced at 7. If the load is to be lowered, the clamp *e*, is loosened, but is again tightened on ceasing. When hoisting, the tension *t* at 2″ is readily overcome. This is in reality a form of running ratchet gear, and as shown it is made with a strap of wedge section, the angle θ being 45°. The wedge portion is made of wood on iron

at least 0.20) increased by $\frac{1}{sm\frac{\theta}{2}}$ which when used to multiply the value of $f \alpha$, requires a very small force to overcome the tension *t*.

§ 306.

CHAIN BRAKES.

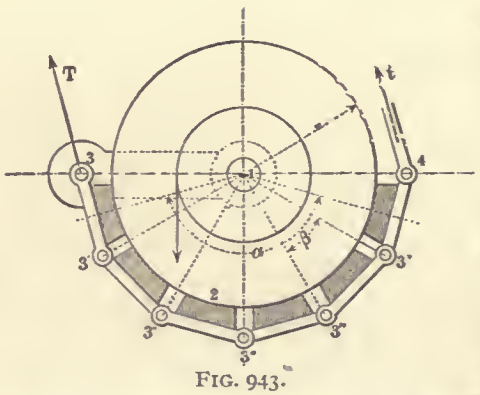


FIG. 943.

Chain may be used as the tension organ in brake construction, in which case it is generally lined with blocks of wood, as in Fig. 943. The tensions *T* and *t*, to be given to the two parts of the chain, are readily obtained from formula (312). The ratio of chain pitch length *l*, to the pulley radius *r*, is increased because of the use of the wooden block. When $l = \frac{1}{3} r$ and the arc of contact is less than 180°, we have:

$$\rho = \frac{T}{t} = \left(1 + \frac{f}{3}\right)^9 \quad \ldots\ldots\ldots \quad (316)$$

For wood on iron we may take $f = 0.3$ (see section 193). This gives:

$$\rho = \frac{T}{t} = 1 \cdot 1^9 = 2.35; \text{ also}$$

$$\frac{T}{P} = \tau = \frac{d}{\rho - 1} = \frac{2.35}{1.35} = 1.74$$

$$\text{and } \frac{t}{P} = \tau - 1 = 0.74, \text{ or } t = 0.74\, P.$$

These proportions should not be strictly followed for heavy brakes such as in Fig. 939, as such should be determined for each case.

§ 307.

INTERNAL STRAP BRAKES.

Strap brakes may be used in internal pulleys, in a manner similar to the internal ratchet gear of Fig. 711, for example. The outside of the strap then acts upon the inner surface of the pulley, the strap being subjected to compression instead of tension,* thus becoming a pressure organ, a subject treated more fully in the following chapter.

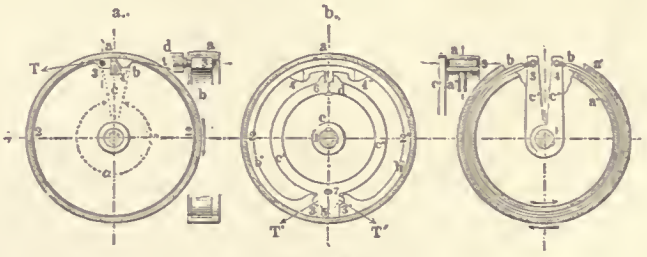


FIG. 944.

The pressure of the internal strap brake is of the same magnitude as with the external brake, but in the opposite direction, so that the previously determined value of ρ from the forces *T* and *t*, may be used. Fig. 944 shows three forms of such brakes, these being used for friction couplings, and not in hoisting machinery (see Fig. 449). Fig. 944 *a*, is Schurman's friction coupling.† The brake lever *c*, acts by means of a wedge 4, upon one end of the strap. The other end of the strap is held by a pin 3, to the member *d*, which is to be coupled to *a* by means of the strap *b*. The lever *c*, is also pivoted to the member *d*. For the forces *T* and *t*, we may use formula (239), and since α is nearly $= 2\pi$, or say $= 6$, we have for $f = 0.1$ the value $f \alpha = 0.6$, which from the table of § 264 gives $\rho = 1.82$, and $\tau = 2.22$, whence $t = 1.22\, P$. The strap must be released by the action of a spring.

Fig. 944 *b*, is Adyman's coupling,‡ which is made with a heavy cast iron ring. The ring *b*, is made in halves, *b'* and *b''*, fitted with projections 4′ and 4″ which engage with an intermediate sheave keyed on the shaft.

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* See Theoretical Kinematics, p. 167; p. 548.
† Zeitschrift des Vereins Deutscher Ingenuiere, Vol. V. p. 301.
‡ Made by Bagshaw & Sons, Batley, Yorkshire.

The levers c' and c'' have a common axis at 5, and when separated by a wedge at 6, they press upon the ends of the ring at 3′ and 3″. A pin at 7, keeps the levers from sliding in the direction 7 . 1, as well as the ring b' b''.

The coupling shown in Fig. 944 *c*, acts both ways, as an internal and external strap brake, and is used on a shaping machine by Prentiss.* The steel strap *b*, is covered with leather. When the arms c' c'' are drawn together it acts as an external strap on the pulley a'', and when they are forced apart it becomes an internal strap in the pulley a'. The arms c' c'' are carried on sleeves and are rotated to or from each other by a screw action.

CHAPTER XXIII.

PRESSURE ORGANS CONSIDERED AS MACHINE ELEMENTS.

§ 308.

Various Kinds of Pressure Organs.

In distinction from the various kinds of tension organs which have been considered in the four preceding chapters, there exists another group of machine elements of which the sole or principal characteristic is that they are capable only of resisting forces acting in compression. This group includes fluids, both liquid and gaseous, whether limpid or viscous, such as: Water, oil, air, steam, all pasty substances, clay, molten metals; also granular materials, all kinds of grain, meal, gravel, etc. In all these materials the principal feature lies in the fact that the particles are subdivided to such an extent that they can be separated from each other by a very small force, while on the other hand they are capable of opposing more or less resistance to compression, this resistance in many instances, as, for example, in the case of water, almost equalling that of metals. These materials may be used as machine elements in a great variety of ways, and in the following discussion they will be included under the general title of Pressure Organs. Like their counterparts the tension organs already discussed, they are used largely for the transmission of motion in various manners, but are of still greater importance on account of the great variety of physical conditions in which they appear.

§ 309.

Methods of Using Pressure Organs.

The distinction which has been made between tension and pressure organs enables various points of contrast and comparison to be made as regards the methods of utilizing them, and pressure organs may be divided in the same manner as tension organs (see § 262) into standing and running organs. These divisions have but little practical application in this instance, but the three following subdivisions in § 262, viz.: Guiding, Supporting (*i. e*, raising or lowering), and Driving are here applicable also. We may therefore distinguish pressure organs, when considered as machine elements, into the following classes:

1. For Guiding.
2. For Supporting (including raising and lowering).
3. For Driving.

These various methods of action may be used either separately or in combination, and are found in most varied forms in many machine constructions. The great variety of possible combinations makes it desirable for a general view of the entire subject to be taken before discussing details.

§ 310.

Guiding by Pressure Organs.

In order to use a pressure organ for guiding, *i. e.*, to compel a more or less determinate succession of motions, it is necessary to use also two other machine elements formed of rigid materials. These latter are for the purpose:

a, Of resisting the internal forces of the pressure organ and keeping it within the desired limits.

b, Of connecting the pressure organ with the external forces to be received and opposed.

Tubes, Conduits, Canals.—The tube *a*, Fig. 945, limits the boundary of the particles of the pressure organ, and retains it in the desired form and controls its direction. A bend in a tube corresponds to a pulley around which the pressure organ is bent, and thus has its direction changed. Even when no change of direction is made, the tube is necessary to oppose resistance to the particle of the pressure organ, and hence at every section it must offer resistance to tension as well as compression. Conduits, or channels, as at *b*, are tubes with one side left open, the force of gravity or the so-called "living force" of the pressure organ serving to retain it within the desired limits. Canals are merely conduits of larger dimensions, as at *e*, and natural streams of water often serve the purpose.

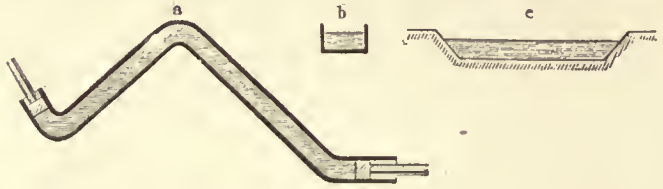


FIG. 945.

Driving Organs, Pistons and Cylinders.—The bodies by means of which the pressure organ is connected with the external forces and resistances with which it is intended to act mechanically may be called generically, Driving Organs, and are very varied in character. Among these are movable receptacles, also moving surfaces or moving conduits (as in turbines), and also moving pistons in tubes or cylinders. A piston serves to oppose the stress in the pressure organ in the direction of its motion, while the walls of the tube oppose their resistance at right angles to the direction of motion. The inclosure in which a piston acts is called, in general terms, the cylinder, and details of construction will be given hereafter. The principal types will here be considered briefly.

A complete working contact between piston and cylinder can only be obtained when both surfaces are alike, and this is only geometrically possible with three forms of bodies; *i. e.*, prismatic bodies, bodies of rotation, and spirally formed bodies. Of these the prismatics are most useful, and among the prismatic bodies the form most extensively used is the cylinder.

The fit of a piston in its cylinder, entirely free from leakage, is very difficult of attainment, and is rarely attempted in practice.

In steam indicators the piston is very accurately fitted directly into the cylinder, but in most cases a practically satisfactory result is obtained by the use of some intermediate packing device.

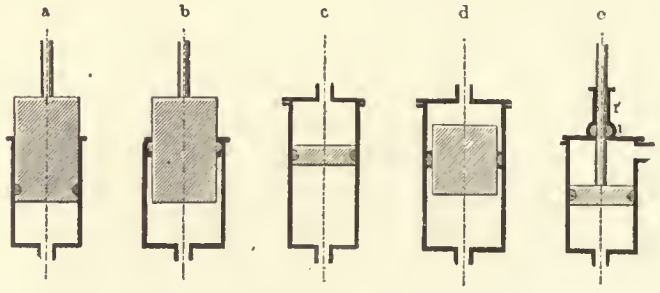


FIG. 946.

In many cases a soft packing of hemp or leather is used, Fig. 946. At *a* is shown a piston with external packing, at *b* an internal packing. In these cases one entire end of the cylinder is open, the piston filling the entire cylinder and acting upon the inclosed pressure organ on one side, this constituting a single-acting position. At *c* and *d* are similar double acting pistons. Pistons of the forms shown in *a* and *b* are sometimes called plungers, and the shorter inclosed pistons, as *c* or *d*, are also called piston-heads. At *e* is a double-acting piston used in connection with a rod and stuffing box, the rod being connected with external mechanism, and the stuffing box made either with external or internal packing, as indicated at 1 and 1′. In many instances pistons are made with openings which are fitted with valves, and hence may be called "valved" pistons, while those here shown are termed closed or solid pistons.

The tightness of the packing is usually produced by the application of some external force, but in the so-called forms of self-acting packing the necessary pressure is supplied by the confined fluid. This is shown in the following illustrations.

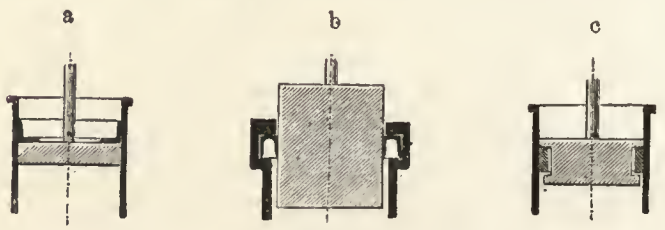


FIG. 497.

Fig. 947 *a* and *b*, Cup packing for piston or stuffing box; metal

*See Mechanics Feb., 1884, p. 140.

packing, usually for pistons, but also used in stuffing boxes. The fluid in all three cases enters behind the packing rings and tightens the joint in proportion to the increased pressure.

In the class of self-acting packing may also be included the various forms of liquid packing, some of which are given in Fig. 948. The forms at *a* and *b* are practically plungers, while

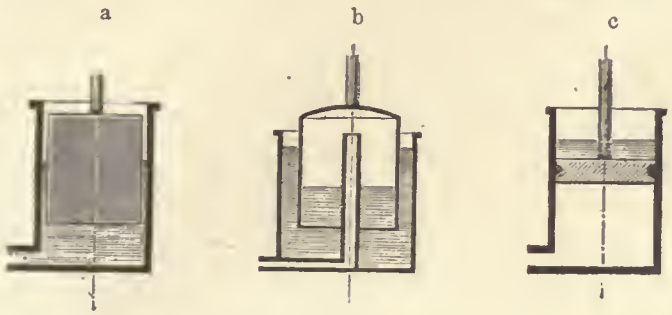


FIG. 948

in many cases an ordinary packing has its tightness increased by a layer of water or oil upon the piston, as shown at *c*.

Another variety occurs when the connection between cylinder and piston is made by means of a membrane or diaphragm, as in Fig. 949.

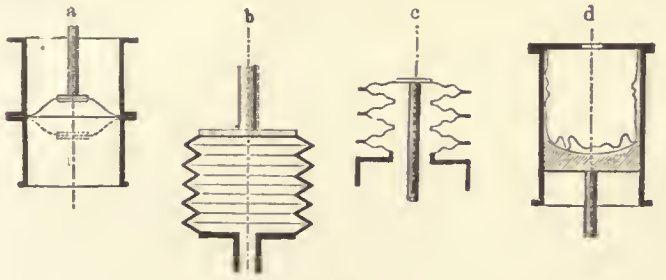


FIG. 949.

These are among the oldest forms of transmission organs, but are practically true pistons in principle and action. At *a* is a single diaphragm, known as the monk's pump: *b* is the so-called "bellows" form; *c* is a series of flexible metal diaphragms, usually of steel, brass or copper, used for pressure gauges or other similar purposes involving but little movement. At *d* is the so-called "bag" pump, in which the liquid does not come in contact with either cyliuder or piston, but is confined within a flexible bag.

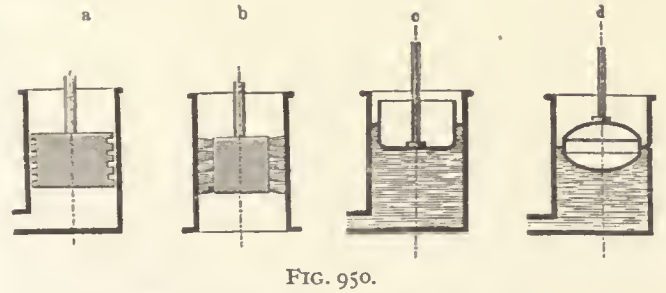


FIG. 950.

Another class of pistons is that in which a tight packing is not attempted, these usually being used only for air. Fig. 950 *a* shows a deep piston with grooves formed in it, the fluid endeavoring to pass the piston in the opposite direction to the motion of the latter, becomes entrapped in the grooves, and before it can pass, the direction of motion is changed and this action reversed.* At *b* is a piston with a brush packing, used for a blowing cylinder at Sydenham. In this class of pistons we may also include floats which rise and fall with the motion of the liquid. Such floats are shown at *c* and *d*, the former being open and the latter closed. A solid block may also be used for this purpose, if its weight is nearly counterbalanced by another weight.

Details of piston and cylinder construction will be given in Chapter XXVI. The corresponding machine elements to pistons in tension organs will be found for ropes in Figs. 825–826, and for chains in Figs. 831 to 834. The change of direction from compression to tension dispenses with the necessity for a cylinder.

* See Weisbach, Vol. III., Part 2, § 410.

§ 311.

GUIDE MECHANISM FOR PRESSURE ORGANS.

The combination of a pressure organ and its accompanying guide mechanism forms a pressure transmission system. Ex-

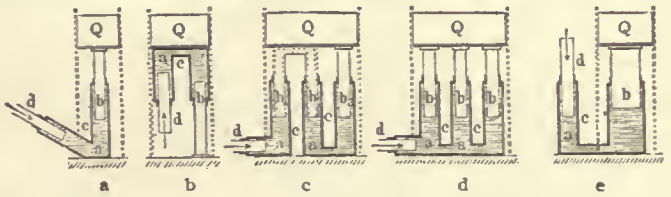


FIG. 951.

amples of such systems are given in outline in Fig. 951. At *a* is an arrangement for raising the load *Q* vertically. The plungers *b* and *d* are of the same diameter; the pressure on *b* must be the same as *Q*, neglecting friction. The column of water is the same diameter as the plungers, and the direction is changed an angle of 120°. It is desirable that distinguishing names should be given to the various arrangements. If we compare these with the corresponding parts in tension organs, Fig. 784 and Fig. 785 *a*, we may properly call such an angle transmission a hydraulic pulley, or water pulley, but a still better name is the "hydraulic-lever" or "water-lever," which will be hereafter adopted.

At *b* is shown a free water-lever. The plungers *b* and *d* are equal in diameter, the load *Q* is supported on two columns of water, hence, if friction is neglected, the force on each plunger will be ½ *Q*, the angle of change of direction is 180°.

At *c* is a combination of case *a* with case *b*. The plungers b_1, b_2, b_3, are of the same diameter, and the load *Q* is supported on these columns. These three cases correspond in principle with the similar cases *a b c* of Fig. 784. Since the three plungers b_1, b_2, b_3, of case *c* all exert the same force, they may also be made to give the same result when made as shown at *d*, or if the three plungers are combined in one, form *e* is obtained. The latter form is well known in practice as the hydraulic press. The principle involved in all these devices is the same as appears in the various pulley systems of tension organs.

A comparison of Fig. 951 *a* with *e* shows that the same principle exists in both, and case *a* may be considered as a water-lever of equal arms, and case *e* as a lever of unequal arms.

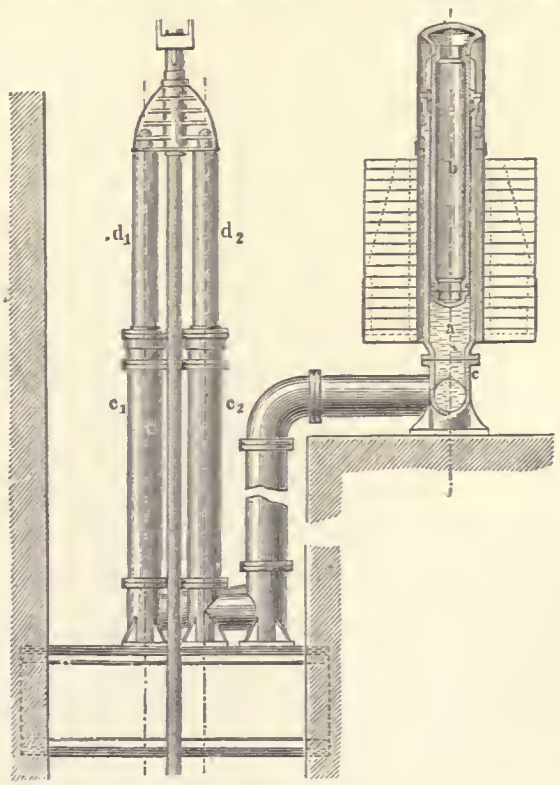


FIG. 952.

The water-lever has been used in more or less complete devices for balancing the weight of pump rods in deep mine shafts. Fig. 952 shows Oeking's water counterbalance.† The

† Zeitschrift Deutscher Ingenieure, 1885, p. 545. Oeking incorrectly call the device *a b* an accumulator.

pump rod is carried on the two plungers d_1 d_2, and its weight counterbalanced by the weighted plunger and cylinder a–b.

In the Emery scales and testing machines water-levers of unequal arms are used in connection with metallic diaphragms.

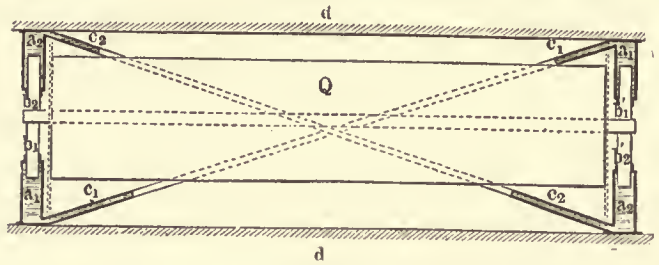


FIG. 953.

Fig. 953 shows a combination of two hydraulic levers, each of the form of Fig. 951 *a*. The weight Q travels in a straight line, being kept parallel by the four equal plungers $b_1 b_2 b_3 b_4$, and crossed pipe connections. This construction is similar to the cord parallel motion of Fig. 784 *d*.

In all of the devices described the rigid body is guided by the motion of the pressure-organ. It must be remembered that motion is merely a relative term, and the rigid body may move through the fluid. An example of the latter is the rudder of a vessel, which acts in one plane; or in the case of the Whitehead torpedo several rudders are used, guiding the torpedo in any direction.

§ 312.

RESERVOIRS FOR PRESSURE ORGANS.

Reservoirs are used in connection with pressure organs in order to enable a number of applications to be operated collectively, and also to enable the pressure to be stored for subsequent service, and in this respect they correspond to the various forms of winding drums used with tension organs, and shown in Fig. 787. The following illustrations will show the use of such reservoirs.

Fig. 954 shows a tank for use with petroleum distribution, as

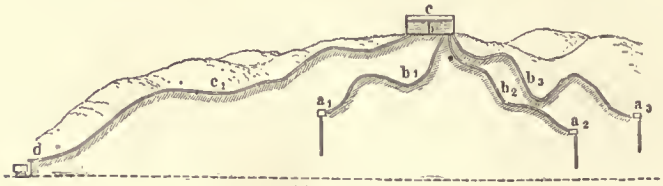


FIG. 954.

used in the American oil fields, and more recently in the oil district of Baku. The oil wells are at a_1, a_2, a_3, and the oil is forced to the elevated reservoir at c by pumps. From the reservoir the oil flows to the point of shipment d, and the supply is gauged by the fluctuations of level in the tank.*

The reservoirs used in connection with the water supply of cities are similar in principle. Where the configuration of the land demands it, the pipes are run in inverted siphons connecting intermediate reservoirs. An illustration of this arrangement is given in Fig. 955, which shows the waterworks system of Frankfurt-am-Main designed by Schmick.

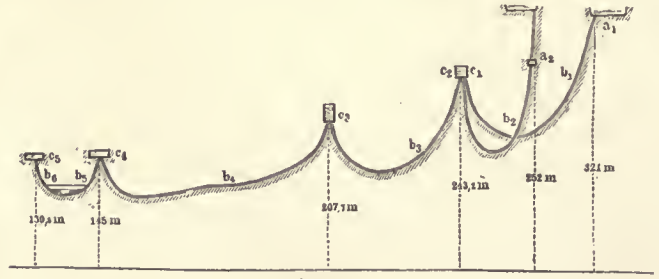


FIG. 955.

The highest spring is at a_1, Vogelsberg, and the next at a_2, Spessart. These both deliver into the reservoir c_1, c_2, at Aspenhainerkopf. The next reservoir is at c_3, Abtshecke, from which the water flows through b_4 to the reservoir c_4 and c_5, from which the city is supplied. The elevations above sea level are given in metres. The flow between the various reservoirs is controlled by suitable valves.†

Small tanks are in very general use at railway stations; and the various ponds and mill dams used in connection with water-wheels are other examples. In many cases the water ways are large enough to serve as reservoirs also, as in the case of canals.

Natural reservoirs are found in the case of many mountain lakes, the Swiss lakes affording many numerous instances.‡ Such basins are also formed artificially by constructing dams across narrow outlets, and so storing the water for use. Noteworthy examples found in France, the basin at St. Etienne, formed by damming the river Furens, being over 164 feet (50 metres) deep.§

Water may also be stored in accumulators at high pressures from 20, 50, as high as 200 atmospheres, and can then be used for operating hydraulic cranes, sluice gates, drawbridges, etc. These accumulators may be considered as a form of releasing ratchet mechanism (see § 260). To this class of mechanical action also belongs the system, used in the Black Forest, by which the streams are temporarily dammed and then suddenly released in order to float the logs down with the sudden rush of the current.

In using high pressure water transmission it is sometimes desirable to transform a portion to a lower pressure in order to operate a lower pressure mechanism, or by a reversal of the same principle, to convert a lower to a higher pressure. This can be done by means of the apparatus devised by the author, and shown in Fig. 956. ‖

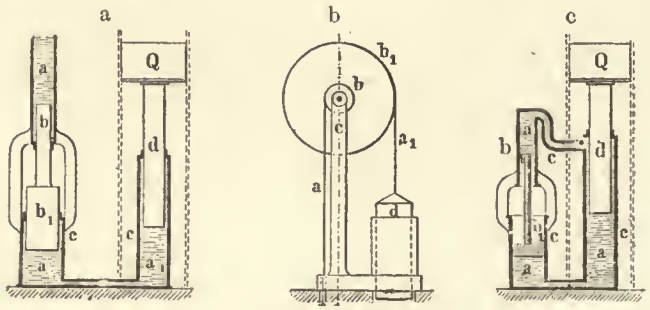
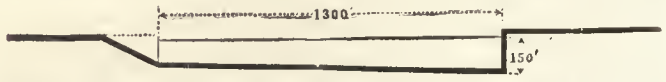


FIG. 956.

This is a form of hydraulic lever of unequal leverage, but is different from those shown in Fig. 951. Referring to Fig. 956 *a*, the high-pressure water is delivered at *a*, and connected with the lower pressure water a_1 by means of the plungers b, b_1, the latter being in one piece of two different diameters. The difference in pressure, neglecting friction, will be inversely as the areas of the two plungers, or if they are of circular section, inversely as the squares of their diameters. In this case the lower pressure then acts in the cylinder c upon the plunger d. The action of this arrangement may be considered as if the plungers b and b_1 were upon the same axis and rigidly connected, and the leverage compounded in a manner similar to that of the rope crane of Fig. 792 *a*; this comparison being more clearly shown by referring to Fig. 956 *b*. This device may also be used as a supporting hydraulic lever, similar to Fig. 951 *e*. If a communication is made between the two different water columns, as shown in Fig. 956 *c*, the pressure will be equalized. This gives a differential hydraulic lever similar in principle to the Chinese windlass of Fig. 790 *a*, or the Weston Differential Block of Fig. 796 *e*.

* A system of this sort was built in 1887 from Baku to Batoum on the Black Sea. The length of line is 1005 kilometres (603 miles), 6 in. diameter, and the reservoirs 3000 feet above sea level.

† A large inverted siphon is formed by the new Croton Aqueduct, which passes under the Harlem River at a depth of 150 feet below the surface of the



river, and a tunnel of 10½ feet in diameter driven through the solid rock. See Mechanics, Nov., 1886, p. 241.

‡ This is examined in detail in a memorial on the better utilization of water, published at Munich in 1883 by the German Society of Engineers and Architects.

§ For further discussion of this subject the following references may be consulted: Jaubert de Passa, Recherches sur les arrosages chez les peuples anciens, Paris, 1846; Ditto, Memoire sur les cours d'eau et les canaux d'arrosages des Pyrénées orientales; Nadault de Buffon, Cours d'agriculture et d'hydraulique agricole, Paris, 1853–1858; Ditto, Hydraulique agricole, application des canaux d'irrigation de l'Italie septentrionale, Paris, 1861–1862; Baird-Smyth, Irrigation in Southern India, London, 1856; Dupuit, Traité de la conduite et de la distr. des eaux, Paris, 1865; Scott-Moncrieff, Irrigation in Southern Europe, London, 1868; Linant de Bellefonds Bey, Memoire sur les principaux travaux d'utilité publique en Egypte etc., Paris, 1873; Krantz, Etude sur les murs de reservoirs, Paris, 1870; F. Kahn, Ueber die Thalsperre der Gileppe bei Verviers, Civil ingenieur, 1879, p. 1; also an article by Charles Grad in "la Nature," 1876, p. 55; also a brief article by the author "Ueber das Wasser," Berlin, 1876.

‖ See Glaser's Annalen, 1885, Vol. XVII., p. 234.

The opposite extreme to a high pressure accumulator is found in those pools or receptacles of water far below the natural sea level, such as are found in mines, and in the polders or drainage pools of Holland, Lombardy, and parts of Northern Germany.

Reservoirs are not confined to use with liquids. Examples of other fluids are found in the gasometers of gas works, in the receivers for compressed air, so extensively used in mining and tunneling, and in making the so-called pneumatic foundations. Smaller reservoirs are found in the air-chambers on pumping machinery, and the like.

The sewage system of Berlin, designed by von Hobrecht, consists of ten drainage pits, with the water level below the natural level, arranged on the so-called radial system. The sewage is pumped from these pits and delivered by means of pipes to sewage farms at a distance from the city.

Negative receivers, so-called, may be used for air, as in the case of the coining presses of the English mint, where a vacuum chamber is used to receive the air already used for driving the machines, and kept pumped out by steam power. The ventilating apparatus for mines also often contains such negative reservoirs for air.

Reservoirs are also used for granular materials, such being extensively used in connection with grain handling machinery.

A steam boiler may be considered as a physically supplied reservoir, as well as a physical ratchet system (see § 260). A combined physical and chemical reservoir is found in the electrical accumulator, which may properly be called a current-reservoir. A combined physically and mechanically operated negative reservoir is found in the various forms of refrigerating machines.

A modern application of pressure organs, and one which is rapidly extending in use, is that of the distribution of power in cities. Following the impulse given by the introduction of the high pressure water system of Armstrong, the use of gas in motive power engines by Otto followed, and many other methods of meeting the problem have been applied.

In long distance transmissions of this sort, special reservoirs are often used, in which force may be stored, so to speak, and from thence distributed in a manner similar to the ring transmission system for rope (see § 301). In this method the pressure organ after use is returned to the reservoir to be compressed and used again, or it may be used as in the line transmission and allowed to escape at the end of the line.*

The following cases are given as applications of pressure organs in long distance transmission:

1. The London Hydraulic Power Company distributes 300 H. P. by means of water at a pressure of 46 atmospheres (675 pounds). A similar and earlier installation is in use at Hull.

2. The General Compressed Air Company distributes power by means of air at a pressure of 3 atmospheres (45 pounds) in Leeds and Birmingham. The system is an open line, and 1000 H. P. are used in Leeds, and 6000 H. P. in Birmingham.† In Paris the Compagnie Parisienne de l'air comprimé, procédés Victor Popp, distributes power from three stations in quantities varying from a few foot pounds up to 70 or 80 H. P., a total of some 3000 H. P. The use of compressed air appears to be destined to a widely extended use for this purpose.

3. The distribution of power in New York by means of steam mains is extensive and well known.

4. The vacuum system is used also in Paris by the Société anonyme de distribution de force à domicile. This is an open line transmission, operating in 1885, about 200 H. P.

5. Transmission by highly superheated water has been used in Washington, by the National Superheated Water Co., distributing heated water at pressures from 26 to 33 atmospheres (400 to 600 pounds), the water being converted into steam at the point of utilization.

6. The distribution of power by means of gas holders has already been referred to, and the distribution by electric currents is rapidly being developed.

§ 313.

Motors for Pressure Organs.

The methods of applying pressure organs to the development of motive power are even more varied as in the case of tension organs. For this reason a general view of the subject will be taken in order to obtain a classification which will simplify the discussion. The main distinctions are those of the character of the motion of the mechanism, and of the method of applying the pressure organ to the motor.

The great difference in the character of the motion of the mechanism lies in the fact that it may be either continuous or intermittent, so that the motor may be either:

A running mechanism, or

A ratchet mechanism (compare § 260). The ratchet pawls for pressure organs are the various forms of valves (see Chapter XXVI).

The various forms may also be classified according to the following important distinctions based on the method of driving.

The pressure organ may drive, or

It may be driven, or

The impelling mechanism may itself be propelled.

There is also a third distinction to be made, namely, whether the pressure organ acts merely by its weight, or whether it acts by its living force of impact. This last distinction cannot be sharply observed in practice, but is especially to be considered in discussing the theory of action of the various machines.

In the following pages the various applications will be shown in a manner similar to that employed in § 262 for tension organs, following the system of classification outlined above, and beginning with running mechanism as the simpler of the two great divisions.

A. Running Mechanism for Pressure Organs.

§ 314.

Running Mechanism in which the Pressure Organ Drives by its Weight.

With a few unimportant exceptions the motors of this class are operated by liquids, which at moderate velocities practically follow the laws of gravity.

In Fig. 957, *a* is an undershot water-wheel, and *b* is a half-

Fig. 957.

breast water. The water is guided in a curved channel and the buckets are radial, or nearly so. The wheel is so placed that the buckets pass with the least practicable amount of clearance over the curved channel. At *c* is shown a high-breast wheel, and at *d* an overshot wheel (compare § 47). In these latter wheels the buckets are so shaped that they retain the water in the circular path, being closed at the sides also, while on account of the moderate pressure they are left open above. At *e* is shown the side-fed wheel of Zuppinger.

Fig. 958, *a* is an endless chain of buckets, and *b* a similar arrangement, using disks running with slight clearance in a vertical tube.

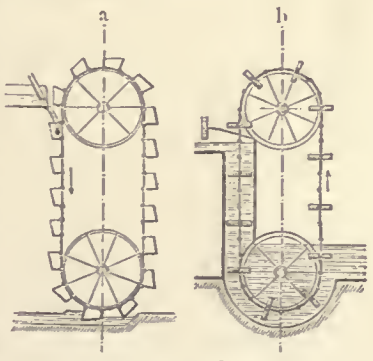


Fig. 958.

In the wheels shown in Fig. 957 the water acts on the wheel much in the same manner as a rack acts when driving a pinion, and in this sense a water wheel may be considered as a gear wheel.

When the water acts only by gravity these constructions are only practical when the wheel can be made larger in diameter than the fall of water, and where small diameters must be used the arrangements of Fig. 958 are available. Very small wheels acting under high pressures may be employed by making use of the so-called "chamber wheel work," ‡ of which some examples are here given.

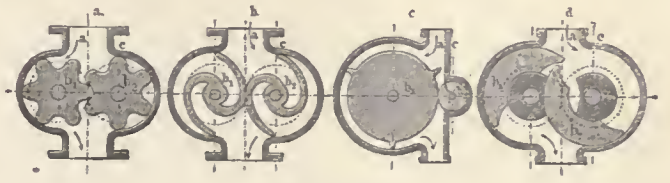


Fig. 959.

Fig. 959*a* is the Pappenheim chamber wheel train. In this the tooth contact is continuous, the teeth being so formed that the continuous contact of the teeth at the pitch circle prevents

* See a paper by the author in Glaser's Annalen, 1885, Vol. XVII, p. 226.

† See Lupton and Sturgeon, Compressed Air *vs.* Hydraulic Pressure, Leeds, 1886: Sturgeon, Compressed Air Power Schemes, London, 1886; also The Birmingham Compressed Air Company, Birmingham, 1886.

‡ See Berliner Verhandlungen, 1868, p. 42.

the water from passing, while the points and sides of the teeth make a close contact with the walls of the chamber. The downward pressure of the water enters into the spaces between the teeth and drives both wheels. The axes of the wheels are also coupled by a pair of spur gear wheels outside the case, thus insuring the smooth running of the inner wheels. This is the oldest form of chamber train mechanism known, and can also be used as a pump, operating equally well in either direction. Fig. 959 *b* is Payton's Water Meter, with evolute teeth. The flow is intermittent, but one contact begins before the action of the previous one ceases.

Fig. 959 *c* is Eve's chamber gear train. The ratio of teeth is 1 to 3, and the flow is also intermittent. The theoretical volume of delivery for all forms of chamber gear trains, whether continuous or intermittent in delivery, is practically equal to the volume described by the cross section of a tooth of one of the two wheels for each revolution.

Fig. 959 *d* is Behren's chamber train. In this case each wheel has but one tooth, as is also the case with Repsold's train (described hereafter), and the gears belong to the class of disc wheels or so-called "shield gears" (see § 211). This arrangement possesses the great advantage of offering an extended surface of contact at the place between the two wheels where, in the previous forms, there is but a line contact. This permits a sufficient degree of tightness to be obtained without requiring the parts to press against each other. Behren's chamber gear makes an excellent water motor if the impurities of the water are not sufficient to injure the working parts.

The flow of water through chamber gear trains is not uniform, and the inequality of delivery increases as the number of teeth in the wheels is diminished, hence they should be driven only at moderate velocities when used as motors, in order to avoid the shocks due to the impact of the water.

§ 315.

Running Mechanism in which the Pressure Organ Drives by Impact.

In driving running mechanism by impact, fluid pressure organs, both liquid and gaseous, may be used, as will be seen from the following examples.

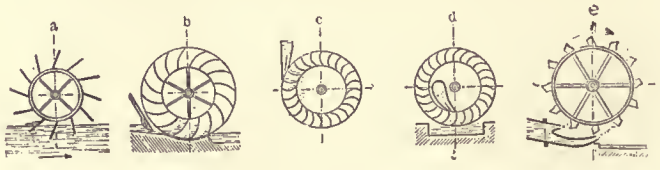


Fig. 960.

Fig. 960 *a* is a current wheel, or common paddle wheel. The paddles are straight, and either radial, or slightly inclined toward the current, as in the illustration. The working contact in this case is of a very low order.

Fig. 960 *b* is Poncelet's wheel. The buckets run in a grooved channel, and are so curved that the water drives upwards and then falls downwards, thus giving a much higher order of contact. At *c* is shown an externally driven tangent wheel. The buckets are similar to the Poncelet wheel, but with a sharper curve inward. The discharge of the water is inwards, its living force being expended. At *d* is an internally driven tangent wheel, similar to the preceding, but with outward discharge. The form shown at *e* is the so-called Hurdy-Gurdy wheel. The water is delivered through curved spouts, and this form is practically an externally driven tangent wheel of larger diameter and smaller number of buckets. This wheel, from a crude makeshift, has become one of the most efficient of motors.*

Wheels with inclined delivery as made in the forms shown in

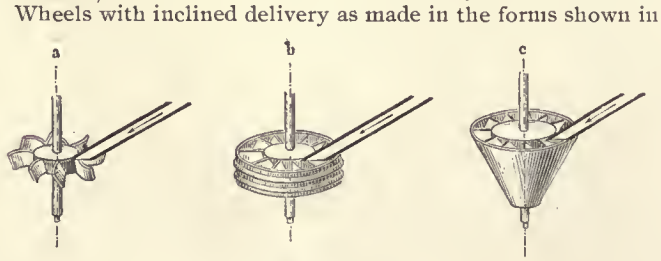


Fig. 961.

Fig. 961. At *a* is shown a crude form, used on rapid mountain streams as a simple expedient, but of low efficiency; *b* is the Borda turbine, consisting of a series of spiral buckets in a barrel shaped vessel; *c* is the so-called Danaide, the spiral buckets being in a conical vessel, this form being mostly used in France.†

In the wheels which have been shown in the preceding illustrations from Fig. 958, the living force of the water acts by direct impact through a single delivery pipe.

The following forms differ from the preceding, in that the water acts simultaneously through a number of passages around the entire circumference of the wheel. This form gives the so-called hydraulic reaction in each of the inclosed channels, and hence wheels of this class are commonly called reaction wheels, or reaction turbines.‡

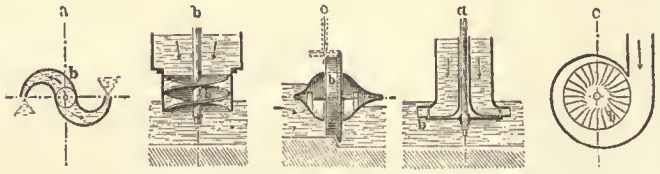


Fig. 962.

Fig. 962 *a* is Segner's wheel, the water entering the vertical axis and discharging through the curved arms; *b* is the screw-turbine, entirely filled with water; *c* is Girard's current turbine, with horizontal axis, and only partially submerged; *d* is Cadiat's turbine, with central delivery, and *e* is Thomson's turbine with circumferential delivery and horizontal axis, the discharge being about axis at both sides. In all five of these examples the column of water is received as a whole, and enters the wheel undivided until it enters the wheel; in the following forms the flow is divided into a number of separate streams.

Fig. 963.

Fig. 963 *a* is the Fourneyron turbine, acting with central delivery; the guide vanes are fixed and the discharge of the water is at the circumference of the wheel; *b* is a modification of the Fourneyron turbine, the water being delivered upwards from below, and sometimes called Nagel's turbine; *c* is the Jouval or Henschel turbine, the guide vanes *c* being above the wheel, which is entirely filled by the water column; *d* is Francis' turbine, with circumferential delivery through the guide vanes *c**; *e* is the Schiele turbine, a double wheel with circumferential delivery and axially directed discharge. In the latter three forms a draft tube may be used below the wheel, to utilize that portion of the fall, as indicated in forms *c* and *d*.

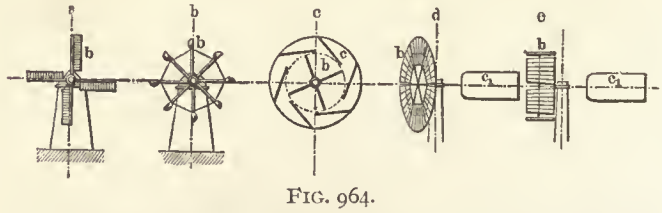


Fig. 964.

For gaseous pressure organs, of which wind is the principal example, some forms are here given. Fig. 964 *a* is the German windmill, with screw-shaped vanes; *b* is the Greek and Anatolian windmill, with cup-shaped vanes. Both forms are similar in action to the above described pressure wheels. At *c* is shown the so-called Polish windmill, with stationary guide vanes;‖ *d* is Halladay's windmill, made with many small vanes, which place themselves more and more nearly parallel with the axis as the force of the wind increases, the rudder c_1 keeping the wheel to the direction of the wind. The extreme position of the vanes is shown at *e*. Anemometers and steam turbines are examples of wheels in which other pressure organs than wind are used.

* This is the Pelton Water Wheel, built in sizes as great as 300 H. P. See Mining and Scientific Press, 1884, p. 246, and 1885, p. 21. This wheel is built in Zurich, by Escher, Wyss & Co., with a casing, and used for driving dynamos.

† See Weisbach-Hernman, Mechanics of Engineering, Part II., Section 4, p. 558.

‡ This use of the term reaction is hardly desirable for this construction, nor is the proposed name of "action turbine," and the name "pressure turbines" is to be preferred.

§ This form is well made by J. M. Voith, of Heidenheim, Würtemberg.

‖ Recueil des Machines avantageuses, Vol. I., No. 31, 1699, also from thence shown in Henning's Sammlung von Machinen und Instrumenten, Nürnberg, 1740.

§ 316.

RUNNING MECHANISM IN WHICH THE PRESSURE ORGAN IS DRIVEN AGAINST THE ACTION OF GRAVITY.

Running mechanism for the purpose of elevating liquids, and especially for lifting water, are of very early origin, and the various machines for this purpose form the very oldest of machine inventions.

FIG. 965.

Fig. 965 *a* is a bucket wheel, the vessels on the circumference lifting the water; this is driven by the power of men or animals, or in many instances by a current wheel (as in Fig. 960 *a*).* At *b* is the Tympanon of Archimedes, used down to modern times, the sections deliver the water through openings into the axis; *c* is a paddle wheel, only adapted to raise the water a small height, much used in the polders of Germany, Holland and Italy. The paddles are made either straight, or curved, or sometimes slightly crooked at the end.† At *d* is the Archimedian screw, which, when placed at an augle as shown, is well adapted to elevate water. The Archimedian screw is extensively used in all positions for the granular and pulverized materials, in which cases the outer cylinder is omitted and a stationary channel substituted, as shown at *c*, in Fig. 965 *e*, and if the transportation of material is in a vertical direction the screw is entirely surrouuded by a stationary tube. A still later form is made with a wire spiral, by Kreiss of Hamburg.

FIG. 966.

Fig. 966 *a* is the spiral pump, in which the screw of Archimedes is replaced by a channel formed in a plane spiral. In this form the inclosed air becomes compressed by the speed of revolution of the mass, and the water can be forced quite a considerable height.‡ Fig. 966 *b* is a conical spiral pump called after its inventor, Cagniard Latour, a Cagniardelle. The Cagniardelle is usually placed entirely in a trough, but the illustration shows how the end of the spiral may be modified so as to require no enlargement of the delivery channel. The diameter of the cone is adapted to the height to which the water is to be lifted. The Cagniardelle may also be used as a blower, the inclosed water driving the entrapped air before it.

The chain and bucket devices already shown in Fig. 958 as motors are also well adapted to drive the pressure organ, and are in practical use in numerous modificatious. Fig. 958 *a* is extensively used in dredging machinery, grain elevators and the like, and Fig. 958 *b* is much used for lifting water.

The various forms of chamber gear trains already described, give by inversion corresponding forms of driving mechanism, some examples of which are here given.

Fig. 967 *a* is Repsold's pump; each wheel has one tooth, the profiles being formed as described in § 207; *b* is Root's blower, the wheels having two teeth each, and the actiou being the same as the Pappenheim machine, Fig. 959 *a*. This device has been very extensively used as a blowing machine. Since the action of these machines in drawing air against pressure is similar to that of lifting water against the resistance of gravity, there is no necessity for distinguishing in classification between them as pumps for liquids or for gaseous fluids. Fig. 967 *c* is

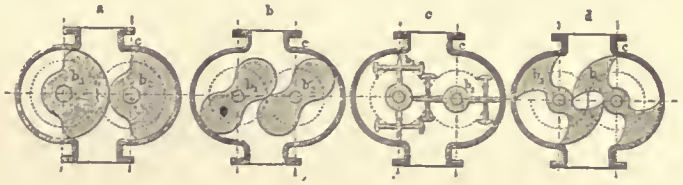


FIG. 967.

Fabry's ventilatiug machine for mine ventilation, consisting of a double-toothed combinatiou chamber train, with unequal duration of contact. Root has also used the form shown at *d*, which has unequal contact duration, and which has since been made by Greindl as a pump.§

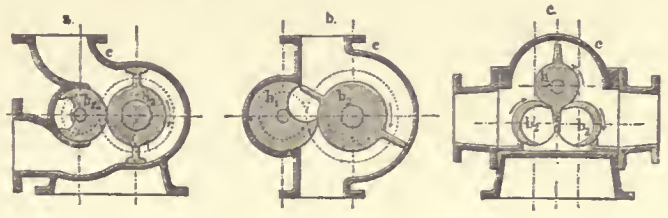


FIG. 968.

Greindl also makes the form shown iu Fig. 968 *a*, with gears of one and two teeth, and rightly claims it to possess the advantage of a greater freedom from leakage. The form shown at *b* has been used by Evrard as a blower, but it does not differ iu principle from *a*. Baker's blower, shown at *c*, is a triple chamber train, also used by Noël as a pump.

It has already been stated that Behren's pump, Fig. 959 *d*, has also been used as a steam engine. As long ago as 1867 a steam fire engine has been constructed by putting two of these machines on the same axis, one being driven by steam, the other forcing the water.

Chamber gear trains may also be used to be worked in connection. Fig. 969 shows an arrangement in which the chamber

FIG. 969.

train *A* delivers water to a distant one *B*, driving the latter and receiving the discharge water from *B* through a return pipe to be used again. The combination forms a transmission system of the second order (see § 26), and is similar to a belt or chain transmission. The loss in efficiency in this device is not an unimportant consideration.

An important class of machines consists of those made with tension organs for transporting granular materials. For this purpose belts, chains, etc., are used, aud when the transmission is horizontal, or nearly so, grain is successfully transported on wide belts.‖ Another application is that of Marolles, using au iron belt, 40 in. wide, 0.06 in. thick, for transporting mud. Twelve such machines were used on the Panama Canal work, the distance being 200 feet, and the speed of the band 12 to 40 feet, according to the nature of the material. Similar apparatus at the Suez Canal handled material at a cost of 7.6 cents per cubic yard.

§ 317.

RUNNING MECHANISM IN WHICH THE PRESSURE ORGAN IS DRIVEN BY TRANSFER OF LIVING FORCE.

The method of driving pressure organs by a transfer of living force is one which admits of numerous applications, as the following examples show.

Fig. 970 *a* is a centrifugal pump for moving liquids. The driving mechanism consists of the curved blades, which in

* Large wheels of this sort have been in use in Syria for many centuries, as at Orontes, north of Damascus. The town of Hamath, of 40,000 inhabitants, receives its water supply from twelve such wheels.

† A recent installation of such paddle wheels has been made at Atfeh, on the Mahmudieh Canal, in Egypt. Eight wheels 32.8 feet diameter, each driven by a separate steam engine lifting water from the Nile 8½ feet to the canal. The eight wheels deliver 115,000,000 cubic feet in 24 hours. See Engineer, 1887, p. 57.

‡ Such pumps, made by Klein, Schanzlin & Becker, at Frankenthal, deliver water from 2 to 30 feet, the revolutions being from 15 to 22 per minute, and diameters from 20 to 70 inches.

§ The firm of Klein, Schanzlin & Becker, at Frankenthal, make a line of pumps similar to Fig. 967 *d*, of a capacity of 1.77 to 177 cubic feet per minute, and discharge openings from 1.18 to 11.8 ins. diameter. These are driven by belt and used beer-mash oil, acids, paper pulp, syrup, etc., as well as water.

‖ An excellent transmission is in use at Cologne. See also Trans. Am. Soc. Mech. Engrs., Vol. VI., 1884–85, p. 400. At the Duluth elevator a rubber belt 50 inches wide, running 600 to 800 feet per minute, carries grain from 600 to 900 feet horizontally. A 26" belt has carried 14,000 bushels per hour.

many instances are made in one piece with the wheel itself, this adding to the efficiency. These pumps have been most successfully made by Gwynne, Schiele, Neut and Dumont among

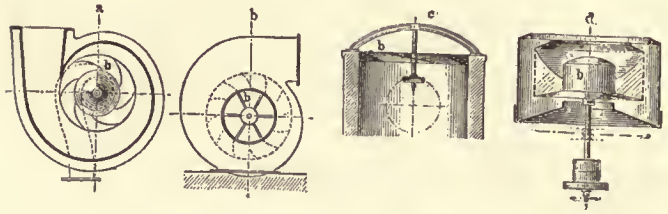


FIG. 970.

others.* Centrifugal pumps have been successfully used as dredging machines for lifting wet sand, gravel and mud, instances among others being the North Sea Caual at Amsterdam, and the harbor at Oakland, California.

Fig. 970 *b* is the well known fan blower used everywhere for producing a blast of air, and acting by centrifugal force. When used as exhaust fan this is widely used in connection with suitable exhaust pipes for removing foul air, sawdust, and other impurities in workshops, as well as for the ventilation of mines.† At *c* is shown a form of spiral ventilator, known as Steib's ventilator; it is similar to some of the preceding forms, but is of limited application, aud is better adapted for lifting water, a service to which it has been applied in the polders of Holland. At *d* is a centrifugal separator, a device of numerous applications for separating materials of different specific gravity by centrifugal force. A notable example of this machine is the centrifugal separator for removing cream from milk.

Another variety of machines for driving pressure organs by a transfer of living force, is that in which another pressure organ, either liquid or gaseous, is used instead of a wheel as the impelling mechanism. To this class belong the various jet devices, injectors, etc.

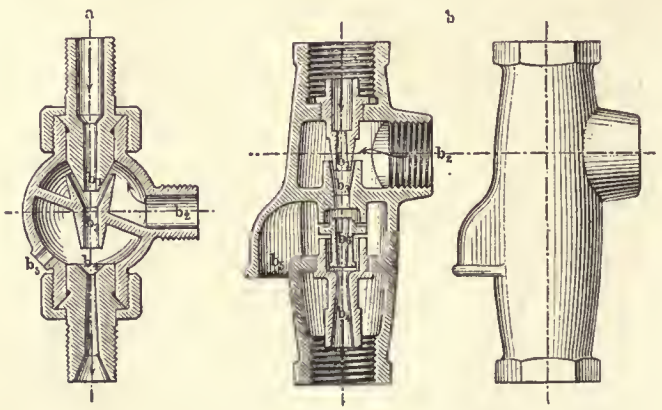


FIG. 971.

Fig. 971 *a* is Giffard's injector in the improved and simplified form made by the Delaware Steam Appliance Co. In this case steam is used to drive a jet of water into a vessel already containing water under pressure. The jet of steam rushing through the nozzle b_1 draws the water in by the suction tube b_2, and both pass through the mixing tube b_3, and are discharged through the outlet tube b_4; the outflow at b_5 provides for the relief of the discharge at starting, before the jet action is fully established. The regulation of the flow of steam is effected by a steam valve attached above b_1. At *b* is Gresham's automatic injector, which is so made that should any interruption occur in the supply of water at b_2, the suction action is automatically started, and the entering columu of water is lifted again. This is done by the introduction of a movable nozzle b_6 between b_3 and b_4, which adjusts its position with regard to b_3 according to the variations in pressure above and below.

Fig. 972 is Friedmann's jet pump. The mixing tube b_3 is divided into a number of sections, which permits a very free entrance to the water, and gives au excellent action; *b* is Nagel's jet pump, used for lifting water from foundations by means of another jet of water. The entrance jet is at b_1, the

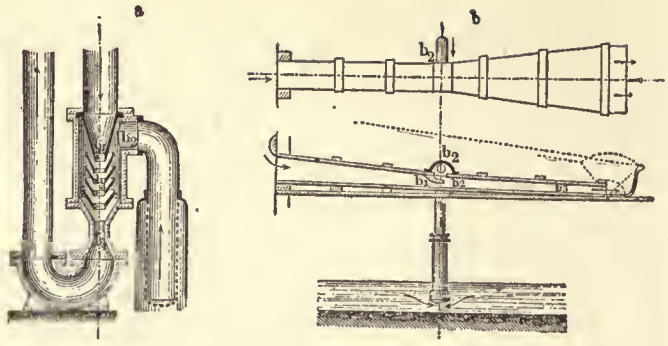


FIG. 972.

suction tube at b_2, and the mixing tube at b_3; the regulation is effected by a valve at the end of b_3.

Steam jets are also used to produce a blast of air, or compressed air may be used for the same purpose, as can also water under pressure. A reversal of the last mentioned arrangement occurs in Bunsen's air pump, in which a jet of water is used to produce a vacuum. Recent devices for utilizing jet action are numerous. Among others, a jet of air has been used to feed petroleum into furnaces as fuel. Dr. W. Siemens proposed to carry the petroleum in the hold of a vessel in bulk, and substitute sea water, as it was consumed, in order to maintain the ballasting of the ship undisturbed. Granular materials have been handled by means of jet apparatus, usually impelled by compressed air, sometimes by water jets.

An especial feature of jet pumps, and one which should not be overlooked, is that they act either by guiding the pressure organ stream, or that the driving action of the pressure organ stream itself produces a guiding action, and that the existence either of a reservoir or some external means of driving must be presupposed. The use of a pressure organ in motion for driving mechanism, is in this respect similar to the so-called inductive action of an electric current.

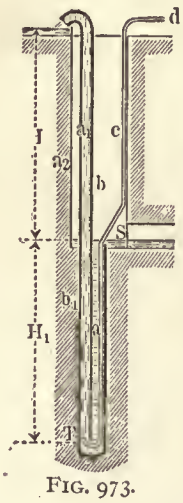


FIG. 973.

An example of pure guiding action is found in the "Geyser Pump" of Dr. W. Siemens, Fig. 973. The water is to be raised from a depth H, and the tube *b* is prolonged downward to a depth H_1 below the sump *S*. The prolonged tube b_1 is open at the lower end, and in the bottom opening T an air tube *c* is introduced, and air is admitted at a pressure slightly under that of a column of water of height equal to H_1. The air mingles with the water and forms a mixture in a_1 which is lighter than water, and the air pressure is then capable of forcing the light mixture up to the surface. The lifting action is assisted by the expansion of the ascending air. Siemens found that it was possible to produce this action when H was equal to H_1, that is, the specific gravity of the mixture of air and water = ½.

§ 318.

RUNNING MECHANISM IN WHICH THE MOTOR ITSELF IS PROPELLED.

The third division, in which the motor itself is propelled in the liquid pressure organ, contains fewer varieties than the preceding ones but is of the greatest importance since to it belongs the entire subject of marine propulsion.

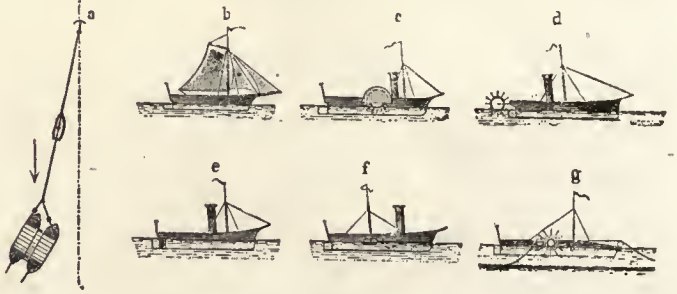


FIG. 974.

Fig. 974 *a* is the so-called "flying bridge," the current flowing in the direction of the arrow, causing the boats to swing across the stream, describing an arc about the anchor to which

* A recent installation of magnitude is that of five centrifugal pumps built by Farcot, of Paris, in 1887, for supplying the Katatbeh Canal in Egypt. The wheels are 12 ft. 6" dia., and each deliver 17,660,000 cubic feet in 23 hours, the lift varies from 1 to 12 feet.

† Fans for these purposes are made in great variety by J. B. Sturtevant. Boston, Mass.

they are held by a chain; *b*, is a sail-boat, the sail being the driving organ transferring to the boat a portion of the living force of the current of wiud. At *c*, is a steamboat with side paddle-wheels, and *d*, a stern-wheel boat; *e*, is a screw propeller. A screw driven by a steam engine pressing the water backward and the reaction of the water impelling the boat. At *f*, is a so-called jet propeller, the reaction heing produced by jets of water forced through tubes at the side of the boat, the water beiug driven by centrifugal pumps.* At *g*, is shown a current wheel motor. The side paddle wheels are caused to revolve by the action of the current, and by counection with a cable or chain gearing (See Figs. 787 and 794) the boat is propelled up the stream.

Direct acting reaction jets have been used for torpedo boats, using carbonic acid gas, hut this method has been superseded by twin screw propellers driven by compressed air. Rockets and rocket shells are examples of direct acting pressure organs.

B. RATCHET MECHANISM FOR PRESSURE ORGANS.

§ 319.

Fluid Running Ratchet Trains.

The pawls in a fluid ratchet train are the valves. They may be divided into two great classes,† similar to those existing in ratchets of rigid materials, viz.

Runuing Ratchets, or Lift Valves, and

Stationary Ratchets, or Slide Valves.

In the first class we have flap valves, also conical and spherical valves, and iu the second, the various forms of cocks, cylindrical and disc valves and flat slide valves. In both kinds of valves there exists an analogy to toothed and to friction ratchet gearing, since by use of contracted openings the effect of friction is produced, and with full openings it is ohviated. This gives a division which does not exist in the case of friction and toothed ratchet gearing.

Viewed according to the precediug classification, piston-pumps, and piston machines are properly ratchet trains.‡ This idea does not seem to offer any practical difficulties, since it can be made to include all the numerous variations without creating more confusion than the former methods of classification. It is not practicable to distinguish between the devices acting by gravity and those acting by transfer of living force, since both are frequently combined.

The oldest devices are those using air, and the oldest piston is the membrane piston, (Fig. 949) in the form of a bag of skin used as a bellows. In this primitive device the earliest valve was the human thumb, and in the larger bellows the heel of the operator, these being followed at a later date by valves of leather.§ The working part of the bag was next strengthened by a plate, (See Fig. 949 *a*.) and developed into the common bellows, next followed the disc piston, a very early improvement‖ and later the plunger, from which the numerous modern forms have grown. The following examples will illustrate.

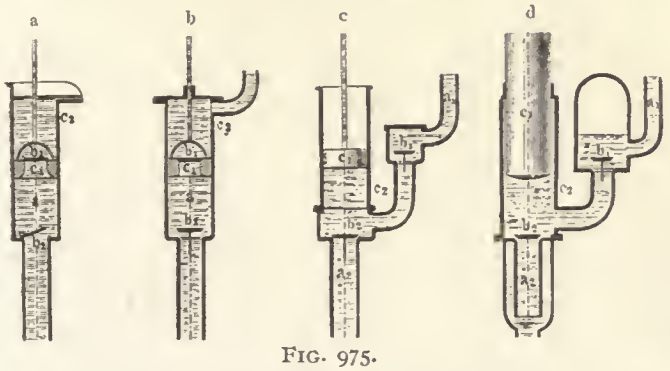


Fig. 975.

Fig. 975 *a*, is the common lift and suction pump, a ratchet train similar to Fig. 749; *a*, is the pressure organ stream (corresponding to the ratchet wheel *a*) b_2, the holdiug pawl in the form of a valve, c_2, is the receiver or cylinder for the water and piston, c_1, is a pawl-carrier in the form of the piston, b_1, the other pawl, or lift valve. The water here overflows at the top of the cylinder, and if it is to be lifted to a greater height the cylinder may be prolonged upward and the rod proportionately lengthened. If the rod is to be kept short, the form shown at *b*, is used. The top of the cylinder is closed and the rod brought out through a stuffing box, and the discharge tube only is prolonged. At *c*, is the so-called force pump with a disc piston, and at *d*, the same form with plunger. In these the discharge valve is in a separate chest. The water column *a*, is divided into two divisions a_1 and a_2, the lower heing impelled iu the up-stroke, and the latter on the dowu-stroke of the piston. A blow or shock is produced at each stoppage of the motion of the water column and to reduce this action the speed of flow must be kept down, and also the shock cushioned by means of air vessels. At *d*, air vessels are showu both on the suction and force pipes.

The preceding pumps are all single acting, discharging one cylinder of water for each complete double stroke of the piston. By cylinder of water is here meant the product of the piston area by the length of stroke.¶ The space between valves aud piston is not included, this being merely clearance or water space.

The piston may be so constructed that it remains stationary and the cylinder slides upon it, this forming an inversion of the common form and possessing many applications.

Fig. 976 *a* is Muschenbrœck's pump (1762) for moderate lifts, *b*, is Donnadieu's pump for deep wells, especially adapted for

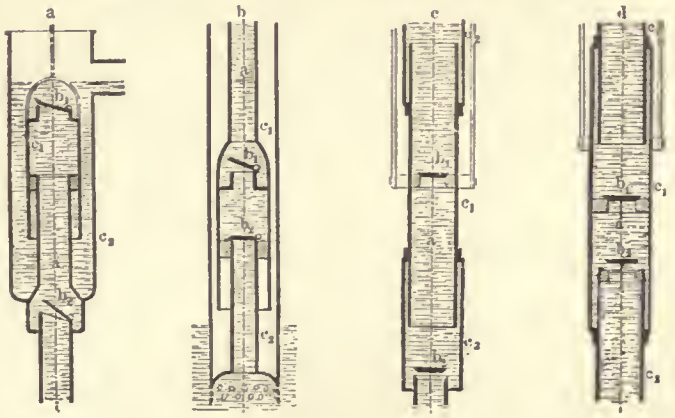


Fig. 976.

artesian wells.** This latter form possesses the peculiarity that cylinder and discharge pipe move, and the piston is stationary while action is not chauged. (See Fig. 749) At *c*, is Althaus so-called telescope pump, which does not differ from Fig. 975 *a*, except that the piston is longer and is operated by two side rods instead of a single central one.†† The form at *d*, is a modification of *c*, with external packing.

In the pumps shown in Fig. 975 *a*, *b*, and Fig. 976 *a*, the piston rod plunges into the water on the downward stroke and hence acts as a piston, lifting water by its displacement. On the upward stroke the water flows into the space again, and so the volume of delivery is not altered but a slight portion of the delivery takes place on the down stroke. This action can be utilized, however, as was very early done in mine pumps, by increasing the diameter of the rod, or forming it into a plunger so as to cause the delivery to be divided equally between the two parts of the stroke. This form may be called a double delivery pump, or briefly a double pump, since it is practically two pumps, using the same set of valves. Some examples follow.

Fig. 977 *a*, the plunger c_2, is connected to the piston c_1, the latter being twice the diameter of the former, this being the so-called "differential" pump. In *b*, two plungers are used, both valves being in separate chests ‡‡ At *c*, two telescopic pistons are used, this being by Rittinger, and well adapted for a mine pump. The form shown at *d*, has an auxiliary piston and cylinder parallel to the main cylinder, (designed by Trevethick in 1802.§§

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* Used by Von Seydell in the Albert in 1856; by Ruthven in the Waterwitch, 1866, and recently in torpedo boats by Thornevcroft.

† See the author's Theoretical Kinematics, p. 459, *et seq.*

‡ This treatment of the subject was first published by the author in Berliner Verhandlungen, in 1874, p. 228 *et seq.*, but had previously been used in his lectures since 1866.

§ Contrary to Wilkinson and Ewbank, the bellows shown in the Egyptian wall paintings have not flap valves, but the inlet opening is closed by the heel of the workman, and the bellows used to-day in India use the heel or thumb of the operator as an inlet valve.

‖ See Belidor, Arch hydraulique, Paris 1739, II., p. 62.

¶ In small and medium sized pumps the loss of cylinder capacity diminishes with the increase of speed. Experimental researches show

at 27 to 40 strokes per minute	92	per cent.	
" 50 "	"	95	"
" 60 "	"	98	"

of the theoretical capacity (Konig, Pumps, Jena, 1869). In very large pumps the momentum of the water shows an increase over the theoretical capacity; the pump in the Beryberg mine, 1 metre diameter giving 4 per cent. excess. See Portfenille John Cocquerill.

** See Poillon, Traité th. et prat. des pompes, Paris, 1885, Plate 27.

†† See the design of the Spaniards, Barnfet, Viciauain & Poillon, Plates 33 and 34, and p. 193.

‡‡ See Poillon, Plate 7, Saigun Waterworks.

§§ See Ewbank's Hydraulics, New York, 1870, p 280.

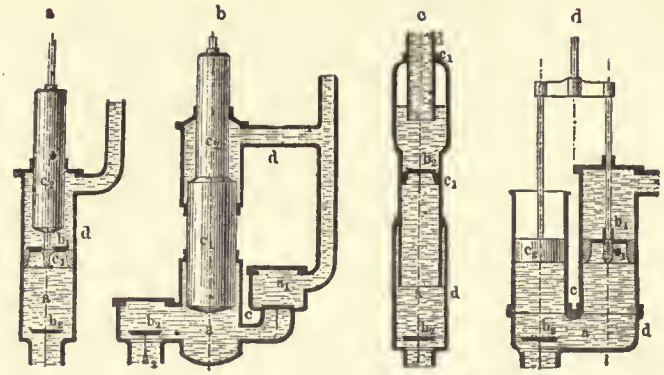

FIG. 977.

By making the suction valve also a moving piston, both the water columns may be kept in motion for both movements of the rod. This is a double acting ratchet mechanism (Fig. 750,) and hence also a double acting pump.

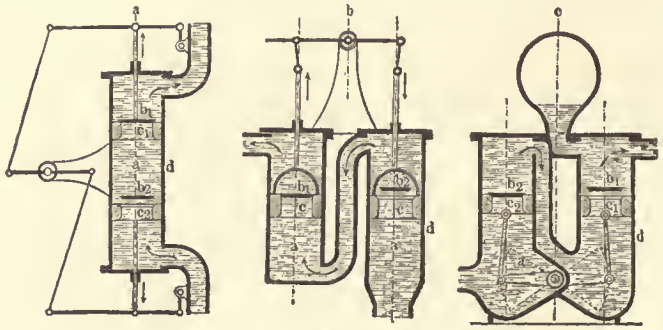

FIG. 978.

Fig. 978 *a*, is a double acting pump with two opposing valved pistons, described by Fourneyron, but much older; this corresponds to the ratchet work of Fig. 750 *a*.

The pumps shown in Fig. 978 *b*, and *c*, are similar, the first by Stolz, the second by Amos & Smyth.*

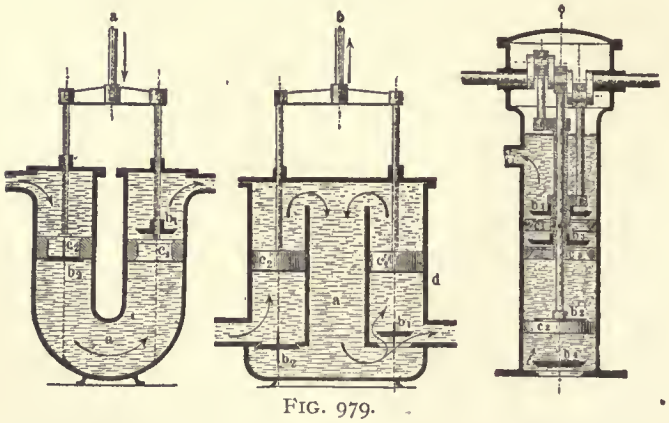

FIG. 979.

Fig. 979 *a*, is Vose's pump, in which the two pistons are placed parallel to each other. This corresponds to the Lagarousse ratchet, Fig. 750 *b*. Similar double acting pumps may be made with solid pistons, if it were desirable; the form of Fig. 979 *b*, designed by the author, being an example, and others might readily be devised.†

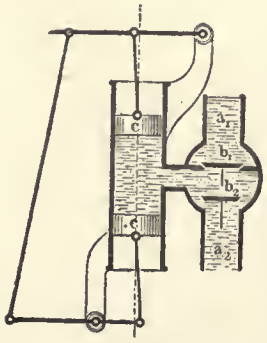

FIG. 980.

Fig. 979 *c*, is Downton's pump. The three pistons c_3, c_2, c_3, keep the water in constant flow, which is further assisted by the air chamber. The foot valve b_4, may be omitted if desired. The annexed sketch of a pump by Lippold, (See Bach. Fire Engines, Stuttgart, 1883, p. 41,) is not double acting but contains practically one piston split in two, and equivalent to one of half the area and same stroke, or two of the same area and half stroke. This is also the case with Franklin's Double Pump, (See Konig, p. 55).

* See Theoretical Kinematics, p. 462. † See Poillon, Plate 29.

By combining two complete fluid ratchet trains in such a manner that they have a common cylinder and piston, a form of pump is obtained which gives two full discharges for each cycle, and which may hence properly be called a double acting pump.

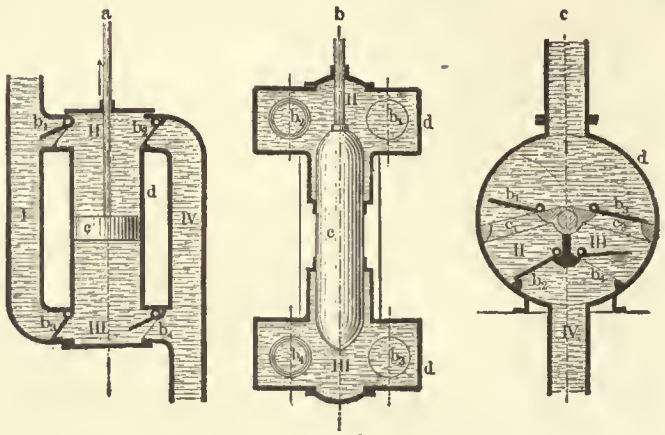

FIG. 981.

Fig. 981 *a* is a double-acting pump with disk piston, and Fig. 981 *b*, the same form with a plunger. In both cases the suction pipe is at IV, and the discharge pipe at I. In double-acting pumps it is usually not convenient to put a valve in the piston; this is, however, done in Fig. 981 *c*, in which we see two single-acting pumps combined in one.

In Fig. 982 *a*, is shown Stone's Pump,‡ which is much used

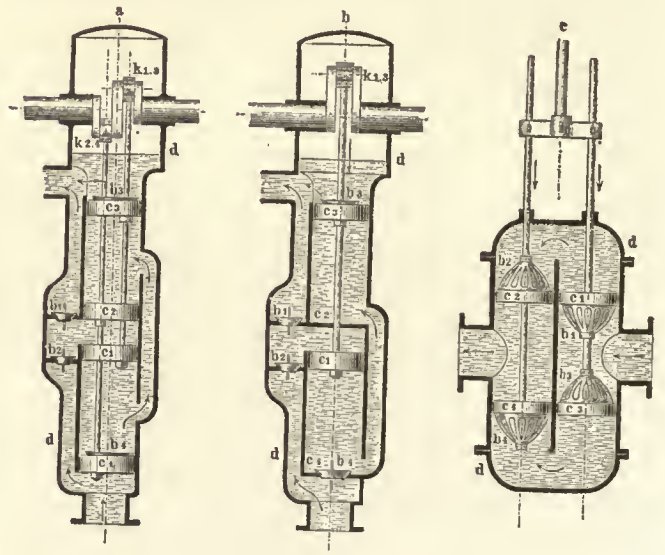

FIG. 982.

for ships, as is also Downton's Pump. In this case there are four pistons, operating in two cylinders, the latter being placed one below the other on the same axis. The pistons c_1 and c_3 are connected by one rod and connected by the same crank *k* 1.3, and the other two pistons are, in like manner, connected and operated by the crank *k* 2.4, which is set opposite the other crank. The action may be more readily understood by examining Fig. 982 *b*, which is similar to the preceding one, if we suppose the pistons c_2 and c_4 to be held stationary and the other pair c_1, c_3 driven by a single crank of double the length of arm of those shown. This will obviously not alter the volume of delivery, and it will be evident that the lower pump is really a double-acting force pump and the upper one a single-acting lift pump, hence each revolution of the cranks will deliver three cylinders of water, two on the up stroke and one on the down stroke. In Stone's pump the pistons c_2 and c_4 are so disposed that for each half revolution ½ cylinders of water are discharged, and in other respects the pump is a double-ratchet tram. Fig. 982 *c* is Audemar's Pump. In this form two double pumps similar to Vose's Pump (Fig. 979 *a*) are combined to make a double-acting pump.§

‡ See Poillon, Plate 26.
§ See Poillon, Plate 6, p. 93.

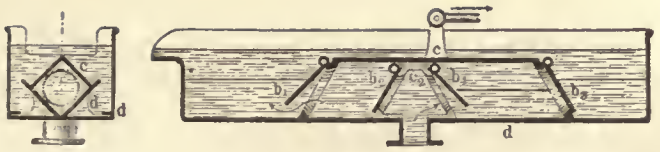

FIG. 983.

Fig. 983 is Norton's so-called V shaped pump. In this device the pistons c_2 aud c_4 form a single stationary piece, and the cylinder and valves b_1 and b_3 is the moving part. It will readily be seen how easily the lift pump may be made double-acting.

A double-acting lift pump as used for a steam engine air puuip, by Watt, is shown in Fig. 984. This is practically a combiuation of two different pumps. It has three valves, the foot valve b_2, piston valve b_1 and upper valve b_3. On the downward stroke the mixed air, water aud vapor passes through the piston from the lower to the upper part of the cylinder, and on the up stroke this is discharged through b_3 and a fresh cylinder full drawn in through b_2. This pump is double acting, since the piston valve acts both in the up and down stroke. This works the same whether pumping liquid or gaseous fluids, the action being the same as if two valves only were used. The upper valve is required for other reasons, *i. e.* to control the discharge, as for boiler feediug, etc.

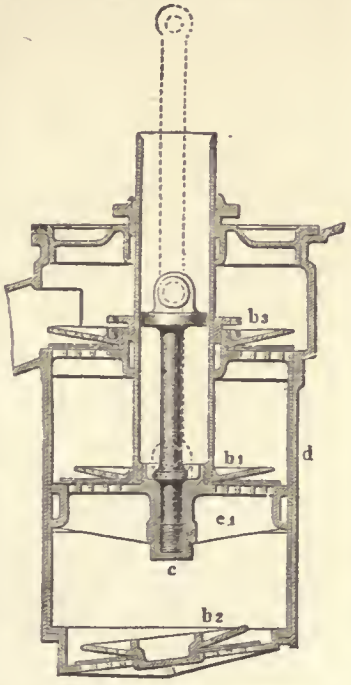

FIG. 984.

The preceding examples will serve to illustrate the application of fluid ratchet trains with ruuuing ratchets. It is important in all cases, and especially with the higher velocities, that provision should be made to have the valves close without shock, or in other words, that the engagement of the pawls should be quiet. This problem has already appeared in some forms of ratchet mechanism (see § 240) and here offers still greater difficulties, especially when heavy moving masses are to be controlled. The question is daily being considered in practical problems of construction* and a great variety of valves has been designed. The present indications appear to be leading toward the use of valves operated mechanically by the pump, instead of those operated by the fluid itself, but a final solutiou of this problem has not yet been reached.

§ 320.

FLUID RATCHET TRAINS WITH STATIONARY RATCHETS.

As already shown in § 255, it is necessary, in ratchet trains with locking teeth, to effect the engagement and disengagement of the pawls by some additioual mechauism. This is also the case in those fluid ratchet trains which used stationary pawls, *i. e.*, slidiug valves. An example is found in the case of the simple single-acting air pump used iu physical laboratories, which siuce its invention by Otto von Gerike† has been made with statiouary pawls, and is shown in a crude form in Fig. 985. The "receiver" d', and its pipe connection forms a negative reservoir, the pump $a\ c\ d\ b_1\ b_2$ a ratchet train for the propulsion of the column of air a. The suction valve is at b_2, and the discharge valve at b_1, both being in the form of stop cocks.

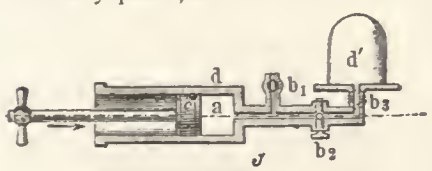

FIG. 985.

The suction valve b_2 is operated by hand when the piston is drawn out, and when the end of the stroke is reached the valve b_1, which had previously been closed, is opened, and the first one closed, and the air expelled on the return stroke. A stop cock, b_3, is also placed close to the receiver.

There is but little difficulty in applying slide valves to single-acting pumps, and they are also readily arranged for double-acting cylinders. By examining the arrangement of flap valves in the compress double-acting pump, Fig. 986, it will be seen that the valves b_1 and b_4 open and close simultaneously, and that the same is true of b_2 and b_3, and that the two actions alternate with each other. The operation of the valves is such that the four spaces *I* to *IV* are conuected alternately in the order *I–II* aud *III–IV*, and *I–III* and *II–IV*. From this it will be seen that if sliding valves are used they may all be connected together, or uuited in the same construction. This may be done as shown in Fig. 987 *a*, which represents the so-called "four-way" cock. As here shown, all four of the passages are closed, this position correspondiug to the end of the piston stroke. When the plug is turned 45°, as shown by the dotted lines, *I* and *III* are connected, and also *II* and *IV*; and if it is turned the same amount in the other directiou, *I* and *II* and *III* and *IV* are

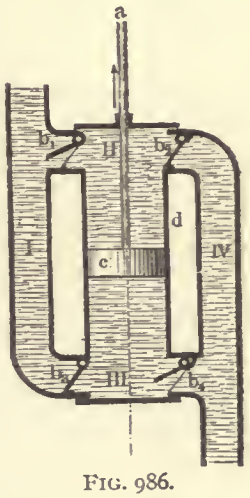

FIG. 986.

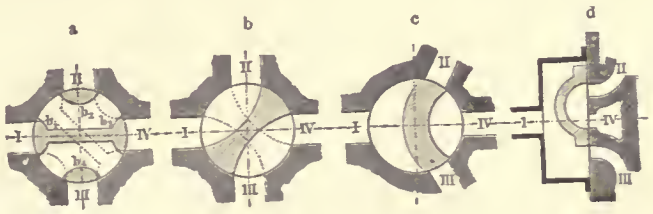

FIG. 987.

connected. The portions b_2 and b_4 may be omitted, as in Fig 987 *b*, and the passages *II*, *IV* and *III* brought closer together, as shown at *c*. From this form it will readily be seen how the passage *I* can be converted into a mere delivery pipe, and the radius of curvature of the bearing surfaces, made of infinite length, giving the well-known slide valve, Fig. 987 *b*. In like manner other forms may be developed. It must not be forgotten that this device really consists of four valves combined in one, and in fact recent forms of steam engines contain the four valves made separately, these often again being lift valves.

A noteworthy peculiarity in the forms shown in Fig. 987 *a* and *d* must be considered. In both instances the valve overlaps the port on both sides, this being technically known as "lap." It is also apparent that the lap on the two sides of one port may differ, and that different laps may be used for different ports. By use of this expedient the opening and closing of the ports need not be simultaneous, but may occur successively.

From the preceding considerations the following propositions may be laid down; the latter applying to all, and the former to uearly all, lift valves:

The application of slide valves in all fluid ratchet trains depends upon two priuciples:

1. The combination of several valves into one piece.
2. The coutrol of the time of action of these valves by means of the lap.

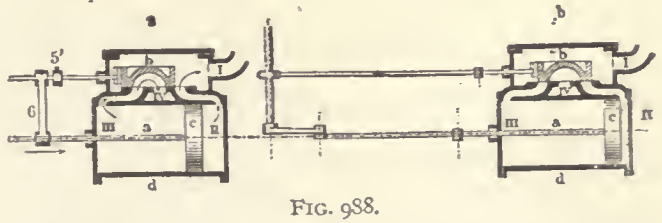

FIG. 988.

The application of a slide valve to a pump is shown in Fig. 988 *a*. In this case *I* is the discharge outlet, and *IV* the suction connection. In such pumps it is necessary to provide some mechanism to operate the valve, and such mechanism is termed the "valve gear." This valve gear may be arranged in a great variety of ways.

A simple form of gear is that shown in the figure, 988 *a*, in which an arm 6, attached to the piston rod, moves the valve by striking against tappets 5′ and 5″ on the valve stem. This

* See Fink. "Konstruction der Kolben-und Zentrifugalpumpen," Berlin, 1872; also Bach, "Konstruction der Feuerspritzen."

† This name is spelled as given above in the earliest records, and not "Guericke," as is often given.

arrangement is similar to the locking ratchet of Fig. 753. It has the defect, however, of requiring the piston to move rapidly, or else the valve will not be carried past the middle position, and the pump will stop. This defect can be met by using a trip gearing device such as shown in Figs. 742 and 743, to continue the condition of the valve when started by the impulse of the piston rod.

A somewhat simpler method is that in which the reciprocating motion of the pump rod is used to revolve a shaft by means of a crank, Fig. 988 *b*, from which the valve may be operated by means of a return crank or eccentric. This arrangement is often used, especially for blowing engines, etc.* It will be apparent that a four-way cock device, Fig. 987, may be arranged so as to be operated by continuous revolution, instead of a reciprocating motion, and hence the eccentric may be omitted and a rotary valve device substituted.

In Fig. 988 *b* the crank and crank shaft are used merely for the purpose of actuating the valve gear. It is practicable, however, when a crank is once admitted, to use it still further as one of the parts of the pump, such as in chamber trains. Many such devices haae been proposed,† although but few of these have been put to practical use. The three following devices will illustrate.

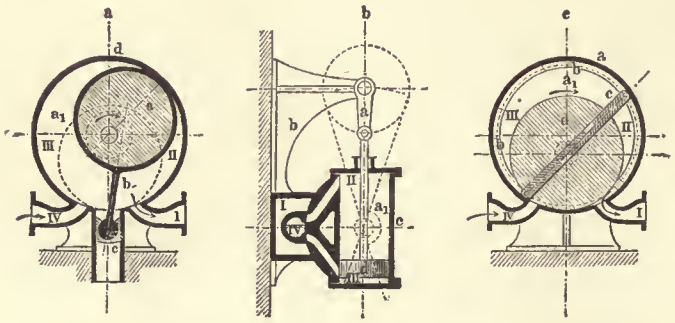

FIG. 989.

Fig. 989 *a* is Pattison's pump, a form of chamber-crank train. The crank *a* here assumes the form of an eccentric, the rod *b* becomes a flat pistou, the edges of which form a tight joint with the ends of the cylindrical chamber *d*. In the position shown in the illustration the spaces *II* and *I* and *III* and *IV* are in communication. In the dotted positions *III* is connected with *I*, followed again by *II* and *I* and *III* and *IV*. This transfer of communication is produced by the action of the crank, and hence no other valves are necessary.

The form shown at Fig. 989 *b* is made with an oscillating cylinder. The piece *c*, which plays an inconspicuous part in Fig. 989 *a*, is now used for the chamber, and its oscillating motion with regard to *b* supplies the necessary valve action. Oscillating pumps are used in a variety of forms.

Fig. 989 *c* is Beale's gas exhauster, made with a so-called "sliding crank" *c*, which acts at the same time as crank and piston. Without the use of special valves, the spaces *II* and *III* interchange with *I* and *IV* by the revolution of *d*. Beale's exhauster is in successful and extensive operation in various gas works.

In the examples cited and in the numerous modifications of them, it will be noticed that the checking or ratchet action of the liquid is invariably performed by slide valves.

One of the objections to the use of slide valves for ordinary water pumps is the wear upon the surfaces due to impurities in the water. When the water is free from such objectionable impurities, it is to be considered whether slide valves might not be much more generally employed than has hitherto been the case. If this form of valve were given the benefit of practical study and experience, it ought to be possible to avoid the shocks due to concussion existing in pumps made with lift valves when operated at high speeds.‡

A great number of valve forms have been designed, § using combinations of single valves on the principle of the multiple ratchet (see § 242), the action of the valves being assisted by weights, springs, etc., but these have not completely attained the desired end. ‖

When the pump is used for pure water, as for drinking supply, the question of wear upon slide valves is not so important as with pressure pumps. A fair comparison can hardly be made, however, between pumps with slide valves and those with lift valves, as the former have been but little used and also not practically designed.

It is a matter of surprise that when occasional applications of slide valves are made in pumping machinery, that such devices should be considered as something new. The difference between the action of water and air is well known, and yet even with the slight weight of an air column the shock in blowing machines is most apparent. It can hardly be supposed that the other form would remain uninvestigated.

The pumps shown in Fig. 989 *a* and *c* are commonly known as rotary pumps, which title is manifestly incorrect, since in form *a* there is an oscillating piston which does not rotate, while in form *c*, notwithstanding the rotary motion the action is similar to form *a*. Other so-called rotary pumps have been devised with curved piston action, some of these being as early as the 17th century. In some designs a radial slide acts in the pump case as a ratchet, and is drawn in and out by a cam of appropriately curved profile. A large number of rotary pumps have been made on this principle, many of which will be found in Poillon's treatise. These pumps are usually made with metallic packing only, and are used in Italy and France for pumping wine and olive oil; they are also adapted for brewery pumps.

The undeniable predilection in favor of rotary pumps on the ratchet train principle is worthy of consideration. It is claimed that they have a higher efficiency, but this remains to be established; also the rotary motion gives a continuous uniform motion to the water column, but this is equally accomplished by the forms shown in Figs. 982 and 989. This uniform flow can only be approximately attained, as must be the case from the nature of the mechanism. The principle is that of a ratchet train which is intermittent in principle, and hence differs from a continuous running movement. The idea that such pumps give a continuous and uniform discharge is due to the fact that the column of water is operated directly from the part which is driven continuously, but this by no means follows. This combination of a continuous running motion, with an intermittent ratchet action which is not apparent to the eye, will be shown in other cases hereafter.

§ 321.

ESCAPEMENTS FOR PRESSURE ORGANS.

Ratchet trains found with pressure organs also include escapements as completely as is the case with the preceding forms of rigid ratchet mechanism. The ratchet of § 258, shown again in Fig. 990 may be considered as an escapement if we assume the checking of *a* by *b* to be uniformly opened and closed.

If now, in Fig. 991, the checked member *a* is made a pressure organ, such as water, in communication at *H* with a pressure reservoir, or with a negative reservoir at *T*, or both, the regular lifting and closing of the valve *b* produces an escape-

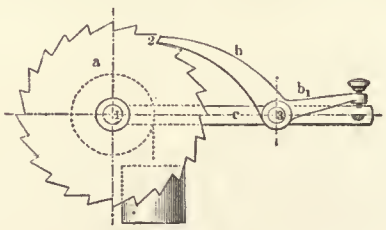

FIG. 990.

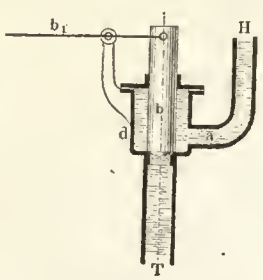

FIG. 991.

ment acting in a similar manner to Fig. 990. By means of such a device the pressure organ *a* can be constrained in performing mechanical work. The range of such an escapement is not determined by the teeth of a wheel, but on the contrary, is similar to a friction ratchet, and can be varied at will.

The applications of escapements with fluids are in principle the same as those formed of rigid bodies, but in practice their nature is very different. We have already distinguished between watch escapements and power escapements, and in the present instance the power escapements are by far the most important. For this reason the latter will be considered first. Unperiodical escapements are shown in the simple form of Fig. 991, in which the time of releasing and checking is regulated by hand; a form very seldom found in rigid escapements. Periodical forms, similar to watch escapements, are used with pressure organs for measurement, but not for measurement of time, but

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* See Zeitschrift Deutscher Ingenieure, 1885, p. 929; also Herrmann's Weisbach's Mechanics, Vol. III, Part 2, p. 1089.

† See the author's "Theoretical Mechanics," in which over 90 chamber crank trains are described and analyzed.

‡ Poillon refers to the fact that the automatic action of mechanically operated slide valves enables high speeds to be obtained with less noise than when lift valves are used, but also notes the wear of the slide valves as an objection to their use.

§ See Riedler, Zeitschrift d. Deutscher Ingenieure, 1885, p. 502 *et seq.*

‖ See Bach, "Konstruktion der Feuerspritzen," also in Zeitschrift d. Deutscher Ing., 1886, p. 421.

of volume. To these we may add the adjustable escapements on the principle of those described in § 259, and we have the following classification:

a. Unperiodical Power escapements.
b. Periodical Power escapements.
c. Adjustable Power escapements.
d. Escapements for measurements of volume.

A. UNPERIODIC POWER ESCAPEMENTS FOR PRESSURE ORGANS.

§ 322.

FLUID ESCAPEMENTS FOR TRANSPORTATION.

One of the simplest practical applications of the principle of Fig. 991 is Felbinger's Postal Tube, shown in diagram in Fig. 992. The line tube *d* is connected with a reservoir of compressed

FIG. 992.

air at *H*, and at *T* with a similar negative reservoir. At *b* is a sliding pawl, here shown open; the piston, or carrier *c*, in the form of a leather box containing letters, telegrams, etc., being driven through the tube. A valve *b'* enables the end of the tube to be thrown into communication with a second negative reservoir, and this mechauism can be arrauged at both ends of the line so that the tube cau be used for transmission in either directiou. Such postal pneumatic tubes are well known and widely used.*

An atmospheric escapement operated by a negative reservoir is found iu the so-called "atmospheric railway," invented by Pinkus in 1834, and put into practical operatiou somewhat later in England by Clegg and Samuda. This was operated on the Kingston-Dalby road with a vacuum of 1/6 atmosphere in the exhausted receiver, but it is no longer in operation.

When an escapement is intended to control the back and forth movement of a piston in the same path, the single valve shown in Fig. 991 is not sufficient, but at least a second must be used, as is already indicated in Fig. 992. One of the most practical of all fluid escapements is found in the lock used on canals and shown in diagram in Fig. 993.

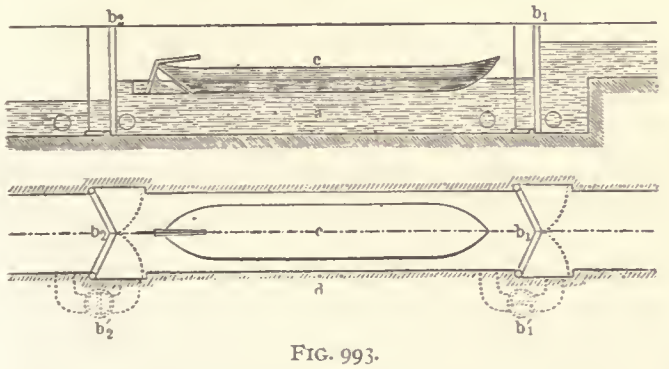


FIG. 993.

The canal is open on the upper side (see Fig. 945 *b* and *c*); the valves b_1 and b_2 are of the running ratchet form, and are in reality double gates. Smaller by-pass valves b_1' and b_2' are used in order to enable the inlet and outlet of the water to be started gradually. The boat *c* forms the piston, and when the motion is upward, b_1 is the escapement valve, and when downward, b_2 is used.

The above canal lock device, while extremely useful, possesses a very low efficiency, since it not only uses a volume of water equal to the displacement of the boat plus the necessary clearance, but also discharges the whole lock chamber of water each time it is used. Later devices have been made for the same purpose, involving a less waste of water. If it is arranged for the service to be doubled by making two lifts adjacent to each other, it is evident that the descending boat can counterbalance an ascending one of the same weight, the only requirement being that there must be some connecting mechanism involving the overcoming an additional resistance, aud capable of a reversal of 180°. This may be accomplished either by the use of tension organs or pressure organs.

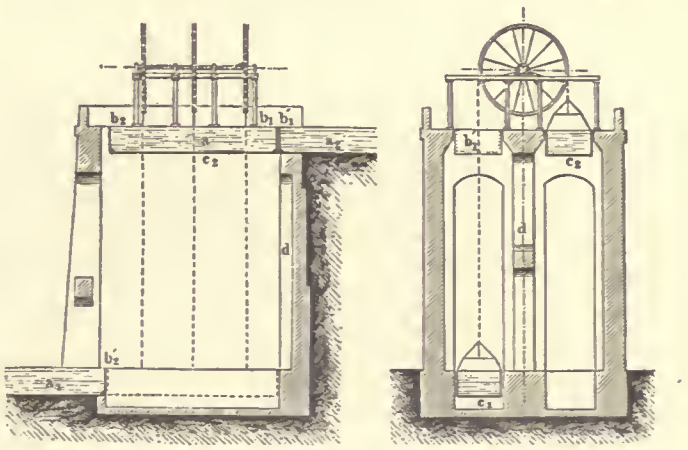


FIG. 994.

Fig. 994 shows a double canal lift constructed by Green for the Grand Western Canal in England in 1840, the connecting mechanism being tension organs in the form of chains. The boats are carried in tanks c_1 c_2, the ends of which are closed by valves or gates b_1 and b_2, and similar gates b_1' and b_2' also close the ends of the canal sectious. A small addition to the weight on the descending side is sufficient to raise the other tank †

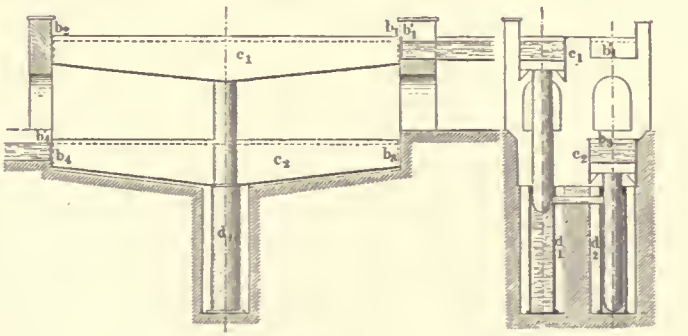


FIG. 995.

The substitution of a pressure organ for the chain was first made by Mr. Edwiu Clark on the Mersey Caual in 1875, in the form of a hydraulic lever, as shown iu Fig. 995. This shows clearly the equivalence of the cord or chain and pulley aud the water lever, already referred to in § 311. The tanks c_1 and c_2 are carried on plungers 3 feet in diameter, and are 75 feet long and 15½ feet wide. A head of 6″ of water is sufficient to overcome the resistance of motion, and a lift of 50 feet is effected in three miuutes.‡ Smaller installations have been made by Clark and by Stanfield, and other large ones at the La Louviere Canal in Belgium, and the Neufosse Canal at Les Fontinettes, iu France. The lifts are 43 ft. and 50 ft. respectively, and the plunger diameters 6½ feet. The loss of water with these lifts is only about $\frac{1}{300}$ of the quantity used by common locks of the same capacity. §

The preceding escapement devices are made for open canals, but escapements may also be constructed with closed tube connections. This latter type includes numerous hydraulic elevators for lifting burdens of all kinds.

An example of a direct-acting hydraulic elevator is given in Fig. 996. The two valves are combined in one cock. The water under pressure enters at *H*, and the discharge against the atmospheric pressure is at *A*. The weight of the plunger is counterbalanced by two counterweights *G* with chains and

* In 1887 the length of the postal pneumatic tubes in Berlin was over 26 miles.

† See Weisbach-Hermann, Vol. III., Part 2, p. 633.

‡ See Duer Trans. Inst. C. E., 1876; Colyer, Hydraulic Machinery, London, Spon., 1881, p. 17; also Robinson, Hydraulic Machinery, London Griffin & Co., 1887, p. 64.

§ See Colyer, p. 29; Robinson, p. 69: also Zentralbl. der pr. Bauverwaltung, 1882, p. 395; Hensch, Schiffshebung in Frankreich; also Schemfil, Kanal and Hafenwerkzeuge in Frankreich und England, Wien, Gerold, 1882, p. 15; also Ernst, Hebezeuge, Berlin, Springer, 1883, p. 630. In Green's lift the loaded boats descended and the empty ones ascended, hence an excess of water was raised, which was permitted to overflow. These lifts enable much greater differences of level to be overcome than do the ordinary locks, and make it practical to use long stretches of canal and make an entire lift at one operation. It may be here noted that pneumatic lifts for canals were designed in 1863 by the Swiss engineer. Seyler.

pulleys, and the plunger operates the valve automatically by means of the rod b', when the highest position is attained. This form of lift has been much used, sometimes of very large dimensions. The great passenger elevator of the Hamiltou St. Station of the Mersey Tunnel has a plunger 18″ in diameter, with a lift of 87½ feet, the car holding 50 passengers.*

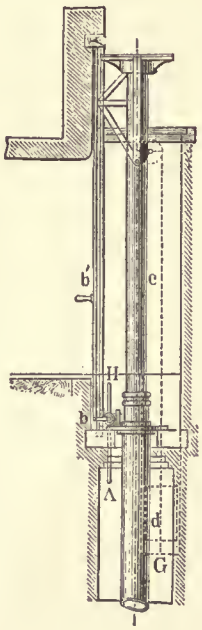


FIG. 996.

A practical objection to direct-acting lifts of this form lies in the heavy counterweights required, and also in the depth to which the cylinder must be sunk. A different form has therefore been designed in which a piston travel of moderate length is multiplied by use of a tension organ system, such devices being extensively used for passenger elevators, notably by the Otis Elevator Company.

Hydraulic cranes are also forms of high pressure escapements, first designed by Armstrong, and since used by many others, especially in connection with Bessemer Steel plaut, in which hydraulic cranes have proved most valuable.

Fig. 997 shows the mechanism of a hydraulic crane by Armstrong. The piston is double acting, and there are four valves b_1, b_2, b_3, b_4, of the type shown in Fig. 986, the external connections also being necessary in order to complete the escapement. The high pressure water enters at H, and passes through the pipe I, and is discharged to the atmosphere at IV. The rod c_2 is made of half the area of the piston e_1

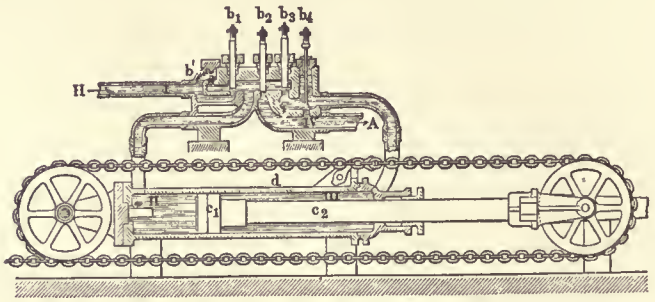


FIG. 997.

(compare Fig. 946 e). When b_1 and b_3 are open, as in the illustration, the forward stroke is made with one-half the full force; when b_1 and b_4 are open, the forward stroke is made with full force. By opening b_2 and b_3, the return stroke is made by the pull of the load upon the chain. At b' is a safety valve which comes into action should the load descend too rapidly, by the opeuing of b_3 alone.*

§ 323.

HYDRAULIC TOOLS.

Hydraulic escapements, similar to those used for lifting loads are also applicable to machine tools. Among these may be noted the devices of Tweddell, for rivetiug, punching, bending, etc. (see § 54).

Figs. 998 and 999 show the arrangement of Tweddell's riveting machine; d is the piston, b_1, b_2 the valves, one of which connects with the pressure reservoir at H, and the other with the atmosphere at A. When b_1 is opened by the lever e, the hydraulic pressure enters above the piston d, and the stroke is made. The return stroke is effected by means of the auxiliary piston d_1, which is fast to d, and under which the water pressure is acting at all times. Closing b_1, and opening b_2, enables this to act and lift the main piston. This gives practically a hydraulic lever of unequal arms, the shorter arm always being loaded with H, and the load on the longer arm varying between H and A. The lever mechanism d', d'', d''', controls the length of stroke of the die, by meaus of the tappets d'' and d''', which are connected with the lever e. This is also used on the lift of Fig. 996, and shows the complete escapement. The arrangement of valves is shown in detail in Fig. 999.‡

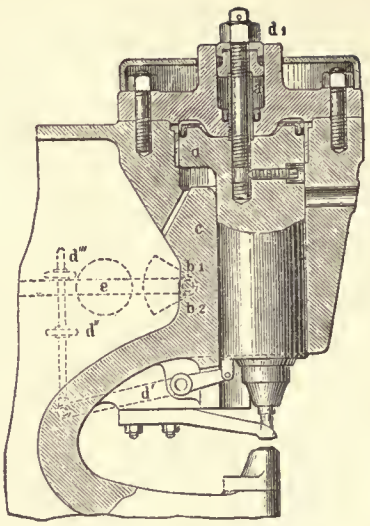


FIG. 998.

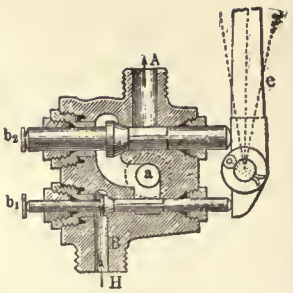


FIG. 999.

The preceding apparatus resembles the hydraulic press. It is in fact quite different, being a genuine ratchet train, capable of all the modifications of such mechanisms as to speed, distance, and arrangement. On account of these points the applications of pressure organ escapements are becoming rapidly more important.

§ 324.

PRESSURE ESCAPEMENTS FOR MOVING LIQUIDS.

The use of unperiodic pressure escapements for moving liquids in machine construction has been practiced from au early period, and at the present time improved devices for this purpose are much used.

Au almost forgotten device of this kind is Brindley's boiler feeding apparatus, Fig. 1000, this being based upon the principles already giveu in Fig. 991.

The necessary openiug of the valve b is made by the float c, and the closing by the counterweight c_1 (compare Fig. 950). This apparatus was first applied to Watt's boilers, the feeding of the boilers of Newcomen's engines being effected by a cock operated by the attendant.

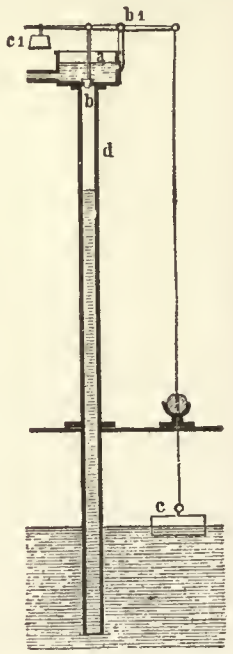


FIG. 1000.

Fig. 1001 is Kirchweger's steam trap for the removal of water of condensation. The escape valve b is opened by the float c, which, in this instance, is open at the top, so that the water flows over the rim until it sinks, and thus opens the valve, This valve motion is in itself a ratchet train, checked and released by the action of the float. When the valve is opened the water in the float is forced out by the pressure of the steam.§

The slow moving float device, as in Fig. 1000, has also beeu advantageously used for operating steam traps, by Tulpin, of Rouen; Handrick, of Buckau; Püschel, of Dresden; Dehne, of Halle, and others. Similar escapements have been designed to separate air from steam, or air from water, as in the devices of Andral, Kuhlmann, Klein and others.‖

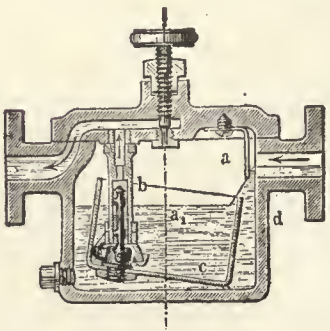


FIG. 1001.

Other examples of escapements of this kind are found in the so-called Montejus, used for elevatiug syrup iu sugar refineries, in the return traps of steam heating systems, and in various other forms of boiler feeders, such as those of Cohnfeld, Ritter & Mayhew, and others.¶

* See Robinson.
† See Weisbach-Herrman, III., 2, p. 240; Colyer, p. 11; Robinson, p. 52.
‡ For fuller descriptions of Tweddell's machine see: Proc. Inst. C. E. LXXIII., 1883, p. 64; Engineer, July, 1885, p. 88; August, p. 111; Revue Industrielle, 1884, p. 5; 1885, p. 493; Mechanics, 1885, p. 272; also Robinson, as above, and Zeitschr. Deutscher Ing., 1886, p. 452.

§ This form of trap is made in many varieties, the one shown being by Losenhausen, of Dusseldorf. A similar one by MacDougal is much used in England, and a feed pump on this principle is made by Korting in Hanover; German Patent No. 36, 332.
‖ For illustrations of these devices see Scholl's Führer des Maschinisten, 10 Edit., p. 493.
¶ See Scholl, p. 235.

B. PERIODICAL PRESSURE ESCAPEMENTS.

§ 325.

PUMPING MACHINERY.

Periodical fluid escapement trains have a wider application than unperiodical trains, since it is practicable, as already shown, to use a fluid ratchet train to operate the valves in a simple manner. This makes it possible to produce the opening and closing of the valves in a periodical succession mechanically, instead of by the fluid column. In this construction the fluid column may therefore drive the piston, instead of being driven by it. This idea seems very simple, and yet pumps had been known for two thousand years, and had occupied the inventive energy of the preceding centuries before the simplest forms of the modern steam engine were devised. It is therefore all the more important in the study of machine design to investigate the fundamental principles involved.

It is impossible, in the limited space which can here be given, to go into this subject in its entirety; the arrangement of the valve gear of the Newcomen engine with tumbling bob gear, is an instructive example.

In Fig. 1002 is shown Belidor's single acting water pressure engine.*

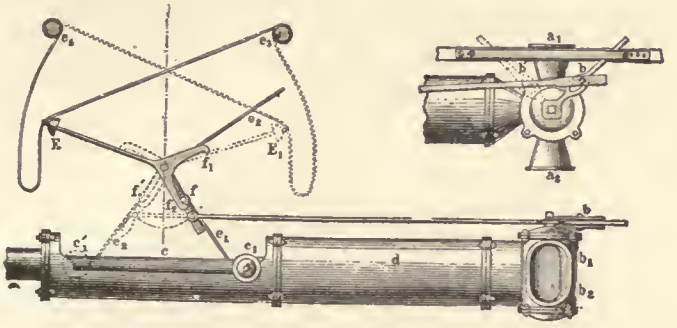


FIG. 1002.

In the cylinder d is a piston; a_1 is the entrance of the water, a_2 the discharge outlet. The valves b_1 and b_2 are united in a three-way cock (see Fig. 987). This valve is operated from the pistou rod c by a tumbling-bob gear (see Fig. 742). The tumbling lever $E\,e_1\,e_2$, weighted at E, is connected with the piston rod at c_1, and moves about its axis independently of the lever f. When the end of the piston stroke is nearly reached, the lever E passes the middle point, and tips over, when the arm f_1 strikes the lever f and carries it to the position f', moving the lever of the three-way cock from b to b'. The arm e_1 is behind E. The return stroke of the piston moves the arm e_2 of the tumbling gear towards the right, and as the end of the stroke is reached, the tumbling bob is again tripped, and the three-way cock moved again into the position b. A cord secured at the ends to the points e_3 and e_4, and fastened to E, limits the travel of the latter. The piston rod is connected directly to the pump to be operated.†

It will be observed that this machine is a ratchet train of the second order, the piston and valve forming an escapement, and the valve gear a releasing ratchet train each operating the other.

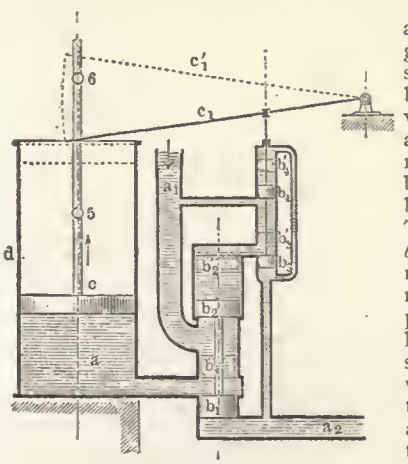


FIG. 1003.

Fig. 1003 is the single acting water pressure engine of Reichenbach. Instead of using a tumbling bob gear to operate the valve, Reichenbach uses a second water escapement, operating the valve by a piston, the valve being itself a piston valve. The double piston valve $b_3\,b_4$ of the second escapement is operated by the main piston rod, the tappets 5 and 6 striking the lever c_1 as each end of the stroke is reached. The water under pressure enters at a_1 and is discharged at a_2. The tappet 5 moves the auxiliary valve into the position $b_3'\,b_4'$, which places the space above b_1 in communication with the discharge, and since b_2 is larger than b_1, the pressure between them moves them into the position $b_1'\,b_2'$. This puts the main cylinder in communication with the discharge, and the piston sinks by the weight of the load upon it. At the close of the stroke the tappet 6 moves the arm c_1' into the position c_1 again, and places the auxiliary valve in the first position and a new stroke is made.‡

This machine constitutes an escapement of the second order, since the small and large escapements alternately release each other; the lever device 5-6-c_1 forms a third mechanism, so that the machine, as a whole, is of the third order.

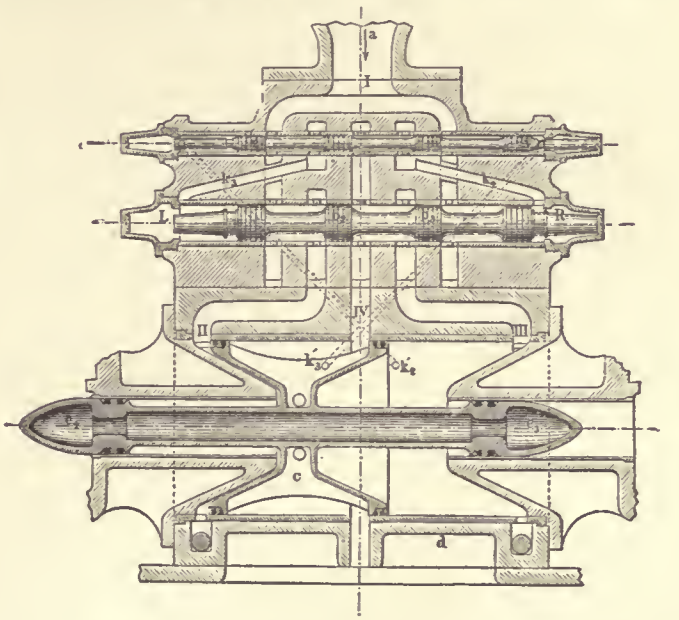


FIG. 1004.

Fig. 1004 shows the double acting water pressure engine of Roux.§ The double action is obtained by combining the four valves in one, and by communicating the admission and discharge alternately with both sides of the piston. In this case the lever connection c_1 is replaced by an escapement. The small pistons $b_2'\,b_3'$ are acted on at the outer ends by the pressure water through the small passages k_2', k_3'. This gives an escapement of the third order. The cup-shaped ends c_2, c_3, of the main piston c form the pump plungers. This machine should operate satisfactorily.

It is readily apparent that the piston steam engine may also be considered as an escapement. The valve gears differ from the preceding forms only on account of the conditions of expansion and condensation. These are reducible to a limited number of simple cases.

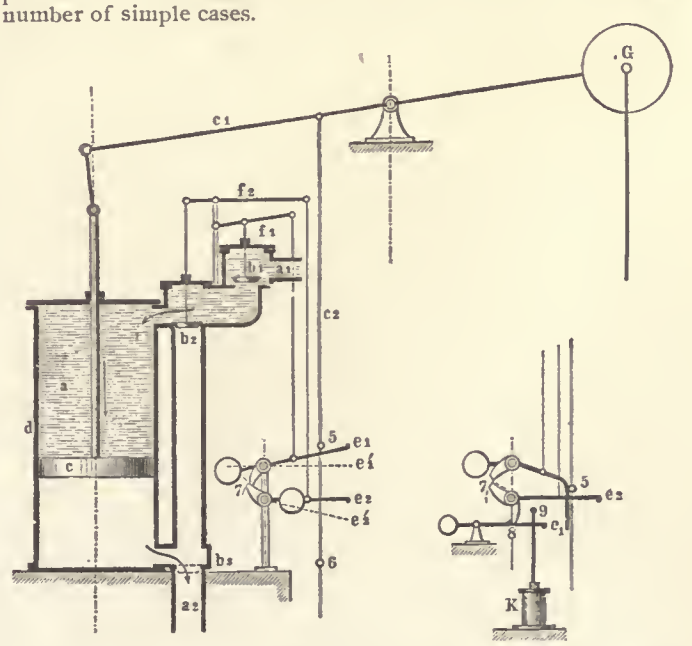


FIG. 1005.

Fig. 1005 is a single acting high pressure engine. The steam

* Belidor, Architecture hydraulique, Paris, 1739, Vol. II., p. 238.

† The above described machine, designed by Belidor in 1737, for the water works at the bridge of Notre Dame, does not appear to be altogether practicable. It has been here given on account of the valve motion, which is historically interesting and doubtless good, and has been re-invented several times since. It was not new in 1737, having been in Newcomen's steam engine, as was already known to Belidor, since it is described by him in the same volume of his treatise.

‡ See Weishach-Herrman, II., 2, p. 536; also Ruhlmann, allgem Maschinen Chre., I., p 348

§ See Revue Industrielle, 1884, p, 114. Built by Crozet et Cie.

enters at a_1, and the discharge to the atmosphere is at a_2. The opening of the valve b_1 permits the steam to enter, forcing the piston c down, and raising the weight G. The valves b_1 and b_2 are operated by a ratchet train released by the tappets 5 and 6 on a rod moved by the main piston. The pawls are double-acting, and are of the form shown in Fig. 671. When c reaches the bottom of the cylinder the tappet 5 releases the ratchet 7, and closes the valve b_1 by means of the connections f_1 e_1. The release of 7 opens the valve b_2 by means of the connections e_2 f_2, and permits the escape of the steam from below the piston. This equalizes the pressure above and below the piston, from which the valve b_2 is called the equalizing valve. The upward stroke of the piston causes the tappet 6 to reverse the ratchet 7 and operate the levers $e_1 f_1$, closing the equalizing valve and its connections.

The device differs from the preceding in that the principal escapement a b_1 b_2 c d changes in character with the stroke. The two ratchet trains can be seen in principle in the double acting tumbling gear of Fig. 1002. The mechanism, when lifting the valve, is of the third order, and when closing, of the second order. The gear as shown is Farey's; Fig. 779 shows this principle in a rigid escapement train, the corresponding form in single-acting train is the chronometer escapement, Fig. 769.

If the engine is a condensing one, a condenser valve b_3 is added, this being opened by the closing of b_2, as is also a jet valve in the condenser. When the steam is to be expanded, the lever e_1 is so arranged, the closing of b_1 is produced earlier (see the smaller diagram) by the position of the tappet 5, and the corresponding counterweight lifted. This only operates the ratchet 7, and f_2 is released by a second train 8, which is effected by the tappet rod or by the so-called cataract K, released by a tappet 9, see § 260.

The condenser is a negative reservoir, and was the principal invention of Watt. It involves the use of two fluid ratchet trains; the air pump, and the cold water pump, and also usually includes a boiler feed pump. The entire engine is composed of a collection of ratchet trains.

Steam pumping engines are by no means always made with lift valves, and a great number of more recent designs are made with slide valves (see Fig. 987). Rittenger has applied slide valves successfully to single-acting engines, and they are especially applicable to double-acting non-rotative engines. In the last decade especially have valve motion for steam pumps with slide valves been multiplied, and some illustrations are here given.

Fig. 1006 is Tangye's direct-acting steam pump. The steam

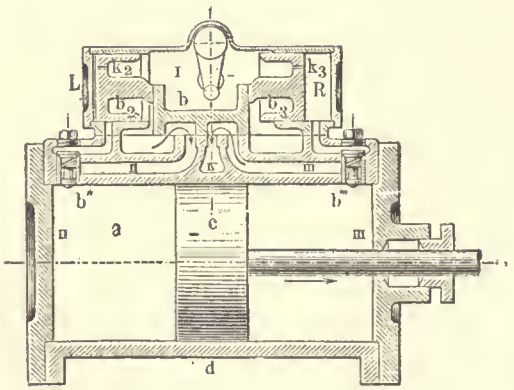


FIG. 1006.

entrance is at I, and the exhaust at IV. The slide valve b is the so-called E form, combining the four valves of Fig. 986 in one; b_2 and b_3 are the auxiliary pistons to move the valve, and form part of an escapement of which the valves b'' and b''' are operated by the main piston c at each end of its stroke. The latter valves communicate with the cylinder posts II and III. When b''' is lifted by the piston, the space R is in communication with the exhaust, and the pressure in L throws the valve over, equilibrium being soon after established through the aperture k_2. The reverse action occurs on the return stroke. This is a steam escapement of the second order, with an independent starting lever, the whole forming a combination of the third order. This has been much used by Tangye for steam pumps.

Fig. 1007 shows the valve motion of the Blake pump, which is very extensively used in the United States. In this case there is a movable seat b_o under the valve b, the opening through the seat always being in communication with the posts II, III, IV, although b_o is moved a short distance at each end of the stroke by tappets on the piston rod. In the position of the parts shown the steam entering at I will pass through III and move the main piston to the left, as indicated by the arrow. Just before the end of the stroke is reached the seat b_o is moved as much to the left of the centre as it now stands to the right. In

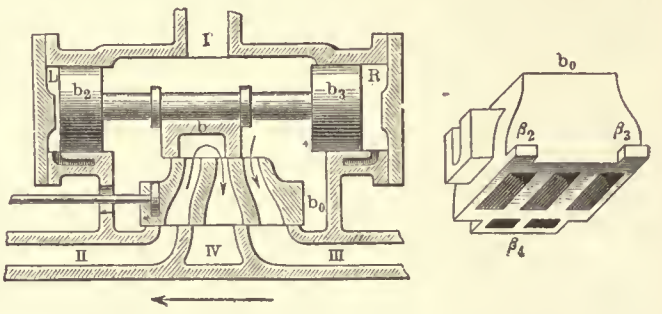


FIG. 1007.

the seat b_o, as shown in the figure to the right, there are additional valves formed, β_2, β_3, β_4, which act to operate the auxiliary pistons b_2, b_3, under which latter the small steam passages can be partly seen. When b_o is moved to the left, a small post is uncovered by β_2, and live steam enters the cylinder L behind b_2, while at the same time β_4 connects R with the exhaust. This causes b_2, b, b_3, to move to the right and reverses the pump. The reverse action takes place at the other end of the stroke, the whole forming a combination of the third order.

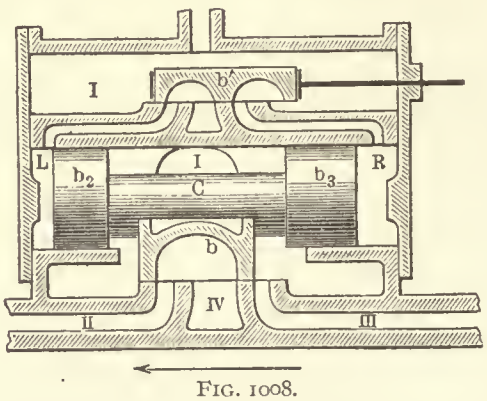


FIG. 1008.

Fig. 1008 shows the valve gear of Deane's steam pump, which has also been extensively used. The main valve is moved by means of auxiliary pistons, as in the preceding instance. The auxiliary pistons are controlled by a separate valve b', which itself is operated by lever connections with the main piston rod. This combination b', b^2, b, forms again a mechanism of the third order.*

If the last three devices described are compared with the Reichenbach water-pressure engine, it will be seen that the fundamental principle is the same in all. The constructive arrangements which may be adopted are clearly shown in the preceding examples, which may be modified in a variety of ways. Among other widely used arrangements, that of Knowles may also be mentioned; in it the action of the auxiliary pistons is controlled by a slight twisting motion given to the valve stem.

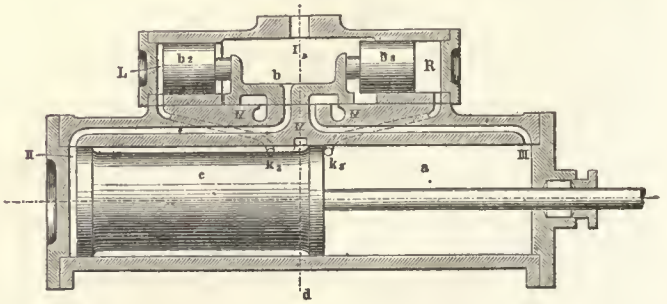


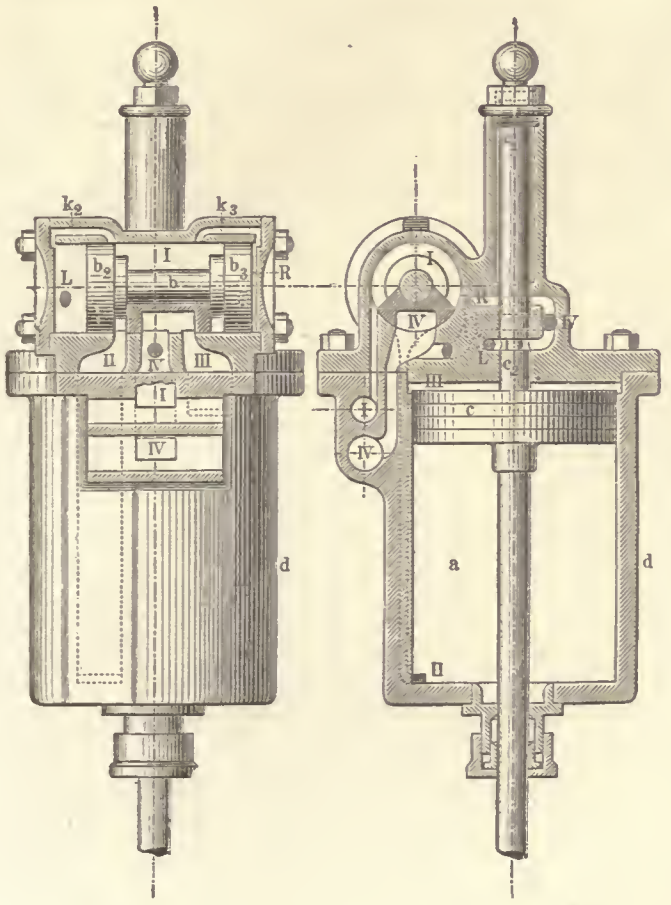
FIG. 1009.

Fig. 1009 shows Pickering's steam pump.† In this design the main piston c acts also as the valve for the auxiliary pistons b_2, b_3, so that the spaces R and L are placed alternately in com-

* See Am. Machinist, Feb. 17, 1883, p. 4. For an excellent steam pump by Dow, see Mining and Scientific Press, 1885, March, p. 169, and May, p. 313.
† See Poillon.

munication with *I* and *IV*. The whole forms a steam escapement of the second order.

Fig. 1010 shows Harlow's valve gear, also used for pumping machinery.* This is also a steam escapement of the second order, similar to the preceding. The valve action for the auxiliary pistons is formed in a prolongation of the piston rod, the grooves c_1 and c_2 placing the spaces *R* and *L* alternately in communication with *IV*.



59*

FIG. 1010.

By comparing the preceding designs with the water pressure engine of Roux, Fig. 1004, the similarity will be apparent. All the examples given show the fundamental relation existing between these devices and the mechanical escapements of watch movements. The escape wheel is replaced by the fluid column; the anchor, by the valve; the vibrating member, whether pendulum or balance wheel has here not a free movement but a determinate one against an external resistance. Similar arrangements include steam hammers, also hammers and rock drills, usually driven by compressed air, these latter consisting of mechanism of the second, rather than the third order. An example will serve to illustrate the general arrangement of such devices.

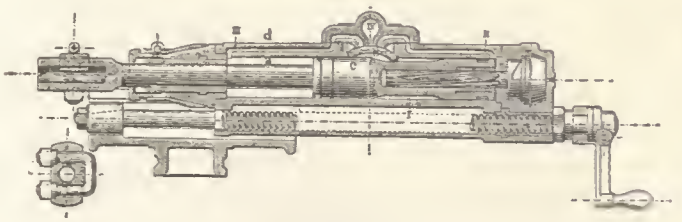


FIG. 1011.

Fig. 1011 shows the arrangement of Githen's rock drill.† The curved valve *b*, is operated by the action of the curved outline formed in the piston *c*. The middle position of the valve is a dead point, but this is overcome by the momentum of the heavy piston.

The devices of the third order are capable of a very important modification which can be considered by examining for instance the Deane gear, Fig. 1008, or either of the two preceding it. An inspection will show that it is entirely practicable to use the auxiliary piston to operate a pump cylinder, as independently of that operated by the main piston *d*. It is only necessary to make it larger in diameter and of proper length of stroke; and there is nothing to prevent making it of the same diameter and stroke as the main piston.

The valve of each cylinder will then be operated by lever mechanism connected to the rod of the other piston. This arrangement involves the replacing of the *E* valve by the common *D* valve, which is not important, but is nevertheless an advantage. The two escapements are conveniently placed side by side for constructive reasons, and the double arrangement is known as a "duplex" machine, this term being given to two combined cylinders, of which the valve of each is operated by the piston movement of the other. This type is now frequently met, having been made for small apparatus very early, in France by Mazelline and yet earlier, in 1859, in the United States by Worthington.

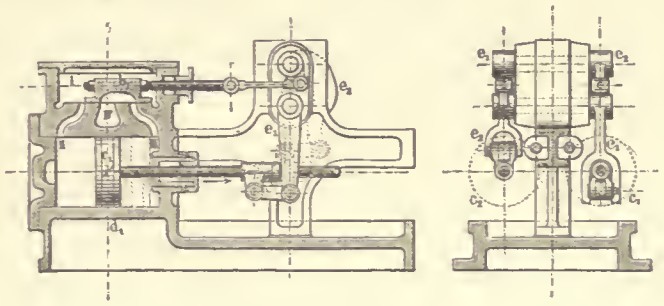
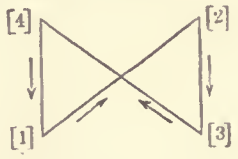


FIG. 1012.

Fig. 1012 shows a duplex pump by Mazelline.‡ The illustration shows one piston c_1, at mid-stroke with its valve b_1, at the end of its travel, and connected to the rod of the other cylinder by the lever e_2.

The work is divided into two portions which is provided for by the doubling of the parts. If the two piston escapements (cylinders, pistons, valves, steam, etc.) are indicated by [1] and [3], and the valve movements by [2] and [4] the action will be as shown in the following lines,



whence we have

[1] [2] [3]

and [3] [4] [1],

both being of the second order.

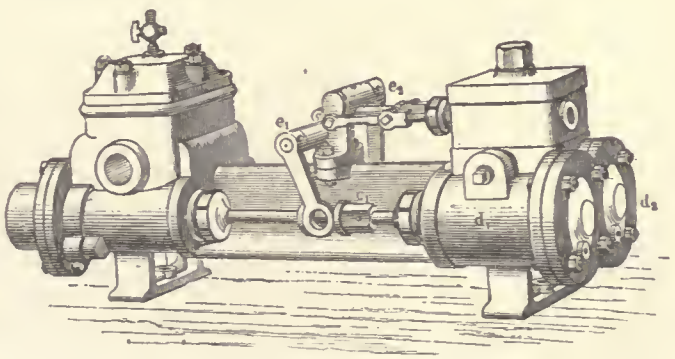


FIG. 1013.

Fig. 1013 shows a perspective view of Worthington's Duplex Pump, the arrangement of which is apparent from inspection. The duplex steam cylinders are at the right, and the double acting pump cylinders on the left.

The advantages obtained by using this form of pumping machine practically outweigh the objections which might be made against the duplication of parts. In double acting pumps of the forms shown in Figs. 1006 to 1010, the motion of the water columns is interrupted, at low speeds, at each reversal of

* See Engineering and Mining Journal, Oct., 1884, p. 23

† See Eng. and Mining Journal, March, 1887, p. 107. Also Halsey's rock drill, Trans. A. S. M. E., 1884–5, p. 71.

‡ See Poillon, Plate IX

the piston, while with the duplex pump the discharge is practically continuous, because each cylinder begins its stroke just before the other comes to rest.

An objection to all the other forms of direct acting pumps already described lies in the fact that to obtain uniform pumping action it is necessary to carry the initial steam pressure for the entire stroke of the piston, or in other words, the best action of the water end is obtained by means of the least economical action of the steam cylinders.

This defect was overcome in the earlier pumping engines, such as the Cornish mine engines, by using the steam to lift heavy weights, pump rods, etc., the living force of the mass permitting an early cut-off and high expansion, and the uniform descent of the weight being used to force the water. By this method the Cornish engines attained a high degree of economy. This method being single acting, caused the entire column of water to come to rest during the time required for the up stroke of the pump rod, and hence the Cornish type of pumping engine gives a most economical action of the steam at the expense of a defective action of the pumps.

In the larger sizes of Worthington pumping engines the expansion of the steam has been for a long time effected by using compound cylinders, and excellent results attained in steam economy. The efficiency, however, was by no means so high as was desired. In 1886 the so-called Worthington equalizer was introduced with a view of enabling the desired high duty to be attained.

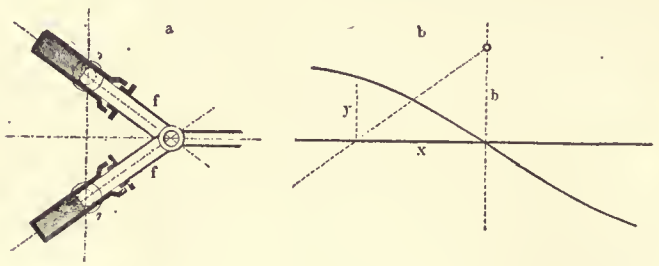


FIG. 1014.

This device, shown in Fig. 1014, is a ratchet train of the tumbling type, similar to that shown in Fig. 743, the springs being replaced by water pressure from a high pressure air chamber.* The air chamber forms a periodical storage reservoir.

The plungers f, f, are attached to a cross-head connected to the prolonged piston rod, and the cylinders are carried on trunnions 7, 7. During the first half of the stroke the plungers are forced into the cylinders the latter swinging about the centres 7, 7; and during the second half they are forced out by the action of the stored energy.†

The resistance and assistance which the pistons f give to the steam piston is shown by a curve of the form of Fig. 1014 b, as has also been obtained by the indicator.

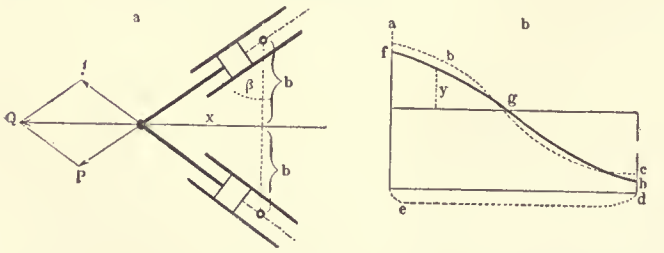


FIG. 1015.

If in Fig. 1015 a, we make P equal the component on each portion of the pressure Q on the main piston rod, we have:

$$Q = 2P \sin\beta = \frac{2P\tan\beta}{\sqrt{1+\tan\beta^2}}$$

in which

$$\tan\beta = \frac{x}{b}$$

This gives

$$Q = \frac{2\frac{x}{b}P}{\sqrt{1+\left(\frac{x}{b}\right)^2}}$$

or if we make Q the ordinate y, of the desired curve:

$$y = \frac{2Px}{\sqrt{x^2+b^2}}$$

and substituting c for $2Px$ we have

$$\frac{y}{c} = \frac{x}{\sqrt{x^2+b^2}} \quad \ldots\ldots\ldots (317)$$

which equation is readily expressed graphically.

If this curve is drawn upon the rectangle which represents the resistance of the water, as in Fig. 1015 b, we get the actual resistance curve $f\,g\,h$, and this resembles closely the expansion line for a high degree of expansion, or in other words, the impelling force and the resistance are practically made equal to each other throughout the stroke. The dotted curve $a\,b\,c\,d\,e$, is that of an actual indicator diagram.‡ This shows that with the Worthington high duty pumping engine the most efficient action of the steam is obtained at the same time as the best action of the water end.§

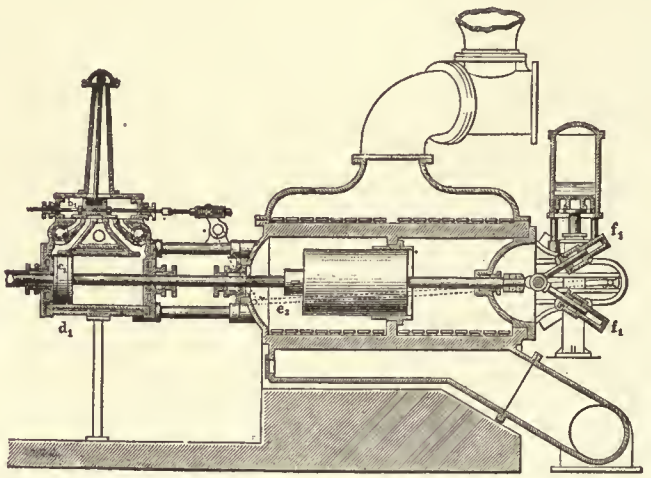


FIG. 1016.

Fig. 1016 shows a longitudinal section of a Worthington high duty pumping engine. The equalizing cylinders and their air chamber are seen on the right; the dotted lines e_2 show the rod of the second cylinder, which operates the valve b_1.

As it has already been seen that many forms of the third order can be reduced to the second order, it may be inquired as to the possibility of obtaining a pumping mechanism of the first order. This has already been accomplished by uniting the steam escapement with a water ratchet train. The device is the Hall Pulsometer, shown in diagram in Fig. 1017.

The steam enters at a, at b, is the anchor shaped pawl, and d, is the vessel corresponding to the framework of a rigid escapement, (compare Fig. 775). If the vessel d is closed as shown by the dotted lines and a volume of water c, included, we obtain an action of the first order. The efficiency is very low; about ¼ to ⅓ that of a piston pump, but the simplicity and convenience is so great that this may often be neglected.

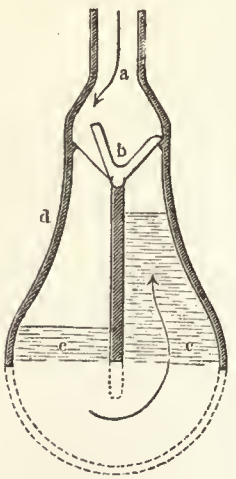


FIG. 1017.

Another escapement of the first order is Montgolfier's hydraulic ram, which is a water checking-ratchet train, the efficiency of which is low. A more recent device is the application of a water ratchet train to drive a pneumatic ratchet train, first used on a large scale by Sommeillier in the construction of the Mont. Cenis tunnel, and by means of which the efficiency was brought up nearly to 50 per cent.‖ Pearsall has recently improved the hydraulic ram and raised its efficiency to nearly 80 per cent., either for water or for air, but this

* An equalizer of this type was patented in Germany, by the Berlin Anhalt Works in 1885.

† In kinematic notation this action is expressed by CP+CP⊥), as shown by b. See Theoretical Kinematics, pp. 322, 325.

‡ See Mair, Experiments on a direct-acting steam pump. Proc. Inst. C. E. London, 1886.

§ The Worthington equalizer accomplishes an end sought by designers of steam pumps for the past 200 years, for since Papin's first machines in Cassel (1690) the desired aim has been to combine the action of a variable elastic driving medium, and a uniform, non-elastic resistance.

‖ See the author's paper, Ueber die Durchbohrung des Mont-Cenis. Schweiz. polyt. Zeitschr, 1857, p. 147.

has been done by the introduction of a valve gear, making it a device of the second order.*

§ 326.

Fluid Transmission at Long Distance.

When the motive power is intended to operate the piston of a pump situated at a distance, some connecting mechanism must be interposed between the two cylinders. Formerly this was accomplished by using long rod connections; instead of this a pressure-organ transmission may be employed. When water is used as the medium for transmission, this may be termed a "water rod" connection. This is used in connection with water levers (see § 311).

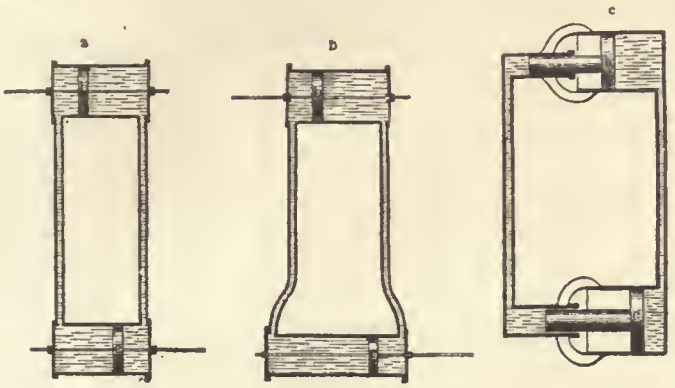


Fig. 1018.

Fig. 1018 shows three devices for this purpose. At *a* is shown a closed system with pistons of equal diameter; *b* is a similar one with unequal pistons; and *c* is a form with combined pistons. Such water-rod connections are adapted for use in mines, and the following example will illustrate.

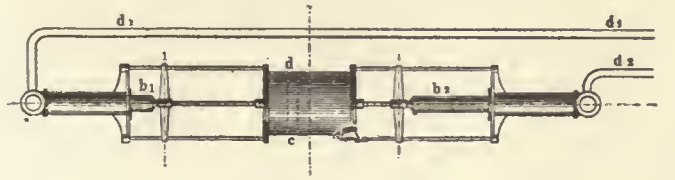


Fig. 1019.

The arrangement of transmission in the Sulzbach-Altenwald is shown in Fig. 1019, which represents the engine above ground, while Fig. 1020 shows the mechanism in the mine.

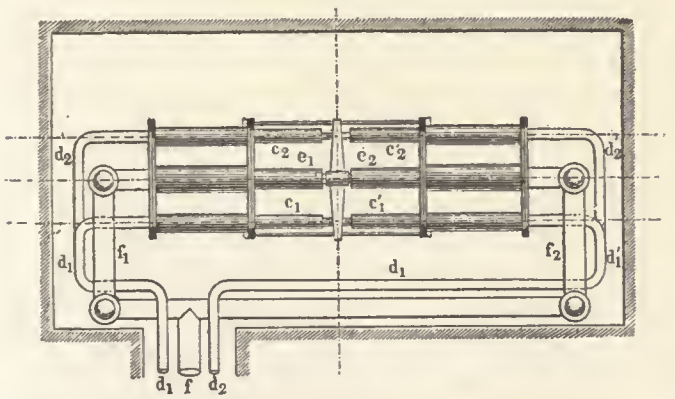


Fig. 1020.

The arrangement is of the same form as Fig. 1018 *b*. The steam piston *c* operates the two plungers b_1 b_2, which in turn operate the plungers c_1 c_1', and c_2 c_2' in the mine, the pump plungers e_1 e_2 being placed in the middle †

* See Engineering, Vol. XLI., 1886, p. 47, also H D. Pearsall, Principle of the hydraulic ram applied to large machinery. London, 1886.

† See Zeitschrift für Berg, Hütten und Salinenwesen, XXII. p. 179; XXIII., p. 6; XXIV., p. 35. The depth is 820 feet, the speed from 6 to 16 double strokes per minute, with a pause of one second, giving about 420 feet piston speed per minute. This engine, built by the Bayenthal Machine Works at Cologne in 1858, has operated regularly for 29 years without any interruption worth mentioning.

§ 327.

Rotative Pressure Engines.

An effective method of obtaining an advantageous action of the steam is to substitute for the reciprocating mass of the Cornish engine a rotating mass. This is accomplished by using the reciprocating motion of the piston to operate a crank shaft upon which a fly-wheel is placed. Since it is practicable to give the rim of the fly-wheel four to six times the velocity of the crank pin, the magnitude of the moving mass can be much smaller, and since the value varies as the square of the mean velocity, the mass is reduced at least 16 times. It is therefore possible by this means to give even small pumps an efficiency equal to that of large pumping engines.‡

It is not practicable to construct single-acting pumping engines into fly-wheels, because the piston speed *v* is too variable. If we draw a curve representing *v*, the ordinates being the positions of the piston, we have for a connecting rod of infinite length a circular curve, as in Fig. 1021 *a*. When the

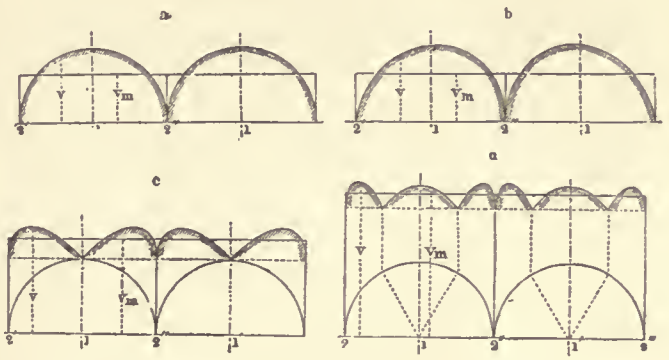


Fig. 1021.

length of the rod is taken into account these curves are modified, as shown in Fig. 1021 *b*, which is drawn for a rod four times the length of the crank. This curve also shows the ratio of the tangential force on the crank pin to the pressure on the piston. §

The variations in the value of *v*, which often differ widely from the mean value v_m, must necessarily be communicated to the mass of water, and hence great variations occur in the stresses. For this reason the velocity of the column of water must be kept within moderate limits, notwithstanding the use of air vessels. These variations become much less serious when two pumps are connected by cranks set at right angles with each other. The corresponding velocity curve is shown in Fig. 1021 *c*, and many pumping engines are now so made. More recently triple cylinder engines are made with cranks 120° apart. The velocity curves in this case are shown at *d*. It is evident that both these forms involve complications in construction which compare unfavorably with the direct-acting pump with equalizing cylinders (see § 325).

Instead of using a revolving fly-wheel, the mass of metal may be arranged to swing in an arc of a circle of large radius. An ingenious application of this principle has been made by Kley, in his water works engine with auxiliary crank motion. The proportion between the steam pressure and the vibrating mass is so arranged that the auxiliary crank comes to rest either a little before, or a little beyond the dead point, so that the return stroke in each case can be effected by the action of a cataract. In the first case, the fly-wheel swings backward after

‡ The Gaskill pumping engine is a duplex pump with fly-wheel, and cranks at right angles, and has given excellent results. See Porter's "Report of the Gaskill Pumping Engine at Saratoga."

§ Referring to the designations in Fig. 1022, we have $\frac{v}{c} = \sin\omega + \tan\alpha \cos\omega$. Since $P\,dz = P'\,r\,d\omega$ and $Pv = P'c$, the ratio $\frac{P'}{P}$ is also equal to the same expression. Hence the curves above given also show the ratio of the force in the path of the crank-pin to the pressure on the piston.

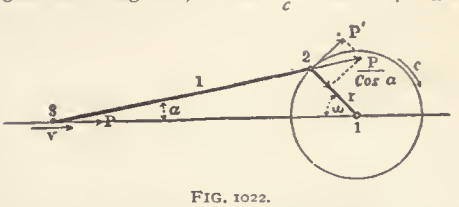


Fig. 1022.

In Fig. 1022 *a* and *b*, *P* and *c* are represented by 1 . 2′; in Fig. *c*, by 2′ 2″; in Fig. *d*, by 2′ . 1″; in *c* and *d*, the ratio of connecting rod to crank is again taken as infinitely great. The curves are adapted for double-acting pumps. When two single-acting pumps connected to right-angled cranks are used, the second half of the curves of Fig. *b* become the same form as the first.

the pause, and in the second case, forward.* The valve motion of this form of engine is considered in the following section.

§ 328.

Valve Gears for Rotating Engines.

Rotative engines are distinguished from pure reciprocating pressure organ escapements in that they deliver their effort in the form of rotary motion adapted to be used for driving running machinery. Between the two forms there is also the intermediate kind, with merely auxiliary rotative mechanism, such as have been already referred to. The translation of reciprocating and rotary motion may be accomplished in a variety of ways, but by far the most useful and best known is that by which the rectilinear motion of a piston is transmitted to the shaft by crank connection.

The variations in the tangential component of the pressure P' on the crank pin, Fig. 1021, becomes still greater when the pressure P, on the piston also varies by reason of the expansion of the steam. For this reason some form of equalizer is required in the form of a fly wheel. This latter becomes a reservoir for the storage of living force. Extreme examples of this action are found in rolling mill work in which within a brief time a 1000 H. P. engine may be called upon to deliver 2000 H. P., a demonstration of action of the fly wheel as a reservoir of power.

The valve gearing for rotative engines is an important and extensive subject. In the preceding sections a series of valve gears have already been described. These have all been based upon the principle of operating the valves by a direct reciprocating motion, taken either from the piston or piston rod. With rotative engines another method is used, the motion being taken from the revolving portion of the machine, and this method may also be adopted for pumps with auxiliary crank action. We may then distinguish between :

Reciprocating valve gears, and
Rotative valve gears.

Rotative valve gears are desirable even for direct acting pumps, but in a still greater degree are they desirable for rotative engines. Watt's rotative engine was made with a reciprocating valve gear,† and this form has one advantage in that it is adapted for rotation in either direction.

Hornblower, the inventor of the compound engine, also used a reciprocating valve gear. The slide valve, invented by Murdock, in 1799, led the way to the introduction of the rotative valve gear in 1800, but the old reciprocating gear still continued to be used, and is even re-invented at the present time. The later direct acting steam pumps with auxiliary rotative mechanism are almost always made with rotative valve gear. Kley's pumping engine, referred to in the preceding section, is made with reciprocating valve gear, since its motion is both before and behind the dead points of the crank.

The use of the slide valve, combining four valves in one member, enables a very simple valve gear to be made for the ordinary double acting escapement, as the diagram of a plain slide valve engine, Fig. 1023, clearly shows.

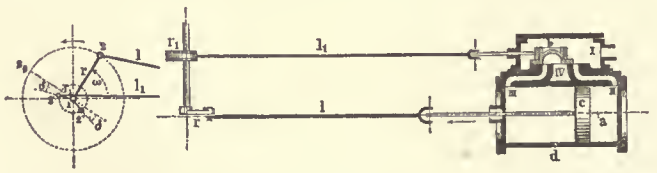


Fig. 1023.

The use of an eccentric r_1 and rod l_1 to operate the valve b, is not the earliest form of gear, the first method being the use of an irregularly shaped cam which brought the valve to rest except at the time of opening or closing.‡ A feature of the slide valve which was long overlooked was the fact that the time of closing the steam ports *II* and *III* could be regulated so as to effect the proper expansion of the steam. In order to accomplish this result without impeding the exhaust of the steam, the eccentric r_1 must be given the so-called angle of advance 2° 1 . 2′ beyond the mid-position. The direction of rotation of the crank is then governed by this angle, the arrangement above giving rotation to the left, and the position 1 2″ for r_1, giving right-hand rotation.

The action of the slide valve may readily be represented graphically.§ The angle of advance and lap being given the point of cut-off can be determined by the following method.

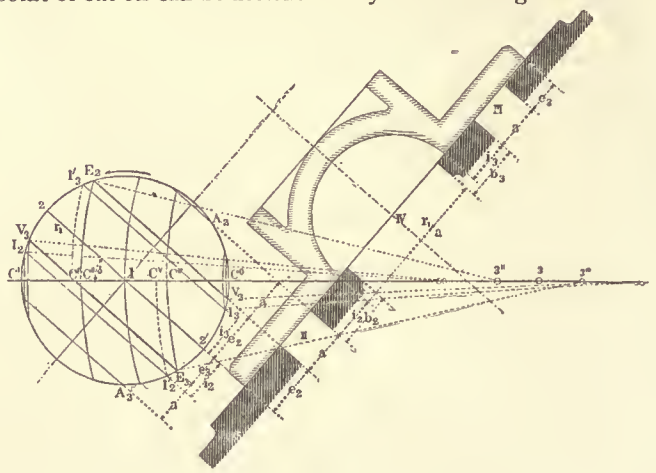


Fig. 1024.

Fig. 1024. The circle 1 C_o represents the circle of the eccentric and may also be taken as the crank circle on a reduced scale. C'' and C''' are two symmetrically placed positions of the piston at which it is desired that the cut-off shall take place. Through these points with a radius 1 . 3 = l describe arcs from centres 3″ and 3‴ ; their intersections E_2 and E_3 with the circle give the angles at which the expansion $C_o\,C''$ and $C'\,C'''$ occurs, in this instance $\frac{7}{10}$ of the stroke. We now select the point v_2 of the crank circle at which the admission shall begin, join $V_2\,E_2$ and draw the equator 2 . 1 . 2′ parallel to it, and the angle 2 . 1 . C' will be the angle of advance S, and the distance of 2 . 1 from $E_2\,V_2$, the outside lap e_2 for the port *II*. The width of port a must also be chosen, and must be so taken that it is less than $r_1 - e_2$, and is represented by the parallel A_2. When the crank reaches I_2, in this instance at $\frac{98}{100}$ of the stroke, the exhaust begins, and the distance $i_2\,i_2$ of the parallel $I_2\,I_2$ from the equator is the inside lap.

The construction is similar for the other half of the stroke. The angle δ is already known, and hence the parallel $E_3\,V_3$ from E_3 can be at once drawn, and the admission point V_3 determined. The outside lap e_3 is somewhat less than e_2, thus giving a correspondingly wider port opening. The inside lap i_3 is made equal to i_2, and the bridges b_3 and b_2 are made equal, thus giving a symmetrical valve seat. A certain amount of discretion is permissible in the selection of $b_2 = b_3$; care being taken that there is sufficient bearing at the extreme valve stroke to insure tightness. The points I_2' and I_3' are also of importance, as they determine the closing of the exhaust. The corresponding piston positions C_{IV} and C_V are not symmetrical, because $i_3 = i_2$, but the inequality in the compression is not serious.

The above method of considering the influence of the ratio $\frac{l}{r}$ is very simple. It is easy to substitute any desired ratio $\frac{l_1}{r_1}$, but the variation is slight. It must be noted that the distance 1 . 3 must be laid out to the actual scale of construction.

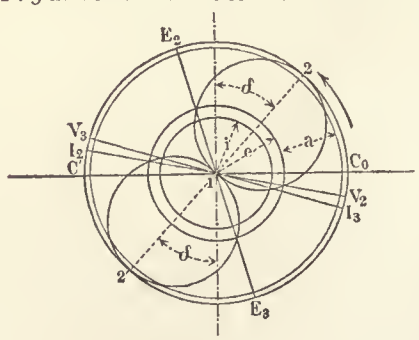


Fig. 1025.

The application of Zeuner's diagram to the same case is made in the following manner, Fig. 1025. The circle 1 C_o represents as before, the eccentric circle and the crank pin path. The angle C_o . 1 . 2 = C' . 1 . 2 = 90 − δ. With 1 as a centre, describe circles with radii e and i, here made alike for both ends of the valve, also one of radius $e + a$. Upon 1 . 2 and 1 . 2 as diameters, describe circles, called the valve circles.

* For a fuller account of this interesting engine (German Patent No. 2345), of which more than fifty are in operation, see: Berg-u. Hüttenm. Zeitung Glückauf, 1877, No. 18, 1879, No. 98 ; Moniteur des int. matériels, 1877, No. 20 ; Compt. rend. de St. Etienne, 1877, June ; Berggeist, 1879, No. 85 ; Z. D. Ingenieure, 1879, p. 304, 1881, p. 479, 1883, p. 579. Dingler's Journal 1881, p. 1, 1882, p. 349 ; Maschinenbauer 1881, p. 63 ; Oesterr. Ztg. f. Berg-u. Hüttenwesen 1882 ; Kohleninteressent (Teplitz) 1882, No. 34 ; Revista metalurgica (Madrid) 1883, No. 968.

† See Farey, Treatise on the Steam Engine, London, 1827, p. 524. Engines with slide valves were only made by Boulton and Watt, after James Watt retired to private life.

‡ See Farey, p. 677.

§ Formerly the so-called "valve ellipses" were used ; since 1860 Zeuner's diagram has superseded these, see Zeuner, Schiebersteuerungen, Freiburg, Englehardt, first published in Civil Ingenieure, Vol. 2, 1856.

The intersection of radii from 1, with these circles, give the distance of the valve from its middle position for various crank positions. For the position 1 V_2, for instance, the admission for the left stroke begins, at 1 E_2 the expansion, at 1 I the exhaust, etc.*

The Zeuner diagram gives the valve position by means of polar co-ordinates, while the writer's diagram is based on parallel co-ordinates. To be strictly correct, the valve circles 1 . 2 and 1 . 2′ of the Zeuner diagram should fall upon each other. The arrangement shown has been adopted by Zeuner as more convenient in practice.

It will be seen from the preceding that the rate of expansion can be varied by altering the eccentricity and the angle of advance. This may be carried so far that the direction of rotation is changed, giving what is termed a reversing motion. A variety of reversing motions have been devised, which accomplish the desired relation of parts by shifting a reversing lever. Of these the most practical are the so-called link motions, of which a number will here be briefly shown.†

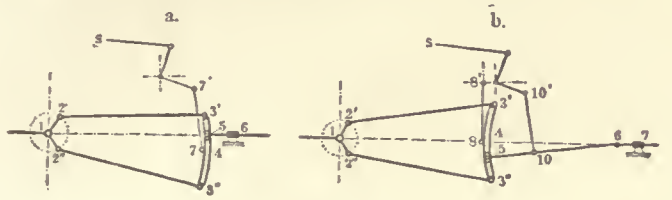


FIG. 1026.

Fig. 1026 *a*, is an outline diagram of Stephenson's link motion. The link 3′ 3″, of convex curvature towards the valve, is given an oscillating motion by means of the two equal eccentrics 1 . 2′, 1 . 2″, and is suspended from its middle point 7, from the bell crank lever S 7′. The motion of the link is transmitted to the valve by means of the sliding block 5, and rod 6. Fig. 1026 *b*, is Gooch's link motion. The link 4 is driven by two eccentrics as before, but is curved in the opposite direction with a radius 5 . 6, and is suspended from its middle point 8 to a fixed pivot 8′, while the rod 5 . 6 is shifted by means of the lever connection S 10 . 10′.

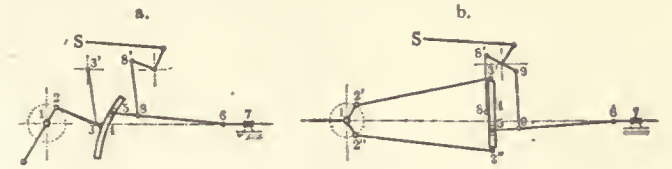


FIG. 1027.

Fig. 1027 *a*, is the link motion of Pius Fink. In this form the link is operated by a single eccentric instead of two, as in the previous forms. This simple mechanism is not as widely used as its merits deserve.

Fig. 1027 *b*, is the link motion of Allan, or Trick. In this design the link 4, is straight, and both the link and the radius rod are suspended and shifted by the lever connections 8′ . 8, and 9′ . 9.‡

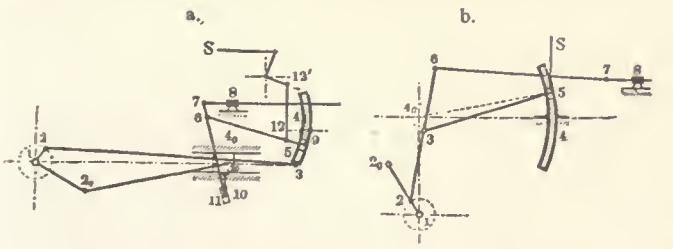


FIG. 1028.

Fig. 1028 *a*, is Hensiuger's link motion. The link 4, vibrates upon a fixed centre 9, and is operated by an eccentric 1 . 2. The valve rod is moved from the main cross head by the connections 10 . 11 . 6 . 7, and also by the radius rod 5 . 6, which latter is suspended from the bell crank S . 12′.

Fig. 1028 *b*, is Klug's valve gear, known in England as Marshall's. The curved link 4, is rigidly secured and does not move. The eccentric 1 . 2, moves the valve connection 6 . 7, by means of the lever 2 . 3 . 6, which vibrates about the point 3, on the end of the radius rod, the other end of the rod being held by the link block 5. Instead of the link 4, a radius arm 4_0 5, is often used, the centre 4_0 corresponding to the centre of curvature of the link, the action being the same in both cases.§

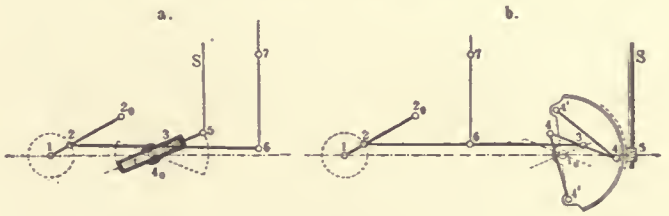


FIG. 1029.

Fig. 1029 *a*, is Brown's valve gear, which differs from the preceding by the substitution of a straight link of adjustable angle, for the curved guide link.

Fig. 1029 *b*, is Angström's valve gear. The point 3 of the preceding gear is guided by a parallel motion, and the point 6 is between 2 and 3, instead of beyond.

The eight preceding valve gears operate the valve approximately in the same manner as if a single eccentric of variable eccentricity and angular advance were used, the eccentric rod being assumed of infinite length as compared with r. The path of the successive positions of the middle point of this imaginary eccentric is called the central curve of the valve gear.

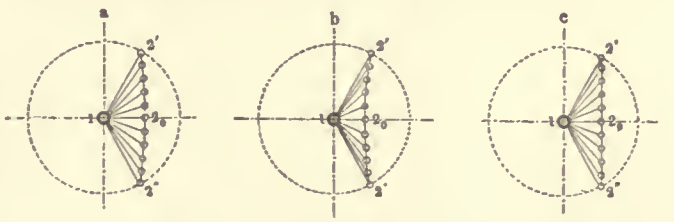


FIG. 1030.

Fig. 1030 shows the form of this curve for link motions in general. Form *a*, is that for cases 1, 4 and 5; form *b*, for case 1, when the eccentric rods are crossed, and form *c*, in which the curve becomes a straight line, is for cases 2, 3, and 6 to 8. In the latter instance, the lead, or opening for admission of steam at the beginning of the stroke is constant, a point considered by many to be of much importance.

It is possible to arrange the mechanism in such a manner that the centre of the valve motion may move directly in the desired central curve, as is shown in Fig. 1031.

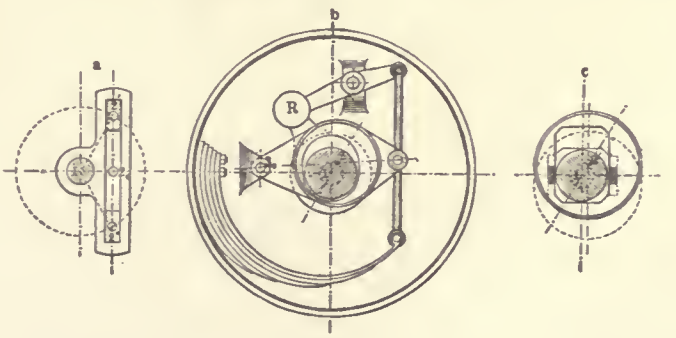


FIG. 1031.

This construction involves the rotation of the link about the crank axis. The only point to be accomplished is to guide the centre 2′ in the path 2′ . 2 . 2″. Fig. 1031 *c*, is a direct guide for the eccentric with wedge adjustments; *b*, is Sweet's valve gear, in which the position of the eccentric is determined by a centrifugal governor.‖ This only uses the central curve from

* It is usual to make the valve symmetrical, *i. e.*, $e_3 = e_2$, which necessarily causes the cut-off to take place at different points for the back and forward strokes.

† See also Zeuner, as above; Gustav Schmidt, Die Kulissensteurungen, Zeitschr. d. österr. Ing. u. Arch.-Vereins, 1866, Heft. II.; also Fliegner, Ueber eine gelr. Lokomotiv-steuerungen, Schweiz. Bauzeitung, March, 1883, p. 75.

‡ See Reuleaux, Die Allanische Kulissen steuerung, Civ. Ing., 1857, p. 92.

§ For a further account of this gear, see: Berliner Verhandl, 1877, p. 345, 1882, p. 52. Engineering, Aug. 13, Oct. 1, Dec. 3, 1880; Nov. 4, 1881; June 23, 1882; Feb. 6 and 27, 1885; Jan. 12, 1886; Sept. 9, 1887. Engineer, May 26, 1887; Feb. 23, Mar. 30, April 27, June 29, 1883; June 5, 1885. Marine Engineer, 1885, No. 1., Civ. Ing. Heft, 7 and 8, 1882; Zeitschr, D. Ing., 1885, p. 289, 1886, pp. 509-625; Revue universelle, 1882, p 421; Busley, Schiffsmaschine, I, p. 454; Hartmann, Schiffsmaschinendienst, Hamburg, 1884, p. 53; Blaha, Steuerungen der Dampfmasch, Berlin. 1885. p. 65.

‖ See Rose, Mech. Drawing Self-taught, Philad. Baird; for similar gears, see Am. Machinist, Grist, Oct. 5, 1883; Ball, ditto Aug. 18, 1883: Harmon, Gibbs & Co., ditto Nov. 24, 1883. Also Sturtevant, The Engineer, New York, Jan. 1888.

2′ to 20, and the path is a curve produced by a radial arm as in Klug's valve gear. The valve is balanced, in order to reduce friction to a minimum.

The last described valve gear possesses the advantage of great simplicity but retains the disadvantage of all single valve gears when used for a high expansion ratio, *i. e.*, the admission and exhaust of the steam do not remain uniform, and are often unsatisfactory. For this reason many valve gears with independent expansion valves have been designed.

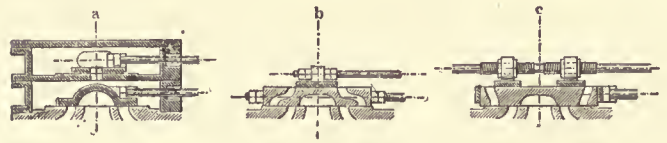


FIG. 1032.

Three forms of gear with separate expansion valves are shown in Fig. 1032. The form *a*, is known as Gonzenbach's; that of *b*, by various names; *c*, is the widely used Meyer valve gear.*

In France, Farcot's gear is used, having two loose cut-off plates carried on the back of the main valve, and in America, the excellent Porter-Allen engine with double valves operated by two eccentrics, is much used. Rider's valve gear is a modification of Meyer's, Fig. 1032 *c*. The two cut-off plates form reverse spirals, and slide in a concave seat on the back of the main valve, the admission parts being also spiral shaped and cut-off varied by twisting the cut-off valve axially.†

Instead of using eccentrics to operate the valves, cams of irregular outline may be adopted, these permitting a rapid opening and closing of the parts. Noteworthy examples of cam valve movements are to be found in the steamboats of the western and southern States in America. In its original form of a cock or cone a slide valve may be operated by an alternating motion as well as by continuous rotation. Such valves have been used in steam engines by various builders, among them the firm of Diugler in Zweibrücken, but the cost prevents wide use. A most extensive use of oscillating cylindrical valves has been developed by Corliss and his followers.

The forms of oscillating and rotating chamber gear trains already described involve other means of operating the valve than are used for reciprocating engines, and shown in Fig. 1023. As an example, the water pressure engine of Schmid, of Zurich, Fig. 1033 is given. In this instance the valve *b*, is formed in the frame of the machine, and is of the type shown in Fig. 987 *c*. The regulation of speed of rotative water pressure engines is a much more difficult matter than is the case with steam engines, partly on account of the lesser fluidity of the water and also because of its slight elasticity. An air chamber in the admission pipe as shown in Fig. 1033 is therefore desirable, and when extreme changes of load are anticipated the valve gear should be modified. If it is desired to cut off the admission of water before the end of the stroke is reached, it is necessary to arrange a special valve to permit the discharge to continue. Excellent engines with this arrangement have been made by Hoppe, of Berlin, for the Mansfeld mines, and for the Frankfurt railway station.

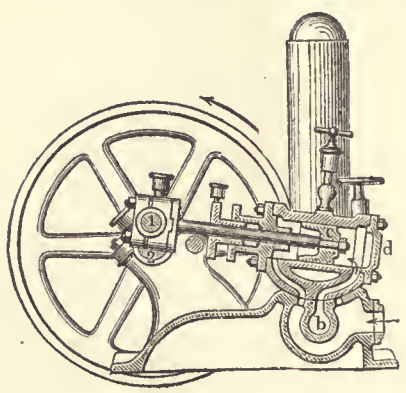


FIG. 1033.

Another method is applicable to power driven pumps, an illustration of which may be found in the design of Franz Helfeuberger, of Rorschach.‡ This is made with a hydraulic ratchet mechanism arranged in the crank disk in such a manner as to move the crank pin to or from the centre, the ratchet being operated by tappets which strike each time the crank passes the dead centres. The throw of the crank is thus varied to correct for variations of speed, the mechanism being controlled by a regulator. The action is very satisfactory, giving results varying from 90 to 82 per cent. for a change of power from 1 to $\frac{2}{3}$, according to the investigations of Autenheimer, Buss and Kuratli, in 1885. §

A later device is that of Rigg, which also acts by varying the stroke. The machine is a so-called "chamber crank train" (described in Theoretical Kinematics, p. 359: English ed., p. 361), with four single acting cylinders carried on the revolving wheel in the same manner as the machines of Ward, Schneider and Moline. The length of stroke is controlled by a regulator, similar to Sweet's governor, Fig. 1031 *b*, which operates a hydraulic escapement and adjusts the radius. This device is used by Rigg for steam and air engines to control the degree of expansion. These latter machines are operated as high as 2000 revolutions without producing trembling.‖

Besides the various forms of valve gear which have already been described, there are also the numerous "trip" gears, of which some examples have been given in § 252. These gears are made in many forms. The valve is made in four parts, as indicated in Fig. 986, on account of the facility with which the release can be controlled by the regulator.

The varieties of trip valve gears are most numerous, and there can be little doubt that the subject has been overdone, when it is considered that in many instances the entire mechanism of the engine has no other aim than to determine the opening and closing of the valves. In America, where this form originated, the reaction has already set in, and there is a disposition to return to the single slide valve, especial care being taken, however, to secure relief from pressure and to produce correct motion.

There is to be found in some parts of Germany a form of valve gear which may be called an "inner" and "outer" gear. This form does not possess sufficient merit to meet general application, but may be briefly noticed. The mechanical action of parts is not different, whether the "inner" or "outer" construction is used, and either arrangement may be adopted, at the discretion of the designer.

The following examples will make the arrangement apparent, as well as the illustration already given of Schmid's water pressure engine, Fig. 1033. Fig. 1033 *a* is an "outer" valve gear for a blowing cylinder, and Fig. 1034 *b* is a valve for a vacuum pump.¶ Another example is Cuvelier's valve, which is placed entirely outside of the steam cylinder, as is also the gear of Leclerq.** The ordinary slide valve is partly without and partly within the machine, being outside the cylinder and within the steam chest.

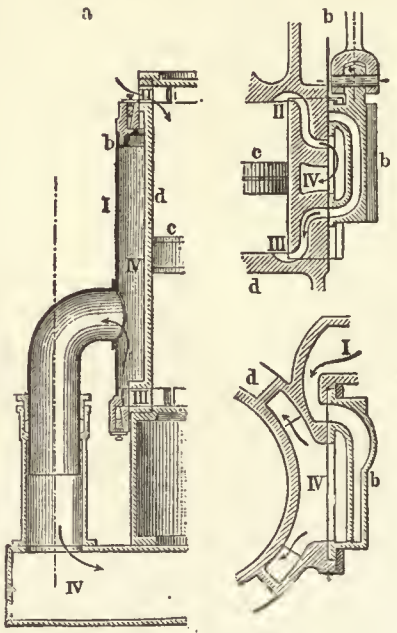


Fig. 1034.

C. ADJUSTABLE POWER ESCAPEMENTS

§ 329.

ADJUSTABLE PUMP GEARS.

The principles of adjustable escapements have already been discussed in § 259, and examples of rigid construction given.

*An excellent gear with two valves operated by a single eccentric, is Bilgram's. See Bilgram, Side Valve Gears. Philadelphia, 1878.

†This is an excellent problem in kinematics, the action of the valves and spiral ports forming a kinematic chain. See Theor. Kinematics, p. 333.

‡German Patent, No. 12,018, Jan. 27, 1881.

§A third system is that devised by Hastie, of London (see Engineer, August. 1878, and April, 1880, p. 304). Hastie controls the position of the crank pin by means of a pair of curved cams which act against increasing external resistance, and opposed by a spiral spring for diminishing resistance. This device does not give complete regulation for the following reasons: 1. In order that the statical moments of the increasing resistance and driving force on the crank pin shall be equal, the spring must act on the curved cams in such a manner as to cause the crank pin to move outward. This can be approximately accomplished by a careful arrangement of parts, but only approximately. Under this arrangement, however, there is a tendency for the pin also to move outward when the driving force increases, instead of moving promptly inward as should be the case. An attempt to correct for this error, reverses it. 2. The angular velocity of a motor is not a function of the impelling force, *i. e.*, the machine will run fast or slow as the case may be, this fact also appearing in practice. This is a common error into which inventors of "dynamometric governors" have fallen, even Poncelet, himself, having done so in his dynamometric regulator.

‖See Rigg, Obscure Influences of Reciprocation in High Speed Engines, Trans. Soc C. E., 1886; also Engineer, June 4, 1886. These engines are used on the compressed air systems of Birmingham and Leeds.

¶See Oppermann, Portfeuille écon., Feb. 1888, p. 18.

**See Genie ind. 1864, also Schweiz. polyt. Zeitung, 1864, p. 83.

Their action consists of the two following operations: 1. By the adjustment of one part the release of another part to the action of an impelling force is accomplished; 2. By the attaining of motion of the checked member, the checking is again produced either directly or indirectly.

These principles also obtain for escapements for pressure organs, and include a great number of important applications.

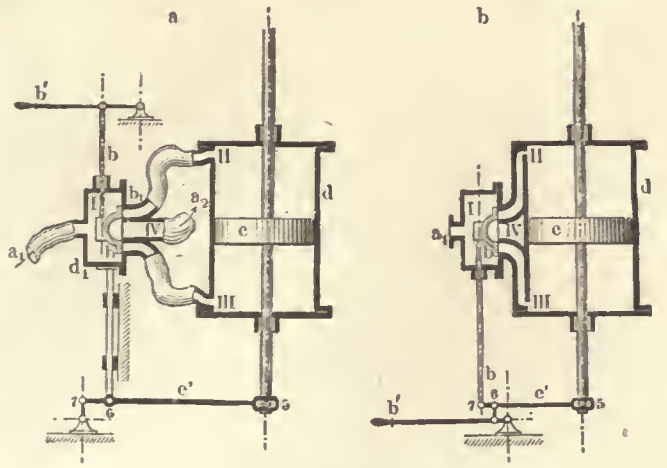


FIG. 1035.

The general scheme of such an adjustable gear for a steam pump cylinder is shown in Fig. 1035 *a*. The valve chest d_1 is made separate from the cylinder *d*, and is capable of movement parallel to it, the connections a_1 a_2 being made flexible. The slide valve *b* is operated from above by the lever *b'*. When the lever *b'* is lifted the pressure is admitted under the piston *c* through the port *III*, while the space above the piston is in communication with the exhaust *IV*. This causes the piston to move upwards and hence the lever *c'* moves the steam chest d_1 also upwards, thus closing the valve ports and checking the movement of the piston. If the lever *b'* is again lifted this action will again take place, and so on until the upper limit is reached. A reverse motion of the valve lever produces a corresponding reverse motion of the piston.* The same action may be obtained by using the arrangement shown in Fig. 1035 *b*. In this case the valve chest is fast to the steam cylinder, while the valve is so arranged as to be moved both by the hand lever *b'* and by the piston rod *c*. When the valve is moved by *b'*, the piston also moves and closes the valve by the lever *c'*, thus bringing itself to rest again. The piston *c* follows the motion of the lever *b'* in either direction; starting when the lever is started, and stopping when it is stopped. Any resistance not exceeding the force of the pressure at a_1, can thus be overcome while the resistance to the operator is only that due to the fracture of the valve and connections. The practical value of this device in many directions will be evident, and the examination of the above simple forms will explain the action of the various modifications.

Two constructions, designed by the author in 1866 for regulators will be found described in the *Civil Ingenieur*.† The lever is connected with the valve by means of a double parallel motion which is moved by the piston motion back into a position parallel with the base line.

The operation was satisfactory but the apparatus was cumbrous. In 1868 Farcot constructed a similar device, using an approximate parallel motion, but the apparatus was too complicated to be practically satisfactory.‡ A somewhat simpler construction was afterwards made by Farcot, but this was also too complicated for practical use.§ Other designs have been made by Farcot. Some recent constructions are here given.

Fig. 1036 shows the hydraulic steering gear of Bernier-Fontaine & Widmann,‖ which is similar in principle to Fig. 1035 *b*.

In this case the controlling gear *b'*, consists of a small hydraulic piston. The water pressure is admitted to it through the pipe *a'*, and is opposed by the spring *a''*. The two plungers C_1 and C_2, act as a double acting piston, the hydraulic pressure being supplied from an accumulator. The valve *b* is operated by the plunger *b'* against the pressure of the springs *b'''*, and again reversed by the pistons C_1 C_2 and connection 5. The piece at 6 is not a lever but is a cross head connected with

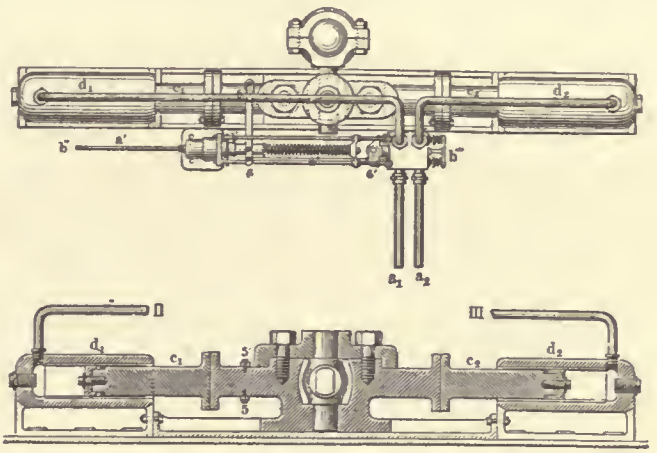


FIG. 1036.

the valve. The admission and release of water pressure through *a'* forms a long distance transmission involving the use of another escapement; the whole thus forming a gear of the second order.

Fig. 1037, is a neat regulator for steam engines by Guhrauer & Wagner.¶ In this, as in Fig. 1035 *a*, the valve seat is capable of movement in a direction parallel to the piston *c*, and is made concentric with the piston rod, the valve *b*, being a piston valve or rod moved by the governor. The piston *c* is subjected to full steam pressure from a_1 on both sides through the ports II and III, but as soon as the valve *b* is moved up or down, the holes b_0 relieve the pressure on one side or the other, the equilibrium is disturbed and the wiredrawing of the steam through the small ports II or III prevents sudden action and the piston moves until the holes are closed. As might be expected, this device is very satisfactory in practice.

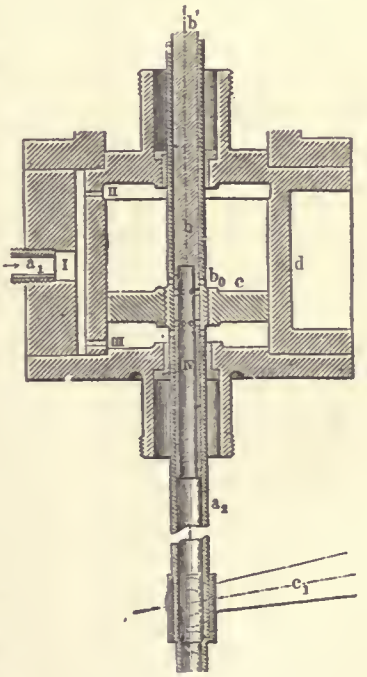


FIG. 1037.

Devices of this type are well adapted for steering gears as well as for regulators and a very delicate application of the principle is found in the Whitehead torpedo, in which the steering gear which determines the depth of the torpedo beneath the water is thus controlled by a barometric device.

§. 330.

ADJUSTABLE GEARS FOR ROTATIVE MOTORS.

The principles of the gears described in the preceding section, are also applicable to rotative motors although the arrangement is not so simple as with direct reciprocating cylinders, since the motion of the valve gear has also to be controlled. At the same time it must be noted that the application of adjustable gears to direct acting reciprocating motors is the more recent of the two. The earliest rotative gear of this sort, so far as the author has been able to ascertain, is that designed by F. E. Sickles, of Providence, R. I., in 1860 (See also §. 252).**

* The escapements shown in § 259 are only single-acting, and do not admit of a reverse motion.

† Civ Ingenieur, 1879–1880, Prof. Rittershaus, Ueber Kraftvermittler. Also the models at the Royal Technical High School in Berlin.

‡ See Civ. Ingenieur, 1879, 1880, Rittershaus, Ueber Kraftvermittler. (Intermediate Power Mechanism.) A model of the design of 1866 is in the cabinet of the Royal Technical High School at Berlin.

§ See Annales industrielles 1873, p. 518; also Oppermann, Portfeuille écon. 1874. p. 113.

‖ See Revue Industrielle, 1886, p. 373; 1887, p. 148.

¶ Built by Ganz & Co., of Budapesth, with Meyer's and with Rider's valve gears.

** According to the catalogue of the Centennial Exposition, in Philadelphia, in 1876, Vol. II. p. 52, Sickles made his first application in 1849, his patent was granted in 1860, and his first machine exhibited at the London Exhibition of 1862.

Sickles' machine was made with two oscillating cylinders. Both eccentrics were fastened together and were loose on the crank shaft and operated by a hand wheel and spindle. The steam chests oscillate with the cylinders. The crank shaft revolved in the same direction as the hand wheel is turned, but as soon as the motion of the latter was stopped, the valve seat moved under the valve and the ports were closed.

The more recent forms of adjustable valve gears for rotative engines are made after two distinct and important principles. The first form is that in which a double engine, without a fly wheel and with ordinary slide valve gear without angular advance is used, in order to permit rotation in either direction. The ports I and IV are then made so as to be interchangeable so that I can be connected either with the admission a_1 or exhaust a_2; and IV with the exhaust a_2 or admission a_1, at will. This change of connection is effected by means of an auxiliary valve sometimes known as a "hunting valve." This hunting

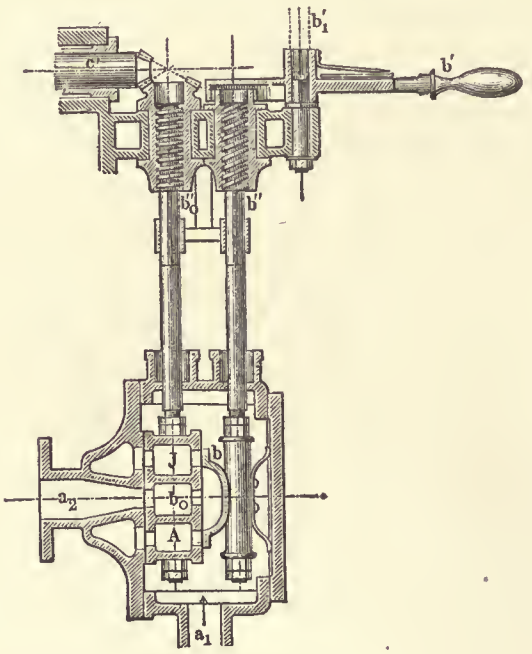


FIG. 1038.

valve can readily be controlled by hand for a steering engine, for which it is well adapted, since the angular motion of the rudder pin is limited, seldom exceeding 90°. The adjusting valve can then be arranged according to either of the principles of Fig. 1035 *a* or *b*. The following designs will illustrate the construction.

Fig. 1038 shows the steering gear of Dunning & Bossière.* The adjusting valve *b* rides upon a moveable valve seat b_o. The lower port *A* is always in communication with chests of the two engine cylinders, while the upper port *J* is in communication with the central port under the valves. The lever *b′* is connected with the spindle *b″* by an internal gear. This spindle

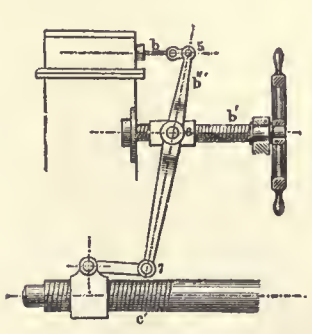


FIG. 1039.

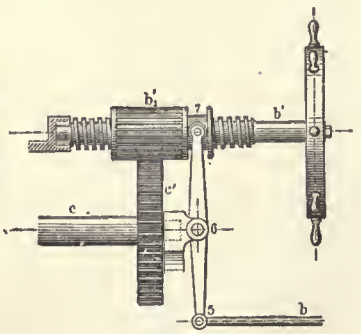


FIG. 1040.

has a screw thread of steep pitch, and is connected to the adjusting valve *b*. The moveable valve seat b_o is connected to a spindle b_o'', which has on it a much slower screw thread, and is also geared by bevel wheels to the axis *c′* of the drum of the tiller chains. Whenever the engines are started by moving the lever *b′*, the chain drum revolves and shifts the moveable seat b_o until the ports are again closed. The parts are so proportioned that the angle through which the rudder is moved is equal to the angle through which the lever *b′* has also been moved. This enables the position of the rudder to be determined at once by noticing the position of the adjusting lever. The moveable valve seat b_o will be recognized as the same in principle as the moveable steam chest of Fig. 1035*a*. The spindle b_1' can be prolonged to operate an indicator on the bridge for the inspection of the officer in charge of the ship.†

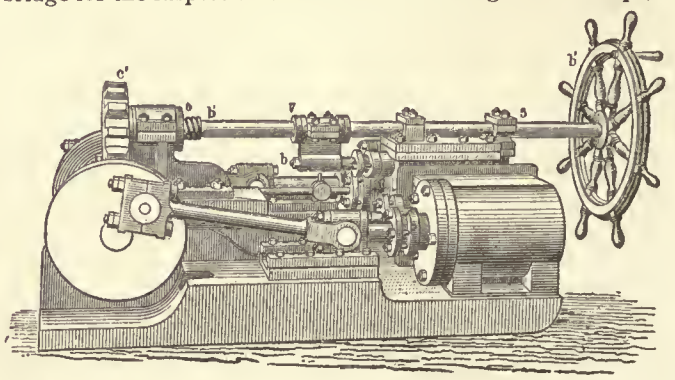


FIG. 1041.

Fig. 1039 shows Britton's steering gear.‡ The adjustment is effected by a hand wheel and screw *b′* operating the lever *b″* at 6, and thus moving the valve *b*. At 7 this same lever is connected to a nut on a screw thread cut on the axis *c′* of the chain drum, so that the motion of the latter closes the valve after it has been opened by *b′*. This corresponds in principle to Fig. 1035 *b*.

Fig. 1040 shows the steering gear of Douglas & Coulson.§ This is another application of the same principle as the preceding device. When the adjusting screw *b′* moves the nut, lever and rod *b* out of the mid-position, the revolving axis *c* of the chain drum turns the nut b_1' by means of the spur gearing until the dead position is again reached.

Fig. 1041 is a steering gear designed by Davis & Co.‖ This is a simple application of the principle of Fig. 1035*b*. The hand wheel shaft *b′* has a screw thread at 6, the nut being in the hub of the worm wheel *c′*, the latter being driven by a worm on the crank shaft. Any adjustment of the valve rod *b* by turning the hand wheel results in a corresponding readjustment by the motion of the worm wheel and nut derived from the engine.

The second kind of adjustable gear for rotative engines is much less frequently used than the first form. In this arrangement the adjustable valve is not connected with the main valve gear, but is operated independently, so that the crank will make any desired number of turns in either direction, according to the motion which is given to the adjusting valve.

Fig. 1042 shows Hastie's steering gear.¶ This is based on the principle of Fig. 1035*a*. The movable valve seat b_o is operated by the piston *c*, the eccentric c_1 being so placed that b_o has a reduced motion coincident with that of the piston *c*. The valve *b* is operated by the eccentric *b″*, which

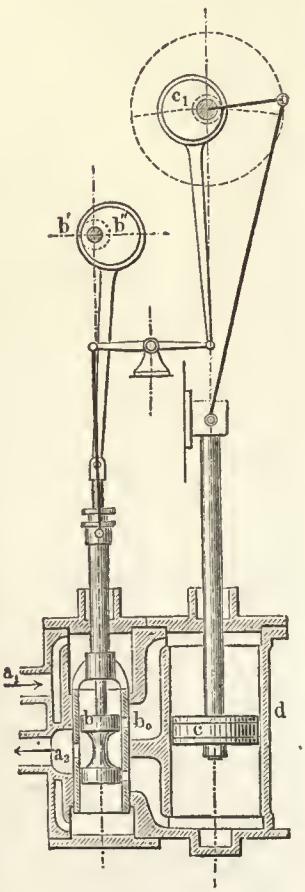


FIG. 1042.

* See Revue Industrielle, 1866, p. 401.

† A steering gear of similar design, with moveable valve seat b_o and adjusting valve, is that of Hastie, English Patent No. 1742, 1875. Also that of Holt; see Engineer, Sept., 1877, p. 221.

‡ See Revue Industrielle, 1884, p. 435.

§ See Engineering, April, 1882, p. 281.

‖ See Engineering, April, 1882, p. 398.

¶ See Rittershaus in Civil Ingenieur, already cited.

has the same throw as c_1, and is moved by a hand wheel on b'. The action which follows is that the crank shaft follows the movement which is given to the hand wheel both in direction and in revolutions. This action is similar to that of duplex pumps. Any number of revolutions may be made in either directions, and the device is a geuuiue rotative gear, as was also the earliest type, *i. e.* Sickles' gear, already described at the beginning of this section. It would not be difficult to design a similar gear ou the principle of Fig. 1035 *b*, but the author has no kuowledge that this has been attempted.

Adjustable valve gears for rotative engines have generally been designed for steam steeriug engiues, and in some of the recent powerful marine engines they have also been used to shift the link motion. There are many other applications which might be made. The speed can also be coutrolled by the adjusting wheel or lever b', if desirable, by suitable connections to the steam valve. As simplicity in construction is most important, steam economy is not considered in designing machiues of this kind.

D.—ESCAPEMENTS FOR MEASUREMENT OF VOLUME.

§ 331.

Running Measuring Devices.

In the classification of running mechanisms operated by pressure organs, it was noted that these devices could be used for measurement of volume. As already shown in § 321, fluid escapements are better adapted for measurements of volume than for measurement of time; but there is a close resemblance between the two operations, and many fluid meters might properly be classed as time-pieces. When the fluid to be measured is a homogeneous liquid, the quantity and volume are in direct proportion. With fluids which are uot homogeneous, such as gases and the like, a knowledge of the density is necessary iu order to determine the quantity from the measured volume; if the deusity is also to be determined at the same time as the measurement, the problem becomes much more complicated, as will hereafter be seen.

Liquids are frequently measured by means of continuous runuing devices; but the choice of construction is very limited. Among the open wheel devices there is available practically only the form shown in Fig. 957 *d*, and that only when the liquid is under moderate pressure. If, then, the liquid is slowly conveyed off below the horizontal plane of the axis so that the acceleration of the wheel is uuiform, then will the continuous rotation of the wheel be proportioned to the volume of the liquid passing through it.

An instance of this construction is the measuring drum in Siemen's meter for spirits.* This is made with three ouckets, and has inward delivery. Since the question of the density is in this case important, Siemens has devised a very ingenious float arrangement by which the counting mechanism is regulated to the volume of flow.

When liquids under high pressure are to be measured by such a device, the wheel must be inclosed in a case in which also a gaseous fluid must also be contained under a correspondingly high pressure, which is usually inconvenient. For measurements of high-pressure liquids, the chamber gear trains already described are preferable, especially since it is practicable to pack the working joints reasonably tight.† Wheels in which the living force of the water acts are also adapted for this service, either as bucket wheels or in the form of reaction wheels (see § 315). Siemens has constructed a meter of this kind, in which a reaction wheel is used, and the error of which does not exceed 2 per ceut., plus or miuus.‡

Another form, giving fair results for opeu channels, is based ou Woltmann's fan.

Gaseous fluids of small and only slightly varying density can be well measured by modifications of bucket water wheels; the conditions beiug practically reversed from those already considered, and the water now being the surrounding medium, and the gas the one to be measured.

One of the best known and most widely used devices for this purpose is the "wet" gas meter of Clegg and Crosley, shown in Fig. 1043 *a*. The revolving drum is a wheel with four buckets, which is driven by the passage through it of the gas. The gas is introduced just above the horizontal plane through the axis, and the liquid used is water; or, if there is danger of freezing, glycerine may be substituted. If the level of the water is lowered through evaporation or leakage, the volume of gas passing through the meter at each revolution will be increased, and to avoid this a float is so arranged that the gas will be shut off if the water level falls too much.

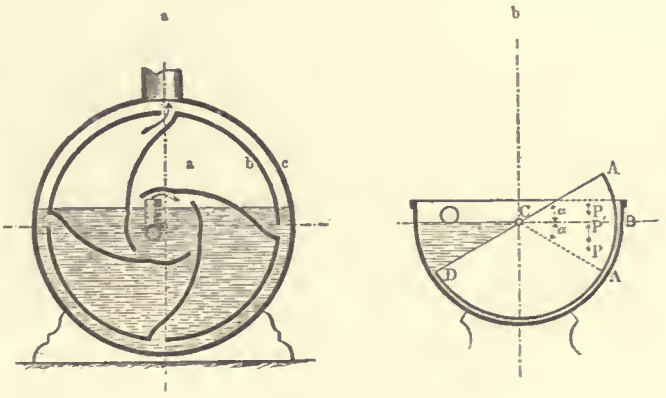


Fig. 1043.

For very accurate measurements of gas, Sanderson's meter is used. Fig. 1043 *b* in the water level remains unaltered so long as the vessel is kept supplied. The semi-cylindrical float is pivoted on the axis *C*, and is so constructed that the centre of gravity of all the sectors is the same as if the sheet metal body were homogeneous. If the float moves through an angle a with a sector ACB, the thrust of the sector $A'CD$ of an angle $180°-2a$ will pass through the axis, with force P' for the sector $A'CB$. The weights P of the two equal sectors ACB and $A'CB$ act downward through their centres of gravity, and are also equal to P'. In order that there may be equilibrium, if for any chosen value of a, P' shall equal $2P$, the specific gravity of the float (assumed to be homogeneous) must be equal to one-half that of the liquid iu the trough, *i. e.*, $= ½$ for water, or $= 0.63$ for glycerine.

The preceding meters have heretofore been used only for gases under low pressure, but are equally well adapted for gases at high pressures, such as compressed air for power transmission, simply by increasing the strength of the case. This has been done at the author's suggestion in connection with the compressed air system at Birmingham, as has also been the case with the "dry" meter described in the next section.

Anemometers, used for measuring the flow of air, generally belong to that class of running devices which are driven by the living force of the pressure organ (see § 315). They are usually screw turbines, or some modification of them. It is always necessary to take into account the stress of the gaseous medium, in order to obtain the desired measurement, since the apparatus ouly determines the volume.§

§ 332.

Escapements for Measurement of Fluids.

There exist certain defects in running devices when used as fluid meters, such as the journal friction, and in chamber gear trains the surface friction, which render the results inaccurate for fluids of weak flow. For this reason piston meters have been devised, these also utilizing the power of the fluid, and for these the application of escapements is necessary. Meters constructed on this principle have been used especially for measuring water. Among these may be meutioned Kennedy's water meter, a form which has been extensively used.‖ This is usually made with a vertical cylinder, the valve being a fourway cock operated by a tumbling gear similar to that of Bélidor's water-pressure engine (§ 325).

Jopling's water meter¶ is a piston escapement of the second

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* See Zeitschr. D. Ing., 1874, p. 108 This meter is in extensive use in Russia, Swedeo aod Germaoy for measuring brandy and alcohol, and is very satisfactory. Io Swedeo, in 1883, a comparison showed that a most careful hand measuremeot gave 15,365,931 litres of 50 per cent alcohol, aod the meter gave 15,450,775 iitres, or an excess of only ½ of 1 per cent.

† See the Crowu Water Meter io Schweizerische Bautzeitung, March, 1883, p. 81; also Payton's Water Meter.

‡ The older form (1857, Z. D. Ing., p 164) was on the priociple of Segner's wheel, Fig. 962 *a*; the more recent design is made like the turbine of Fig 963 *d*. At the end of 1886 Siemens & Halske had made 88,500 meters, aod the English house of Siemeos Bros., 130,000 of the old and new patterns.

§ Runniog devices may also be used to measure time as well as volume, aod in fact the oldest constructions for this purpose, the clepsydra, the saod glass, etc., belong to this class. Escapemeot clocks were iotroduced only in the middle ages. There have been numerous recent attempts to make ruoniog time-pieces. (See Redtenbacher, Bewegungs-mechanismeo, Heidelberg, 1861, p. 34, pl. 79; also Rühlmann, Allgemeine Maschinenlehre I., Braunschweig, 1862, p. 62.) The problem is a difficult ooe. since it involves the constructioo of a running device which shall operate both with a uniform aod a determinate velocity. Examples are found in the driving mechanism of astronomical instruments, in which the motion is transmitted hy frictioo. controlled by revolving fly-wheel devices. With these may be included the fan regulators for the striking mechaoism of clocks, aod similar applications.

‖ See Revue Industrielle. 1881, p. 205.

¶ See Z. D. Ing., 1857, p. 164; Maschinenbauer, Vol. XVI., 1881, p. 324; Technologiste, 1882, Vol. 42, p. 95.

order. There are two parallel horizontal double-acting cylinders, each operating the valve of the other.

Fig. 1044 shows a section of Schmid's water meter. This is made with two single-acting pistons, each also being the valve of the other, and the whole forming with the crank connection an escapement of the third order.

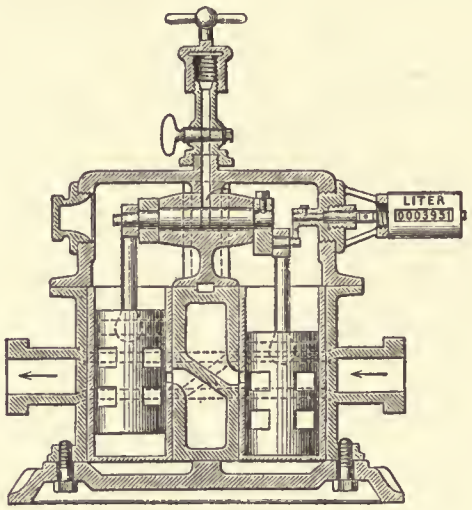

FIG. 1044.

Escapement meters are also used for gaseous fluids. A very extensively used form is the so-called "dry" meter used for measuring illuminating gas. These have, in many cases, superseded the "wet" meter, since the use of the liquid seal is avoided. In order to prevent friction, these meters are constructed with flexible diaphragms instead of pistons, much like the diaphragm pumps shown in § 317. A good example is Glover's dry meter, which is an escapement of the second order connected to a crank shaft which operates the counting mechanism. The diaphragms are made of linen, made impervious to gas by a gelatine sizing. These meters do not show a higher degree of accuracy than the wet forms.

§ 333.

TECHNOLOGICAL APPLICATIONS OF PRESSURE ORGANS.

The applications of pressure organs for technological uses are not so numerous or important as those in which they act in connection with the help of various machines. These applications are not dissimilar from those of tension organs, which have already been discussed in § 263. A general survey will be of value for the better understanding of the whole subject.

The use of a pressure organ from a technological standpoint consists in so using it that the result is a modification in form or shape either of another body directly by the action of the pressure organ or of the pressure organ itself by the other body. This "forming" action adds a fourth manner of action for pressure organs to those already classified in § 309, so that we now have:

1. Guiding,
2. Supporting,
3. Driving,
4. Forming,

as the four methods of action or application. Of these the first three belong to all classes of machine elements used in construction; the fourth falls within the domain of technology.

In order to speak comprehensively of the action of pressure organs, we will arrange them in five groups, according to the method of action, viz.: by Filling, Discharging, Internal Flow, Jet Action and Inclosing or Covering.

a. Filling.

1. The ease with which pressure organs can be led into desired channels on account of their fluidity is applied in the operations of casting. Metals which it is desired to make into given forms are rendered fluid by heat, and thus converted into pressure organs which can readily be run into moulds. In similar manner wax, stearine, paraffin, etc., are cast, in making candles and the like, the formed material resuming its solid state on cooling. Plaster, cement, magnesia or similar materials may also be made fluid by mixing with water, and then cast into forms which afterwards become hard by combination with water and carbonic acid. Other and similar methods are adopted for other materials.

2. Glass, in a plastic state, is formed by pressure in moulds or by passage between rollers.

3. In cases where complete fluidity is unnecessary, the material may be softened by heat, and then shaped in suitable presses, the slight fluidity of the material being overcome by the mechanical pressure of the machines.

4. Lead is sufficiently soft to be readily formed by the action of a plunger press, and is thus formed into bullets in arsenals, and also made into pipe.

5. The forming of a pressure organ by cooling is shown in the action of an ice machine, by means of which water may be given the form of sheets, rods, blocks, etc.

6. Copper, tin, zinc, etc., and also gold and silver are formed under the drop press in dies. Steel and wrought iron are heated for this purpose; but sheet steel is stamped cold.

7. Wire, already considered as a tension organ, may also be treated as a pressure organ, having great similarity to a flowing stream with its curves. Examples are found in the ingenious machines for making hooks and eyes, and also wire chains, made by William Prym, of Stolberg. Another illustration is the machine of Hoff & Vogt for rolling spiral springs.

8. Hydraulic or lever presses are used to press clay in a plastic condition into various dies to make bricks. Bricks are also forms of compressed turf, culm, etc. Chocolate and cocoa are also compressed in moulds.

9. The so-called art work in pressed wood is composed mixture of sawdust formed into a solid mass by great pressure in suitable moulds.

10. Papier maché is formed into shape from paper pulp reduced to a doughy consistency, and then subjected to heavy pressure.

11. In the use of moulding machines the pattern is first pressed into the moist sand, this being a granular pressure organ, this being followed by the casting of the liquid metal in the mould thus formed. This gives two applications of forming,—the first in moulding, the second in casting.

12. Compresses, such as are used for packing merchandise of powdered or fibrous nature, are also examples in point. These are used for baling hay, cotton, wool and similar materials under very great pressure.

b. Discharging; Formation of Jets.

When a pressure organ is contained in a guiding inclosure, and by properly directed pressure is forced out through a suitable mouthpiece, the jet which is emitted is formed with a cross section corresponding to that of the mouthpiece used. Jets may be formed in this way not only from materials which flow readily, but also from those which are of a tough or doughy consistency, so that even moderately dense substances may be thus formed:

1. The clay presses made by Schlyckersen and others are used to form tiles, drain pipes, etc., by this jet method of forming, the issuing stream being cut off at regular intervals by a wire cutter. The clay in such machines is effectively forced through the discharge opening by an arrangement of screw-propelling blades.

2. In the model press the dough is forced by a piston up through a die plate in which various shaped holes (such as stars, circles and the like) are made, and the issuing streams are sliced off by a wire cutter and dried.

3. The so-called artificial silk of De Chardonne is a jet formation of nitro-cellulose. This is made into a semi-fluid mass with iron or tin chloride and alcohol, and forced through a tube of glass or platinum of about a sixteenth of an inch bore drawn to a fine aperture, whence it issues in a hair-like thread. It is then toughened by passing through acidulated water, after which it is wound on a reel.

4. In the manufacture of paper the liquid pulp is discharged in a broad, flat sheet by its own pressure and the superfluous water first removed by absorption, after which the paper is dried and made into sheets.

5. Lead pipe is made by a process of jet formation in a pipe press. The mass, which is only moderately heated, is forced by piston pressure through the die in a continuous stream.

6. The insulating covering of gutta percha is formed upon wires used for electrical conductors by a jet action.

7. The common punching press, used for punching rivet holes in plates, really works with a jet action, as has been shown by the celebrated researches of Tresca upon the flow of metals.

8. The drawing press for forming various cups, pans and other household articles, also cartridge shells, from sheet metal, operates by a kind of jet action, one part of the mouthpiece being forced against the other. The powerful presses built by Erdmann Kircheis at Aue, and by the Oberhagener Machine Works, operate by means of cranks and cams, while those of

Lorenz, of Carlsruhe, work by hydraulic pressure. Drawing presses are much used in the United States.*

9. The drawing bench for the manufacture of wire as well as rods is an example of jet action. The wire acts both as a tension and a pressure organ, since it is pulled through the die in which it is formed. Drawn brass tubing is found in a similar manner and of various shaped sections.

10. The manufacture of shot is a variety of jet action, the melted alloy of lead and arsenic being poured through a sieve and permitted to fall in streams from the top of a shot tower, the drops assuming a spherical shape during the fall.

11. In gas lighting, the shape of the flame is formed by the jet tip on the burner, the flat flame in one form being made by two round inclined jets impinging against each other.

c. Internal Flow.

There are a number of pressure organs which are not homogeneous, being composed of granular and fluid materials, or of fluid materials of different density. It is a frequent problem in technology to separate such substances so as to divide the liquid from the solid, the large from the small, the light from the heavy, etc. In general, this can only be done by some application of the method of internal flow in the mass of the pressure organ. The methods include the use of artificially produced high pressures, the natural gravity of the material, or in some instances by vibratory or other motion, *i. e.*, by the action of the living force of the material, or rather by the unequal action of the various portions. The following examples will illustrate the various methods:

1. Presses used for the extraction of liquids (such as wine presses), presses for seed oil, olive oil, also for oil cake, stearine, beet root, yeast, etc., all act to separate the liquid from the solid portion by the action of internal flow.

2. Filter presses act to separate the fluid from the more sluggish portion of the mass, the liquid passing through the minute openings of the filter under the action of the high pressure, while the slimy mass remains behind. Filter presses are used for separation of colors, stearine, yeast, starch, sugar, potters' clay, etc.

3. The purification of water under natural pressure is effected by conducting it through settling and filtering tanks; also by special devices (as that of G. Niemax, of Cologne, German patent No. 30,032), by which the water is rendered harder or softer, as may be required.†

4. In mining and machine shop operations, the separation of mingled pressure organs by difference of internal flow is effected in various ways, showing very effective applications of the laws of hydraulics.‡

5. Various applications of sieves are used to separate granular materials of different sizes, as are also different devices which act by shaking or jigging the material, the separation thus being effected by differences of living force.

6. Centrifugal machines are used for drying yarn, wet clothes, etc., although the action in this case might be more properly termed external, rather than internal flow.

7. Another application of the centrifugal machine is for the separation of materials by their difference in specific gravity, as in the case of the machines for separating cream from milk.

8. In the Bessemer process the molten fluid mass of iron is penetrated by a gaseous pressure organ, *i. e.*, air, under high pressure, producing a violent internal flow and agitation, and burning out the carbon of the iron.

d. Jet Action.

A considerable amount of living force may be stored up in a fluid jet. This may be utilized in a limited number of ways, a few of which are here given:

1. The system of hydraulic gold mining used in California, to a great extent, is an important application of the jet.§

2. Tilghman's sand blast acts by means of a jet of air which sets a stream of sand particles in motion. This sand blast is used to cut glass, surface metals, sharpen files, clean castings, and has many other useful applications.

3. Machines for cleaning grain are made to throw the grain against frictional intercepting surfaces, thus removing dust and other impurities.

4. The impinging of a rapidly issuing jet of steam against the bell of a whistle causes a series of rapid vibrations, producing sound.

5. In the reed pipes of organs and similar musical instruments, the notes are produced by the action of a jet of air causing the reeds to vibrate.

6. The "Siren" used for fog signals acts by setting a column of air in vibration by rapidly succeeding jets of steam, causing a shrill note to be emitted.

7. In the simple organ pipe a column of air is set into musical vibration by a jet of air. The church organ is probably the oldest type of pressure organ escapement, the release being effected by the hand of the performer. The modern church organ consists of a series of the fifth order, namely: a water motor (hydraulic escapement), bellows (escapement) and regulator (checking escapement), stops (escapements), and keyboard with pneumatic action (escapement). In an organ manual of 10 octaves there are 120 escapements, each with n stops, with n different pipe connections arranged together. In this connection also may be mentioned barrel organs, in which the closing and releasing of the escapements is effected mechanically.

e. Inclosing or Covering.

As a counterpart to the inclosing of a pressure organ in a pipe or vessel, we have the inclosing or covering of a solid body by a pressure organ. This occurs when a body is submerged in a liquid, when its surface at least is covered with the liquid. A partial covering may also take place, as, for instance, one side of a flat piece, or by distribution after any particular plan. These conditions appear in a number of technical operations, as will be seen:

1. In the operations of dyeing, the articles are immersed in a liquid containing the coloring matter, many machines having been devised to assist in the operations.

2. In the various operations of finishing fabrics, heavy flowing liquids are used, distributed by various brush devices, this forming at least a combination of the second order, since the liquid must first be distributed to the brushes, and then to the fabric.

3. In coating paper with gum, the gum is distributed in the form of a liquid solution.

4. In the manufacture of colored papers and leathers, the color is distributed in liquid form over the desired surfaces.

5. The various operations of printing from surfaces of stone, copper, zinc, steel, etc., involve devices of the third and fourth order for the distribution of the printing material before it is finally transferred to the paper.

6. In the operation of printing fabrics and paper hangings, the printing surfaces are charged with color by a distributing system usually of the third order, and then impressed upon the fabric. The printed fabric is dried, usually, by a current of warm air, which is merely a gaseous pressure organ. In some instances the printed surfaces are dusted with felt dust while yet sticky, and then finally dried.

7. In printing woven fabrics, the processes of (5) and (6) are used with a mordant liquid, and the material then immersed in dye, and finally the color washed out of the unprinted portion with water.

8. The operations of electro plating surfaces with gold, silver, copper, brass, zinc, nickel, etc., involve the use of a physical apparatus, *i. e.*, the galvanic battery. The disposition of the covering may be modified by covering portions with a non-conducting material. Another operation in electrotechnics consists in the decomposition and deposition of minerals by means of electric currents generated mechanically.

9. In the use of illuminating gas, the method of making a gas poor in carbon, and then enriching it either with a rich hydrocarbon gas is a form of combination in the line of inclosing. The incandescent gas lamps operate by the surrounding of a network of magnesia or zircon with the flame of a weak illuminating gas.

10. The jet condenser acts by surrounding the discharge of exhaust steam with cold water.

11. In the surface condenser the tubes are surrounded with water and filled with the steam to be condensed, an arrangement of the second order.

12. The apparatus for cooling beer consists of an arrangement of parallel surfaces of sheet metal between which the cooling water flows, thus forming an apparatus of the second order.

* * * * * * * *

The above outline of technological applications of pressure organs is only an indication of the systematic treatment of which the subject is capable, but cannot here be carried farther, as it does not properly belong to the subject of machine design. The fifty examples given might each be made the sub-

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* See address by Oberlin Smith (Jour. Frank. Inst., Nov., 1886, "Flow of Metals in the Drawing Press"). The American presses are made for rapid duplicate work, the German for more general service with various special attachments, which latter Mr. Smith commends. We may be permitted to accept and return the compliment as regards the other side of the question.

† See Z. D. Ingenieure, April, 1888, p. 377.

‡ For an account of the separation of pulverized minerals by means of currents of air into portions of various fineness, see Z. D. Ing., April, 1888, p. 381.

§ See Appleton's Cyclopedia of Applied Mechanics, New York, 1880, Vol. II., p. 434.

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ject of a chapter, many of entire books. Even these can lay no claim to be a complete survey of the subject; rather are they merely a beginning. They will serve, however, to indicate at least how great a number of machines and mechanical appliances are involved in the use of pressure organs, and how these may all be considered to rest upon the same foundations.

CHAPTER XXIV.

CONDUCTORS FOR PRESSURE ORGANS.

§ 334.

Empirical Formulæ for the Thickness of Cast Iron Pipes.

Among the various forms of conductors for pressure organs mentioned in § 310, the most important are the various kinds of pipes. These are made of a great variety of materials, such as cast iron, steel, copper, bronze, brass, lead, wood, clay, paper, etc. For underground pipes for water, air and gas, cast iron has been most extensively used, and it is yet a question whether wrought iron or steel will successfully displace it.

Cylindrical cast iron pipes, which will first be considered, are subjected to so many varying conditions in the course of manufacture that the determination of the proper thickness to resist moderate internal pressures cannot be made upon strict theoretical principles, and recourse must be had to empirical formulæ. It is customary to subject cast iron pipes to a hydraulic test both at the foundry and also at the place where they are to be used, to a pressure of 1½ to 2 times the working pressure to which they are afterwards to be subjected. As a protection against rust, the pipes are also coated with asphaltum, applied at a high temperature, or in special cases where the expense is permissible, they may be enameled.

Let the internal diameter be D, and the thickness δ, we may make:

For cast iron pipes for water, air or gas,

$$\delta = 0.315'' + \frac{D}{80} \qquad (318)$$

For cast iron steam pipes and air pump cylinders,

$$\delta = 0.472'' + \frac{D}{50} \qquad (319)$$

For bored cast iron steam cylinders and pump barrels,

$$\delta = 0.787 + \frac{D}{100} \qquad (320)$$

Example 1.—A pipe 12 inches bore, according to (318), should be of a thickness $\delta = 0.315 + \frac{12}{80} = 0.465''$, or in practice say ½ in., while for a steam pipe, according to (319), we have: $\delta = 0.472 + \frac{12}{50} = 0.722''$, say ¾ in.

Example 2.—The pipes used in sections δ_1 and δ_2 of the Frankfort water system, already shown in Fig. 955, and subjected to a pressure of 270 pounds (18 atmospheres) are 15¾ inches (400 mm.) and 20 inches (508 mm.) diameter, and according to (318) the corresponding thicknesses should be: $\delta = 0.315 + \frac{15.75}{80} = 0.55$ in., and $\delta = 0.315 + \frac{20}{80} = 0.565$ in. The pipes for the water systems of Salzburg, Bamberg, Carlsbad, Goslar and Iserlohn are all of thicknesses conforming to formula (318).*

Example 3.—The pipes for conveying the compressed air in the construction of the Mont Cenis tunnel were 7⅞ in. (200 mm.) bore, and subjected to a pressure of about 75 pounds, and were exposed in lengths of over 2000 feet long during winter and summer. The thickness of these pipes was 0.39 in. (10 mm.), and by (318) the thickness would be 0.41 in.

Example 4.—The thickness of a locomotive cylinder 15¾ in. diameter would be, according to (320):

$$\delta = 0.787 + \frac{15.75}{100} = 0.944'', \text{ say 1 inch.}$$

The use of cast iron pipes has greatly increased during the past twenty years. Manufacturers have been disposed to make them of excessive thickness, not only to obtain the increased security, but to add to the cost,—a matter which public officials are sometimes not disposed to discourage, but which has frequently caused such installations to be excessively expensive. Hundreds of thousands of pounds of metal have thus been uselessly buried in the earth,—a waste which the co-operation of hydraulic and gas engineers might join in reducing.

* The foundry of Roll, at Solothurn, using a high grade, make still lighter pipes, using the formula $\delta = 0.24'' + \frac{D}{80}$.

§ 335.

Table of Weights of Cast Iron Pipe.

Diameter, D. Inches.	Weight per Foot, for Thickness δ.						
	¼	⅜	½	⅝	¾	⅞	1
2	5.53	9.69	. . .	. . .	. . .	. . .	. . .
4	10.30	16.10	. . .	. . .	. . .	. . .	. . .
6	15.40	23.50	32.0	. . .	. . .	. . .	. . .
8	20.3	30.9	41.8	. . .	. . .	. . .	. . .
10	25.1	38.3	51.7	65.3	. . .	. . .	. . .
12	30.0	45.7	61.5	77.7	94.1	. . .	. . .
14	35.0	53.1	71.4	89.4	109	128	. . .
16	39.1	60.4	81.2	102	124	145	. . .
18	44.8	67.8	91.0	115	139	163	187
20	. . .	75.2	101	127	153	180	207
22	. . .	82.6	111	139	168	196	227
24	. . .	89.9	121	152	183	214	246
26	. . .	. . .	131	164	198	231	266
28	. . .	. . .	140	176	212	249	286
30	. . .	. . .	150	188	227	266	305
32	. . .	. . .	160	201	242	283	325
34	. . .	. . .	170	213	257	300	345
36	. . .	. . .	. . .	225	271	318	364
38	. . .	. . .	. . .	237	285	336	384
40	. . .	. . .	. . .	249	299	353	403
42	. . .	. . .	. . .	262	315	370	423
44	. . .	. . .	. . .	274	329	387	443
46	. . .	. . .	. . .	286	344	405	462
48	. . .	. . .	. . .	298	359	422	482

Sockets and flanges should be calculated separately.

§ 336.

Pipes for High Pressures.

In determining the thickness of pipes which are to be subjected to an unusually high pressure, Lame's formula (see § 19) may be applied to advantage, *i. e.*:

$$\frac{S}{D} = \tfrac{1}{2}\left(\sqrt{\frac{S+p}{S-p}} - 1\right) \qquad (321)$$

in which p is the internal pressure per unit of area, S the permissible stress in the material; the external pressure being so small as to be neglected.

If in the preceding formula we substitute the external diameter, $D_o = D + 2\delta$, we get:

$$\frac{D_o}{D} = \sqrt{\frac{S+p}{S-p}} \qquad (322)$$

This shows that the internal pressure p should in no case exceed the permissible stress S of the material. If we make S equal to the modulus of rupture for tension, and make $p \geq S$, the pipe will be burst according to either formula however great the thickness δ be made.

For given dimensions and pressures we have for the stress S in the walls of a pipe:

$$\frac{S}{p} = \frac{D_o^2 + D^2}{D_o^2 - D^2} = \frac{1+\psi^2}{1-\psi^2} \qquad (323)$$

in which the ratio $\frac{D}{D_o}$ is indicated by ψ, as already discussed in § 90. From this we have the followiug values:

ψ	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70
$\frac{S}{p}$	1.67	1.74	1.82	1.91	2.01	2.13	2.25	2.39	2.54	2.72	2.92
ψ	0.72	0.74	0.76	0.77	0.78	0.79	0.80	0.81	0.82	0.83	0.84
$\frac{S}{p}$	3.15	3.42	3.73	3.91	4.11	4.32	4.56	4.81	5.11	5.43	5.79
ψ	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95
$\frac{S}{p}$	6.26	6.68	7.23	7.86	8.62	9.52	10.63	12.01	13.80	16.17	19.51

Example 1.—In the case of the compressed air pipe of the Mont Cenis tunnel, already mentioned, we have $\delta = 0.39$, $D = 7.875$, $D_0 = 8.66$, whence $\psi = \frac{D}{D_0} = 0.91$. For a pressure $p = 72$ pounds, we have from the above table $S = 10.63 \times 72 = 765$ lbs., or, if tested at double the working pressure, the metal would be under a stress of 1530 pounds per square inch.

When p is small, we may use instead of (321) for a sufficient approximation (compare Case I., § 19)

$$\frac{\delta}{D} = \frac{1}{2}\frac{p}{S} \text{ and } \frac{S}{p}\frac{D}{2\delta} \qquad (324)$$

Example 2.—Applying these formulæ to the data of the preceding examples, we have $S = 0.5\frac{pD}{\delta} = 0.5\frac{72 \times 7.875}{0.39} = 710$ lbs., as an approximate value.

Example 3.—A pipe 4 inches diameter is subjected to a water pressure of 1500 pounds. It is required that the stress S shall not exceed 3200 pounds. This gives $\frac{S}{p} = 2.13$, which from the above table gives $\psi = 0.60$. From this we have $D_0 = \frac{D}{0.60} = \frac{4}{0.60} = 6.66$ in. In some instances the pressure may reach as high as 2250 pounds, in which case the stress would reach $3200 \times 1.5 = 4900$ lbs. A 4 in. pipe at the Frankfurt Railway Station at 1500 lbs. has an outside diameter of 6.4 ins.

Example 4.—The Helfenberger Water Pressure Engine at Hersbrugg near Rheineck, of 40 H. P., has a cast iron feed pipe under a head of 1312 feet giving a pressure of 569 pounds. The pipe is 14,760 feet long, and 4.6 ins. diameter, and the thickness in the lower third of its length is 0.43 ins. This gives $D_0 = 5.46$ ins., $D = 4.6$, and from (333):

$$S = 569\,\frac{5.46^2 + 4.6^2}{5.46^2 - 4.6^2} = 3357 \text{ lbs.}$$

Example 5.—Using the empirical formula (318), we have for the following sizes under the assumption of 150 lbs. pressure:

$D =$	4″	6″	12″	30″	48″
$\delta =$	0.36	0.39	0.46	0.68	0.915
ψ	0.89	0.88	0.92	0.95	0.96
$\frac{S}{p}$	8.62	7.86	12.01	19.5	24.51
S	1293	1179	1800	2925	3176

The above values of S are taken as acting upon the longitudinal section of the cylinder, which is the case when a pipe is open at both ends. When the ends are closed, there is also to be considered the stress on a section at right angles to the axis, which is equal to ½ S. This, combined with the previous value, gives for the inclined resultant, $\sqrt{S^2 + (0.5\,S)^2} = 1.12\,S$ as the minimum. These conditions exist in the case of a cylinder for a hydraulic press. These are usually made of cast iron, and the increased thickness adds greatly to the weight. It is therefore important to use material capable of withstanding a high stress, and to take great care in construction and in the disposition of the material. Repeated meltings of the iron give more homogeneous castings. Good results are also obtained by adding wrought iron in the cupola. By thus improving the quality of the metal, the permissible stress may be increased. A stress as high as 10,000 lbs. may be permitted when the casting is assuredly sound. Similar conditions obtain when bronze is used. With good bronze, if no alteration of form is to occur, the stress should not be greater than 5000 lbs. If it is desired to go higher, some harder composition, such as manganese bronze, must be used.

FIG. 1045.

A few practical examples will be given:

Example 6.—In raising the tubes of the Conway Bridge, a hydraulic press of the following dimensions was used:* Diameter of ram, $K = 18$ inches; bore of cylinder, $D = 20$ inches; thickness, $\delta = 8\frac{1}{4}$ in. The load was 650 tons = 1,456,000 pounds. The pressure in the cylinder being 5900 lbs. per square inch, we get from (323) the stress $S = 10,500$ lbs. The cylinder is shown in Fig. 1045.

Example 7.—In the construction of the Britannia Tubular Bridge several forms of hydraulic presses were used. One of these was a double press with cylinders of the same dimensions as in the preceding example. The load on each ram was only 460.5 tons, and the cylinder pressure 4190 lbs., giving a stress on the metal $S = 7460$ pounds.

Example 8.—The press which sustained the heaviest load on this great work was one which lifted 1144 tons, or 2,562,590 pounds. This was made with a single cylinder with a ram 20 inches diameter, cylinder 22 in. bore, and 10 inches thick. The water pressure was 8400, and the stress in the metal, according to (323), was 14,500 lbs.! When the tube of the bridge had been raised 24 feet, the cylinder gave way, and the load dropped upon the safety supports, but was seriously damaged. The fracture was not longitudinal, but in the cross section near the bottom of the cylinder, as shown in Fig. 1046. The fracture was doubtless due to the sharp angle at the bottom. A new cylinder was made and successfully used, the bottom being altered in shape as indicated by the dotted lines. The first cylinder which was cast for this press was moulded with the bottom up, but was rejected as being porous; the second was cast bottom down, and gave way in use, as above described; the third, for which the iron was melted twice, was successfully used to the end, while a fourth, which was made as a reserve, was not required.

Example 9.—A press designed for making compressed emery wheels has the following dimensions: $D = 28.35$ in., $D_0 = 40.94$ in. $K = 27.56$ in. $P = 2,640,000$ lbs., from which $p = 4425$ pounds. We have $\psi = \frac{28.35}{40.94} = 0.69$, whence $S = 12,134$ pounds, which must be considered a high stress.

More recently the cylinders for hydraulic presses have been cast of steel, permitting stresses as high as 20,000 to 28,000 pounds. Modifications in the method of construction may also be made to enable cast iron to stand higher pressures.

The danger due to casting the bottom in one with the cylinder may be avoided. The method used by Hummel, of Berlin, is to make the cylinder as a ring, and the bottom as a separate plate (Fig. 1047).

Lorenz, of Carlsruhe, makes the bottom separately and screws it in, as shown in Fig. 1048.

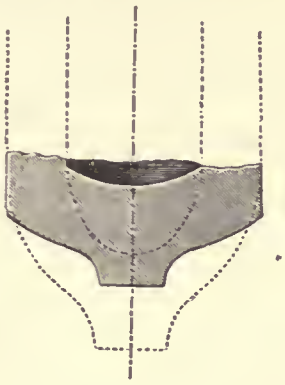


FIG. 1046.

By increasing the diameter of the ram to exert a given force, the pressure of the water required will be reduced, and the stress S will be less. This is not attended with a proportional increase in the amount of metal required, but on the contrary with a reduction. If the cross section of the cylinder be F, we have $F = \pi\,(D + \delta)\,\delta$. Substituting the value of δ from (321), we get $F = \frac{1}{4}\,\frac{\pi D^2 p}{S - p}$ and introducing K,

$$F = \left(\frac{D}{K}\right)^2 \frac{2P}{S - p} \qquad (325)$$

which, for any chosen value of S, diminishes as p is reduced.

Example 10.—In a hydraulic press by Hummel, for making rollers of compressed paper, there are two cylinders of the form shown in Fig. 1047, placed side by side. The diameter K of the ram is 23 inches, and the cylinder diameter D 24 inches, the thickness δ being 8½ inches. The load P on the ram is 2,200,000 pounds. The water pressure is 5174 pounds, and the stress on the material about 10,000 pounds. If we increase K to 26 inches, we have, since this is ⅞ the preceding value, the value of p reduced to $(\frac{7}{8})^2 = 0.79$ of the previous amount, or 4087 lbs. If we now make the relation between the inside and outside diameters of the cylinder the same as before, we have the same relation between S and p, hence $S = 10,000 \times 0.79 = 7,900$ lbs., which is quite practicable. By leaving the relation $\frac{D}{K}$ unchanged, we find the relation between the cross sections of the two cylinders will be also as 0.79 to 1. Hence this alteration in dimensions which reduces the pressure in the cylinder also causes a reduction of about 20 per cent. in the amount of material.

§ 337.

WROUGHT IRON AND STEEL PIPES.

Wrought iron pipe is in very extensive use for conveying gas, water, air, petroleum, as well as steam. These pipes are made either by the process of welding during passage between rollers or are riveted while cold. The former method produces either a butt- or lap-welded joint, the seam being parallel to the axis of the pipe, and more recently pipe has been made in America with a spiral lap-welded seam.† After welding the outside of wrought pipe is generally made smooth by passing between another set of rolls after re-heating, whence it is sometimes called "drawn" pipe. Pipe is also made of mild steel in the same manner as if wrought iron. The Mannesmann system is also used for rolling tubing from the solid rod of steel, copper, delta metal, etc., the product being without any seam.

Welded tubing possesses a great resistance to external pressure and to tension, but a less resistance to internal pressure. Butt welded pipe should not be subjected to a greater stress than $S = 1500$ lbs.; but for lap-welded pipe S may reach 8000

* See Clark, The Britannia and Conway Tubular Bridges. London, 1850.

† See Engineering and Mining Journal, April 7 and 14, 1888; also Scientific American, June, 1888, p. 377.

to 12,000 pounds. Spiral lap-welded tubing has been tested to pressures corresponding to stresses from 30,000 to 40,000 pounds, according to the quality of the material used; but in practical service lower values are used. The Mannesmann tubes have been used without deformation almost to the elastic limit of the material, which, with cast steel and with Siemen's open-hearth steel, reaches 25,000 to 50,000 pounds, and therefore possess a utility to which welded tubes have not attained.

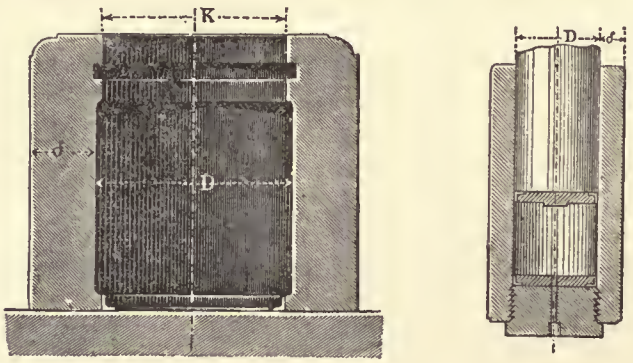


FIG. 1047.

Example 1.—In the oil pipe line shown in Fig. 954, 6 inch lap-welded pipe is used, $\frac{6}{16}$ in. thick, at a pressure of about 1000 pounds. We get from (324)

$$S = \frac{6 \times 1000}{2 \times 0.3125} = 9600 \text{ lbs.}$$

From (323) we have more accurately $S = 1000$

$$\frac{(6.625)^2 + (6)^2}{(6.625)^2 - (6)^2} = 9887 \text{ lbs.}$$

Example 2.—If a spiral lap welded pipe had been used for the preceding example, the thickness δ need only have been $\frac{5}{64}$ in.

Example 3.—If a Mannesmann tube of Siemens steel had been used for the high pressure water service of Example 3, § 336, and the stress put at the moderate limit of 15,000 pounds, we get from (324) $D = 4''$, $p = 1500$ lbs.,

$$\delta = \frac{1500 \times 4}{15{,}000 \times 2} = 0.2'',$$

and from (323) we get more accurately

$$S = 1500\ \frac{(4.4)^2 + (4)^2}{(4.4)^2 - (4)^2} = 15{,}750.$$

The steel pipe would weigh only about $\frac{1}{2}$ that of the corresponding cast iron pipe.

As an example of the efficiency of this construction, Mr. Hamilton Smyth cites an installation over two miles long and under a head of 550 ft., the pipe lying on the surface of the ground and only protected from changes of temperature by a roof of roughly nailed boards, and in which the total loss by leakage was only 3 to 4 cubic feet per minute.

As a consequence of the successful use of these pipes for mining purposes, they were next used for more permanent service as for water supply of cities, and with excellent results. Two such pipes were put in for the supply of drinking water for San Francisco, and a third pipe, many miles long, was subsequently added. For large diameters in permanent installations the sections should be riveted together, while for smaller diameters the joint may be made with lead, as hereafter described. The following table will illustrate some important constructions of this kind.

NAME.	Date.	Length.	Diameter.	Head of Water.	Stress S.	Description of Pipe.
		ft.	in.	ft.	lb.	
Cherokee . . .	1870	12,800	30	887	17,500	Sheet Iron, double riveted.
Virginia City {	1872	37,116	11	1722	14,000	" " " "
	1873	37,116	10	1722	14,000	Lap welded, screw connections.
Texas Brook .	1878	4,440	17	781	17,000	Sheet Iron, double riveted.
Humbug . . .	1868	1,194	26	120	11,500	$\frac{1}{16}$" sheet iron, single riveted.

Also may be noted the Kimberley water works in England, 14 inches diameter, $\frac{1}{4}$ in. thick and eighteen miles long. The superior economy of wrought pipe over that of cast iron is worthy of greater attention.

In order to illustrate the arrangement more fully of an installation of such pipe the inverted siphon in the valley of the Texas Brook, constructed by Mr. Hamilton Smyth, is given, Fig. 1049. The difference in the level is 303.6 feet, and the total length 4438.7 ft. The pipe is in lengths of 20 feet and the figures in the diagram indicate the gauge thickness of the sheet iron in the various portions.

The average diameter of the pipe is 17 inches and the highest value of the stres S was calculated as equal to 16,500 pounds; some of the plates were too thin and the stress in such places reached 18,000 pounds. The inlet is so shaped that the coefficient of contraction reaches 0.92. The pipe is bedded in gravel 12 to 18 inches deep, and passes entirely under the bed

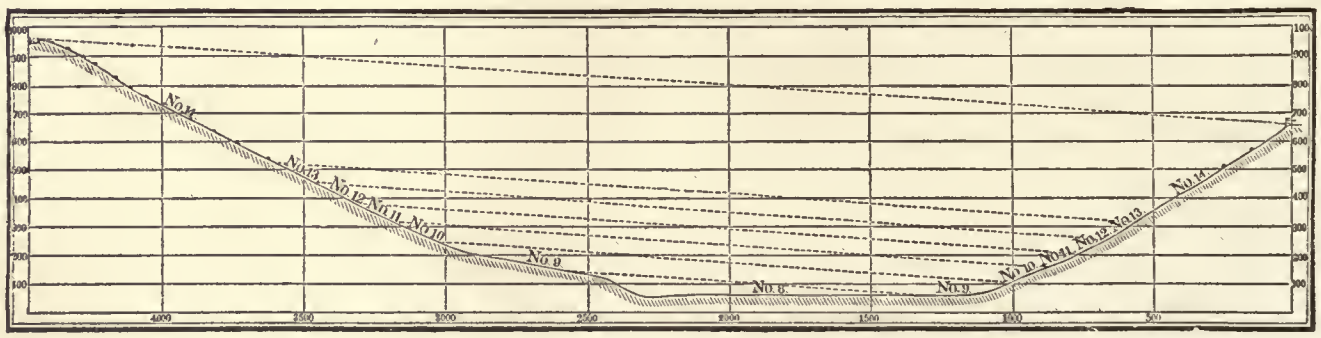


FIG. 1049.

Riveted pipe of wrought iron have been successfully used in America for conducting over long distances, and valuable information has been furnished by Mr. Hamilton Smyth, Jr., upon this subject.*

Wrought iron riveted pipes were first used in California, made steel metal $\frac{1}{16}$ in. thick, to take the place of the canvas hose then extensively used in the operations of hydraulic mining. The pipes were made of ordinary sheet iron, there being a single row of rivets, driven cold, and the joints made simply by inserting the end of one section into the next, as in the case of stove pipes. These first attempts succeeded beyond all expectations and were followed by numerous installations, in sizes reaching as high as 22" to 30" diameter and sections 18 to 25 feet long. A satisfactory protection against rust was obtained by immersing the finished pipes for a few minutes in a boiling mixture of asphaltum and tar. If the fit of the ends was too loose to make a good joint the smaller pipe was wrapped with tarred cord, leaky places being stopped with wedges of wood and the small leaks being checked by sawdust admitted with the water.

*See Engineering and Mining Journal, May and June, 1884; also Journal of the Iron and Steel Institute, 1886, No. I, p. 133.

of the stream. During a large part of the year the siphon is not full of water and hence entraps much air. In order to permit this to escape, air valves of the construction shown in Fig. 1050, are attached at suitable points, fourteen in all being used.

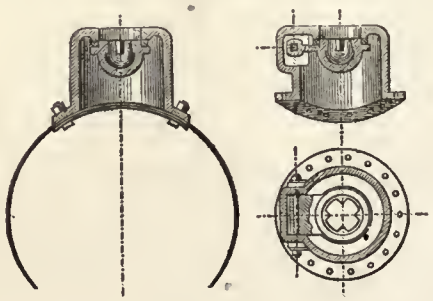


FIG. 1050.

These are simply heavy cast iron flap valves with rubber ring packing. When the chamber is filled with air the valve falls open by its weight but is closed by the action of the water

when the air has escaped. In case of a rupture in the lower portion of the pipe, the air valves in the upper portion prevent the collapse of the pipe from atmospheric pressure.

§. 338.

STEAM PIPES.

When steam is to be conducted to considerable distances, the condensation which is due to loss of heat through the walls of the pipes becomes so great that it is necessary to surround the pipes with a non-conductiug covering. Materials for covering steam pipes play quite an important part in the science of steam economy and their manufacture constitutes an extensive industry. The importance of this subject has long been appreciated, having been considered, among others by the Industrial Society of Mulhouse more than sixty years ago. In these investigations the measure of effect is the amount of water condensed by a unit of surface, as one square metre per second. The following table will indicate some of the results obtained.*

Material of Covering.	Grammes condensed per sq metre per second.	Material of Covering.	Grammes coudensed per sq. metre per second.
Uncovered Pipe . .	2.84 gr.	Clay Pipe	1.12 gr.
Pimont's Mass. . . .	1.56 "	Cotton Waste . . .	1.39 "
Straw	0.98 "	Felt	1.35 "

The so-called Pimont's Mass, consists of loam and cows' hair, 60 mm. (2⅜ in.) thick. The straw was first laid on longitudinally 14 mm. ($\frac{9}{16}$ in.) thick, and then wrapped with straw 15 mm. (⅝ in.) thick. The cotton waste was 25 mm. (1 in.) thick covered with canvas. The felt was saturated with rubber. Straw shows the best results, the condensation beiug only one-third that given by the uucovered pipe.

These experiments have not great present value, partly because the comparisou by condensation of water is not altogether reliable, and partly because new material for covering pipes have since come into use. The Society of German Engineers (*Verein Deutscher Ingenieure*) has uudertaken a series of experiments from which results of value are to be anticipated. In the United States, Prof. Ordway, of Boston, has made some very beautiful investigations, the results being iu two series, the first by the method of measuring the condensed water, the second by the calorimetric method.† The unsatisfactory character of the method of condensation is apparent, as it was found, for example, that a portion of pipe 2 feet long condensed 328 grammes of water per square foot per hour, while 30 feet of pipe gave only 140 grammes per square foot per hour. It is also to be noted that Prof. Ordway's first researches showed much less condensation for the uucovered pipe than appeared in the Mulhouse experiments, so that no definite couclusions could be deduced. The calorimetric method appears to be much more reliable, as the results appear to be more consistent. From the great number of experiments the two following tables have been selected.

TABLE I.

MATERIAL.	Per Cent. Solid Material.	Kilo-Cent. Heat Units.
Air Space	0.0	1302
Carded Cotton	1.0	310
Feathers	2·0	321
Wool	2.1	301
Calcined Magnesia	2.3	335
Cork Charcoal, coarsc	3.1	343
Calcined Magnesia	4.9	340
Wool	5.6	220
Lampblack	5.6	266
Carbonate of Magnesia	6.0	371
Fossil Meal	6.0	393
Wool	7·9	238
Asbestos	8.1	1329
Zinc, White.	8.8	466
Fossil Meal	11.2	426
Pine Charcoal.	11.9	376
Carbonate of Magnesia	15.0	416
Hair Felt	18.5	177
Lampblack	24.4	286
Chalk	25.3	560
Graphite	26.1	1922
Calcined Magnesia	28.5	1156
Zinc, White.	32 3	1164
Pumice Stone	34.2	845
Plaster of Paris	36.8	839
Common Salt	48.0	1983
Anthracite Coal	50.6	968
Fine Sand	51.4	1690
Coarse Sand	52.9	1684

Temperature of steam 155° C. All coverings 1 inch thick = 25.4 mm.

This table gives noteworthy, and in many cases unexpected results. It is important to note that in all cases the transmission of heat bears a definite relation to the percentage of solid matter. For instance, calcined magnesia gives off 335 to 1156 heat units when the percentage of solid matter ranges from 2.3 to 28.5. Asbestos makes an unfavorable showing, and lampblack gives good results but is inconvenient to use; wool, is excellent. In practice the cost is of course an important consideration.

TABLE II.

Temperature of steam 155° C.

MATERIAL.	Thickness. Milli-metres.	Per Cent. Solid Matter.	Kilo-Cent. Heat units.
Glazed Cotton Wadding	50	1.0	129.1
" " "	40	1.3	193.4
" " "	30	1.7	205.5
" " "	20	2.5	326.4
" " "	15	3.4	424.2
" " "	10	5.1	502.4
Wool Wadding	25	5.6	219.8
Calcined Magnesia, loose . . .	25	2.3	335.2
" " crowded . .	25	4.9	340.1
" " compressed	25	28.5	1155.9
Carbonate of Magnesia, loose .	25	6.0	370.9
" " crowded	25	9.4	386.7
" " compressed	25	15	416.5
Fossil Meal, loose	25	6.0	393.4
" " crowded	25	11.2	425.8
‡ Cork in Strips	15	?	87.1
⁂ Silicated Cork Chips	30	?	59.2
Paste of Fossil Meal and Hair .	9	1.0	69.4
Carded Cotton	50	?	157.7
Rice Chaff, straw board	12	?	71.9

This table gives a comparison of fibrous and granular materials. In the first cases the same material was successfully compressed, reducing the thickness and increasing the density, showing and increasing loss of heat. Ordway recommends cork as the best material, especially in the form of cemented chips, which may be formed into semi-cylindrical sections, as has already beeu done in Germany.‖

Ordway does not advise air space uuder the covering, but rather recommends the filling such space with a light powder. Of all the materials tried he recommends in the order given: Hair Felt, Cork, Fossil Meal, Magnesia, Charcoal and Rice Chaff.¶ Prof. Ordway remarks that "it is useless to make the testing apparatus of cumbrous dimensions, for as in chemical analysis we use a gramme or less of the sample, instead of kilogrammes, so in physical experimeuts increase of size does not necessarily enhance the accuracy of the results."

In long stretches of steam pipe the expansion from the heat demands the use of some compensatory device or expansion joiut.**

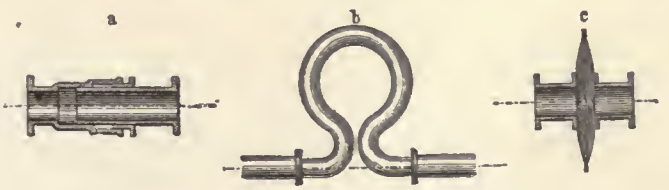


FIG. 1051.

Some of the forms in general use are shown in Fig. 1051. As *a*, is a packed expansiou joint; *b*, is a bent copper pipe; *c*, a drum with flexible steel diaphragms.

* This table has been kept in the metric system, as it is only available for comparison.—*Trans.*

† See Trans. Am Soc. Mechanical Engineers, Vol V. p. 73; Vol. VI. p. 168.

‡ The cork was put on like barrel staves, with a slight air space beneath.

⁂ The cork was chopped into small chips and mixed with 1½ times its weight of water glass at 30° Beaumé.

‖ See Z. D. Ingenieure, 1886, p. 38.

¶ A new, and efficient as well as cheap material made of common flour paste and saw dust, is described in the Revue Industrielle, Sept. 1888, p. 346.

** This "compensation" does not neutralize the expansion, as in a pendulum, but only renders it harmless.

Fig. 1052 shows a U joint with packed connections. The forms given in Fig. 1051 generally require one position of the pipe to be held fast; that in Fig. 1052 permits both lengths of pipe to remain free.

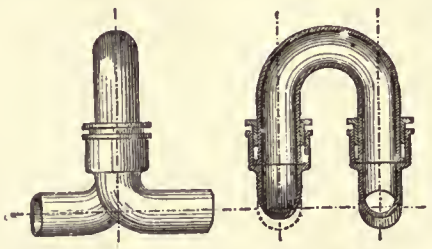


FIG. 1052.

In calculations the actual amount of expansion due to any given temperature we may put the expansion, if t, be the difference of temperature in degrees for:

Material.	Centigrade.	Fahrenheit.
Cast Iron	$\frac{t}{90,100}$	$\frac{t}{162,180}$
Wrought Iron	$\frac{t}{84,600}$	$\frac{t}{155,280}$
Copper	$\frac{t}{58,200}$	$\frac{t}{104,760}$
Brass	$\frac{t}{53,500}$	$\frac{t}{92,300}$

Example. A cast iron pipe 98.4 ft. long, (= 1181.1 inches). At a temperature of 50° F. is filled with steam at 63 pounds pressure, =310° F. The expansion will then be

$$\frac{1181.1 \times 260}{162,180} = 1.89 \text{ in.}$$

§ 339.

PIPES OF COPPER AND OTHER METAL.

Brazed pipes of copper when used as conductors of steam, should not be subjected to higher stresses than 1500 to 2000 pounds, since the brazed joint is not reliable and reduces the strength of the cross section of the metal about one-third. The heat due to the temperature of steam at pressures from 60 to 100 pounds also reduces the strength of the copper from 10 to 12 per cent.*

Seamless pipes made from the solid metal, or rolled by the Mannesmann process, can stand stresses from 8,000 to 10,000 pounds, and when made by forcing in the hydraulic press (see § 333, b. 5) only a stress of about 700 to 800 pounds.

Wooden pipes for water conductors, made water-tight with cement, have been made by Herzog of Logelbach with excellent results; the most recent being 71 in. diameter, and 5900 feet long.

Pipes made of paper coated with asphalt have been used to a limited extent, but do not stand the heat of the sun.

§ 340.

RESISTANCE TO FLOW IN PIPES.

The resistances which oppose the motion of a liquid in a pipe are due either to changes in the direction of motion, to changes in the rate of motion, or to the resistance of friction. We can only here consider a few cases, and those will be limited to the flow through pipes.

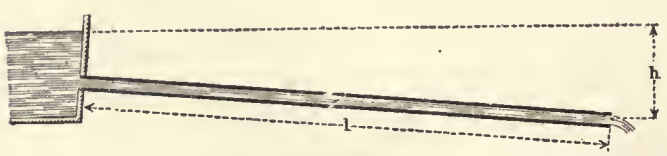


FIG. 1053.

Frictional Resistance.—When a flow of water takes place in a vessel with flat walls, through a cylindrical tube, Fig. 1053, the difference of level between the surface of the water and the mouth of the discharge pipe being h, we have, according to Weisbach:

$$h = \left(1 + \zeta_0 + \zeta \frac{l}{d}\right) \frac{v^2}{2g} \quad \ldots\ldots\ldots (326)$$

in which l is the length and d the diameter of the tube in feet, and v the velocity in feet per second. The volume of flow will be:

$$Q = \frac{\pi}{4} d^2 v \quad \ldots\ldots\ldots\ldots\ldots (327)$$

per second.

In (326) ζ_0 is the coefficient of friction for the orifice of influx, and ζ the coefficient of friction for the rest of the tube.

The coefficient ζ_0, when the entrance is a sharp angle, becomes considerable, having a mean value of 0.505, but when the entrance is carefully rounded it may fall as low as 0.08. In the latter case, for long tubes, ζ_0 may be neglected.†

For the coefficient of friction ζ in the pipe various deductions have been made. The conditions which affect the flow of water in pipes are numerous and variable. In cylindrical pipes the particles arrange themselves in such a manner that those in the axis move with the greatest velocity, and each successive annular sheet moving slower, while the particles in contact with the walls of the tube remain practically at rest, so that the velocity of each annular film, from the wall to the axis is a function of the distance from the wall to the centre, increasing from zero to the maximum.

In the case of gases the velocity of adjacent rings approximates much more closely than with liquids. In both instances the resistance is the sum of the friction of the successive annular layers upon each other.

In practice, the variation of the section of the pipe from the circular form must be taken into account, and also the roughness of the walls. The mathematical expression of these relations cannot be a simple one. In practice also many disturbing influences exist, such, for instance, as ice, weeds, etc. In all comparisons with calculated resistances it is therefore essential that the walls of the pipe should be ascertained to be smooth and clean.

The Society of German Architects and Engineers has in progress modern investigations conducted by several of its members with a view of determining the most useful formula for finding the value of ζ for water. The value which such a formula would possess is undoubted, but before it can be satisfactorily determined the fundamental principles of the subject must be determined.‡ We are at present obliged to use formulas previously determined. Among these the formulas of Weisbach and of Darcy are most available. If we express the loss of head in the height h by friction h_1, in feet, we have for water, according to Weisbach:

$$h_1 = \zeta \frac{l}{d} \frac{v^2}{2g} = \left(0.01439 + \frac{0.017155}{\sqrt{v}}\right) \frac{l}{d} \frac{v^2}{2g} \quad \ldots (328)$$

all dimensions being expressed in feet, and g being the acceleration of gravity.§ We have for:

$v =$	0.1	0.2	0.3	0.4	0.5
$\zeta =$	0.0686	0.0527	0.0457	0.0415	0.0387
$v =$	0.6	0.7	0.8		0.9
$\zeta =$	0.0365	0.0349	0.0336		0.0325
$v =$	1	1¼	1½	2	3
$\zeta =$	0.0315	0.0297	0.0284	0.0265	0.0243
$v =$	4	6	8	12	20
$\zeta =$	0.0230	0.0214	0.0205	0.0193	0.0182

According to Darcy we have for water:

$$h_1 = \zeta \frac{l}{d} \frac{v^2}{2g} = \left(0.01989 + \frac{0.00166}{d}\right) \frac{l}{d} \frac{v^2}{2g} \quad \ldots (329)$$

which gives somewhat greater values than does Weisbach' formula for the higher velocities.‖

*Investigations made after the explosions on the Elbe and the Lahn will be found in *Engineering* for August, 1888, pp. 113, 116, 125. These gave for the modulus of rupture K for tension; for hard brazed pipes $K = 33,400$; for seamless electrically deposited pipes, $K = 50,000$. The reduction in strength due to heat is given according to the old but reliable experiment of the Franklin Institute.

† If the tube starts from another tube instead of from the side of a reservoir, the coefficient of resistance becomes much greater and much care must be given to the shape of the entrance. See Hertel, Zeitschr. D. Ingenieure, 1885, p. 660, also W. Roux, Jenaische Zeitschr. fur Naturwissenschaften, Vol. XII., 1878.

‡ See a Memoir of the D. Arch- u Ing.-Vereine, edited by Otto Iben, published by Meissner, Hamburg, 1880. This must be used with caution on account of numerous typographical errors.

§ This and the two following formulas may also be used when in addition to the height h_1 a second height h_1' is to be added due to the contraction of discharge. This is only of importance in the case of high pressure water transmission, and experimental researches are to be desired.

‖ Recherches Experimentales, etc. Paris, Mallet-Bachelier, 1857.

The formula of M. de Saint Venant, which gives lower results than either of the above, is:

$$h_1 = (0.0321\ v^{-\frac{2}{7}})\ \frac{l}{d}\ \frac{v^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (330)$$

If we insert in equation (327) the value for v from the equation $h_1 = \zeta\ \frac{l}{D} \cdot \frac{v^2}{2g}$ we get:

$$Q^2 = \left(\frac{\pi}{4}\right)^2 d^4 h_1\ \frac{d}{l} \cdot \frac{2g}{\zeta} = \left(\frac{\pi}{4}\right)^2 \frac{2g}{\zeta}\ d^5\ \frac{h_1}{l} = \frac{39.72541}{\zeta}\ d^5\ \frac{h_1}{l}$$

By assuming ζ constant, as proposed by Dupuit,* we may state the practical formula:

$$Q^2 = C\ d^5\ \frac{h_1}{l}$$

Dupnit makes $\zeta = 0.03025649$, whence C becomes 1313, and we have:

$$d^5 = \frac{l}{h_1}\left(\frac{Q}{36.23}\right)^2 \quad \ldots\ldots\ldots (331)$$

and hence for an approximate formula:

$$Q = \sqrt{\frac{h_1}{l}\left(\frac{d}{0.237}\right)^5} \quad \ldots\ldots\ldots (332)$$

These formulas can be applied so that first from the given values of Q and l and the friction loss of head h_1, the diameter D may be determined, and then by making D somewhat larger, and applying the formula of Weisbach or Darcy, the excess of head over friction determined. This will be illustrated by a few examples:

Example 1.—The large inverted siphon described in § 337, Fig. 1049, gives $D = 17'' = 1.417$ ft., $l = 4438$ ft. and $Q = 32$ cu. ft. From th.s we get

$$v = \frac{32}{0.7854 \times (1.417)^2} = 21.2 \text{ ft. per second.}$$

These give in Weisbach's formula:

$$\zeta = 0.01439 + \frac{0.017155}{\sqrt{21.2}} = 0.0181.$$

and from (328) $h_1 = 395.6$ ft.

The actual difference of level is 303.6 feet, and hence the coefficient as determined from Weisbach, is too high. The coefficient determined from the given difference of level is $\zeta = 0.0155$, and as a flow takes place the actual coefficient must be somewhat less. According to Saint Venant's formula (330) $h_1 = 293$ feet, which is slightly under the actual difference of level.

Example 2.—In the work of Iben, already referred to, is given a case in Stuttgart in which $l = 3614$ ft., $D = 0.33$ ft., $v = 2.063$ ft. From these data, Weisbach's formula gives $\zeta = 0.0263$, and thence $h_1 = 19$ ft. The actual value is 23.2 ft., which corresponds to $\zeta = 0.0332$. The difference is probably due to the construction, there being two stop valves and six elbows included in the resistance.

Example 3.—Another instance in Stuttgart is as follows: $l = 302$ ft.; $D = 0.0843$ ft.; $v = 5.897$ ft.

The friction head as determined by observation for $l = 328$ ft., was f2.89 ft. According to Darcy, the friction head would be 76.57, which is quite close to the experimental results. In other instances, however, Darcy's formula has not agreed so well with experiment.

When air is used instead of water, Weisbach gives for the height of a column of water equal to the frictioual resistauce:

$$h_1 = \zeta_1\ \frac{l}{d}\ \frac{v^2}{2g\,e} = 0.025\ \frac{l}{d} \cdot \frac{v^2}{2g\,e} \quad \ldots\ldots (333)$$

in which e is the ratio of the density of the air in the pipe to that of the external atmosphere. Since is e always greater thau unity when the air in the pipe is under pressure, h_1 is smaller than is the case for water, especially when the pressure of the air is great. Valuable experiments upon the transmission of compressed air have been made by Engineer Stockalper at the St. Gothard tunnel.† These showed that Darcy's formula (329) served well for air when the results are multiplied by the ratio of the density of the air to water. Professor Unwin has given some valuable researches upon the frictiou of air, in which he shows the important influence which D exerts upou ζ.‡

Example 4.—At the construction of the Hoosac Tunnel it was observed that the pressure of compressed air fell from 821 pounds per square inch to 801 pounds in being transmitted a distance of about 118,000 feet.

Resistance in Angles and Bends.—The resistance due to an angle, such as Fig. 1054 a is important, and is dependent upon what Weisbach calls the semi-angle of deviation, β, according to the following formula:

$$h_2 = \zeta_2\ \frac{v^2}{2g} = (0.9457\ sin^2\beta + 2.047\ sin^4\beta)\ \frac{v^2}{2g} \quad \ldots\ldots (334)$$

from which we get:

$\beta =$	10	20	30	40	45	50	60	70
$\zeta_2 =$	0.046	0.139	0.364	0.740	0.985	1.260	1.861	2.431

Example 5.—In a right angle bend $\beta = 45°$, the loss is practically eqnal to $\frac{v^2}{2g}$.

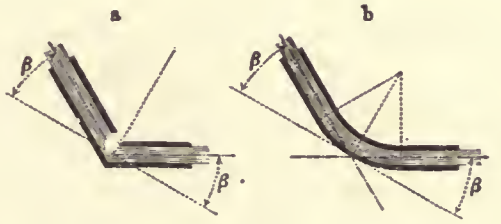


FIG. 1054.

In the case of bends, Fig. 1054 b, the resistance is not so great, but is too large to be neglected since we have:

$$h_2 = \zeta_2\ \frac{\beta}{90} \cdot \frac{v^2}{2g} \quad \ldots\ldots\ldots\ldots (335)$$

The ratio of the radius of the tube to the radius of the curvature of the bend affects the coefficient as below:

$\frac{0.5\,D}{r} =$	0.1	0.2	0.3	0.4	0.5
$\zeta_2 =$	0.131	0.138	0.158	0.206	0.294
$\frac{0.5\,D}{r} =$	0.6	0.7	0.8	0.9	1.0
$\zeta_2 =$	0.440	0.661	0 977	1.408	1.978

Example 6.—For a right angle bend in which $r = D$ we have:

$$h^2 = 0.294\ \frac{45}{90}\ \frac{v^2}{2g} = 0.147\ \frac{v_3}{2g}$$

or only about ½ the resistance of a sharp bend with any curvature.

Resistances due to Sudden Changes of Cross-Section.—When water which is moving at a velocity v_1 suddenly changes to another velocity v, see Fig. 1055 a, it experiences a loss of pressure which, according to Weisbach, is equivalent to a height:

$$h_3 = \frac{v_1^2 - v^2}{2g} = \left(\frac{F}{F_1} - 1\right)^2 \frac{v^2}{2g} = \zeta_3\ \frac{v^2}{2g} \quad \ldots\ldots (336)$$

F and F_1 being the respective cross sections; also $Fv = F_1 v_1$.

Doubling the cross section causes a loss of head equal to $\frac{v^2}{2g}$.

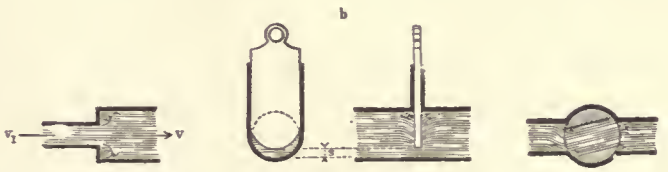


FIG. 1055.

For gate valves, Fig. 1055 b, or cocks, Fig. 1055 c, there is a loss due to the amount of coutraction. For gate valves we have from Weisbach:

Openings =	⅛	¼	⅜	½	⅝	¾	⅞
$\frac{F_1}{F}$ =	0.159	0.315	0.466	0.609	0.740	0.856	0.948
ζ_3 =	97.8	17.00	5.52	2.06	0.81	0.26	0.07

and for cocks:

Angle =	10°	20°	30°	40°	50°	60°	65°	82½°
$\frac{F_1}{F}$ =	0.850	0.692	0.535	0.385	0.250	0.137	0.091	0
ζ =	0.29	1.56	5.47	17.3	52.6	206	486	∞

From the above tables it will be seen how important an influeuce is exerted by valve chests, mud traps and the like upon the flow of water. In all such cases it is important to modify the suddenness of the change of velocity by rounding and curving all angles in the passages, and in this way a large pait of the loss may be obviated. For gaseous fluids the resistance is less, but is at the same time sufficiently important to be carefully considered. For a fuller discussion of the resistances offered to water in canals and streams the reader must be referred to special treatises ou the subject.

*See Dupuit, Traité theoretique et pratique de la conduite et de la distribution des eaux. Paris, Dunod, 1me ed. 1854; 2me ed. 1865.

†Stockalper, Experiences, fates au Tunnel de Saint Gothard, sur l'écoulement de l'air comprimé. Genève, 1879.

‡The coefficient of friction of air flowing in long pipes. Proc. Inst. C. E. London, 1880.

§ 341.

Methods of Connecting Cast Iron Pipes.

One of the most frequently used methods of connecting cast pipes is by means of the common flange joint, Fig. 1056.

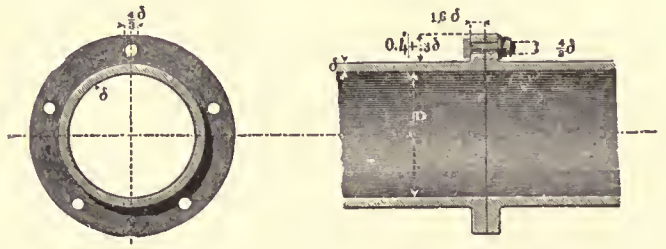


Fig. 1056.

The proportions are given in the illustration. Formerly it was customary to raise a small bearing surface inside the bolt circle, but this is generally omitted now, and the entire surface of the flanges finished, making a much better joint, although a trifle more expensive. In many instances a ring of copper wire, let into a groove, is used to make the joint. For pipes which are not subjected to very high pressures the number of bolts A, is determined from the following:

$$A = 2 + \frac{D}{2} \quad \ldots\ldots\ldots\ldots (337)$$

in which D is the diameter of the pipe inches. This would give for a pipe 4 inches in diameter four bolts, and for one 3 inches diameter 6 bolts. According to (337) an air pump cylinder 60 inches in diameter would have $2 + \frac{60}{2} = 32$ bolts.

When the pressure is known to be great, or for cylinder lids, etc., the following formula is to be preferred:

$$A = \frac{a}{2400}\left(\frac{D}{d}\right)^2 \quad \ldots\ldots\ldots\ldots (338)$$

in which d is the diameter of the bolts, D the diameter of the cylinder, and a the pressure in pounds per square inch. This assumes the diameter of the bolt at the bottom of the thread to be 0.8 d, and the stress in the bolts to be 3500 lbs. as in formula (72).

Example.—A steam cylinder 40 inches in diameter, subjected to a pressure of 60 pounds, would have according to (320) a thickness of $\delta = 0.787 + \frac{40}{100} = 1\frac{3}{16}$ in. This gives from Fig. 1056 for the bolts, $d = \frac{4}{5} \times \frac{3}{16} = 1.58$, say $1\frac{9}{16}$ in., and these values in (338) give for the number of bolts:

$$A = \frac{60}{2400} \cdot \left(\frac{40}{1.58}\right)^2 = 16 \text{ bolts.}$$

(Compare close of Chapter XXVI).

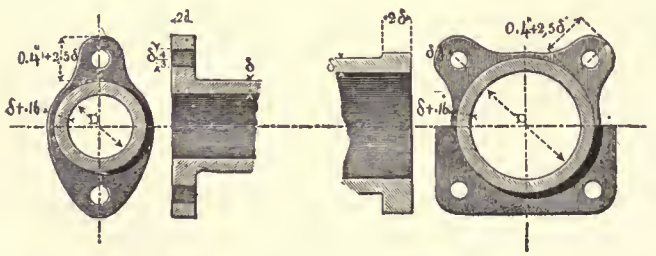


Fig. 1057.

Flanges with ears, as shown in Fig. 1057, are frequently used, the thickness being made 2 to 2.5 δ, instead of 1.6 δ, on account of the smaller flanges.

On the Prussian State railways flange joints are made with a lenticular shaped ring inserted in the joint, as shown in Fig. 1058.

This permits a certain amount of motion and gives good results in practice. The following table of dimensions is based on one used on the Prussian railways:

D	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6
D_1	3	3¼	3½	3¾	4	4½	5	5⅝	6¼	6⅝	7¼
r	2⅛	2⅛	2⅜	2⅞	3	3⅜	3⅝	4	4¼	4⅝	4⅞
s	½	½	½	$\frac{9}{16}$	$\frac{9}{16}$	⅝	⅝	$\frac{11}{16}$	¾	¾	$\frac{13}{16}$

Fig. 1059 a shows a cast iron bend with flange. The bend should not be too sharp, in order to avoid excessive resistance to the flow of the water. (See Example 6, § 340.) Bends of this sort require a separate pattern to be made for every different angle. Brown's joint is more convenient in this respect, Fig. 1059 b. The bolt holes in this form should be drilled in only one of the flanges first, and the other flange marked off in

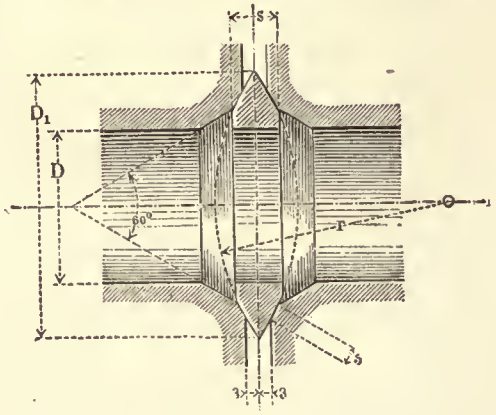


Fig. 1058.

place. For any flange angle α the pipes may be connected for any angle between 2α and 180°. In the illustration $\alpha = 40°$, which answers for most practical purposes.

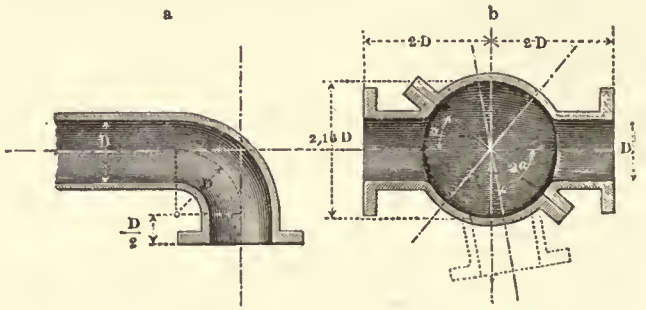


Fig. 1059.

Bell or socket connections are much used for gas and water pipe. The joint is caulked with lead, which may conveniently be made in half rings and driven in, or run in in place, a packing of oakum being first driven in.

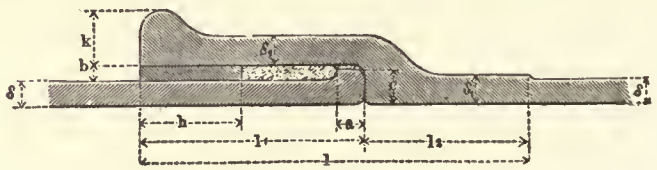


Fig. 1060.

The large end of the pipe is called the bell, the other the spigot. The dimensions of the various parts in Fig. 1060 may be taken as follows, the thickness δ being determined from formula (318), *i.e.*, $\delta = 0.315 + \frac{D}{80}$.

Thickness of bell,	$\delta_1 = 0.375'' + 0.0135\,D.$
Thickness of bead,	$k = 0.7'' + 0.0025\,D.$
Inside length of bell,	$l_1 = 2.625'' + 0.11\,D.$
Length of bell reinforcement,	$l_2 = 2'' + 0.09\,D.$
Outside length of bell,	$l = 4.625'' + 0.20\,D.$
Space for packing,	$b = 0.1875'' + 0.007\,D.$
Depth of lead ring	$h = 1.125'' + 0.07\,D.$
Length of bead on spigot.	$a = 1.2\,\delta.$
Thickness of bead,	$c = \delta + b - 0.0625''.$

Some makers put a bead around the inside edge of the bell to assist in retaining the lead packing, but others consider this but little use, owing to the softness of the metal. More recently the bead has been altogether omitted from the spigot end, a shoulder being cast on the inside of the bell instead.

In Belgium a joint is used in which a gum ring of globoid form (see Fig. 637 a) is used instead of the lead packing, the ring rolling in as the spigot is pushed into the bell.

Fig. 1061 is Petit's pipe joint. A gum ring is inserted in the short bell, and one clamp being connected the pipe is used as a lever to compress the gum ring, the second clamp can be secured. This coupling, which was used in the extensive water system of the camp at Châlons, is cheap and can be rapidly

connected, and possesses a certain flexibility which permits it to be used in running a line of pipe over uneven ground.

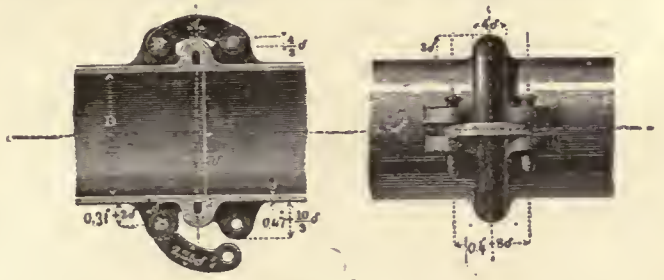


FIG. 1061.

A form of screw connection for cast iron pipe is shown in Fig. 1062. The screw thread is cast on the pipe and a leaden

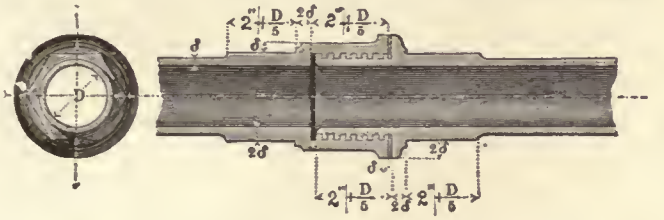


FIG. 1062.

gasket is placed so as to pack the joint outside of the screw connection.* This may be considered as a flange joint with a single central bolt, which latter is made large enough to permit the pipe opening to pass through it (see § 86). Since the pipe must be revolved in making the connection, it is necessary to provide wrenches of suitable size for the purpose.

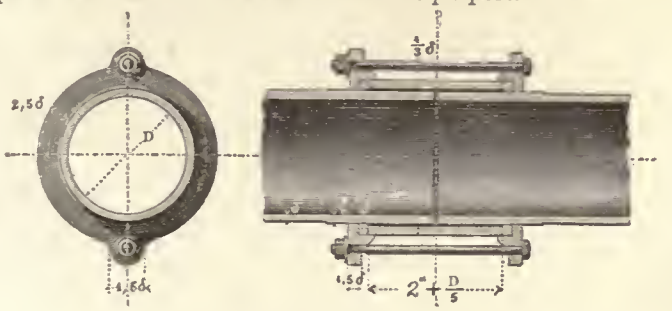


FIG. 1063.

Fig. 1063 shows Normandy's pipe joint. The packing consists of two rubber rings. This very simple joint is very useful under certain circumstances, where the proper packing is available. It possesses the flexibility of Petit's joint and is easily connected and disconnected.

A similar form of joint has been made for water pipes, using packing rings of lead. The sleeve may be considered as a double bell and the pipes are perfectly straight without any bead at either end. The distance from the centre of one joint to that of the next constitutes a "length." With cast iron pipe this is made a minimum of about 4 to 7 feet, being made as long as practicable for extensive lines of pipe. For gas and water pipe with bell and spigot connections the following proportions occur in practice:

$D =$	4 inches,	$l =$	7 to 10 ft.
$D =$	12 "	$l =$	10 to 12 ft.
$D =$	24 " and over,	$l =$	12 ft.

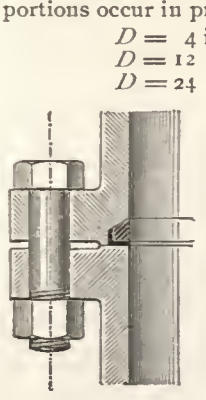


FIG. 1064.

A form of joint used by Riedler for high pressure water connections is shown in Fig. 1064.† The flanges are faced in the lathe and bolted together without any packing in the joint. A leather ring is placed in a channel turned in the pipe and held in place by a spring ring in two parts, or this latter may sometimes be made in one piece.

Joints with spherical contact surfaces have been used by Hoppe for cast iron high pressure pipes when they are to be laid in yielding ground.‡

Three forms of construction are shown in Fig. 1065. At *a* is a single ball joint. The bearing ring is held in position by a ring of bronze divided at right angles to the axis; this form permits a deflection of 5°. At *b* is shown a double joint constructed in a similar manner

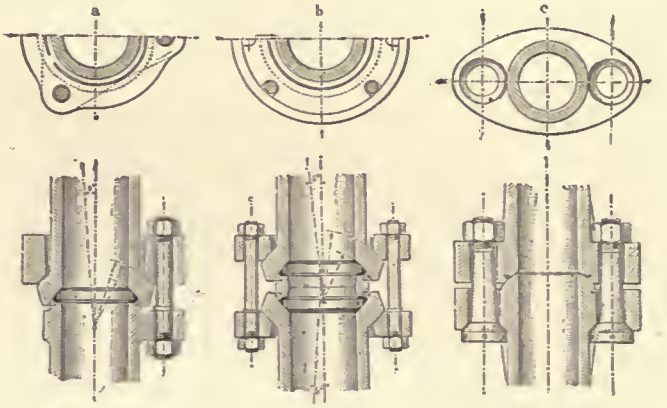


FIG. 1065.

and permitting a deflection of 10°. The third form, which is the most recent, has no packing ring, and the bolts are made with spherical heads to facilitate motion.

§ 342.

CONNECTIONS FOR PIPES OF WROUGHT IRON AND STEEL.

Riveted pipes are often connected by means of wrought or cast iron flanges, as shown in Fig. 1066 *a* and *b*. When no

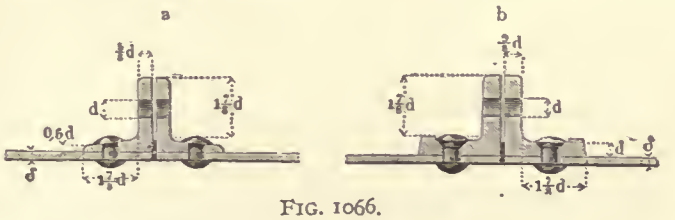


FIG. 1066.

other data are at hand, the diameter and number of bolts may be determined by assuming the pipe to be of cast iron, and using the proportions given in the illustration. The actual thickness δ of the pipe may then be determined independently according to the material, pressure, and other conditions.

Example.—A wrought iron pipe 3 ft. 4 in. in diameter, for delivering water to a turbine, is to be fitted with flanges of wrought iron. A cast iron pipe of this diameter would have a thickness, according to (318)

$$\delta = 0.315'' + \frac{40}{80} = 0.815''$$

whence from Fig. 1056 $d = \frac{4}{5} \times 0.815 = 1.05''$, say $1\frac{1}{8}$. The number of bolts, according to (337), will be $2 + \frac{40}{2} = 22$. If the internal pressure is 30 pounds per square in. we have, according to (324), taking $S = 4200$:

$$\delta = 0.5\,\frac{30 \times 40}{4200} = 0.143'', \text{ say } \tfrac{3}{16}''.$$

FIG. 1067.

For thin pipes a very practical form is that shown in Fig. 1067 *a*. The ends of the pipes are flanged over, and the turned-over ends countersunk into the cast flange rings, the bolt heads also being countersunk. A similar form with wrought iron flange rings is shown at *b*.‖ For the thin pipes described in § 337, when subjected to a high internal pressure, the joint shown in Fig. 1067 *c* is adapted. In this form a short sleeve is riveted into one of the pipe ends and a loose ring slipped over the outside of the joint, forming a space into which lead is run and afterwards caulked. This also serves as a sort of expansion joint (compare § 338).

Many important constructions are made with wrought iron pipe. The connections are usually made by screwing the parts together, and for this purpose many special pieces are made, known by the generic term of "pipe fittings." For straight connections the ordinary "socket" is used, while for angles the so-called "elbows" and "tees" are made.

* This joint is used by the Lauchhammer Works for pipe up to 2½ inches diameter.

† See Zeitschr. D. Ing. Vol. XXXII., 1880, p. 481.

‡ German Patents, No. 42,126.

‖ For description of a flange joint with welded conical rings, by De Naeyer, see Zeitschr. D. Ing., Vol. XXX, 1866, p. 106.

The American practice of making the thread tapering is much to be recommended, since by means of a little cementing material a tight joint may be made. The American Mechanical Engineers have given careful attention to the proportions of pipe fittings, and since 1887 the forms proposed by the late Mr. Robert Briggs have been generally adopted.*

The system is as follows: The thread is of triangular section with the angle $2\beta = 60°$, as in Sellers' system. The top and bottom of the thread are flattened $\frac{1}{10}$ of the theoretical depth t_o, so that the actual depth $t = 0.8t_o$, and hence equal to 0.69 of the pitch s, Fig. 1068 *a*.

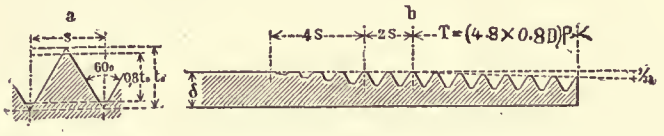


FIG. 1068.

The end of the pipe is given a taper of $\frac{1}{32}$ on each side, the length of the tapered part being $T = (4.8 + 0.8D)s$, D being the outside diameter of the pipe and s the pitch. Beyond the taper portion comes a length $T_1 = 2s$, which threads are full at the root but imperfect at the top, beyond which there is a length $T_2 = 4s$, consisting of imperfect threads blending into the full outside diameter. The thickness δ of the pipe is such that the thickness of metal below the thread at the end of the pipe is $= 0.0175D + 0.025''$. The pitch s is finer than for bolts of the same diameter, there being only five different pitches used, and the various dimensions are given in the following table:

TABLE OF STANDARD PIPE THREADS.

Diameter of Pipe.			Thickness of Metal.	Screwed Ends.	
Nominal Inside.	Actual Inside. D.	Actual Outside. D_o.		Threads Per Inch.	Length. T.
Inches.	Inches.	Inches.	Inch.	No.	Inch.
⅛	0.270	0.405	0.068	27	0.19
¼	0.364	0.540	0.088	18	0.29
⅜	0.494	0.675	0.091	18	0.30
½	0.623	0.840	0.109	14	0.39
¾	0.824	1.050	0.113	14	0.40
1	1.048	1.315	0.134	11½	0.51
1¼	1 380	1.660	0.140	11½	0.54
1½	1.610	1.900	0.145	11½	0.55
2	2.067	2.375	0.154	11½	0.58
2½	2.468	2.875	0.204	8	0.89
3	3.067	3 500	0.217	8	0.95
3½	3.548	4.000	0.226	8	1.00
4	4.026	4.500	0.237	8	1.05
4½	4.508	5.000	0.246	8	1.10
5	5.045	5.563	0.259	8	1.16
6	6.065	6.625	0.280	8	1.26
7	7.023	7.625	0.301	8	1.36
8	8.082	8.625	0.322	8	1.46
9	9.000	9.688	0.344	8	1.57
10	10.019	10.750	0.366	8	1.68

Taper of conical portion of tube 1 in 32 to axis of tube.

It will be observed in the table that the thickness δ agrees very well with the formula $\delta = 0.111\sqrt{D_o}$. This gives for the diameters 0.405, 1.050, 4.000 and 10.750, the thicknesses 0.071, 0.114, 0.222, 0.364, which agree quite closely with the actual values. The shape of the sockets is shown in Fig. 1069, the thread being given a somewhat greater taper than $\frac{1}{32}$, so that the greatest stress will come on the strongest part of the socket.

The increasing use of such pipe in Germany makes it most desirable that a standard of dimensions should be adopted. The American system is manifestly unsuited for use with the metric system. The general arrangement of the American system may, however, be followed with some approximations to adapt it to the metric measurements.

The angle of thread may be the same as in the American system: $2\beta = 60°$. The depth of thread may also be abbreviated $\frac{1}{10}$ top and bottom, making $t = 0.8t_o = 0.68s$, and the taper can also be made $\frac{1}{32}$ on a side. The length T of the tapered portion may be made $T = (5 + \frac{1}{25}D_o)s$, which is about the metrical equivalent of the former expression, the nearest even value being taken. The lengths $T_1 = 2s$ and $T_1 = 4s$ may be retained.

For the thickness of pipe the American formula transformed gives $\delta = 0.555\sqrt{D_o}$ in millimetres. Finally for the pitch we may take

$s =$	1	1.4	1.8	2.2	3.2	mm.
	(0.94)	(1.41)	(1.81)	(2.21)	(3.17)	in.

the values in parentheses being the corresponding equivalents of the American pitches. The following table gives the values from 10 to 325 mm. This system has been submitted by the author to the manufacturers of the Mannesmann tubes in Remschied, Saarbrück and Komotau, and by them adopted.

METRIC PIPE THREAD SYSTEM.

Outside Diameter D_o.	Thickness δ.	Inside Diameter D.	Pitch s.	Length of Thread T.	Length $T_1 = 2s$.	Length $T_2 = 4s$.
10	1.75	6.5	1.0	5.5	2.0	4
15	2.00	11.0	1.4	7.5	2.8	5.6
20	2.50	15.0	1.4	8	2.8	5.6
25	2.75	19.5	1.8	11	3.6	7.2
30	3.00	24.	1.8	12	3.6	7.2
35	3.25	28.5	2.2	14	4.4	8.8
40	3.50	33.0	2.2	15	4.4	8.8
50	4.00	42.0	2.2	15	4.4	8.8
60	4.25	51.5	2.2	16	4.4	8.8
70	4 75	60.5	3.2	25	6.4	12.8
80	5.00	70.0	3.2	26	6.4	12·8
90	5.25	79.5	3.2	28	6.4	12.8
100	5.50	89.0	3.2	29	6.4	12.8
110	5.75	98.5	3.2	30	6.4	12.8
120	6.00	108.0	3.2	31	6.4	12.8
130	6.25	117.5	3.2	33	6.4	12.8
140	6.50	127.0	3.2	34	6.4	12.8
150	6.75	136.5	3.2	36	6.4	12.8
175	7.25	160.5	3.2	38	6 4	12.8
200	7.75	184.5	3.2	42	6.4	12.8
225	8.25	208.5	3.2	45	6.4	12.8
250	8.75	232.5	3.2	48	6.4	12.8
275	9.25	256.5	3.2	51	6 4	12.8
300	9.50	281.0	3.2	54	6.4	12.8
325	10.00	305.	3.2	58	6.4	12.8

In the preceding table the pipe is classified according to its outside diameter D_o, but it is a question whether it would not be better to follow the custom of designating the sizes by the interual diameter D. The former, however, has an important influence upon the dimensions of the fittings, which it is most desirable to reduce to a standard system. It will be seen by refereuce to the table of American pipe dimensions that the actual internal diameter differs frequently from the nominal size, the latter really being only a convenient name. By adopting a strict gradation for the sucessive sizes of D_o it would be practicable to make the thickness δ somewhat less than given in the table, but in some cases it would be greater. When D_o is greater than 325 mm., δ may in ordinary cases be made $= 10$ mm.

The production of the screw threads both in pipe and fittings must be carefully considered in order to insure the interchangeability which is necessary. Powerful and accurate machines have been devised for cutting the threads, as well as devices for producing the taps and dies, and also gauges to insure maintenance of standards. This branch of the art has been carried to a high degree of perfection in America.

Fittings for Wrought Pipe.

The simplest pipe fitting is the socket used for connecting two pipes of equal diameter D_o, and is made of wrought iron

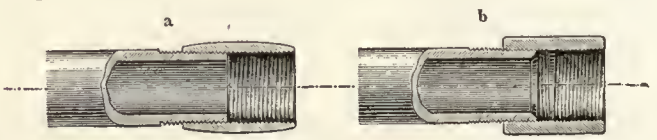


FIG. 1069.

or of steel. It is made of sufficient length to give a thread in each end of length equal to T, as given in the preceding tables, together with a slight clearance between the ends of the pipes, Fig. 1069 *a*. In many cases the socket must be made with right

* See Trans. Am. Soc. Mech. Engineers, Vol. VII, pp. 311 and 414; also Vol. VIII, pp. 29 and 347.

and left hand threads, as in Fig. 1069 *b*, this being necessary to connect two pipes which cannot be turned axially. For other connections a variety of fittings are made, examples of which are showu in Fig. 1070.

FIG. 1070.

In Fig. 1070, *a* is a right angle; *b* an elbow (abbreviated in practice to "ell"); *c* is a T; *d* a cross; and *e* a reduciug socket. These fittings are used as connections for all sorts of gaseous pressure organs. They may also be used for liquids, as water, brine, oil, etc., wheu the velocity of flow is not great. For important iustallations it is becoming more aud more the practice to design the fittiugs in such forms as to produce a minimum of resistance. By making the fittings of cast iron, as is done in England and America, where pipe constructious are very extensively used, it is possible to adhere to accurately designed standard forms.

The most important fitting is the elbow, for the right angle bend occasions far too much resistance to be used in important cases. In Fig. 1071 three forms are shown, all of which are

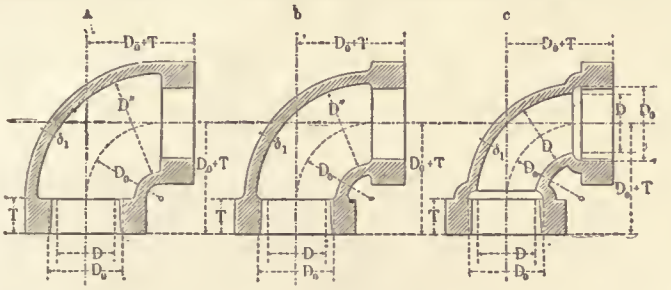


FIG. 1071.

designed to be used with the thread already described. Of these, form *b* is the most popular, although form *a* is frequently used because of the smoothness and neatness of external appearance. Form *c* is here proposed as an additional design. A comparison between the three forms will show a differeuce in resistance which may be calculated as follows: The resistance may be divided into two portious; one due to the curvature, the radius of curvature being made equal to D_0; and one due to the enlargement and consequent contraction of the passage.

Example 2.—In the three forms shown in Fig. 1071 let the radius of curvature $D_0 = 1$ inch, and let the velocity v be taken at 6.56 feet per second. We then have from (335) for the resistance due to the curvature, $h_2 = \zeta_2 \frac{\beta}{90} \cdot \frac{6.56^2}{64.4} = 0.334\,\zeta_2$, and for ζ_2 in the various forms

		a	*b*	*c*
	$\frac{0.5\,D}{r}$ =	0.66	0.54	0.39
	ζ_2 =	0.573	0.352	0.201
whence	h_2 =	0.191	0.118	0.067

We also have from (336) for the loss due to enlargement and coutraction:

$$h_3 = 2\left[\zeta_3\left(\frac{6.56^2}{64.4}\right)\right] = 1.336\,\zeta_3$$

whence		*a*	*b*	*c*
	$\frac{F}{F_1}$ =	$\left(\frac{1.3125}{0.75}\right)^2$	$\left(\frac{1.0625}{0.75}\right)^2$	1.0
	ζ_3 =	3.444	0.904	0 *
whence	h_3 =	4.6	1.207	0
hence	$h_2 + h_3$ =	4.791	1.325	0.067

It will thus be seen that form *a* cannot be recommended, except for steam for which the coefficient of loss is much less than for water; and that form *b* occasions quite a perceptible loss. Form *c* is much to be preferred, both because it offers the least resistance, and also because it is lighter, the proportion of metal in the curved portion of the three forms being as 36 : 30 : 25.

The only dimeusion which is important in connection with a staudard system of fittings is the distauce $D_0 + T$, which should be taken from the preceding table. The thickness δ_1 is mainly dependent upon matters of casting, and is here made $= \delta + 0.04''$ (δ + 1 mm.) the thickness of the collar being $= 2\delta_1$.

An indispensable condition for auy standard system of fittiugs is the constant length from end to centre for each size of elbow, cross, or T, so that at auy time one fitting may be substituted for another without affecting the length of the pipes. This principle can also be observed when the fittings are used to connect pipes of different diameters.† Such fittings are always known by the name of the largest opening, whether T, elbow, or cross, this dimension governing the proportions.

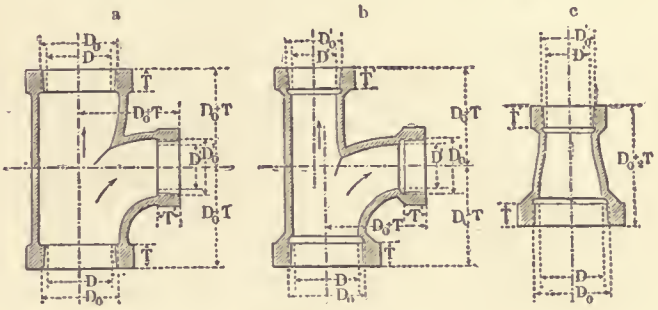


FIG. 1072.

Fig. 1072 *a* shows a T, which is proportioned to permit one-half the flow of water to pass off the side openiug. This is based on the form *b*, of the preceding illustration, and, as is usual, the direct discharge opening is made the same size as the entrauce. D' is made equal to 0.7 D, thus giving one-half the area, and making the velocity the same as in the entrance pipe; if the side opening had been kept full the velocity would have been reduced one-half. The side outlet is shaped like an elbow, with a sharp internal partition to direct the flow. According to Roux, these partitions are of much importance, acting as wedges to split the flow of the water. At *b* is shown another form, in which both discharge openings are reduced, and every precaution taken to give a smooth flow to the water. At *c* is a reducing fitting which will double the velocity of flow, the reduction in diameter being made by gradual curves.

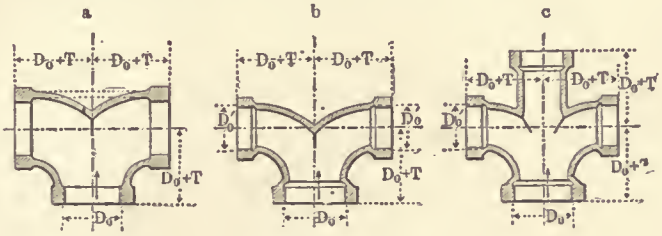


FIG. 1073.

Fig. 1073 *a* shows a T with equal outlets, formed on the plan of the elbow shown in Fig. 1071 *b*. This is made with a dividiug wedge, which is much better than the straight form shown by the dotted lines. The latter forui causes material loss by the sudden reduction of velocity to one-half. The form shown at *b* is inteuded still further to reduce this loss. At *c* is shown a cross with three equal outlets designed on the same principle.

The previously described fittings have beeu given on the assumption that the velocity of flow is to be kept uniform from the point of division both as regards the fittings and in the pipes. In extensive installations, whether in residences, public buildings or manufacturing establishments, this is not ofteu the case. Very often it is found that one portion of a system is possessed of but little velocity of discharge, while a neighboring pipe has a flow of high velocity in it. The resistances in such systems become quite material, but may be somewhat reduced by giviug care to the shape of the fittings.

In adopting standard dimensions for pipe fittings, which may be based either upou form *b* or *c*, especial precautions must be takeu to insure interchaugeability, this being the principal advantage to be obtained. This iuvolves accurate tappiug of the threads both in the sockets and in the right-angle fittings, which is accomplished by special devices which enable all these operations to be performed without releasing the fitting, the accuracy of angles and sizes then being readily controlled by the machiue. The sizes of the fittings are cast upou them in distinct figures, so that they may readily be determined.

§ 343.

CONNECTIONS FOR PIPES OF LEAD AND OTHER METALS.

Lead pipes may be connected by means of separate flanges of wrought irou which draw the expauded ends of the pipes together.

* The small clearance for the screw thread may be neglected.

† See Trans. Am. Soc. Mech. Engrs., IV, p. 273.

A good flange connection for lead pipe is shown in Fig. 1074;* the pipes are expanded and a double cone socket of brass inserted and drawn together by bolts. Fig. 1075 shows

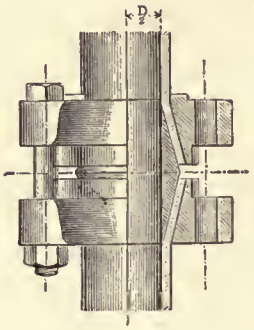


FIG. 1074.

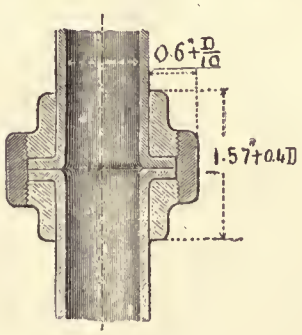


FIG. 1075.

another design, by Louch; the pipes are drawn together by means of screw flanges and a collar, the three pieces all being made of cast iron.

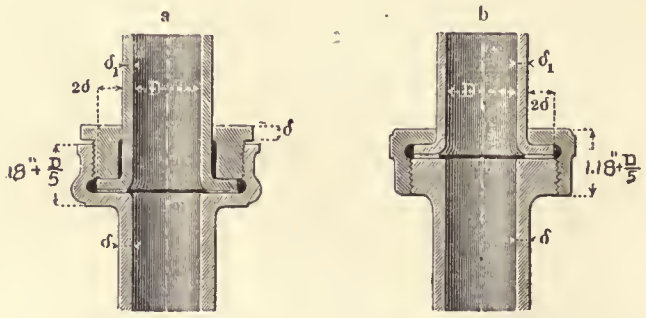


FIG. 1076.

Fig. 1076 *a* shows a connection for joining lead to cast iron pipe, and Fig. 1076 *b* is for lead to wrought iron pipe; the loose collars in both forms are made hexagonal or octagonal externally, so as to be operated by wrenches.

§ 344.

FLEXIBLE PIPES.

For many purposes it is desirable to have a pipe which shall be yielding or flexible, so that, for example, it may follow the inequalities of the ground, or may accommodate itself to yielding supports. In such cases the flange connections may be constructed to permit motion by means of ball and socket bearings, as shown in Fig. 1065, such joints being especially adapted for pipes to be laid under water. An example of such construction is found in the water main built by G. Schmidt, of Carouge, for the water supply of Geneva, laid on the bed of the Lake of Geneva. The pipe is 47¼ inches (1.2 metre) diameter, and is made in lengths of 29½ feet of riveted wrought iron, 0.197 in. thick (5 mm.). The connections are ball and socket flanges, riveted to the pipes.

Instead of making the pipe rigid and the joints flexible, the joints may be made rigid and the pipe flexible. Familiar examples of flexible pipe are various kinds of hose, made of leather, canvas or rubber. Special forms of couplings are made for fire hose. If the hose is to be subjected to heavy pressure, either internally or externally, special methods of increasing its strength are used. This may be done by means of a spiral of wire, or better by two separate spirals, one to resist internal pressure and one to resist external pressure, as shown in Fig. 1077 *a*. The wire spirals furnish the strength and the hose the

FIG. 1077.

tightness. This idea may be still further carried out by making the material which makes the pipe tight, also in the spiral form. This is shown in the flexible metallic tubing of Levasseur, of Paris, shown in Fig. 1077 *b*.† This is composed of a spiral of copper or similar metal, the section resembling somewhat the figure 5. The spiral is wound upon a mandrel in a special machine, a layer of rubber packing being wound in at the same time, as shown in the illustration. This pipe has been found to answer well for gas, water, steam, air, etc., and is adapted to high internal or external pressures, being tested to 180 pounds. Flanges and other fittings are screwed on to the spiral and soldered carefully. This pipe is used, among other purposes, for connections for air and vacuum brakes.

§ 345.

PISTONS.

Next to the various kinds of pipes, as already discussed in § 310, the most important members in pressure organ mechanism are the various forms of pistons, and with these the different methods of packing will be considered. Pistons, properly so called, are fitted with packing which presses outward against the walls of the cylinder, while in the case of plungers the packing presses inward. Both forms will be given consideration.

The most important forms of pistons are those used in steam engines. Some of the low-pressure engine pistons are yet made with hemp packing; but for higher pressures, metallic packing is used, this consisting of metal rings pressed against the walls of the cylinder by springs and by the steam pressure. In some instances a combination packing is used, the metal rings having a backing of hemp instead of springs.

The unit upon which the dimensions of the following pistons are based is determined from the formula:

$$s = 0.368 \sqrt[4]{D - 0.04} - 0.118 \quad \ldots\ldots (339)$$

in which D is the piston diameter in inches.

The following table will aid by giving a series of values for s and D:

s	D	s	D	s	D
0.4	4	0.65	20	0.90	58
0.45	5.7	0.7	24	0.95	70
0.5	8	0.75	30	1.00	85
0.55	11	0.8	40	1.05	100
0.6	14	0.85	48	1.10	120

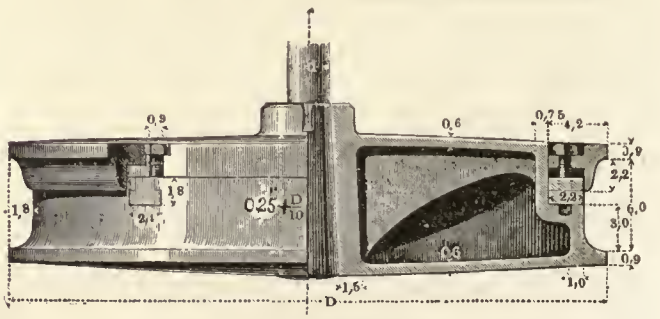


FIG. 1078.

Fig. 1078 shows a hemp packed piston by Penn. This is made of a cored casting with a ring follower secured by bolts, screwing into bronze nuts recessed into the piston. For pistons of large diameter an increased depth is given at the centre; this increase may be made by making the depth in the middle equal to $6s + \frac{1}{10}D$, the depth at the edge being $7.8s$, and the piston being made flat—when the latter value exceeds the former.

Example.—Let $D = 24$ inches—for a hemp packed piston, as Fig. 1078, we then have $s = 0.7$. This gives for the thickness of the packing $0.7 \times 1.8 = 1.26$, say 1¼ in.; the depth of packing $= 0.7 \times 6 = 4.2$ in.; the depth of piston at the edge $= 0.7 \times 7.8 = 5.36 =$ say 5⅜ in. The depth in the middle will be equal to $6 \times 0.7 \times \frac{8}{8} = 6.6$, say 6⅝ ins.

Fig. 1079 shows a good form of piston with metallic packing, by Krauss. The packing consists of two steel rings, each cut at an angle, a ring of white metal being cast on each steel ring. If it is desired to make the cut in each ring tight, some one of

* German Patent, No. 11,535.

† Made by the Metallic Tubing Co., Ld., Port Pool Lane, Gray's Inn Road. London, N. C.

the methods shown in Fig. 1080 may be used. In the first one the overlap makes a tight joint, while in the others the inserted piece is fitted steam tight. By filling the packing rings with white metal the wear comes mainly upon the softer material instead of on the cylinder, a most desirable feature, since the rings are easily and cheaply renewed. For the same reason

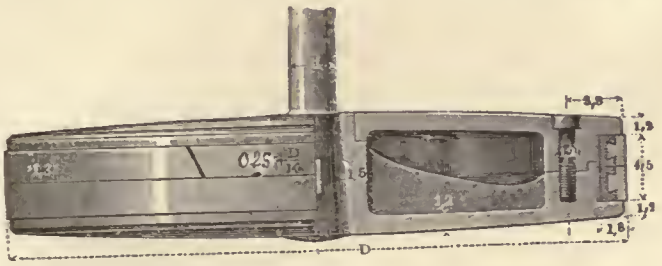


FIG. 1079.

bronze rings are used, while iron or steel are not to be recommended, with the exception of soft cast iron, which works well, the cylinder being made quite hard.

In Fig. 1081 is shown the so-called "Swedish" piston, as used in a large blowing engine by Egestorff. This piston is

FIG. 1080.

made with increased depth in the centre, similar to that in Fig. 1078, and the holes shown in the sectional plan view are for the purpose of removing the core from the casting. The packing rings are made of cast iron, with the joint made as

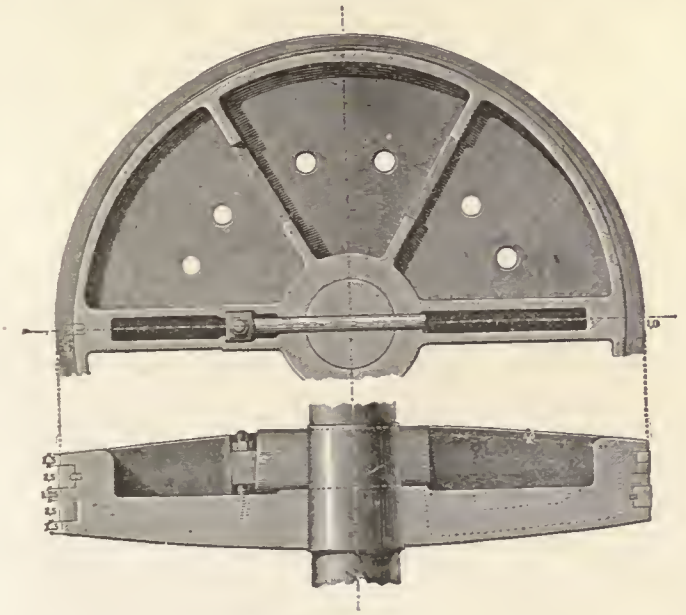


FIG. 1081.

shown in Fig. 1080 *a*. The rings are kept in their proper position by small pins. The method of securing the piston to the rod is worthy of notice. The large key is secured and tightened by a smaller key, the latter being held by a bolt, thus forming a fastening of the third order.

Fig. 1082 shows a metallic piston in which the packing rings are pressed out by an inner spring ring of steel.*

The double cone shape of the inner ring enables the piston to be closely fitted to the cylinder by tightening the bolts when the engine is built. The nuts for the bolts are made of bronze, as in Penn's piston, the thread in this case being carried entirely through the nut and the hole closed by a plug.

* E. Webers & Co., Machine Works, Rheine, Westphalia. This firm makes a specialty of high class steam engines.

A piston for a single acting engine, with combination packing, is shown in Fig. 1083. The metallic packing rings are backed with hemp, this combination presenting the advantage

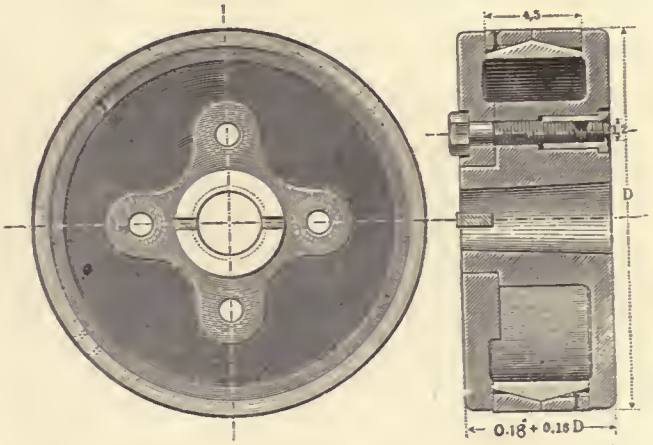


FIG. 1082.

of elasticity together with durability. This style of packing is well suited also for marine engines, as its elasticity renders it less likely to be injured by the pitching and rolling of the vessel than an entire metallic packing.

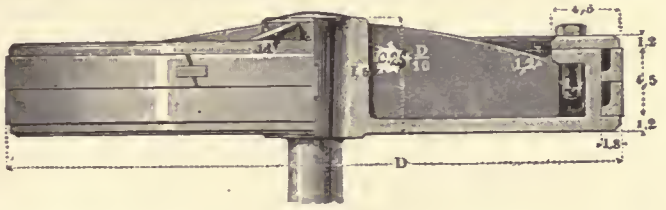


FIG. 1083.

Pistons for pump cylinders may be packed with leather so long as the temperature of the liquid to be pumped does not exceed 88° F. (30° C.).

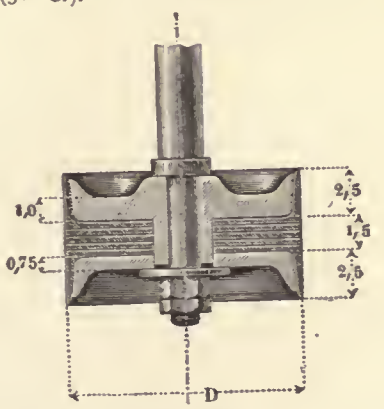


FIG. 1084.

A form of packing for this purpose is shown in Fig. 1084, the principle being the same as the forms shown in the following section. The units for the dimensions are the same as already given.

§ 346.

PLUNGERS AND STUFFING BOXES.

As already observed, the packing for plungers and rods acts from the circumference inward, and such packings, in connection with the necessary parts, are known as stuffing boxes.

Two stuffing boxes for leather cup packing, especially adapted for hydraulic presses and for pumps, are shown in Figs. 1085 and 1086, the former being for small and the latter for large plungers. The double cup in Fig. 1085 is made with a spring ring of iron between the cups to hold them in position before the water pressure is applied. When the form shown in Fig. 1086 is used in the horizontal position, a ring of bronze made in several parts is introduced below the packing, as shown in dotted lines. This is intended to support the plunger and prevent it from rubbing against the cast iron cylinder. The proportions given in the illustrations are all based on the unit *s*, given by formula (339).

The friction existing between a plunger or piston rod in the ordinary stuffing box in which the packing is tightened by screws, cannot well be calculated, as it depends upon the pressure which is put upon the packing. In those forms of stuffing box in which the pressure in the cylinder tightens the packing the friction may be calculated. According to the very elaborate

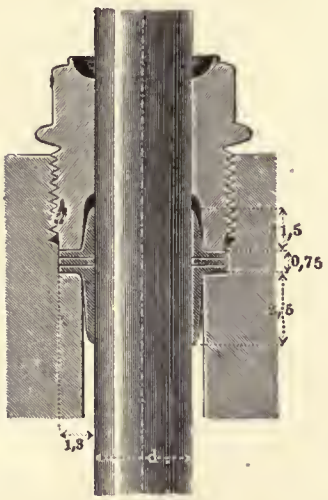


FIG. 1085.

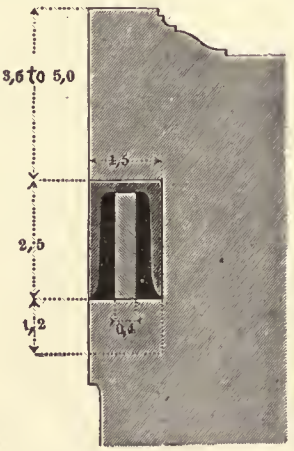


FIG. 1086.

researches of Hick,* the friction of a well-lubricated cup leather packing is independent of the depth of the packing, and is directly proportioned to the water pressure and to the diameter of the plunger. If P is the total pressure, D the diameter of plunger, and F the fractional resistance, we have:

$$\frac{F}{P} = \frac{0.04}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots (340)$$

For a new leather packing the friction is about 1½ times greater. If instead of the total pressure P we use the pressure p, in pounds per square inch we have:

$$\frac{F}{p} = 0.0393 \frac{\pi}{4} D \quad \ldots\ldots\ldots\ldots (341)$$

Example.—For a piston rod 0.4 in. diameter, according to (340) the loss by friction would be $\frac{1}{10}$, or 10 per cent., while for a plunger 24 in. diameter it would be 0.0016, or $\frac{1}{6}$ of 1 per cent. If, for example, the pressure is 4000 pounds per square inch, the friction according to (341) would be

$$F = 4000 \times 0.0393 \times 0.7854 \times 24 = 2963 \text{ pounds.}$$

The total pressure on the plunger would be

$$P = 4000 \times 0.7854 \times 24^2 = 1{,}810{,}000 \text{ pounds.}$$

Stuffing boxes for the piston rods of steam engines must be capable of resisting the action of heat. Hemp packing is still much used for this purpose. The following illustrations show two excellent forms of stuffing boxes to be used with hempen packing.

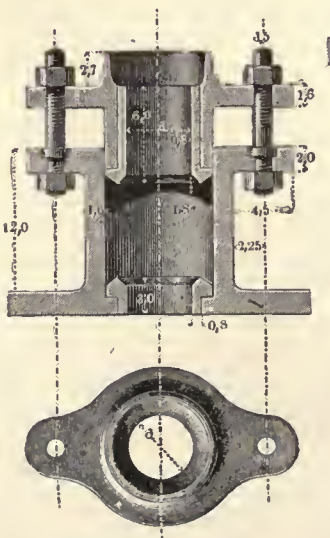


FIG. 1087.

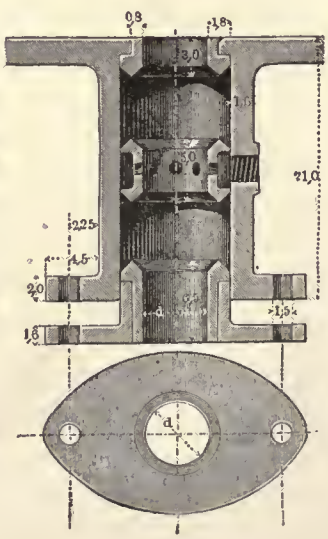


FIG. 1088.

Fig. 1087 is intended to be used on the top of a cylinder; Fig. 1088 is for an inverted cylinder. Both gland and box are fitted with bronze rings, in order to reduce the wear upon the rod. The wedge-shaped edge which is given to these rings was introduced by Farcot, and is an improvement on the older style of beveling the edge in one direction only, the latter method often drawing the packing away from the sides of the box and permitting leakage. In some designs the edge is left square, as in Fig. 1090, or slightly rounded, as in Fig. 1089.

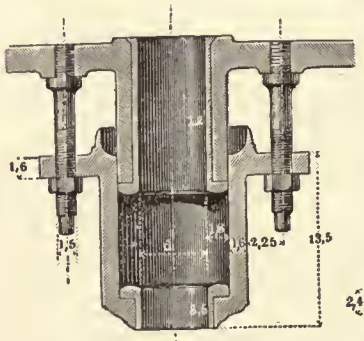


FIG. 1089.

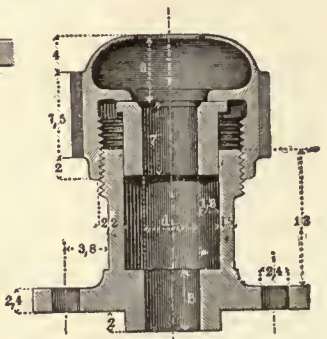


FIG. 1090.

Fig. 1089 shows a form especially adapted to inverted cylinders. The construction will be apparent on examination, and it will be seen that the ordinary arrangement is reversed, and the gland is cast upon the cylinder and the box containing the packing is made separate. This prevents water from the cylinder from readily getting into the box.

In order to prevent the gland from binding on the rod it is important that care should be taken to tighten both nuts equally. In large marine engines, for example, the nuts are made with worm wheels upon a common shaft. For small stuffing boxes this is accomplished by having the screw thread cut upon the outside of the box, as shown in Fig. 1090. This box is intended to be made entirely of bronze. The nut is made with six or eight notches in its circumference, to enable it to be turned by a spanner wrench.

The dimensions of all the preceding figures are based upon the unit s given by the empirical formula (339).

Example.—For a rod 3 ins. diameter, according to (339) we get $s = 0.36$. The thickness of packing will then be $0.36 \times 1.8 = 0.648$, say $\frac{5}{8}$ in. The height of box for Fig. 1087 will be $0.36 \times 12 = 4.32$ ins., and for Fig. 1088 $0.36 \times 21 = 7.56$ ins., and so for the other dimensions.

In horizontal stuffing boxes the length of the bronze collars should be made not less than 8 to 12 s, in order to reduce the wear. The dimensions given in the illustrations may sometimes be modified in order to conform to the thickness of adjoining parts, so as to avoid difficulties in casting and shrinkage.

In some instances the stuffing boxes for valve rods for steam engines are made in two parts, divided in a plane passing through the axis of the rod. The flange of the steam chest is then made in the same plane, so that with this construction the chest can be opened and valve and rod very conveniently removed and replaced.

The large plungers for mine pumps are packed with hemp, the stuffing boxes having 4 to 8 bolts.

More recently metallic packing has been introduced for stuffing boxes of steam engines. An excellent example is

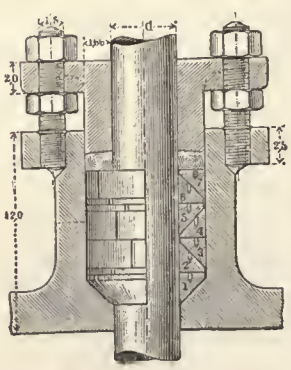


FIG. 1091.

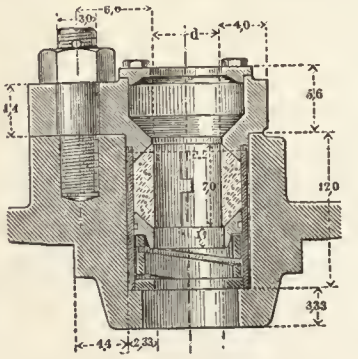


FIG. 1092.

shown in Fig. 1091, which is made by Howaldt Brothers, of Kiel.† The rings are made of white metal, in double cone

* See Verhandl. des Vereins F. Gewerbfleiss, 1866, p. 159.

† German Patent, No. 15,418. Over 9000 such boxes had been made up to 1888: one of these had been running eight years without opening.

pairs as shown, thus causing the pressure to be exerted alternately against the rod and the walls of the stuffing box. An elastic washer is placed between the gland and the first ring to equalize the pressure. Fig. 1092 shows the standard metallic packing introduced on the Prussian State Railways by Superintendent Neumann. This uses a single ring of white metal made in two parts. The pressure is obtained from a steel spiral spring placed in the bottom of the stuffing box, and acting against a bronze pressure ring. The whole is enclosed in a steel cylinder which, together with its contents, can be drawn out by inserting a hook into a T-shaped recess. The form shown in the illustration is intended for a valve rod, but a similar pattern is used for the piston rod.

§ 347.

PISTONS WITH VALVES.

Pistons with valves are used in lift pumps and in steam engine air pumps. An example of such a piston, with leather packing, intended for a mine pump, is shown in Fig. 1093.

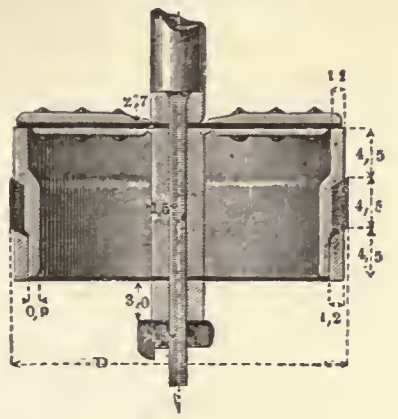


FIG. 1093.

The packing is composed of conical rings of leather and canvas, each three adjoining layers being sewed together. The pressure of the water acts to tighten the packing. The acid mine water often acts injuriously upon the leather packing of the pump pistons, and in such cases metallic packing, with rings of soft cast iron, is used. At Fahlun, in Sweden, after many experiments the best material for packing was decided to be birch wood. The proportions for Fig. 1093 are based upon the unit s. A valved piston for steam engine air pump is shown in Fig. 984.

§ 348.

PISTON RODS.

Piston rods for steam engines are usually made of wrought iron or steel, and recently compound rods of wrought iron surrounded with hard steel have been used. The rod is either subjected to tension only, as in single acting engines, or is alternately subjected to tension and compression, in which case the length and resistance to buckling must be taken into account. For short rods the same results are obtained for both conditions, but in no case should a rod subjected to alternate tension and compression be made lighter than a rod under tension only.

a. Dimensions of Piston Rods, Tension only.

D = diameter of cylinder in inches.
p = pressure in pounds per square inch.

The total pressure P on the piston will be $P = \frac{\pi}{4} p D^2$. In order that the stress on the rod should not exceed 8500 pounds we have for the diameter d of the piston rod when made of wrought iron, and is subjected to tension only:

$$\frac{d}{D} = 0.0108 \sqrt{p} \quad \ldots\ldots\ldots\ldots \quad (342)$$

or for a close approximation:

$$\frac{d}{D} = \frac{57 + 0.5\,p}{1000} \quad \ldots\ldots\ldots\ldots \quad (343)$$

Example.—If $p = 60$ pounds we have from (342), $\frac{d}{D} = 0.0836$, and hence for a 20 inch cylinder $d = 20 \times 0.0836 = 1.67$ in. The approximate formula (343) gives $\frac{57 + 30}{1000} = 0.087$, which for $D = 20$ gives $d = 1.74$ in.

Steel rods subjected to tension only may be made 0.8 times the diameter of wrought iron rods.

If a piston rod is weakened by having a keyway cut through it, or by a screw thread, the reduction in cross section should be provided for by a proper increase in diameter. For this reason the diameter of the rod is sometimes increased in the cross head, an example of which will be seen in the locomotive cross head, Fig. 539. This construction involves the necessity of making the stuffing box gland in halves, as it could not be slipped over the enlarged end of the rod.

b. Dimensions of Piston Rods for Buckling Stresses.

Using the preceding nomenclature and indicating the length of stroke by L, we have:

$$\frac{d}{D} = 0.0295 \sqrt{\frac{L}{D}} \sqrt[4]{p} \quad \ldots\ldots\ldots \quad (344)$$

from which the following table has been calculated:

$\frac{L}{D}$	$p = 50$	$= 60$	$= 70$	$= 80$	$= 90$	$= 100$	$= 120$	$= 140$	$= 160$	$= 180$
1.5	0.0967	0.100	0.104	0.108	0.111	0.114	0.119	0.124	0.129	0.133
2.0	0.111	0.116	0.121	0.125	0.128	0.132	0.138	0.143	0.148	0.153
2.5	0.124	0.130	0.135	0.140	0.144	0.148	0.154	0.161	0.166	0.171

These values will serve both for wrought iron and for steel (compare § 182, and table in § 2).

Example.—For a steam cylinder 16 in. bore, 4 in. stroke, with a pressure of 60 pounds, we have $\frac{L}{D} = 2.5$, and $d = 0.130 \times 16 = 2.08$, say 2 inches diameter, either for steel or wrought iron.

The dimensions of steel keys to secure the piston to the rod are so taken as to give shearing stresses from 5600 to 7500 pounds in the key. Care should be taken that the key be not made too narrow, and the consequent superficial pressure become too great. Pressures of 6000 to 7000 pounds per square inch are found in stationary engines, and 10,000 to 15,000 pounds in locomotive engines.

§ 349.

SPECIFIC CAPACITY OF PRESSURE TRANSMISSION SYSTEMS.

Having discussed the subject of conductors for pressure organs, we return to the consideration of the various mechanical devices which may be operated by pressure organs, although these have already been described in Chapter XXIII. We are now prepared to consider these in connection with the subject of long-distance transmission of power, in a manner similar to that in which tension organs are used in Chapter XXI. For this purpose we may use to advantage the conception of *specific capacity*. This method is especially desirable because its simplicity and general character enables comparison to be made between widely differing systems.

The conception of specific capacity can be extended without difficulty to motors operated by water, air, steam, etc., since for all these we may put the general equation:

$$N_o = \frac{N}{q\,v}$$

deduced in § 280. In this equation q represents the cross section of the pipe or other conductor in square inches; the mean velocity in feet per *minute* $= v$, and N being the horse power. If, for example, in a water pressure engine, h is the available head of water. Q the weight of water delivered per minute, and h' the head equivalent to the resistance against which the water leaves the engine, we have for the work delivered:

$$N = \frac{Q\,(h - h')}{33000}$$

But $Q = 0.0361 \times 12\,q\,v = 0.434\,q\,v$, the coefficient 0.0361 being the weight of a cubic inch of water, and the pressure p with which the water acts $= 0.434\,h$, whence $h = 2.3\,p$. Substituting these values we get:

$$N = \frac{0.434\,q\,v \times 2.3\,(p - p')}{33000} = \frac{1}{33000}\,q\,v\,(p - p')$$

and the specific capacity becomes:

$$N_o = \frac{N}{q\,v} = \frac{1}{33000}\,(p - p') \quad \ldots\ldots\ldots \quad (345)$$

a value of the same form as that previously deduced in § 280 [see formula (262)].

Example.—If the effective pressure $p - p'$ be 320 pounds, the specific capacity will be $N_0 = 0.0097$. If the pipe is 4.75 in. diameter, and the water has a velocity of 236 feet per minute, we have:

$$N = 4.75^2 \times \frac{\pi}{4} \times 236 \times 0.0097 = 40.56 \text{ H. P.}$$

This is only the capacity of the pipe. The effective capacity will be considered later.

Formula (345) can also be used for air pressure or for vacuum, for steam or gas, by expressing the effective pressure in terms of an equivalent head of water. For steam and air it may be considered as an expression of the following form:

$$N_0 = \frac{1}{33000} (p - p') \mu \quad \ldots\ldots\ldots (346)$$

The coefficient μ is very comprehensive; it increases with p and with the rate of expansiou ϵ, and can be calculated frou these data, and also confirmed by observation. For $\epsilon = 2$, it ranges from 1½ to 1⅔, and increases to 3 to 4 for $\epsilon = 20$ to 30, results which conform to the higher pressures and greater efficiency of compound engines in which such high expansion ratios are used.

With some transformations the equation for specific capacity may also be used to solve another important problem, that is the question of the best material to be used for the conductiug pipe.

If we assume the diameter of the pipe, the horse-power N will be:

$$p = \frac{\pi}{4} D^2 \frac{p\, v}{33000}$$

For the thickness of pipe, we have from (321), for a stress S, in the material:

$$2\delta + D = D \sqrt{\frac{S+p}{S-p}}$$

And since $2\delta + D$ is the external diameter D_o, we have for the cross sectiou q_1 of the pipe.

$$q_1 = \frac{\pi}{4}\left((D_o^2 - D^2\right) = \frac{\pi}{4} D^2 \left(\frac{S+p}{S-p} - 1\right)$$

or

$$q_1 = \frac{\pi}{4} D^2 \frac{2p}{S-p}$$

Substituting the value of $\frac{\pi}{4} D^2$ frou its equation iu the above expressiou for N, we have

$$N = \frac{1}{33000}\, q_1 \frac{S-p}{2p}\, p\, v =$$

$$= \frac{1}{33000}\left(S - p\right) q_1\, v$$

whence

$$N_0 = \frac{N}{q_1\, v} = \frac{1}{33000}\left(S - p\right) \quad \ldots\ldots (347)$$

a form similar to the preceding expressions for N_0.

This expression is very instructive. It is applicable to all forms of conducting pipes for power transmission. It shows clearly the importance and value of a high value of S. A high value of S reduces the proportional influence of p, to a degree which practically makes N_0 dependent mainly upon S. It follows that we may cousider that the specific capacity of the pipe in a pipe transmission system, is practically independent of the pressure of the fluid used iu it. In other words, the capacity of a given pipe in horse-power is the same, whether the medium be liquid or gaseous, high or low pressure, provided the stress in the material of the cross section of the pipe is constant.

It is therefore desirable to use pipes of small diameter and fluids at moderately high pressures. The friction in the pipe need not preveut this, as care in avoiding sharp bends and angles can be taken; and as already shown in § 340 the friction is independeut of the pressure of the medium, at least so it appears from such experiments as have yet been made.

The value of the stress in the material of the pipe cannot be taken very high; $S = 7000$ lbs. being about the upper limit, and $S = 6500$ lbs. appears to be quite high enough. Wrought iron and steel, especially in the Mannesmaun rolled tubes, permit the use of high stresses; for wrought iron $S = 17{,}000$ lbs. aud for steel 35,000 to 40,000 lbs., or even higher, if necessary, may be used. By neglecting the value of p iu formula (347) we have for:

Cast Iron,	$S = 6{,}500$,	$N_0 = 0.197$
Wrought Iron	$S = 17{,}000$,	$N_0 = 0.515$
Steel	$S = 35{,}000$,	$N_0 = 1.060$

This gives an indication of the efficiency of the pipe system of power trausmission aud enables comparisons to be made with other systeuis.

§ 350.

THE RING SYSTEM OF POWER DISTRIBUTION WITH PIPE CONDUCTORS.

Before proceeding with the further discussion of the preceding equations it is advisable to investigate further the subject of power trausmission by means of pipe conductors, as already iudicated in § 312. It was there remarked that pressure organs might be used in connectiou with pipe conductors so as to form "ring" transmission systems in a manner similar to those already described for rope. Taking into consideration first, hydraulic systems, especially high pressure hydraulic systems, we find two distinct kinds of "ring" systems which may be used.

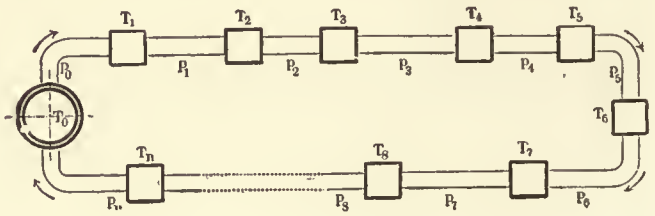


FIG. 1094.

In the first method, Fig. 1094, the flow of water under pressure starts from the power station T_o, with a pressure p_o, and proceeds to the first station T_1, where it operates a water pressure engine, and passes on with a reduced pressure p_1. It has therefore operated at the station T_1 with a pressure $p_o - p_1$. With the pressure p_1 it passes on to the second, third, fourth — — nth station T_n, each time losing pressure until it returns to the power station with a fiual pressure p_n, where it is again raised to the initial pressure of p_o. This is practically a counter part of the rope trausmissiou system of Fig. 917. It is appareut that the water pressure engines (escapements) at T_1, T_2, T_3, — — — T_n, should all be of equal size in order to utilize the entire flow without excessive resistauce. Automatic regulation, such as Helfenberger's, described in § 328, is also desirable.*

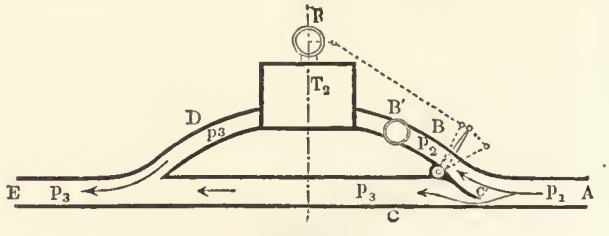


FIG. 1095.

The second system is shown in diagram in Fig. 1095. It will be seen that at each statiou there is a branch or shunt tube, leading through the motor (or escapement) T_2, and then reuniting with the main pipe. The main pipe A, forks at the station iuto the two brauches B and C, of which the first diverts any required fraction of the power of the main flow, as $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{3}$, as the case may be. At the fork is a swing valve C', operated by a speed governor R, driven by the motor. This governor requires the assistance of some form of power reiuforcement, such, for example, as shown in Fig. 1037. The discharge pipe D of the motor uuites with the by-pass C, to form again the main conductor E. At the entrauce in the main pipe A, we have the pressure p_1 of the original flow; the motor T_2 is now supposed to be stationary, the stop valve at B' having been closed by hand. The flap valve C' which has beeu discounected

* The London Hydraulic Power Company has installed separate ring systems, each with a single generator and motor.

from the regulator before stopping the motor, is also closed. The flow of water then passes through C to E with the pressure p_1.

When the motor T_2 is to be started, the valve B' is opened and the flap valve C' gradually opened until the motor begins to move, when it is connected to the governor, which regulates it thereafter so as to keep the motor at its normal speed. When a heavy load is thrown on, the valve is opened so that the pressure p_2 in B, becomes a greater fraction of p_1, and when the work is less it is reduced. The pressure of discharge p_3 acts as a back pressure so that the motor works with an effective pressure $p_2 - p_3$. The flow of water in the by-pass pipe C, also passes the valve C' with a pressure p_3, and unites with the discharge at E to be further utilized at subsequent stations until it returns to the power station, where if it has reached the minimum pressure, it is permitted to flow into a tank, from which it is again drawn by the pressure pumps. If the return water is delivered under pressure it may be allowed to enter the suction pipe of the pressure pumps direct and so form a closed ring system to start anew on the circuit.

This system has not yet to the Author's knowledge been put into practical operation.*

The ring system of hydraulic power transmission is to be recommended when the various stations are distributed over a wide area and are readily connected by a continuous line of pipe. The pipe can be kept from freezing in winter by occa sional gas flames, as has already been demonstrated by experience with Armstrong's hydraulic cranes. The ring system should be carefully distinguished from those forms in which the flow of water passes through the motor and is allowed to flow off at lowest pressure of discharge. A corresponding distinction is to be made with other forms of power transmission.

The author distinguishes as "line" transmissions, those forms in which the transmitting medium does not return to itself in a complete circuit, in contradistinction with the "ring" systems. The older form of rope transmission (§ 297) is therefore a "line" system, while the system devised by the author and discussed in § 301 is a "ring" system. A hydraulic system in which there is a free discharge of water from the motors is in like manner a hydraulic "line" transmission system.

There is, however, an intermediate form possible, namely, that in which water after passing through a series of motors as in a ring system, is discharged freely from the last motor T_n. A similar arrangement is possible with other systems of transmission. We may therefore extend the defiuition of a "ring" system to include those forms in which the medium of transmission returns to the place of starting. The distinction can then be made between "open" and "closed" ring systems, the latter being shown in diagram in Fig. 917.

High pressure hydraulic systems are well adapted for large railway stations where numerous elevators as well as winding hoists and other rotative machines are to be operated. For such installations a combination of "ring" and "line" systems is best suited. The hydraulic elevators are more conveniently arranged on a line system than in a ring circuit. An apparent objection to the use of high pressure water to direct acting elevators lies in the fact that the diameter of the plunger becomes so small as to be hardly stiff enough to support the load on the platform without buckling. This difficulty is readily overcome by use of the hydraulic lever, as shown in Fig. 956 *a*, the construction of which offers no difficulties, and it is unnecessary to go into details.

Up to the present time air has only been used upon line systems, either with direct pressure or with vaacum. Gas engines can only be operated on a line system since the gas is burned in the engiue. Steam has been used in a ring system in New York for some time, on a long distance transmission, and short ring systems exist in most cases of compound or triple expansion steam engines as used in marine and stationary practice.

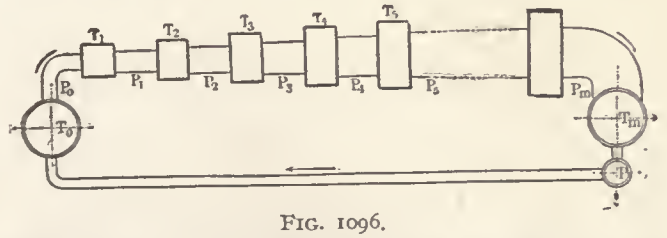


FIG. 1096.

Steam at a high initial pressure is expanded successively in one cylinder after anether, and between the last cylinder or station T_n and the first, or boiler T_0, is placed the surface condenser T_m, where the medium reaches the minimum pressure and is converted into water to be returned to the boiler and start anew on the circuit. In order that the velocity of flow shall be uniform the successive passages for the expanding steam should be made with continually increasing cross section as shown in diagram in Fig. 1096. If a jet condenser is used instead of a surface condenser the circuit becomes an open ring. The high economy which has been attained by the application of the "ring" system with steam in the form of multiple expansion engines, points to the possibility of a similar economy in the application of the ring system to wire rope transmission.

Lehmann's hot air engine, which is a true closed circuit, is an example of the ring system confined within the limits of a single machine.

§ 351.

SPECIFIC CAPACITY OF TRANSMISSION BY SHAFTING.

The subject of the specific capacity of shafting was not considered in Chapter IX, and it is introduced in this place in order to obtain a basis for comparison with the other systems of transmission.

If we have the moment PR and shaft diameter d, we have, if S is the fibre stress at the circumference

$$PR = S\frac{\pi}{16}d^3$$

(see § 144).

If we make the lever arm $R = \frac{1}{2}d$, we have $P =$ the force at the circumference of the shaft and hence $P = S\frac{\pi}{8}d^3$. Taking $v =$ the velocity at the circumference of the shaft and N the number of horse-power transmitted, we have:

$$N = \frac{Pv}{33{,}000} = \frac{\frac{1}{2}S\frac{\pi}{4}d^2v}{33{,}000}$$

But $\frac{\pi}{4}d^2 = q$, the cross sectional area of the shaft, whence

$$N = \frac{1}{2}\frac{Sqv}{33000} \quad \ldots\ldots\ldots (348)$$

and hence the specific capacity of the shaft is:

$$N_0 = \frac{N}{qv} = \frac{1}{2}\frac{S}{33{,}000} \quad \ldots\ldots\ldots (349)$$

This expression, which is of the same form as those already obtained, does not give values numerically great, because S must be taken low enough to avoid excessive torsion of the shaft. If we require, as in § 144, that the torsion shall not exceed 0.075° per foot of length we must have $S \leqq 630\,d$ which gives for shafting from 2 to 6 inches diameter $S =$ about 1200 to 3700 pounds and the specific capacity

$$N_0 = 0.018 \text{ to } 0.056 \quad \ldots\ldots\ldots (350)$$

In other words, such a shaft will transmit, at one foot per minute circumferential velocity, 0.018 to 0.056 horse-power for each square inch cross section.

In the application of shafting to long distance transmission the friction of the journal bearings is a very important consideration. The influence of friction may be determined in the form of a general expression in a similar manner to that of the friction of water in a pipe (§ 340). According to formula (100) we have for the force F, exerted at the circumference to overcome the journal friction $F = \frac{4}{\pi}f$ times the weight of the shaft, that is $= \frac{4}{\pi}f\left(\frac{\pi}{4}d^2\right)12\,L \times 0.28$ in which L is the length of the shaft in feet, and 0.28 is the weight of a cubic inch of wrought iron. It follows that the horse power N_1 required to overcome the friction will be:

$$N_1 = \frac{Fv}{33000} = \frac{\frac{4}{\pi} \times q \times 12\,L \times 0.28}{33000}$$

and if we take the coefficient of friction $f = 0.08$ we have

* See the Author's article in Glaser's Annalen, Vol. XVII (1885), part 12. from his paper to the Verein für Eisenbahnkunde, Nov. 10, 1885.

$$N_1 = \frac{0.08 \times 4 \times 0.28 \times 12\, L\, q\, v}{33000\,\pi} = \frac{L\, q\, v}{96{,}422}$$

or

$$N_1 = \frac{L}{96{,}422}\, q\, v \quad \ldots\ldots\ldots\ldots (351)$$

and if we wish the specific frictional resistance, we have:

$$\left(N_1\right)_o = \frac{N_1}{q\, v} = \frac{L}{96{,}422} \quad \ldots\ldots (352)$$

This resistance is by no means inconsiderable. Expressed as a percentage it will be:

$$pr = \frac{N_1}{N} = \frac{L}{96{,}422} \cdot \frac{q\, v}{N} = \frac{L}{96{,}442} \cdot \frac{1}{N_o} \quad \ldots (353)$$

The value pr, it will be seen, is inversely proportional to the specific capacity. If we apply this to (350) we have for a 2 inch shaft

$$pr = \frac{L}{0.018 \times 96{,}422} = \frac{L}{1735}$$

and for a 6 inch shaft

$$pr = \frac{L}{0.056 \times 96{,}422} = \frac{L}{5400} \quad \ldots\ldots (354)$$

hence 1735 feet and 5400 feet are the limits of length respectively for the two diameters given, at which the frictional resistance will equal the total transmitting capacity. Much higher efficiency is obtained by using hollow steel shafting such as is now produced by the Mannesmann process of rolling weldless tubing. This furuishes a seamless tube, of sufficient truth as to cylindrical shape, the journals of which may be made either entirely of steel or of so-called "compound steel." *

If we take the ratio of outer to inner diameter $\psi = \frac{d_1}{d_o} = 0.9$ (compare § 90) and the thickness of the journal $d' = 0.4\, d_o$ we have for N_o:

$$N_o = \frac{N}{q\, v} = \frac{1}{2} \cdot \frac{S}{33000}\left(1 + \psi^2\right) \quad \ldots\ldots (355)$$

which for $\psi = 0.9$ gives

$$N_o = 1.81 \times \frac{S}{33000} \quad \ldots\ldots\ldots\ldots (356)$$

which is decidedly higher than for the solid shaft. (The value $S \leqq 630\, d_o$ must be retained to avoid too great torsion). For the frictional resistauce at the circumference of the shaft we have:

$$F = \frac{4}{\pi} f \left[\frac{\pi}{4}\, d_o^2 \left(1 - \psi^2\right) 12\, L \times 0.28\, \frac{d'}{d_o}\right]$$

and if $d' = 0.4\, d_o$ we have:

$$N_1 = \frac{0.4\, L\, q\, v}{96{,}422} = \frac{L}{241{,}000}\, q\, v$$

or dividing agaiu by N:

$$pr = \frac{N_1}{N} = \frac{L}{241{,}000} \cdot \frac{1}{N_o} \quad \ldots\ldots\ldots (357)$$

With the values for N_o as given in the two preceding instances, we have for the 2 inch shaft:

$$N_o = 0.0326$$

and for the 6 inch shaft:

$$N_o = 0.101$$

and these give in (357):

for the 2 inch shaft—

$$pr = \frac{L}{0.0326 \times 241{,}000} = \frac{L}{7856}$$

and for the 6 iuch shaft—

$$pr = \frac{L}{0.101 \times 241{,}000} = \frac{L}{24{,}341} \quad \ldots\ldots (358)$$

so that in both instances it is less than one-fourth the resistance of the corresponding solid shafts, as given in (354). Hollow shafting thus greatly extends the capacity of shafting for long distance transmission and also permits an important economy in material.

The subject of shafting made of steel tubing was not considered in Chapter IX, and a brief discussion will therefore be given here.

Let d_o be the outside diameter, d_1 the iuside diameter, let the ratio $\frac{d_1}{d_o} = \psi$. Making $\psi = 0.9$ as is usual in practice with such tubing, the diameter for resistance to torsion, (compare formula (133)) will be:

$$d_o = 0.39 \sqrt[4]{P\, R} = 6.18 \sqrt[4]{\frac{N}{n}} \quad \ldots\ldots (359)$$

This requires that the number of revolutions be known or assumed. If instead of n, the circumferential velocity v, be given, we have for the same shaft:

$$d_o = 7.25 \sqrt[3]{\frac{N}{v}} \quad \ldots\ldots\ldots\ldots (360)$$

v, being expressed in feet per minute at the circumference of the shaft. The number of revolutions will be:

$$n = \frac{3.82}{d_o}\, v \quad \ldots\ldots\ldots\ldots (361)$$

The diameter for strength (compare (131)) will be:

$$d_o = 1.11 \sqrt[3]{\frac{P\, R}{S}} = 5.35 \sqrt[3]{\frac{N}{nS}} \quad \ldots\ldots (362)$$

If ψ is not assumed as above, it may be takeu at will and the following formula used:

$$d_o = \frac{0.79}{\sqrt[3]{1 - \psi^4}} \sqrt[3]{\frac{P\, R}{S}} = \frac{3.75}{\sqrt[3]{1 - \psi^4}} \sqrt[3]{\frac{N}{nS}} \quad \ldots (363)$$

in which, when:

$$\frac{a_1}{a^\circ} = \psi = \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.75 \quad 0.80 \quad 0.85 \quad 0.90$$

$$\frac{1}{\sqrt[3]{1 - \psi^4}} = \quad 1.01 \quad 1.02 \quad 1.05 \quad 1.10 \quad 1.14 \quad 1.19 \quad 1.242 \quad 1.427$$

The weights of tubular and solid shafting are to each other as $\left(\frac{d_o^2}{d^2}\right)\left(1 - \psi^2\right)$.

Example.—If $N = 60$ horse-power, $n = 120$ revolutions per minute, we have from (359)

$$d_o = 6.18 \sqrt[4]{\frac{60}{120}} = 5.2$$

instead of

$$d = 4.7 \sqrt[4]{\frac{60}{120}} = 3.95 \text{ in.}$$

as would be the case for a solid shaft. The hollow shaft, however, weighs only $\left(\frac{5.2}{3.95}\right)^2\left(1 - 0.81\right) = 0.33$ times the weight of the solid shaft. The circumferential velocity $v = \frac{120 \times 52}{3.82} = 163$ feet. If a higher speed be chosen, as may readily be done, on account of the small journal diameter d', we have from (360), making $v = 300$ ft., for example:

$$d_o = 7.25 \sqrt[3]{\frac{60}{300}} = 4.24 \text{ in.}$$

whence $d' = 0.4\, d_o = 1.7$ in.
The number of revolutions will then be

$$n = \frac{3.82 \times 300}{4.24} = 270$$

The weight of shaft will be

$$\left(\frac{4.24}{3.95}\right)^2\left(1 - 0.81\right) = 0.22$$

times that of a solid shaft at 120 revolutions. The loss from friction will be only 0.26 times that of the solid shaft.

* The Mannesmann "compound" steel tubing is made with the interior of soft wrought iron and the outside of hardened steel.

§ 352.

SPECIFIC VALUE OF LONG DISTANCE TRANSMISSIONS.

In the two preceding sections, equations have been given showing comparative relations between various methods of transmissions but at the same time the general equation by which all the various methods of long distance transmission may at once be compared, has not yet been given. The point which yet remains to be determined is the amount of material which the tranimitted force carries in the shape of the transmitting medium. Investigation reveals certain fundamental points which may be applied either to a special case or to a comparative judgment as to the value of different systems.

The amount of material required for the principal transmitting medium of a long distance transmission system, may be considered as a function of the number of horse-power required to transport one pound of the material of which the conductor is composed over the distance between the origin of power and the point of application.

The name "Specific Long Distance Value" may properly be given to this quantity. If it is high, the method is efficient, if low it is less efficient for applications in which the distance plays an important part.

In all the cases considered the medium of transmission may be taken as a form of prism of constant cross section q, having an endlong motion and the length of which is eqnal to the distance A from the point of origin to the point of application. A chosen length A_0 may be selected as a unit. The weight G of such unit will then be:

$$G = 12\, A_0\, q\, \sigma \quad \ldots\ldots\ldots\ldots (364)$$

in which σ is the weight of a cubic inch of the material of which the conductor is made. The work in horse power exerted during the passage over the distance A_0, is expressed by:

$$N = N_0\, q\, v$$

Dividing this value by the preceding we obtained the desired result. It is desirable to select a standard unit for A_0 which shall be generally applicable. Making $A_0 = 1$ inch $= \frac{1}{12}$ ft, and letting the quotient $\frac{N}{G}$ be represented by N_s, we have for the general equation of "specific long distance value:"

$$N_s = N_0 \frac{v}{\sigma} \text{ and also } = C\frac{Sv}{\sigma} \quad \ldots\ldots (365)$$

These values hold good for all the systems which have been considered with the exception of rope and belt transmissions. In these the tension organ moves not only forward, but must be returned back on the slack side, and hence for these cases we must put:

$$N_s = \frac{N_0}{2}\ \frac{v}{\sigma} \quad \ldots\ldots\ldots\ldots (366)$$

When rope is used on the "ring" transmission system, however, the preceding formula (365) is properly used, since a single rope makes the entire circuit back to the origin of power and the last point of application usually lies near the starting point.

The formula for N_s is especially noteworthy because its application reveals great and unexpected differences between the various systems of long distance transmission.

When the conducting medium is operated at a high velocity and also at a high working stress the specific value is very high; when they are both small the specific value becomes lower, since they are both multiplied together. The following table shows both numerically and graphically the "specific long distance value" for the several methods named:

The stresses in the table are taken at the maximum values given in the preceding pages of this work, without approaching too close to the upper limit. The high value of steel cable transmission is most noteworthy, and explains its frequent use. The table does not even give as good a standing for wire rope as might have been done, as when used with a tightening pulley its specific capacity is increased 1½ times, see formula (310).

Equally noteworthy with the value of wire rope is the poor showing made by shafting, especially solid shafting; as it occupies the lowest position of all. The newer system of hollow steel shafting stands somewhat better, but still very low.

Welded iron and steel tubing when used for conductors give fairly good results. To avoid misunderstanding, it must be noted that where pipes are referred to in the table, v is the velocity of the fluid passing through them. No distinction is made between pipes for steam, compressed air or water, since it is only the weight of the tube which is here considered. A circumstance worthy of note is that the reciprocal of N_s is proportional to the weight of the transmitting material, omitting connections, flanges, couplings, etc., so that $G_s = \frac{N}{N_s}$. If, for example, 200 horse-power is to be transmitted by hydraulic pressure over a distance A, of 984 feet = 11,808 inches; the weight of the bare steel pipe will be $\frac{11{,}808 + 200}{1448} = 1631$ pounds, the thickness of the pipe being so made that the stress on the material shall be 34,000 pounds, and the diameter such that the velocity shall be 787 feet per minute. Such calculations are very useful for a general aud preliminary investigation. The designer must be careful not to lose sight of the fact that the stress in the material, and the velocity bear a very important relation to each other.

The values of N_s, given above, are the gross values including the entire work transmitted by the system. The net value $(N_2)_s$ and its relation to the gross value, that is, the quotient $\frac{N_2}{G}$ is the next question to be answered. This question is by no means so simple as the preceding. The actual efficiency of a long distance transmission depends so much upon the resistances of friction, stiffness, centrifugal force, heat, etc., all of which differ for the different constructions, that ouly a very general allowance can be made to include them. A brief glance can only here be given to the method of determining this point.

The greater the number of horse-power which can be transmitted for each pound of material, the less, proportionally will be the load upon the bearings and other points of loss, and hence the smaller, proportionally, will be the loss of friction and other hurtful resistances. In other words: *The greater the specific value of the system, the less, in general, will be the proportion of hurtful resistance.*

The values already given iu the table for the gross specific value, give also, therefore a measure of the net efficiency as well.

While it can hardly be asserted that the above values for N_s are inversely proportional to the losses from hurtful resistances, yet there is a relation existing between them, so that it may be said that the net value $(N_2)_s$ is in all cases higher than the gross value N_s; higher in the sense, that, the greater gross values are accompanied also with a higher net efficiency.

The difference will appear most distinctly by comparing wire cable transmission with solid shafting. Such a comparison is the more readily made because in both instances the resistances can be closely determined.

SPECIFIC VALUE FOR LONG DISTANCE TRANSMISSIONS.

SYSTEM.	v	S	σ	N_0	N_s	PERCENTAGE OF EFFICIENCY.	PER CENT.
Steel Cable, Ring System . . .	5900	21,000	0.32	0.318	5683		100.
Steel Cable, Line System . . .	5900	21,000	0.32	0.318	2631		50.
Iron Cable, Ring System . . .	5900	8,500	0.32	0.129	2380		40.6
Steel Conducting Pipe	787	34,000	0.28	0.515	1448		24.7
Iron Cable, Line System . . .	5900	8,500	0.32	0.129	1190		20.3
Leather Belting, Line System .	5900	540	0.036	0.0068	1114		19.
Wrought Iron Pipe	787	17,000	0.28	0.257	722		12.3
Hemp Rope, Line System . . .	5900	240	0.036	0.0036	295		5.02
Cast Iron Pipe	787	6,400	0.28	0.097	272		4.64
Hollow Steel Shafting	394	5,200	0.28	0.114	160		2.73
Solid Shafting, Iron or Steel .	197	4,200	0.28	0.063	45		0.8

The iron wire cable transmission at Oberursel, discussed in § 300, showed a loss of about 14 per cent. in a transmission of 104 H. P., over a distance of 3168 feet. To transmit the same power over this distance with solid shafting, we get from (353) the frictional resistance:

$$p_r = \frac{3168}{95{,}422} \cdot \frac{1}{N_0}$$

Taking $N_0 = 0.063$, which is amply high enough, we have $p_r = 0.52$ or about $\frac{1}{2}$. The net specific value for long distance will then be for iron cable, $(1 - 0.14)\ 1190 = 1023$; for solid shafting, $(1 - 0.5)\ 45 = 22.5$.

It will be seen that about 52 H. P. is absorbed in the friction of the shaft; so that at periods of low water, when the turbine yields only 40.3 H. P. it would not be able to overcome the friction of the shaft alone.

The correctness of these considerations will be confirmed when it is remembered that a wire cable runs at a very high velocity and operates at a high stress which the journals of the rope pulleys move at a very low velocity (scarcely $\frac{1}{50}\,v$); while on the other hand the shaft can only be subjected to a low stress, and the velocity at its circumference is not only low, but it has to overcome the resistance of friction at the same velocity. This also explains clearly the reason why rope transmission has so frequently superseded shafting in actual practice.

CHAPTER XXV.

RESERVOIRS FOR PRESSURE ORGANS.

§ 353.

Various Kinds of Reservoirs.

Reservoirs form a most important feature in connection with the use of pressure organs, and are divided into tanks, receivers, chambers of various kinds, in which the pressure organs may be stored in greater or less quantity and drawn upon for use as may be required. Such reservoirs may be used either for positive or negative pressure according to the system with which they are used. Both kinds are shown in Fig. 993, in the case of a canal lock. As already indicated in § 312, the various forms of reservoirs are very numerous. From the nature of the subject we can only here discuss that branch of the subject which relates to machine construction, including reservoirs of cast and wrought iron, copper and steel. These are applicable both to gaseous and liquid organs and in most cases are of special construction to meet the circumstances of use.

A reservoir when considered in connection with the apparatus for filling and emptying, as well as for controlling the pressure, whether positive or negative, forms a storage system which may properly be considered as a ratchet train (see Chap. XVIII).

For the present, however, it is here only the intention to discuss the constructive features of the reservoir itself considered as a machine element.

§ 354.

Cast Iron Tanks.

Cast iron tanks with flat sides are used only for very small reservoirs and need not be discussed here; for larger sizes the walls are made cylindrical in order better to resist the internal pressure. Cylindrical cast iron tanks can be advantageously

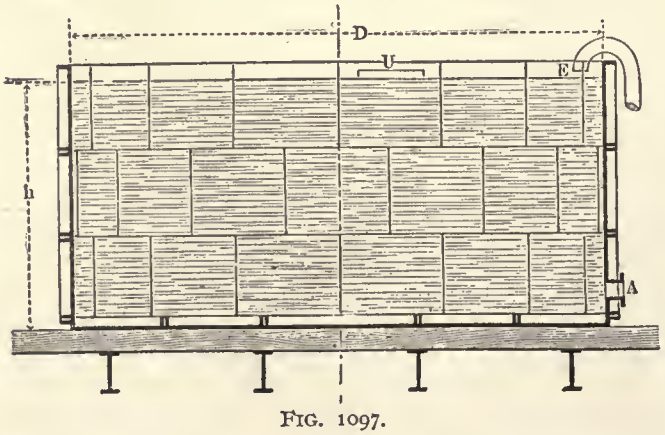


Fig. 1097.

used for water up to 1000 cubic feet capacity. A good construction has already been shown in Chapter IV, as made by Lauchhammer's Iron Works, of Groditz, and used in many places.

Fig. 1097 shows a tank of this sort. The water is delivered at E; A, is the discharge, and U the overflow. The thickness of the walls is made about ¼-inch; the flat bottom rests on a strong floor of wood carried by heavy beams. The flange joints are made as in Fig. 268, 269. If h, is the greates head of water in the tank, the pressure per square inch on the bottom will be $p = 0.434\,h$, h being taken in feet, and we have for the thickness g, when D is the inside diameter according to (324):

$$\frac{\delta}{D} = \frac{1}{2}\,\frac{p}{S} = \frac{0.434\,h}{2\,S} = 0.217\,\frac{h}{S} \quad \ldots\ldots (367)$$

Example.—If $\delta = 0.25$ in., $D = 118$ ins., $h = 9.83$ ft., we have $S = 0.217\,\frac{h\,D}{\delta} = 0.217\,\frac{118 \times 9.83}{0.25} = 1109$ lbs., which is such a moderate value that the tank is amply secure. If we take the diameter of the bolts at ⅝-in. for the joint 4 inches deep at the bottom of the tank, and let n, be the number of bolts, and further put the permissible load upon each bolt at 275 pounds, we have:

$$4\,D \times 0.434\,h = 2\,n \times 275$$

from which $n = \frac{4 \times 118 \times 0.434 \times 9.83}{2 \times 275} = 3.6$ which gives for the distance from centre to centre of bolts, $\frac{4}{3.6} = 1.11$ in. or about 1⅛ ins. For the joint half-way between the top and bottom of the tank the pressure would be but half that at the bottom and the bolts may be spaced proportionately wider, say about 2 inches apart. The total contents of the tank will be = 742 cubic feet = 5550 gallons.

In using cast iron tanks of this sort care must be taken to avoid filling them with liquids which have an injurious action upon the rubber packing of the joints.

§ 355.

Riveted Tanks.

When tanks of large capacity are required, wrought iron or steel must be used in their construction and these involve the use of riveted joints. With tanks of large diameter constructive difficulties arise in connection with the flat bottoms.

In the United States, oil tanks are made with flat bottoms, carefully bedded in cement, and similar tanks are used in Germany for water. It is, however, found that greater facility of construction, as well as economy of material, is obtained by making the bottom convex, as will be shown.

A very frequent and useful form is that in which the bottom

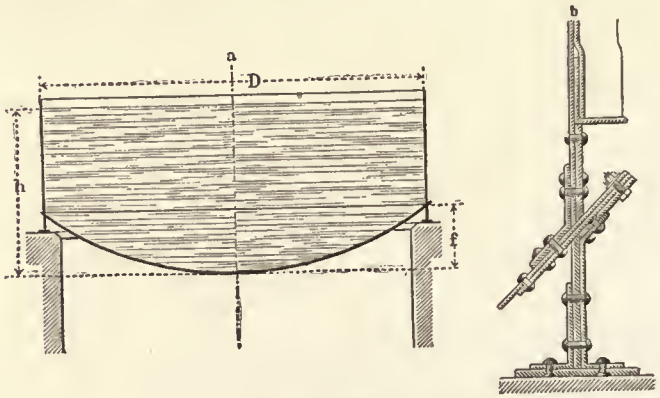


Fig. 1098.

is made in the shape of a spherical segment, Fig. 1098 *a*, the tank being supported on a flanged ring riveted to its circumference and the ring standing on a support of masonry.

The construction of the supporting ring is shown in Fig. 1098 *b*, from the design of Prof. Intze.

The tension in the inclined direction of the bottom of the tank is carried by the lower half of the supporting ring, while the upper portion is subjected to the pressure of the tank at right angles to the vertical. This latter force is well resisted by a ring of angle iron running entirely around the tank.

The calculation of the bottom of spherical segment shape is as follows:

If R is the radius of the sphere of which the segment is a part, we have from § 19, Case II.:

$$\frac{\delta_1}{R} = \frac{p}{2S_1}$$

in which δ_1 is the thickness and S_1 the stress therein due to the pressure p. The pressure is the greatest at the lowest point of the bottom where the height in feet of the column of liquid is

equal to h, so that if σ, is the weight of a cubic inch of the liquid $p = 12\,h\,\sigma$. We then have:

$$\left.\begin{aligned} \frac{\delta_1}{R} &= \frac{12\,h\,\sigma}{2\,S_1} = 6\,\sigma\,\frac{h}{S_1} \\ \text{which for water gives, } \sigma &= 0.0361 \\ \frac{S_1}{R} &= 0.217\,\frac{h}{S_1} \end{aligned}\right\} \quad \ldots\ldots\ldots (368)$$

At each higher point of the bottom the pressure is less, until at the edge of the bottom the height h, is diminished by the depth f, of the bottom. For simplicity, however, it is customary to make the entire bottom of the same thickness δ, which is required for the lowest point.

For the thickness of the cylindrical walls of the tank at the bottom we have the pressure $p = 0.036\,(h - f)$ both h and f, being in inches, and from (367)

$$\frac{\delta}{D} = \frac{1}{2}\cdot\frac{0.434\,(h-f)}{S} = 0.217\,\frac{(h-f)}{S}$$

this gives δ in feet, hence we have for δ in inches:

$$\delta = 12 \times 0.217\,D\,\frac{(h-f)}{S} = 2.604\,D\,\frac{(h-f)}{S} \quad \ldots (369)$$

In order to obtain good proportions it should be considered that as h diminishes, the ratio of $\frac{\delta}{D}$ becomes smaller, while as D increases the size and thickness of the bottom increases.

An approximate formula by which the minimum amount of material will be required is:

$$D = 1.366\sqrt[3]{Q} \quad \ldots\ldots\ldots (370)$$

in which Q is the volume of the material in cubic feet to be contained in the tank.

For the height H of the wetted portion of the surface we have:

$$H + \frac{f}{2} = h - \frac{f}{2} = \frac{D}{2} \quad \ldots\ldots\ldots (371)$$

if we assume, as we may with sufficiently close approximation, the segment of the sphere to be practically that of a paraboloid.

The same remark about the most economical ratio of depth to diameter applies here as in the note to § 354.

Example 1.—For $Q = 42{,}000$ cubic feet we have from (370):

$$D = 1.366\sqrt[3]{42{,}000} = 47.36.$$

A carefully calculated tank at Halle, of this capacity (1200 cu. metres) was made 51.88 feet diameter.

If $Q = 65{,}600$ cu. ft. we have $D = 1.366\sqrt[3]{70{,}000} = 56.3$ ft., while a tank of the same capacity at Essen is 58 feet in diameter.

The water tower at Neustassfurt has a capacity $Q = 21{,}160$ cu. ft., and is 39.36 ft. diameter; according to (370) it would be $D = 1.366\sqrt[3]{21{,}160} = 37.79$ ft. All three cases thus agree well with the formula.

For the depth f, of the concave bottom, we have for any given radius R, the expression

$$2\,Rf - f^2 = \tfrac{1}{4}\,D^2,$$

from which we get

$$\frac{f}{D} = \frac{R}{D} - \sqrt{\left(\frac{R}{D}\right)^2 - \frac{1}{4}} \quad \ldots\ldots\ldots (372)$$

It is found convenient, but not essential, to choose such a value for R, that $\delta_1 = \delta$, when $S = S_1$. To accomplish this result, the conditions which obtain for the equations both for δ_1 and δ must be fulfilled. These are:

$$\frac{R}{D} = \frac{h-f}{h}, \text{ whence, } \frac{h}{D} = \frac{\frac{f}{D}}{1 - \frac{R}{D}} \quad \ldots\ldots (373)$$

The following table gives a series of numerical values for these relations:

$\frac{R}{D}$ =	0.5	0.55	0.60	0.625	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.0
$\frac{f}{D}$ =	0.5	0.32	0.27	0.25	0.23	0.21	0.19	0.18	0.16	0.15	0.14	0.134
$\frac{h}{D}$ =	1.0	0.71	0.68	0.67	0.66	0.70	0.76	0.88	1.07	1.52	2.84	∞
$\frac{h-0.5f}{D}$ =	0.75	0.55	0.54	0.54	0.56	0.59	0.67	0.79	0.99	1.45	2.77	∞
$\frac{2}{3}\left(\frac{f}{D}\right)^2$ =	0.17	0.07	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01

These relations are also shown graphically in Fig. 1099, and the results are interesting. It will be seen that in order to have $\delta_1 = \delta$ when $S_1 = S$ we must always make $R < D$. It also appears that the best ratio of depth to diameter occurs when $\frac{R}{D}$ is about equal to 0.60, for then $h - 0.5\,f$ nearly approaches $0.5\,D$; this, however, is only approximate. It thus appears that the two conditions of greatest economy of material and equality of value δ_1 and δ, cannot be attained at the same time exactly. The most useful ratio in practice will be obtained by selecting a value for D, according to (370).

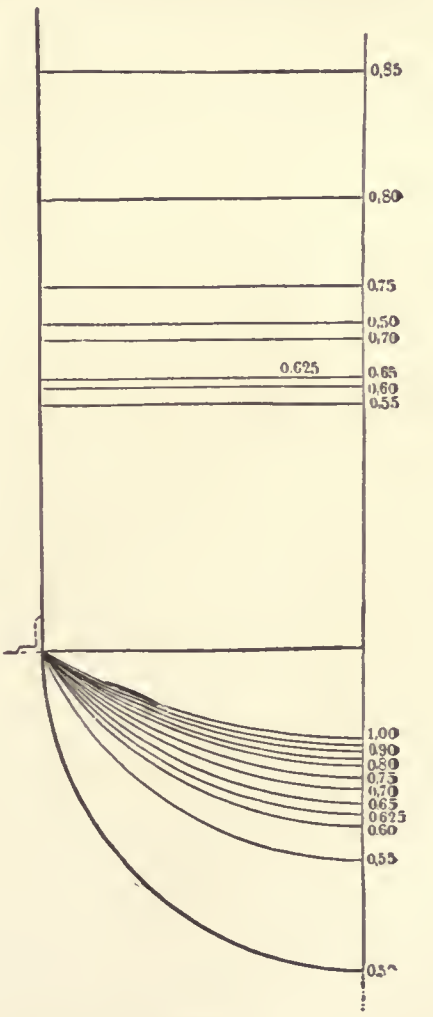


FIG. 1099.

The value $R = 0.5\cdot D$, which corresponds to a hemispherical bottom, is useful to the extent that when the supporting ring is placed at its upper edge there is no lateral pressure produced tending to compress the ring, as there is in all of the other cases. The hemispherical bottom, however, offers too many constructive difficulties to be much used.

Example 2.—Let $Q = 53{,}000$ cubic feet. We have from (370):

$D = 1.336\sqrt[3]{53{,}000} = 50.28$ feet and according to (371), $h - 0.5\,f = 0.5\,D = 25.14$ ft., and combining these again we get: $Q = 25.14 \times 0.7854\,(50.28)^2 = 49.420$ cu. ft., which is a little under the required content, but shows the correctness of the proportions.

If we now make $f = 0.21\ D = 0.21 \times 50.28 = 10.56$ we have from the above table, $R = 0.7\ D = 0.7 \times 50.28 = 35.2$ ft. We have from (371) $h = 0.5\ D + 0.5 f = 0.605\ D = 30.42$ ft. The height of the wetted perimeter will be $H = h - f = (0.605 - 0.21)\ D = 0.395\ D = 19.86$ ft.

Taking for the stress in the metal at the lowest part of the walls of the tank we have from (369):

$$\delta = 2.604\ D\ \frac{H}{S} = 2.604 \times 0.395\ \frac{D^2}{S} = 0.372 \text{ in}$$

For the bottom we have

$$\delta_1\ 2.604\ R\ \frac{h}{S} = 0.7 \times 2.604 \times 0.605\ \frac{D^2}{S} = 0.4 \text{ in.}$$

and $\frac{\delta_1}{\delta}$ 1.07; that is, the thickness of the bottom is 7 per cent. greater than that of the lowest row of plates in the walls of the tank.

If we make the tank with six rings of 3 ft. width and one of 2 ft. we get for the thicknesses:

Depth =	19.86	16.86	13.86	10.86	7.86	4.86	1.86
$\frac{\delta}{}$ Calculated =	0.372	0.315	0.260	0.203	0.147	0.091	0.035
In practice =	3/8″	5/16″	1/4″	1/4″	1/4″	1/4″	1/4″

The latter figures show an excess over the theoretical thickness, but the excess is needed for stiffness and for constructive reasons. The thickness of the bottom, as already calculated is 0.4 in., but in practice would probably be made 7/16″.

The riveting may be made the same as ordinary boiler riveting; and from the table in § 59, we find for $\delta = 3/8$″, $d = 1/8$″ and for single riveting the modulus of efficiency is 0.47. This gives a stress of $\frac{7000}{0.47}$ = 15,000 pounds, which seems rather too high. For this reason the two lower seams at least should be made with double riveting; which gives a stress of $\frac{7000}{0.59}$ = 11,800 pounds. The seams of the bottom should always be made double riveted.

Example 3.—Let Q again be taken as 53,000 cu. ft. We will now proportion the tank so that $\delta_1 = \delta$, and take $D = 50$ ft.

In order that δ_1 shall at least equal δ, we will take $\frac{R}{D} = 0.625$ whence $f = 0.25\ D = 12.5$ ft. We then have $h = 0.67\ D = 33.5$ ft., and $h - \frac{1}{2} f = (0.67 - 0.125)\ D = 0.545\ D = 27.25$ ft. We therefore have

$$Q = 0.7854 \times 27.25 \times (50)^2 = 53{,}500 \text{ cu. ft.}$$

which agrees quite closely enough with the original assumed capacity.

H will be = to $h - f = (0.67 - 0.25)\ D = 0.42\ D = 21$ ft.

We therefore have for the lowest cylindrical portion of the tank:

$$\delta = 2.604 \times 0.42\ \frac{D^2}{S} = 2.604 \times 0.42\ \frac{50^2}{7000} = 0.3906''$$

and for the bottom:

$$\delta_1 = 2.604\ \frac{0.625 \times 0.67\ D^2}{S} = 2.604\ \frac{0.625 \times 0.67 \times (50)^2}{7000} = 0.3894''$$

thus giving practically $\delta = \delta_1$.

The tank will be heavier than the preceding proportions give, as might be expected, but the excess weight will be only about 1 per cent.

§ 356.

Tanks with Concave Bottoms.

The question of the action of the forces upon the bottom of a tank as discussed in the preceding section, was first thoroughly investigated by Prof. Intze, whose valuable researches have practically revolutionized the construction of riveted tanks.* The following discussion is based on Intze's, but the calculations are simplified and abridged.

Fig. 1100 shows two forms in which the spherical segment may be used, *a*, with convex or hanging bottom, as already discussed, and *b*, with concave or reversed bottom. In both forms the pressure of water on the bottom produces a stress at the base of the cylindrical portion of the tank in the direction of the tangent to the curve of the bottom, the stress acting inwards in case *a*, and outward in case *b*. It is desirable to make the construction such that this force is received by the base ring and not by the shell of the tank. In every case, however, an increase is required in the thickness of the bottom of the tank.

There is also a force t, acting at right angles to the tangent or normal to the curve of the bottom of the tank, and the determination of both of these forces is a matter of importance.

If G be the weight of the liquid, and a the angle which the tangents make with the axis we have for case *a*, for the two lateral forces which act, each on one half the circumfe the base ring of the tank:

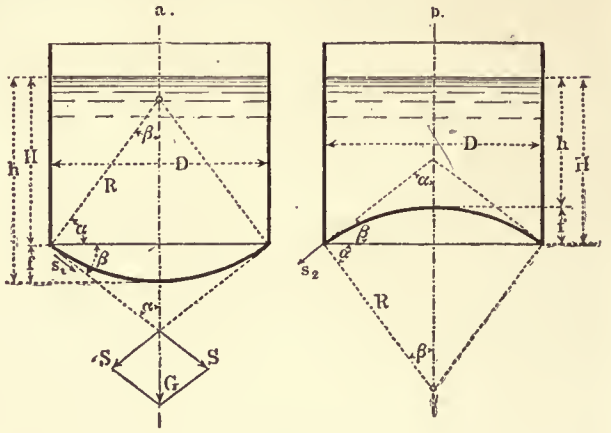


Fig. 1100.

$$S = \frac{G}{2 \cos a}$$

producing a load s_1 per running foot:

$$s_1 = \frac{S}{\frac{\pi}{2} D}$$

Substituting for G, its value

$$\gamma \left[\frac{\pi}{4} D^2 (h - f) + \frac{\pi}{6} f \left(\frac{3}{4} D^2 + f^2 \right) \right]$$

in which γ is the weight of a cubic foot of the liquid, we get:

$$s_1 = \gamma \frac{R}{2} \left[h - \frac{f}{2} + \frac{2}{3} f \left(\frac{f}{D} \right)^2 \right]$$

In this h is the distance from the level of the surface of the liquid to the crown of the curve of the bottom, and for the case *b*, we have:

$$s_2 = \gamma \frac{R}{2} \left[h + \frac{f}{2} - \frac{2}{3} f \left(\frac{f}{D} \right)^2 \right]$$

The last member in the brackets is always very small in value as will be seen by reference to the table in the preceding section. It can therefore generally be neglected, when we have for both cases:

$$s = \gamma \frac{R}{2} \left(h \mp \frac{f}{2} \right) \quad \ldots\ldots\ldots (374)$$

The detailed determination of the forces t_1 and t_2, need not be gone into here, we have for both cases:

$$t = \gamma R (h \mp f) - s = \gamma \frac{R}{2} (h \mp \frac{3}{2} f) \quad \ldots (375)$$

There is also a third force u, acting upon the rim of the spherical bottom in the direction of a great circle at right angles to the plane of the drawing, for which we have per running foot:

$$u = \gamma \frac{R}{2} (h \mp f) \quad \ldots\ldots\ldots (376)$$

and finally for the crown of the curve, where the force u_o in a great circle is:

$$u_o = \gamma \frac{R}{2} h \quad \ldots\ldots\ldots (377)$$

These formulæ will be somewhat simplified if we take the height H, of the wetted portion of the cylinder, whence $h = H \pm f$. This gives:

$$\left.\begin{aligned} s = \gamma \frac{R}{2} \left(H \pm \frac{f}{2} \right), \quad t = \gamma \frac{R}{2} \left(H \mp \frac{f}{2} \right) \\ u = \gamma \frac{R}{2} H, \quad u_o = \gamma \frac{R}{2} (H \pm f) \end{aligned}\right\} \quad \ldots (378)$$

* See the article by Dr. Forchheimer: "On the Construction of Iron Tanks, for Water, Oil and Gas, according to the Calculations and System of Prof. Intze, of Aachen." Schilling's Journal für Gas-beleuchtung, 1884, p. 705.

These are the necessary formulæ for the calculations of spherical bottoms. The following points are to be noted:

1. For the convex bottom (Form. *a*) u_o has the greatest value, that is, the stress must be calculated for the deepest point if δ_1, is to remain constant; 2. For the concave bottom (Form. *b*) t has the greatest value, and must be used to determine δ_1; 3. The supporting rim should be capable of sustaining s, if the shell is to be free from any stress due to the bottom of the tank.

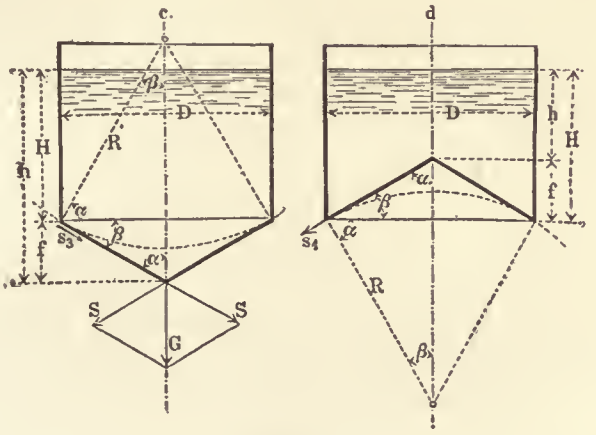
The determination of δ_1 is the same as before.

If we divide the values for u_o and t, by 12, we get the stress per running inch, and by using the weight σ of a cubic inch of the liquid and taking R in inches, we have for the convex bottom:

$$\frac{\delta_1}{R} = \frac{12\,h'\,\sigma}{2\,S_1} = 12\,\sigma\,\frac{H+f}{2\,S_1} \quad \ldots\ldots (379)$$

and for the concave bottom:

$$\frac{\delta_1}{R} = \frac{12\,h'\,\sigma}{2\,S_1} = 12\,\sigma\,\frac{H+\frac{1}{2}f}{2\,S_1} \quad \ldots\ldots (380)$$



[FIG. 1101.

If the bottom is made conical, projecting either within or without as in Fig. 1101, the height of the cone being f, we have for the weight of the body of liquid

$$G = \gamma\left(\frac{\pi}{4}D\,H \pm \frac{1}{3}f\frac{\pi}{4}D^2\right)$$

and taking the component as before in the direction of the angle of the cone, we have:

$$S = \frac{\pi}{2}D\,s = \frac{G}{2\cos\alpha}$$

whence:

$$s = \gamma\frac{\pi}{4}D^2\,\frac{2}{\pi D\,2\cos\alpha}\left(H \pm \frac{f}{3}\right)$$

$$= \gamma\frac{1}{2}\,\frac{D}{2\cos\alpha}\left(H \pm \frac{f}{3}\right)$$

But $\frac{\frac{1}{2}D}{\cos\alpha}$ is equal to the radius R of a sphere inscribed within the cone; whence we have:

$$s = \gamma\frac{R}{2}\left(H \pm \frac{f}{3}\right) \quad \ldots\ldots (381)$$

We also have for t the same value as for u, and

$$t = u - \gamma\frac{R}{2}H \quad \ldots\ldots (382)$$

For the inverted hanging cone bottom, form *c*, the greatest of the three forces is s, while for form *d*, in which the cone projects into the tank $t = u$, is the greatest, and we use in practice for form *c*:

$$\frac{\delta_1}{R} = 12\,\sigma\,\frac{H+\frac{1}{3}f}{2\,S_1} \quad \ldots\ldots (383)$$

and for form *d*:

$$\frac{\delta_1}{R} = \frac{12\,\sigma\,H}{2\,s} \quad \ldots\ldots (384)$$

The conical form of bottom, as will be found upon comparison, requires about 40 per cent. more material thau the spherical, but as will be seen, its use under some circumstances is advisable.

Instead of using a complete tone, the bottom may be made a truncated cone, the tank being formed of two concentric cylinders connected by a ring-shaped bottom, as in Fig. 1102.

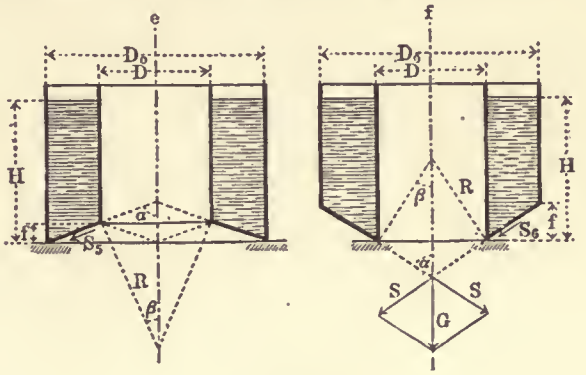


FIG. 1102.

These may be made either projecting inward or outward. Following the same line of investigation as in the previous cases we have for case *e*:

$$G = \gamma\frac{\pi}{4}(D_o^2 - D^2)\,H - \gamma\,f\frac{\pi}{4}(D_o - D)\,H\,(D + \frac{1}{3}(D_o - D))$$

and for case *f*:

$$G = \gamma\frac{\pi}{4}(D_o^2 - D^2)\,H - \gamma\,f\frac{\pi}{4}(D_o - D)\,(D + \frac{2}{3}(D_o - D))$$

This gives for case *e*:

$$s_5 = \gamma\frac{R}{2}\left\{\left[\left(\frac{D_o}{D}\right)^2 - 1\right]H - \frac{f}{3}\left[\left(\frac{D_o}{D}\right)^2 + \frac{D_o}{D} - 2\right]\right\} \quad (385)$$

and for case *f*:

$$s_6 = \gamma\,\frac{D}{D_o}\,\frac{R}{2}\left\{\left[\left(\frac{D_o}{D}\right)^2 - 1\right]H - \frac{f}{3}\left[2\left(\frac{D_o}{D}\right)^2 - \frac{D_o}{D} - 1\right]\right\} \quad \ldots\ldots (386)$$

in which R is the radius of the sphere inscribed within the truncated cone.* The forces t and u are obtained in a similar manner as before.

The subject of truncated conical bottoms will be discussed agaiu.

We have for the weight σ, of a cubic inch of various liquids:

Water	0.0361 lbs.
Petroleum	0.0289 lbs.
Linseed Oil, at 12° C. = 54° F.	0.0339 lbs.
Bisulphide of Carbon, at 0° C. = 32° F.	0.0459 lbs.
Glycerine, at 0° C. = 32° F.	0.0455 lbs.
Beer, at 0° C. = 32° F.	0.0372 lbs.
Alcohol (absolute), at 20° C. = 68° F.	0.0286 lbs.

In the construction of tanks, it is necessary also to consider the peculiar properties of the various liquids. For alcohol no packing should be used in the joints, the tightness only being secured by caulking the riveted seams

* If D_o be made o, the formulæ will become those for complete cones, as indicated in the dotted lines. The formulæ for the weight might also be symmetrically expressed: the form used has been selected because it makes H the higher of the two walls, which is more convenient in numerical calculation.

§ 357.

Combination Forms for Tanks.

In the forms of tanks already described the force $s \sin a$ acts either to press the supporting ring inward or outward in a direction radical to the axis, according as the forms *a*, *c*, *e*, or *b*, *d*, *f*, are used. This circumstance lends itself very fortunately to Prof. Intze's method of construction, since by combining both forms in one bottom the forces may be made to equilibrate each other and thus relieve the supporting ring from all radial stresses.

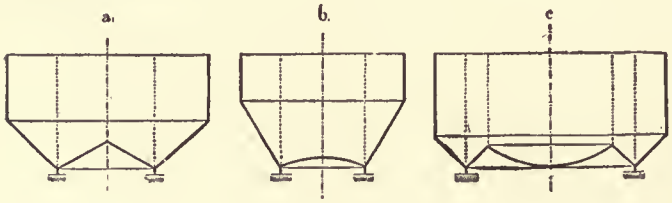


Fig. 1103.

This idea may be carried out in many ways, as by combining forms *d* and *f*, Fig. 1103 *a*, or forms *e* and *b*, Fig. 1103 *b*, or using all three forms as in Fig. 1103 *c*, the inner vertical walls being, in these combination forms omitted.*

The forms shown in the illustration also have the advantage of reducing the diameter of the supporting ring and hence requiring less extensive foundation walls.

In order that the supporting ring may be free from radial stresses, the condition:

$$s' \sin a' - s'' \sin a'' = O \ldots\ldots\ldots (387)$$

must be satisfied. This simple equation cannot be briefly solved numerically, hence an example is here given of its application.

Example.—Given a water tank of the form and dimensions of Fig. 1104, th radius of curvature of the bottom being R''. The first member of the equation belongs to the outer, and the second to the inner portion of the tank

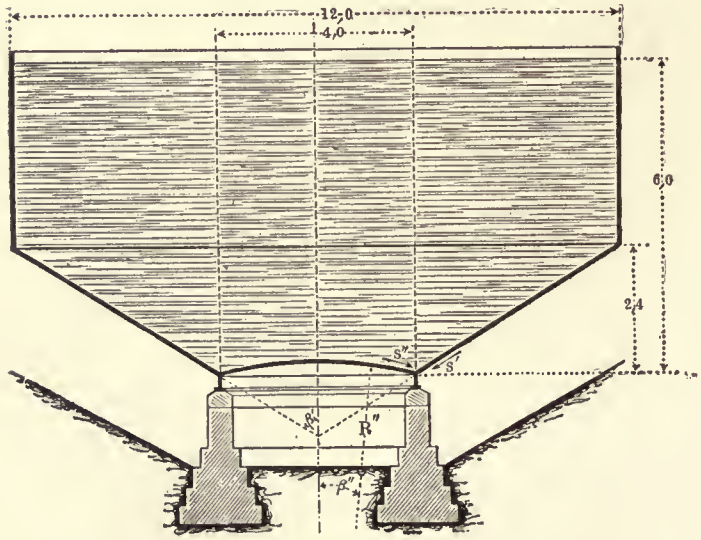


Fig. 1104.

For the first member we have for s', from (385); $D_0 = 12$, $D = 4$, $H = 6$, $f = 2.4$, whence $\tan \alpha = \frac{4}{2.4} = 1.667 = \tan 59°$. This gives $\sin \alpha' = 0.8572$, and $\cos \alpha' = 0.5150$, and

$$R' = \frac{0.5\,D}{\sin \alpha'} = \frac{2}{0.515} = 3.883 \text{ and}$$

$$s' \sin \alpha = 0.8572\ \gamma \times 0.5\ D \left(\frac{R'}{R_0}\right) \left\{ \left[\left(\frac{D_0}{D}\right)^2 H - \frac{1}{3} f \left[2 \left(\frac{D_0}{D}\right)^2 - \left(\frac{D_0}{D}\right) - 1 \right] \right\} = 0.552\,(\gamma \times 0.5\ D \left(\frac{3\,883}{12}\right) (8 \times 6 - 0.8 \times 14) =$$

$$(\gamma \times 0.5\ D)\ 0.8572 \times 0.523\ (48 - 11.2) =$$

$$(\gamma \times 0.5\ D)\ 0.2769 \times 36.8 = 10.19\ (\gamma \times 0.5\ D).$$

For the second member we have from formula (378):

$$s'' \sin \alpha'' = \sin \alpha''\ \gamma\ 0.5\ R''\ (H - 0.5\ f'')$$

in which both R'' and α'' are unknown, hence we introduce β'' and have:

$$s'' \sin \alpha'' = \gamma \cos \beta''\ R''\ (3 - 0.25\ R''\ (1 - \cos \beta'')\ ).$$

Introducing these into the equation of condition, we get:

$$10.19 \times 0.5\ D\gamma - \cos \beta''\ R''\ (3 - 0.25\ R''\ (1 - \cos \beta'')\ )\ \gamma = 0$$

$$\text{But } \frac{0.5\,D}{R''} \sin \beta'' \text{ whence:}$$

$$\tan \beta'' - \frac{3 - 0.25\ R''\ (1 \cos \beta'')}{10.19} = 0$$

We may obtain a first approximation for β'' by neglecting the second member of the numerator. This gives $\tan \beta'' = \frac{3}{10.19} = 0.2954 = \tan 16° 25'$.

The true value must be somewhat less. Assuming it to be $\beta'' = 16° 20'$, the tangent $= 0.2930$, the sine $= 0.2812$, the cosine $= 0.9596$. We then have $R'' = \frac{0.5\,D}{\sin \beta''} = \frac{2}{0.2812} = 7.11$ ft. and $10.19 \times 0.2930 - (3 - 0.25 \times 7.11 \times 0.0404) = 0$ nearly. Numerically this gives:

$$2.986 - 2.928 = 0.058$$

or, since the weight of a cubic foot of water = 62.4 lbs., the unbalanced radial force upon the ring is $62.4 \times 0.058 = 3.62$ lbs. per running foot, which is so small as to be unimportant.

The question may properly be asked, as to the stresses upon the supporting ring when the tank is not full, that is, when H varies. In answer, it is true that the pressure on the ring necessarily changes. Suppose $H = 3$ ft. We then have for the first member of the equation:

$$\gamma \times 0.5\ D \times 0.2769\ (24 - 11.2) = \gamma \times 2 \times 0.2769 \times 12.8 = 7.088\ \gamma$$

and for the second member

$$\gamma \times 0.9596 \times 7.11\ (1.5 - 0.25 \times 7.11 \times 0.404) = 6.823 \times 1.428\ \gamma = 9.743\ \gamma.$$

This gives a pressure of

$$7.088\ \gamma - 9.743\ \gamma = -2.655\ \gamma$$

or $2.655 \times 62.4 = 165.67$ lbs. per running foot acting from without inwards, which is large enough to be worth considering. It is therefore important to base the calculation upon a depth of water which will be usually maintained in the tank. The proportions may also be so made that the forces will be in equilibrium when the tank is half full, when a greater depth will cause an outward pressure and a lesser depth an inward pressure.

Tanks constructed on the combination are well adapted for use with gasholders, the level of the water remaining so nearly uniform that the supporting ring may be kept free from any lateral pressure.

§ 358.

High Pressure Reservoirs or Accumulators.

The forms of tanks already described are intended to be placed at such elevation either in buildings, or towers or on natural elevations that the liquid is delivered through pipes at the desired pressure.

In this way a water tank with a pump and the necessary piping forms a storage system, an overflow being provided as a security against flooding the tank. Systems of oil storage are constructed also in this manner; and on a small scale the water tank stations for railway service come under the same classification. These water stations are usually provided with steam pumps, although windmills are often used, especially in the United States.

It is a question whether the required pressure might not be obtained by the use of compressed air, the tank being closed at the top and the confined air exerting by its elasticity sufficient pressure to obviate the necessity of elevating the tank upon a tower to obtain the necessary pressure.

For high pressure water systems for operating hydraulic machinery the use of weighted devices, as suggested long since by Armstrong, has superseded the open water column, such devices being generally known as Accumulators.

The volume of such accumulators is generally quite small, but the pumping mechanism is so efficiently devised as to enable them to possess a very extensive capacity. The pressure is obtained by means of a weighted plunger, the overflow being replaced by a safety valve.

Fig. 1105 shows an accumulator built by C. Hoppe, Berlin.* This is weighted to a pressure of 20 atmospheres, or nearly 300 pounds per square inch. The plunger is 17¾ in. diameter (450 mm.) weighted with shot which is enclosed in a cylinder. The plunger is shown in the highest position. When it reaches the position the lever and connections *M M'* act to shut off the steam from the duplex pump, and at the same time the rod *s*

* Combination forms of this sort have been patented by Prof. Intze, (German Patents, No. 23,187, 24,951 and built for oil, water, gas, etc., at the works of F. A. Neumann, at Aachen.

* All dimensions in the illustration are in millimetres.

relieves the safety valve. When the use of the water causes the plunger to sink, the steam is turned on and the pump starts. If the pressure should be suddenly released by the bursting of

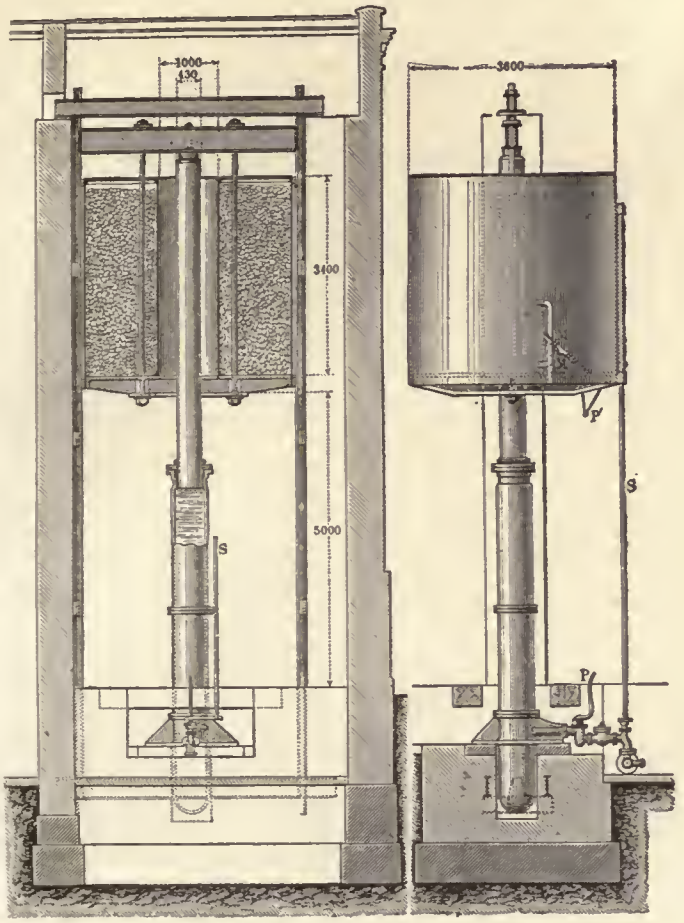


FIG. 1105.

a pipe, the sudden drop is received by heavy beams, and at the same time the stop P' strikes the lever P and checks the water flow in time to moderate the shock.

An accumulator for very high pressures is shown in Fig. 1106.* This is designed by Tweddell for use for operating riveting machines, punches and similar tools. The plunger c, is stationary; the cylinder d, sliding upon it, weighted with rings d_1 of cast iron. In the lowest position the cylinder rests upon vertical buffers of oak. The water is delivered under high pressure at H, while the water is taken off for use through suitable valve gear at A; the safety valve is at b'. The plunger is of the differential variety similar to those shown in Fig. 977 b, and Fig. 981 b. The difference in diameter between the two portions of the plunger is the space to be filled by the entering water, the small annular area bearing the total weight, thus giving a very high pressure per square inch. The pressure attained when the cylinder is stationary is about 100 atmospheres (1420 pounds), but experimental investigation has shown that when the weighted cylinder is permitted to descend rapidly the pressure reaches as high as 193 atmospheres, (2740 pounds), so that it is worthy of note that the attainable water pressure in such devices may reach double the statical pressure.

§ 359.

STEAM BOILERS, VARIOUS FORMS.

Steam boilers may properly be considered as reservoirs for vapor of water, while at the same time they serve as generators of force by the application of heat. The pressure is produced by the heat, the feed is effected either by a pump, as Fig. 975 d, or injector, Fig. 971. The overflow is represented by the safety valve, and the observation of the water level is provided for in a variety of ways.

The forms used for steam boilers are very numerous; the great variations of size, the varying conditions of locality, and the efforts to attain compactness, having led to a vast number of modifications of the original simple forms.

The various boilers used in Germany may be reduced to

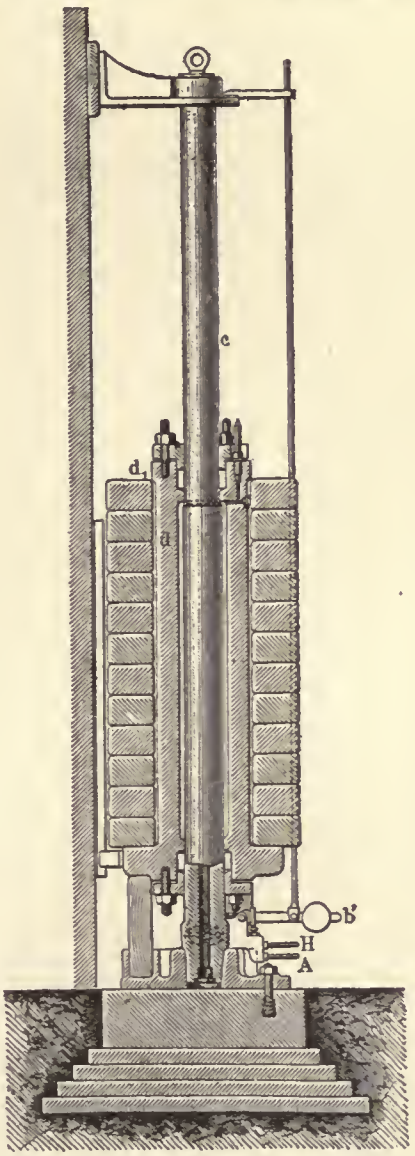


FIG. 1106.

eight principal classes, examples of which will here be given.

1. Plain cylinder boiler, Fig. 1107, usually placed in the horizontal position, and now principally used in iron works where the waste gases from the furnaces are used.

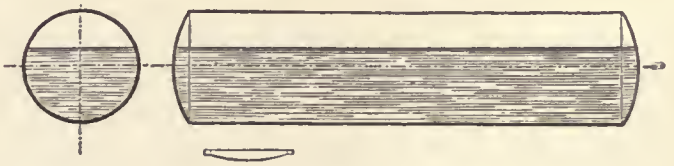


FIG. 1107.

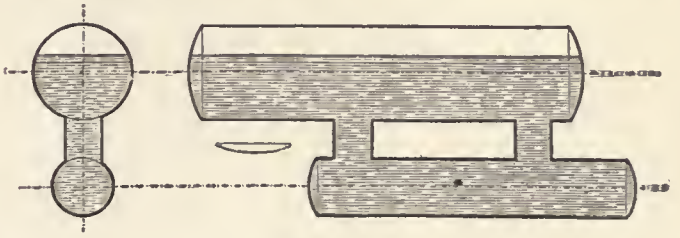


FIG. 108.1

2. Cylinder Boiler with Heater, Fig. 1108. The cylinder has

*See Proc. Inst. C. E. Vol. LXXIII. 1883, p. 92.

added to it a "heater" or auxiliary cylinder placed beneath and forming a part of the boiler, being entirely filled with water and surrounded by heated gases. Besides the usual form, there are Henschel's in which the heater is placed at right angles to the main boiler, and the vertical form.

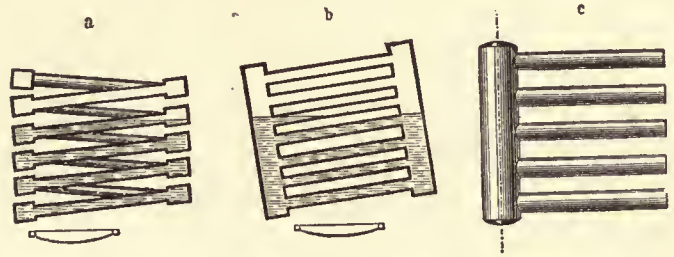


FIG. 1109.

3. Tubulous Boilers, Fig. 1109. This class includes boilers made of tubes 6 inches in diameter aud under. Of the arrangement shown, *a*, is Belleville's; *b*, Root's; and *c*, Howard's.

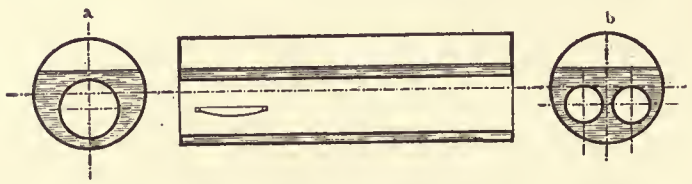


FIG. 1110.

4. Flue Boilers, Fig. 1110. Flue boilers are constructed with internal f··es entirely surrounded by water and containing the furnaces, fire and heated gases. The Cornish boiler, *a*, is made with a single flue, and the Lancashire boiler, *b*, with two flues.

FIG. 1111.

5. Flue Boilers with Cross Tubes, Fig. 1111. This form, also known as the Galloway boiler is constructed with water tubes crossing the flue at various poiuts.

6. Plaiu Tubular Boilers, Fig. 1112. These are made with tubes of 6 inches or less in diameter, through which the heated gases pass. The tubes are lap-welded or seamless, and a distinction is made between direct and returu tubes.

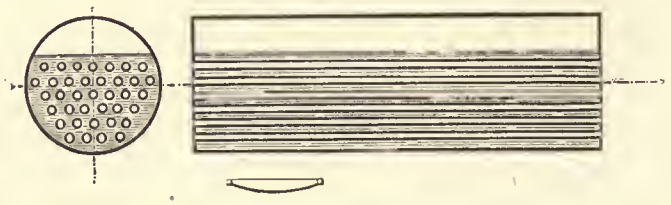


FIG. 1112.

7. Fire Box Tubular Boilers, Fig. 1113. A fire box consists of a box forming a part of the boiler containing the furnace and

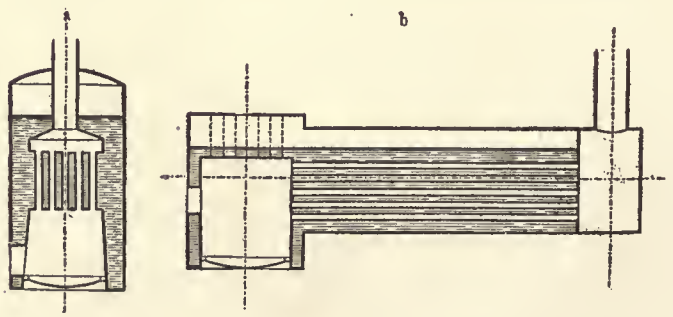


FIG. 1113.

surrounded with water. These boilers are also made with direct and returu tubes. These are made either with vertical tubes as at *a*, or horizontal tubes *b*, and are much used for locomotives and portable boilers.

8. Fire Box Boilers with Water Tubes. Fig. 1114. Of the forms shown, *a*, is a boiler fitted with Field's tubes. These tubes are closed at the lower end, each coutaining a small inner tnbe to produce a circulation by the difference of temperature between the inner and outer tubes. Boilers with large and small cross water tubes are shown at *b* and *c*.

A ninth group might be formed of special combinations of the eight groups above shown.

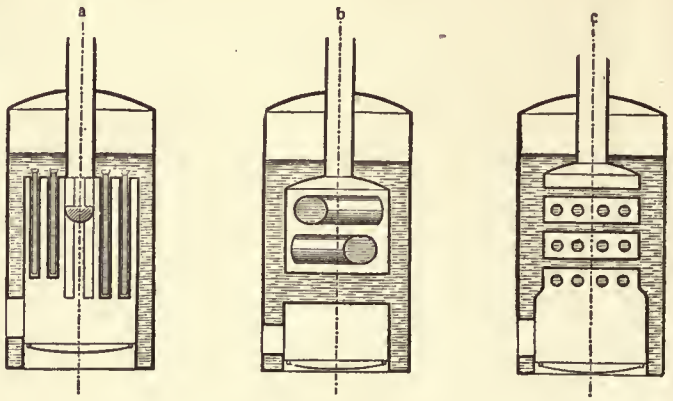


FIG. 1114.

In England and America a different classification is made, the boilers being divided into two great classes, those which consist of a large shell, with the necessary auxiliary parts, and those composed of numerous small elements, the number of elements being governed by the size of the boiler. These two classes are known as "shell" boilers, and "sectional" boilers. The third group shown above, consists of sectional boilers. A popular form in many countries is the Harrison boiler, composed of small spherical elements of cast iron. The relative value betweeu shell and sectional boilers is a question not yet entirely settled. The latter form is incapable of destructive explosious, such as may occur with shell boilers containing large volumes of water. Sectional boilers are also adapted for very high steam pressures, but have the defect in many cases of producing moist steam.

The latest police ordinances in Prussia, which are similar to those of Austria, distinguish between "dwarf" boilers and ordinary boilers. The former are boilers of small volume, less than 18 cubic feet (½ cubic meter) capacity, these together with sectional boilers being permitted for small private industries.

§ 360.

BOILER DETAILS SUBJECTED TO INTERNAL PRESSURE.

The walls of steam boilers are subjected to varied and sometimes complicated stresses greatly dependent upon the method of construction. It will only be practicable here to discuss the ordinary forms, first taking the parts which have to resist the internal pressure.

a. Cylindrical Details.

The Prussian ordinance relating to steam boilers, used the formula of Brix, for cyliuder boilers subjected to iuternal pressure:

$$\delta = \frac{D}{2}\left(e^{0.00 3\,a} - 1\right) + 0.1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (388)$$

in which δ and D are in inches and e is the logarithmic base = 2.71828, a being the pressure in atmospheres. This is closely approximated by the simpler formula:

$$\delta = 0.0015\,a\,D + 0.1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (389)$$

The French formula is much the same but gives a slightly greater thickness:

$$\delta = 0.0017\,a\,D + 0.12 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (390)$$

only ⅔ of this value being used in locomotive practice. On account of the large constant added to provide for deterioration all three formulæ must be considered as empirical. At present there are nearly everywhere government enactments which prescribe the method of determining the thickness of steam boilers and regulate by law the limits of construction.

In most cases the boilers must be subjected to a test pressure which may reach double the workiug pressure.

The stress existiug in the longitudinal seams of a cylindrical

boiler shell may be obtained with sufficient accuracy from (324), as:

$$S = \frac{p}{2}\,\frac{D}{\delta} \quad \ldots\ldots\ldots\ldots (391)$$

p being the pressure in pounds per square inch, and D and δ being in inches. If we calculate δ from (389) and determine S from (391) we have the following results

$a =$	4 = 60 lbs.		7 = 105 lbs.		10 = 150 lbs.		13 = 175 lbs.	
D	δ	S	δ	S	δ	S	δ	S
24	0.24	3000	0.35	3600	0.48	3750	0.58	3700
36	0.31	3500	0.48	3900	0.64	4000	0.80	4000
42	0.35	3600	0.54	4000	0.73	4300	0.92	4000
72	0.43	5000	0.85	4400	1.18	4600	1.50	4200

This table shows that the formula gives for large diameters and heavy pressures, thicknesses which are excessive, and with quite moderate stresses. The stress at the riveted seam will be greater, and from § 59 we have for the stress in the perforated plate:

$$\left.\begin{aligned} &\text{for single riveting } S' = \frac{S}{\phi'} \\ &\text{for double riveting } S'_2 = \frac{S}{\phi'_2} \end{aligned}\right\} \quad \ldots\ldots (392)$$

in which, if d, is the diameter of rivets and a, the pitch,

$$\phi' = \frac{a-d}{a} \text{ and } \phi'_2 = \frac{a_2 - d}{a_2}.$$

Even with this increase the stresses fall below the values which good boiler plate should properly bear. In practice, smaller values are often used for δ than are given by (389) especially since mild steel came into use for boiler plate.

The stress which comes upon the rivets, according to § 59, is greater than that upon the perforated portion of the plate. This, however, should be considered in connection with the fact that rivets are generally made of a still better grade of iron than the plates.

At the present time the disposition is apparent to break loose from set rules for the thickness of boiler shells. Careful designers aim more and more to investigate each case for itself and endeavor to adapt both design and material so as to obtain at the same time the greatest strength and economy. The more recently designed ocean steamers are fitted with boilers as large as 16 feet in diameter, operated at pressure from 160 to 250 pounds pressure. The older formulæ cannot be used for such extreme values, and every resource of the art must be used to reinforce the strength of the plates and the riveting. The method of group riveting, (§ 57) is here found of value and is already used to some extent.

Longitudinal Seams.

For all large steam boilers the longitudinal seams are double riveted. For plates $\frac{5}{16}''$ thick and over a modulus $\phi_2' = 0.76$ to 0.73 is obtained which corresponds to a ratio of $S'_2 : S$ of 1.32 to 1.37. It is more and more made a point of importance that these joints shall not be exposed to the direct action of the fire.

A construction especially intended to meet this point is shown in Fig. 1115 in which the entire shell is made of two sheets, the lower sheet comprising about $\frac{3}{5}$ of the entire circumference.*

Another method which bids fair to become very important, is to weld the longitudinal seams, this being more and more and more used for large boilers. The welding is accomplished either by furnace heat or by water-gas burners, or as more recently by electric welding by the Bernados' method.

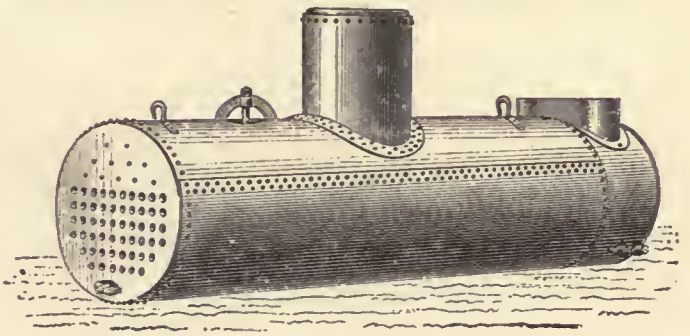


FIG. 1115.

In Fig. 1116 is shown the cross section of a marine boiler, constructed by H. C. Stülken, of Hamburg, the two longitudinal seams being welded.* Both seams are reinforced by double riveted flaps the strength of the plates being reduced by the rivet holes.

The joint, however, is preferable to a lap joint and needs no strengthening. The pressure in this boiler is 180 pounds. The

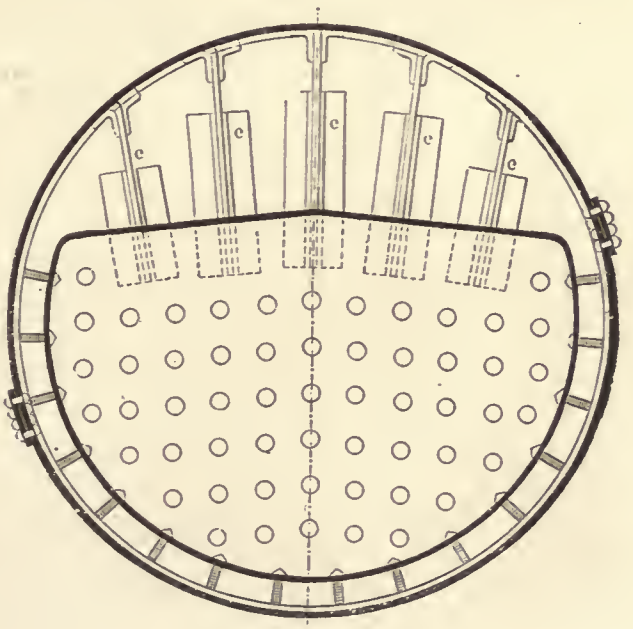


FIG. 1116.

strength may be calculated as follows; the diameter being 76.5 ins. and the plates $\frac{7}{8}$ in. thick. From (391) we have: $S = \frac{180 \times 76.5}{2 \times 0.875} = 7868$ pounds. For the double riveting in both flaps the pitch a_2 is 2.9 ins. and rivet diameter $d = \frac{7}{8}$ in. This gives for the modulus of efficiency $\phi'_2 = \frac{2.9 - 0.875}{2.9} = 0.7$ and the stress in the perforated plate in the longitudinal seams is $\frac{7868}{0.7} = 11{,}240$ lbs. The thickness of plates according to formula (389) would be $0.0015 \times 12 \times 76.5 + 0.1 = 1.5$ ins., instead of $\frac{7}{8}$ ins.

A third method of construction which may become important is to construct the shell in a single piece of mild steel by the Maunesmann process of rolling. This method would be best of all, since the question of the strength of the riveted seam would be entirely eliminated, and the high elastic limit of the material would permit correspondingly high working stresses. At the present time, however, the Mannesmann rolling mills cannot make tubes over 24 inches in diameter.

* Boilers of this sort have been made at the Erie City Iron Works, Erie, Pa. See Trans. Am. Soc. Mech. Engrs. Vol. VI., 1884-5, p. 110. Scheffler, A New Method of Constructing Horizontal Tubular Boilers. The first boiler was 16 ft. by 60 in., $\frac{3}{8}$ thick, of mild steel, 60,000 lbs. ultimate strength, 30,000 lbs. proof strength.

* See Zeitschr, D. Ing., 1886, p. 109.

Circumferential Seams.

The cross section of the boiler shell, when the head is fast to it, is subjected to a force $\frac{\pi}{4} D^2 p = S_2 \pi D \delta$, in which $S_2 = \frac{\frac{1}{4} p D}{\delta}$; that is half as great as the stress S, in the longitudinal seams. For this reason it is deemed necessary to use only single riveting for the circumferential seams. It will also be shown hereafter, that the cross section of the shell can be relieved of this load.

Openings in the Shell.

The openings for the steam dome and manholes weaken the boiler, and in some instances explosions have been caused by cracks radiating from such openings. All such openings should be carefully reinforced by riveting on rings of wrought iron or preferably steel, as shown hereafter in Fig. 1118. The size of a manhole opening should be about 12 by 18 inches, and when practicable the short axis of the oval should be placed lengthwise of the boilers.

b. Spherical Details.

A sphere of the diameter D_1 with an internal pressure p, will be subjected to a force $\frac{\pi}{4} D_1^2 p$, which is the same as already found for the cross section of a cylinder, and one-half that on the longitudinal seams. The thickness, therefore, need be only half so great as that of a cylindrical shell of the same diameter, *i. e.*, $D = D_1$. If, however, both vessels are to have the same content we must have $D_1 > D$. If the cylindrical shell is made with flat heads its content will be $\frac{\pi}{4} D^2 L = \frac{\pi}{4} D^3 \left(\frac{'L}{D}\right)$, and the spherical vessel will have a content $= \frac{\pi}{6} D_1^3$; hence we must have $D^3_1 = \frac{3}{2} D^3 \left(\frac{L}{D}\right)$.

For the thickness of metal we have:

$$\delta = \frac{\frac{1}{2} D p}{S} \text{ and } \delta_1 = \frac{\frac{1}{4} D_1 p}{S_1}$$

and for the respective surfaces:

$$F = \pi D L + \frac{\pi}{2} D^2, \text{ and } F_1 = \pi D^2_1.$$

Assuming the heads of the cylindrical vessel to be made the same strength as the shell, we have for the material required for each case:

$$F\delta = \frac{\pi}{4} D^3 \left(\frac{L}{D} + \frac{1}{2}\right) \frac{p}{S} \text{ and}$$

$$F_1 \delta_1 = \frac{\pi}{4} D^3_1 \frac{p}{S_1}.$$

Making $S = S_1$ and putting for D^3_1 its value $\frac{3}{2} D^3 \left(\frac{L}{D}\right)$ we get:

$$\frac{F_1 \delta_1}{F \delta} = \frac{3}{4} \frac{\frac{L}{D}}{\frac{L}{D} + \frac{1}{2}} \quad \ldots\ldots\ldots (393)$$

for the ratio between the amount of material required for spherical and cylindrical vessels. We have for:

$\frac{L}{D} =$	1	1½	2	3	4	5	6	∞
$\frac{F_1 \delta_1}{F \delta} =$	0.50	0.56	0.60	0.64	0.67	0.68	0.70	0.75

showing that the spherical vessel is in all cases the lighter form.

The earliest boilers were made in the spherical form, but soon abandoned on account of the demand for increased heating surface and small content. The spherical form is, however, well adapted for units for sectional boilers.*

For spherical ends of cylinder boilers, as in Fig. 1107, and for the heads of domes, and auxiliary drums, we have for the thickness, R_1 being the radius of the sphere:

$$\delta_1 = \frac{R_1}{2} \frac{p}{S} \quad \ldots\ldots\ldots (394)$$

which gives, when $S_1 = S$ the same value for the thickness δ, as in the shell when $R_1 = D$. This latter condition cannot always be fulfilled since the curvature of the boiler head is usually controlled by the dies with which the press is provided.

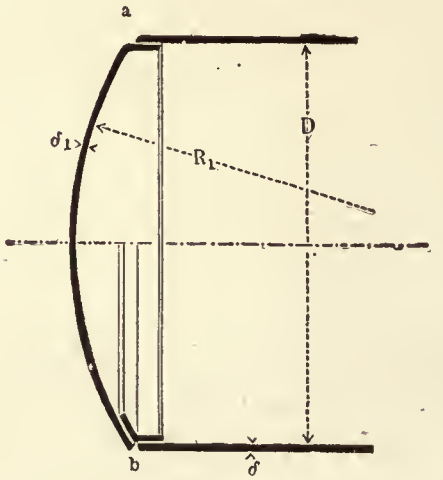


FIG. 1117.

The head is usually joined to the shell by being flanged or turned over around the edge in the flanging press, thus enabling a joint to be made as at *a*, Fig. 1117; or it may be made with a ring of angle iron, as at *b*. Here the circumferential force, as considered in § 355, may be taken into consideration, especially the radial component $s \sin \alpha$, since this acts to draw the shell inward. It is, however, hardly necessary to take this into account as the flange of the head reinforces the shell amply at this point.

c. Flat Surfaces.

Unstayed flat surfaces can only be used in boilers of small dimensions, as already shown in § 19, and should only be used for heads of steam domes, auxiliary heaters, and the like. Where extended flat surfaces are used, it is necessary to adopt some method of staying; or in other words to subdivide the extended surface into supported portions small enough to be of ample strength and at the same time of moderate thickness.

A number of methods of staying flat surfaces are in practical use, those most generally employed being shown in Fig. 1118.

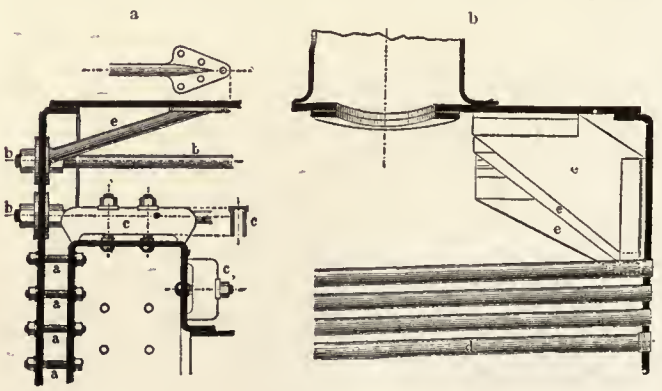


FIG. 1118.

Stay bolts, such as shown in Fig. 1118 *a*, (see also § 61) are used for parallel surfaces which are near to each other. Those shown at *a* are made with nuts instead of riveting the heads as is sometimes done. Flat surfaces which are farther apart are secured by anchor bolts, as shown at *b*; these are practi-

* The Harrison Boiler, the pioneer of modern sectional boilers, is composed of spherical units. Trans.

cally long stay bolts. These are shown reinforced by large riveted washers under the nuts.

Stay bars, as shown at *c*, are used for staying crown sheets of fire boxes in marine and locomotive boilers. Stay tubes, such as shown at *d*, are used to strengthen tube sheets. These are heating tubes about ¼ to $\frac{5}{16}$ in. thick reinforced at the ends and screwed into the tube sheets. Gusset plates *e*, Fig. *b*, are used to stay flat heads to the shell, and are used both in land and marine boilers.*

§ 361.

Boiler Flues Subjected to External Pressure.

The stresses which appear in the case of a boiler flue subjected to external pressure are similar to buckling stresses upon columns, rods, etc., since beyond a certain increase in pressure when a slight departure from the true cylindrical form occurs a sudden collapse follows. The smaller sizes of flues used in the ordinary tubular boiler possess ample strength against collapsing, but for larger flues such as are used in Cornish and Lancashire boilers the question of strength to resist collapsing must be considered. The experiments of Fairbairn have demonstrated that the length of the flue has an important influence upon the resistance to collapsing, practically being inversely as the length of the flue, or rather as the distance between the points at which the flue is reinforced against external pressure.

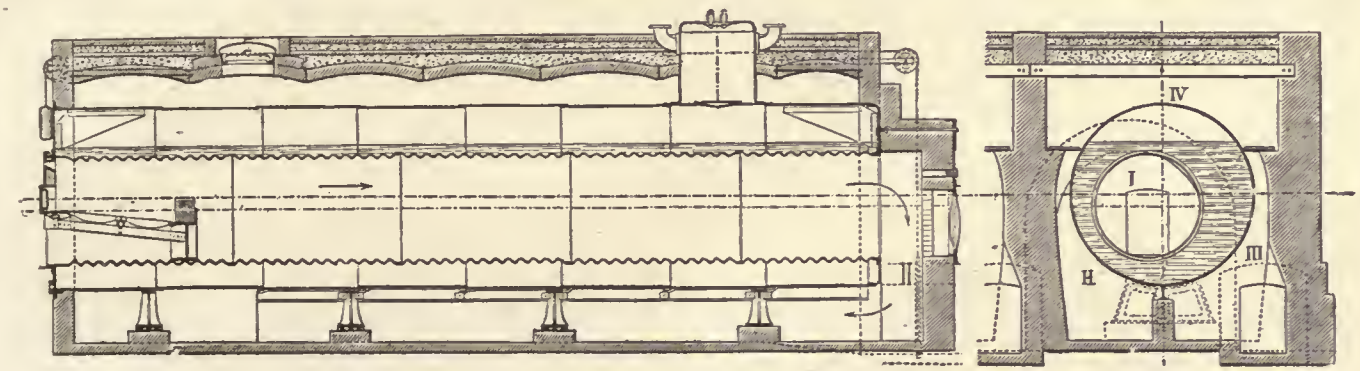


Fig. 1120.

Fairbairn deduced from his experiments for the collapsing pressure of such flues:

$$p' = 806{,}300\frac{\delta^{2.19}}{l\,D} \quad \ldots\ldots\ldots\ldots (395)$$

in which p' is the pressure in pounds per square inch, D and δ are in inches, and l is the length of the flue in feet.

If the dimensions are given in millimetres and p' is the pressure in kilogrammes per square millimetres, this becomes:

$$100\,p' = a' = 367{,}973\,\frac{\delta^{2.19}}{l\,D} \quad \ldots\ldots\ldots (396)$$

Fairbairn's experiments have been discussed more recently, with a view of deducing a formula which should be more convenient to use.† The results of Dr. Wehage in connection with later experiments, ‡ give the following formula:

$$p' = \left\{\begin{matrix}368{,}000\\490{,}000\end{matrix}\right\}\frac{\delta}{D}\sqrt[3]{\frac{\delta}{lD}} \quad \ldots\ldots\ldots (397)$$

in which the upper coefficient is to be used for flues made with lap joints, riveted; and the lower coefficient for flues in which the joints are made with flap plates riveted on.

This formula gives results approximating very closely to Fairbairn's most important experiments. It is best used by selecting the desired dimensions for D, l and δ and then by substituting these in the formula it will be found if the flue is safe against collapsing.

Example.—In a Cornish boiler intended to work at 37½ pounds pressure, the dimensions are $l = 25$ ft., $D = 23$ ins., $\delta = 0.25$ ins., the flue being made with lap joints.

From (397) we have:

$$p = 368{,}000\,\frac{0.25}{23}\sqrt[3]{\frac{0.25}{25\times 23}} = 303 \text{ lbs.}$$

at which pressure the flue actually collapsed. It is evident that should the thickness of the flue be only slightly reduced by corrosion, etc., an explosion might readily follow.

A method of increasing the safety without using a greater thickness of metal in the walls of the flue, is to reinforce it by stiffening rings, thus practically reducing the length l, as noted by Fairbairn.

Fig. 1119.

Two forms of stiffening rings are shown in Fig. 1119, *a* being Adamson's and *b*, Hick's. The first form is the more difficult of construction, but possesses the advantage of removing the rivet heads entirely from the action of the fire. This form of joint and stiffening plate is also frequently used in other parts of boilers for the sole purpose of avoiding the action of the fire on the heads of the rivets.

The use of corrugated iron for boiler flues enables great strength against collapsing to be obtained. Fig. 1120 shows a boiler with corrugated flue, the lengths being welded together. This boiler is made by Schulz, Knaudt & Co., of Essen, and is 86.6 inches in diameter (2.2 metre). Notwithstanding the constructive difficulties the use of the corrugated flues is constantly increasing. In England corrugated flues are made by the inventor, Sampson Fox & Co., of Leeds. The depth of corrugations is usually about 4 inches.

Corrugated fire boxes have been used in locomotive boilers, Fig. 1121, showing Kaselowsky's fire box. In this form the stay bars to support the crown sheet, and the stay bolts at the sides are entirely omitted. The cross section shows the method of supporting the boiler by a cross beam below the grate bars. The corrugated flue is attached to the boiler by a riveted joint, either by flanging as in Fig. 1121, or by the use of angle iron, as Fig. 1120.

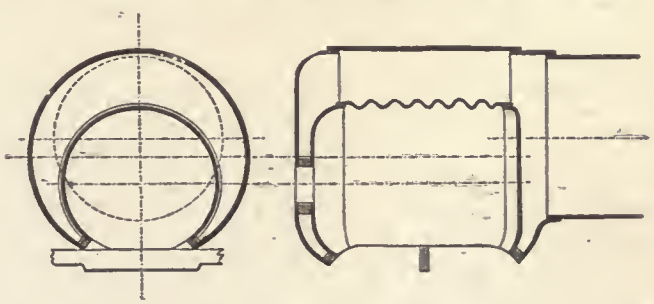


Fig. 1121.

Tubes of small diameter are treated practically as single hollow rivets, the ends being inserted into holes in the tube sheets

* The question has been raised as to whether it is not best to stay only one boiler head to the shell and then tie the other head to the first by means of a number of parallel anchor bolts, thus closing one end of the shell in a manner similar to the cylinder of a hydraulic press, and relieving the shell of any stress due to the pressure on the heads, and permitting the use of packing to make the joint tight. The author recollects such a construction having been used in a portable engine and boiler but without knowledge of any further attempts of the sort.

† See Grashof, Zeitschr. D. Ing. 1859, p. 234, Vol. III.; also Love, Civilingenieur, 1861, p. 238, Vol. VII., discussed by the author for the Hütte Society in the Berliner Verhandlungen, 1870, p. 115.

‡ See Engineer, Vol. 51, 1881, p. 426, also Dingler's Journal, Vol. 242, 1881, p. 236.

and expanded by an expanding tool, the ends beiug riveted over, as shown in Fig. 1122 *a*.

FIG. 1122.

In many establishments, as for example, the Esslingen locomotive works, the tubes are fitted with hard copper ferules which stand the expanding and riveting better than tubes of steel or iron. The form of tube shown in Fig. 1122 *b*, is reinforced at the ends, and one end made conical, thus euabling old tubes to be more readily removed aud replaced. This construction is used by Pauksch & Freund, of Landsberg, in Germany, and by various Freuch builders since 1867.

§ 362.

FUTURE POSSIBILITIES IN STEAM BOILER CONSTRUCTION.

The discussion of the preceding sections has necessarily been limited to a few constructive details, since a complete treatment of such an extensive subject requires a special treatise. It is proposed here to give only a broad general view of the subject of boiler construction in its present and prospective condition.

The descriptions in the preceding sections and in the previous chapter on riveting show that the art of boiler construction has made little or uo advance during the past twenty or thirty years, although there is reason to believe that there is ample room for improvement, especially in the matter of greater economy of fuel. In the author's opinion there are four points in construction which deserve the closest attention and to which efforts at improvement should be directed, while in other directions also serious wastes of force appear.

1. *Expenditure of Material.*—As already shown in § 359, the expenditure of material is considerably greater in the preseut forms of steam boilers than if the spherical form were more generally used. It is questionable to what extent the spherical form may be made practicable, but the possibilities iu this direction have not been exhausted, at least for certain purposes, for example, for boilers used solely for heating purposes. The spherical vacuum pans only serve as reminders that this oldest form of boiler (*i. e.*, that used with Newcomen's engine), is no longer used; but it may be only a question of the increase in the capacity of the flanging press; or, in other words, of the increased command over the working of iron and steel, when the spherical form shall again be used.

Another point in the question of material, is the subject of riveting. One of the greatest sources of weakness in steam boilers is the reduction in strength due to the presence of riveted seams. Even if the very best material obtainable is used for the rivets, the reduction in strength for single riveting is about 40 per cent., and for double riveting, 25 per cent.* This weakening is unimportant so far as the circumfereutial seams of cylindrical shells are considered, but is well worthy of consideration in connection with the longitudinal seams, especially since it concerns the largest and heaviest part of the boiler, *i. e.*, the main shell. It is for this reason that attempts have been made to weld the longitudinal seams.

The meagre results which have been obtained for welded shells subjected to internal pressure, as compared with welded flues for external pressure, may be seen from the case shown in Fig. 1116. The welded seam is there reinforced by a riveted flap, thus reducing the strength practically to that of an unwelded seam. Experimental results with welded joints in the testing machine, justify this distrust of welded seams, and do not warrant the idea that the weld is equal to the full strength of the plate.

This leads to the remark that the coming boiler shell must be without longitudinal seams of any kind, either riveted or welded. Heating flues for external pressure are already made seamless, and the Maunesmann process produces seamless tubes adapted for internal pressure, and of a grade of material far superior to that heretofore used, as experimental researclies have demonstrated. If this process can be so extended as to be made available for boiler shells, an ecouomy of at least one-third of the material can be obtained.

2. *Combustion.*—The subject of economy of combustion of the fuel is even more important than that of material. In the general description given in the preceding sections it will be seen that the present methods of firing are all based upon the principle of exposing portions of the boiler to the direct action of the fire and of conducting the products of combustion into contact with various portions of the boiler, arranged to act as heating surface. This means that in nearly all cases boilers are independently fired. For a long time the advantages of this system have been doubted. It is manifestly impossible for a complete combustion of the gases to be effected when they are almost iuunediately brought into contact with surfaces which have a temperature of 1200 to 1800 degrees lower than the flame. The production of smoke and soot, that is, of unconsumed fuel, is the necessary result of these conditions, and hence a great reduction in efficiency. This subject has been actively worked over, and an almost endless variety of furnaces and systems has been proposed. The true method of solving the problem appears to have been first discovered by Frederick Siemens (Dresden), and for a number of years he has been engaged in developing the practical applications of his researches.†

The previous methods of firing were based upon the idea of bringing the flame into direct contact with the surface to be heated, but since about 1879 the method of construction, especially in glass furnaces, open hearth steel furnaces, smelting furnaces, etc., has been to utilize the radiaut heat from the arched roof of the furnace, and to economize the heat of the escaping gases in the regenerator. An economy in the use of the heat of as much as 80 to 90 per cent. has resulted. This has been followed by a still more marked separation between the two principal periods of combustion, and by the application to steaui generators where such a high economy cannot be expected, although a saving of about 25 per cent. has been shown in actual practice.‡

It is therefore strongly recommended to use such furnace constructions as shall not bring the direct flame of the fire in contact with the heating surface of the boiler, but to use radiating surfaces and also to conduct the highly heated but fully burned gases through the flues, both of which can be accomplished in various ways.§

The application of the principle to stationary boilers is not difficult, and experiments have shown that it uiay also be successfully applied both to marine and locomotive boilers. In all cases it has been demoustrated that the fuel should be burned in a combustion chamber lined with refractory material, and the discharge of the heated gases retarded by a fire brick bridge or screen before coming in contact with the boiler. It will be seen from the preceding, that by using the Siemens' method instead of the older methrod of burning the fuel directly in the boiler, an economy of about 25 per ceut. can be obtained, and this fact should always be kept in mind in future designs.

3. *Heating Surface.*—The third point concerns not so much a variation in construction, as it does the lack of knowledge of the fundamental principles, this subject having beeu much less fully investigated than other portions. Recent investigations show conclusively that the axiom that the heating surface is a magnitude proportional to the desired efficiency of the boiler, cannot be sustained. It is evident that there must be a very considerable difference in the heating value of portions of the surface which are at greatly different distances from the fire. A very high temperature of the gases at the beginning, and a comparatively low temperature near the end, must mean a rapid formation of steam uear the fire and a weak production over

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* When the rivets are made of no better material than the plates, the reduction for single riveting is about 53 per cent., and for double riveting about 41 per cent. Triple riveting, as shown in Fig. 155, is too expensive to come into general use.

† The following list will serve for those who oesire to refer to the original and fundamental publications upon this subject:—Friedrich Siemens, Heizver fohren mit freier Flammeut faltung, Berlin, Spriuger, 1882; Siemens' Regenerativöfen, Dresden, Ramming, 1854; Vortrag von Friedrich Siemens über Ofenbetrieb mit ausschliesslicher Benutzung der strahlenden Wärme der Flamme, Gesundheitsingenieur, 1884; Vortrag von demselben über eiu neues Verbreunungs-und Heiz-system, Busch, Journ. f. Gasbeleuchtung, etc., 1885; Vortrag von demselben in der Ges. Isis in Dresden über die Dissociation der Verbrennungsprodukte, Dresden, Blochmann, 1886; Vortrag von demselben im Sächs. Ing. u. Archit Verein über die Verhütung des Schornsteinrauches, Civ. Ing. Bd., 32, Heft 5, 1886; Vortrag vom demselben im Bez Ver. D. Ing. in Leipzig am 8 Dez. 1886 über den Verbrennungsprozess, 2 Aufl, Berlin, Springer, 1887; Vortrag von demselben, gehalten in Hamburg im Ver. D. Gas-und Wasser fachmänner über Regenerativ—Gasbrenner, etc., Dresden, Ramming, 1887; Ueber die Vortheile der Anwendung hocherhitzter Luft für die Verbrennung, etc. 2 Aufl., Berlin, Springer, 1887.

‡ For example, a test by K. H. Kühne & Co., of Dresden Löbtau on Feb. 16, 1884, showed a gain of 26 per cent. due to the substitution of a Siemens furnace for one of the usual kind; the conditions of draft and cleanuess of flues being alike in both cases.

§ Two methods have been described by Dr. Siemens, both of which have been applied by him to flue boilers. In the first, the combustion of the fuel takes place upon a grate in a combustion chamber which is directly over the grate. A bridge wall of fire brick is placed about half the length of the grate further back, and beyond this are two ring shaped screens of fire brick, which are so placed as to direct the products of combustion toward the axis of the boiler flue; after passing through the flue the gases return about the outside of the shell and are then sufficiently cooled to be permitted to pass over the portions of the shell unprotected by water on the way to the chimney. In the second method the fuel is burned to gas in a gas producer separately constructed from the boiler, and the gas mixed with heated air and thus delivered to the boiler flue, where it follows the same course as in the first case.

distant portions of the surface. It has been shown that in some instances the heating surface of one and the same boiler may be reduced one-half without causing any reduction in the steam production. The usual method of proportioning the heating surface in all kinds of boilers appears to be based upon previous results with similar forms, and hence is often one-sided and unsuited for systematic investigation. A new departure in the discussion of this important subject has been made by the chief director and engineer of the Swedish railways, Mr. F. Almgren. He has made the subject of the proportioning of heating surface the object of a series of experiments extending over a number of years, and has placed the matter upon a much higher plane of investigation than heretofore. The practical results are of much importance, aud in advance of the publication of the whole the following general discussion has kindly been placed in the author's hands by Mr. Almgren, and is here given in his own words.*

PRACTICAL RESEARCHES UPON LOCOMOTIVE BOILERS WITH SMALL TUBES.

BY F. ALMGREN.

"According to the investigations of Geoffroy, as given by Couche,† the amount of steam produced by tubular heating surface depends upon the volume of heated gases passing through the tubes per hour. The heating surface under experiment consisted of portions 0.9 metre long of tubes, the total length of which was 3.6 metres long each.

"I have found that the volume of gases may be considered as a function of the length l of the tubes, the latter being considered as a variable, according to the following general expression; in which i is the number of tubes, and L the number of heat units given off by each tube per second.

$$\frac{a}{i\,L} = 1 + \frac{b}{l} \quad \ldots\ldots\ldots\ldots \quad (398)$$

"In this formula a and b are constants which depend upon the mean temperature Te of the gases, upon the temperature δ of the water, and upon the weight $i\,G$ of the gases passing through the tubes per second.

"As the result of a series of experiments I have found these constants as follows:

$$\left.\begin{aligned} a &= 0.357\; i\,G\,(T_e - \delta) \\ b &= 7.15\;\; G^{0.217} \end{aligned}\right\} \quad \ldots\ldots \quad (399)$$

in which G is the mean weight of gases or products of combustion for one tube. For the number of heat uuits L given off by a single tube of a set, the following expression is given:

$$L = \frac{0.357\; G\;(T_e - \delta)}{1 + \frac{7.15}{l}\; G^{0.217}} \quad \ldots\ldots\ldots \quad (400)$$

"In order to show the utility of these formulæ, a table is here given of the results of twenty-one experiments upon a locomotive boiler, the walls of the fire box having been made non-conducting by means of brick-work. A second table is also given to show the great advantages resulting from these experiments. The quantities given in the table are as follows:

i = the number of tubes.
G = the weight iu kilogrammes of the products of combustion passing through each tube per second.
$T_r - \delta$ = the difference between the temperature of the smoke and the water, the former being measured in the smoke box.
$T_e - \delta$ = the difference between the mean temperature of the gases in the tubes and the water $= T_r - \delta + \frac{L}{0.24\; G}$.
L_e = the mean value of L determined by experiment.
L_b = the value of L determined by formula (400).

TABLE I.

Locomotive boiler: pressure 4 atmospheres, tubes of brass, 2.934 metres long, 42 mm. diameter, somewhat scaled.

No.	i	G	$T_r - \delta$	$T_e - \delta$	L_e	L_b
1	110	0.00713	210° C	901° C	1.184	1.248
2		0.00601	185 "	916 "	1.035	1.090
3		0.00733	222 "	969 "	1.304	1.370
4		0.00827	230 "	1009 "	1.531	1.570
5		0.00900	235 "	1000 "	1.648	1.700
6	55	0.01795	275 "	1067 "	3.360	3.330
7		0.01871	285 "	1091 "	3.600	3.520
8		0.01832	278 "	1115 "	3.660	3.530
9		0.01479	290 "	1421?	4.000?	3.750
10		0.01514	240 "	1221° C	3.510	3.290
11		0.01303	255 "	1312 "	3.300	3.080
12		0.01091	235 "	1328 "	2.860	2.700
13	88	0.00466	90 "	682 "	0.650	0.646
14		0.00448	95 "	724 "	0.670	0.660
15		0.00405	95 "	781 "	0.660	0.652
16		0.00360	95 "	709 "	0.530	0.530
17	110	0.00586	75 "	462 "	0.542	0.534
18		0.00529	70 "	368 "	0.376	0.388
19		0.00640	83 "	466 "	0.591	0.586
20		0.00715	95 "	522 "	0.734	0.721
21		0.00668	90 "	529 "	0.695	0.686

"*Remarks.*—Between each set of experiments the boiler was blown off and both boiler and tubes cleaned. The 110 tubes of the fourth set were only partially the same as those of the first set. In the ninth experiment one of the cast iron plugs which were used to close the tubes not in use was melted out.

"The correspondence between the experimental value L_e and the calculated value L_b is very striking. A formula for special practical cases has also been deduced, being adapted for the special number of tubes as given in the preceding table, and without the variation in G and T_e which occur in single experimental cases.

"Equation (400) shows that for a given length l for the tubes, the production of steam is nearly proportional to the weight of gases flowing through them, and that it also increases nearly in direct proportion to the quantity of heat $0.24\; G\; (T_e - \delta)$. This indicates that for a constant blast opening, the amount of steam produced by the heating surface of the tubes will almost exactly equal the amount of steam passing through the blast nozzle, that is the amount of steam used by the engine. If it is also remembered that an increase of draft also increases the temperature of combustion, it will be seen that the tubes and the blast nozzle of a locomotive boiler bear a most intimate relation to each other, and that great and sudden variations in the production of steam occur almost hourly.

"Now the researches of Geoffroy show that the walls of the fire box have a much less favorable action. In this portion of the heating surface the production of steam responds much more slowly to variations in the draft. The larger the fire box, the more marked is its action in this respect, and consequently the less effective will be the blast. Equation (400) shows that for a given tube length, the production of steam of each tube increases with the increase of the draft, and hence the number of tubes and consequently the weight of the boiler may be kept at a determinate minimum, which depends upon the permissible force of blast and limit of size of grate. The formula also shows that with a strong draft and high temperature even the latter portion of long tubes is of excellent steaming value.

"Since also a given amount of tube heating surface is lighter and cheaper than the same amount of fire box surface, and since by the reduction of the latter the products of combustion will be cooled less and so enter the tubes at a higher temperature, it will readily be seen that a material advantage can be gained by removing that portion of the fire box surface which is of the least value (that is, the side walls), and adding an equivalent proportion by lengthening the tubes. As an example may be cited the case of a locomotive boiler with 125 tubes, 3 metres long and 45 millimeters inside diameter in which a reduction of 7 square metres of fire box surface was made up by an increase in length of tubes which gave 14 square metres of surface, the force of draft being 40 millimetres water pressure. This change removed the expensive stayed fire box walls, which were replaced by a fire brick lining, and the reduction in weight and cost amounted to about 700 kilogrammes and 1500 marks.

* It has been thought best to leave the formulæ and tables in the metric system, and temperatures in the Centigrade thermometer, also keeping the French thermal units, and thus retaining the discussion in Mr. Almgren's own figures, as the principles are equally well shown, and the uuity of this preliminary presentation thereby retained.—*Trans.*

† Voie, matériel et exploitatiou des Chemins de fer, Tome III.

"The latest boilers for the Swedish State Railways have been constructed with the preceding principles in view as shown in Fig. 1123. The fire brick lining of the fire box is shown at *a*, *a*,

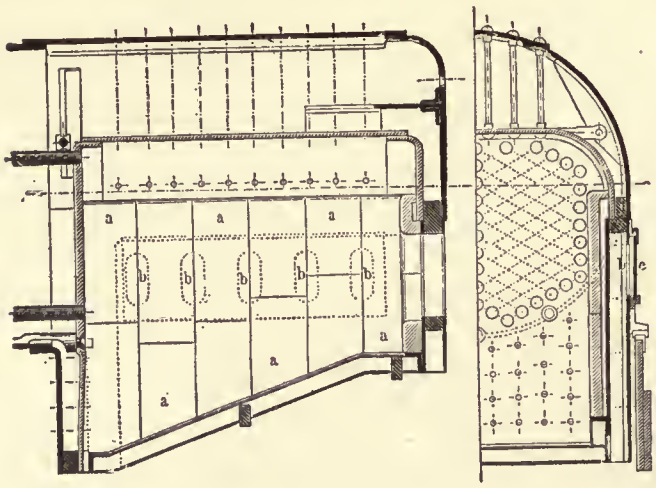

FIG. 1123.

while at *b*, *b*, are openings for the admission of air, which can be closed by sliding dampers *c*. A year's experience with this construction has given satisfaction, as the following table shows. It will be seen that the new form of boiler produced the same amount of steam per unit of heating surface as the old form, the force of the draft and the temperature in the smoke box being nearly the same in both instances.

"The external length of this fire box is 1.485 metres, and the internal width is 1 metre. The diameter of the shell is 1.103 metres, with 144 tubes, 45 millimetres inside diameter, and the fire brick lining is 74 millimetres thick.

"TABLE II.

A. Dimensions.

Boiler.	Tubes.			Heating Surface.		
	Length.	Diameter.	No.	Tubes.	Fire Box.	Ratio.
Old Style.	3.111 m.	46 mm.	184	77.28 sq. m.	7.82 sq. m.	9.9
New "	3.305 m.	46 mm.	102	50.83 " "	2.19 " "	23.1

B Performance.

Evaporation per sq. meter per hour	Draft Pressure in millimetres of water.		Temperature in Smoke Box.	
	Old Style	New style	Old Style.	New Style.
24 *kg.*	20 mm.	24 mm.	310° *C*	315° *C*
30 "	30 "	35 "	340°	340°
37–45 "	40–50 "	50–60 "	410°	395°
55 "	80 "	90 "	470°	470°

"A patent has been applied for by Herm. Von Storckenfeldt for the construction shown in Fig. 1123, and made from my calculations and directions."

The preceding brief description shows the nature and importance of Almgren's researches and appears to form a starting point for a change in methods of locomotive boiler construction. Further investigations may develop a theoretical foundation for this empirical formula. Especially interesting is the conformity of Almgren's observations with the above described results of Siemen's. We also see the previous remarks upon the subject of economy of material confirmed in the advantages resulting from the replacing of flat stayed and riveted surfaces by cylindrical welded tubes. A corresponding gain would be attained were it possible to produce a shell free from riveted seams.

4. *Artificial Draft.*—The use of forced draft has been common for many years in locomotives and portable engines, and by this means a much greater quantity of steam produced from a unit of heating surface than with natural draft. More recently forced draft has been applied to marine boilers, the blast generally being produced by fan blowers. Especially has this been necessary in the case of torpedo boats, in which the highest speed is demanded. By the use of multiple expansion engines operated by greatly increased steam pressure speeds of 18 to 20 knots are attained without an excessive increase in the consumption of fuel. This, however, involves a much greater increase in the steaming capacity of the boilers in proportion to their weight, and this result is accomplished by the use of artificial draft.

This has been discussed very completely in a paper presented before the Royal United Service Institution, by Naval Engineer H. J. Oram, upon the subject of the motive power of modern war ships. The large boilers of the English war ships "Blenheim" and "Blake" are 15 feet in diameter and 18 feet long, with four furnaces at each end These are worked with closed ash pit, and an air pressure of two inches of water, and at a steam pressure of 150 to 165 pounds, the engines indicating 3,350 horse power. The air pressure of two inches is ample for the desired rate of combustion, and by reference to the preceding table it will be seen that it is no greater than has long been common in locomotive practice. The combustion is more complete under this pressure than with natural draft, being more uniform and producing less smoke. It may be remarked that the efficiency of the boilers may also be increased by proper heating of the feed water and by use of the double distilling apparatus. The use of forced draft also makes it practicable to cool and ventilate the stokeholds.

The latest examples of construction, of American design, are made to work at pressures as high as 250 pounds per square inch, with boiler shells 16 to 17 feet in diameter. Mr. Oram considers that there is a limit to increase in this respect due to the increase in weight beyond practical limits, both of the boilers and of the engines.

It is worthy of note that in the recent express steamers of the French "Societè des Messageries Maritimes" the use of shell boilers has been abandoned, and sectional boilers of the Belleville type introduced. The increase in speed also appears to have its limits, but the advantages of forced draft, however, as regards the reduction in size and weight of the boiler, should at least lead to its introduction in the future for stationary practice.

Taking into consideration all the points of the preceding discussion, it appears that an application of them to practical boiler construction should result in an economy both of construction and of operation of 25 to 33 per cent. with entire safety.

§ 363.

RESERVOIRS FOR AIR AND GAS.

In the use of compressed air now so general in mining and tunneling operations, cylindrical reservoirs similar to steam boilers are used In tunnel construction, portable reservoirs are sometimes found mounted upon tram locomotives, the engines of which are operated by the compressed air instead of steam. Compressed air locomotives have only been used to a small extent, however, for general tram service. The so-called pneumatic method of sinking shafts and construction piers involves the use of air reservoirs. In this case the air reservoir is the caisson within which the work is carried on, the water being kept out by the air pressure, and the workmen entering and leaving by an air lock chamber with a double system of doors.

In the case of power transmission in cities by means of compressed air, the entire system of piping is included in the reservoir capacity. Negative reservoirs for mingled air and steam are found in the case of condensers for steam engines. These are usually made of cast iron and are from one to two times the capacity of the steam cylinder. The regular removal of the contents by the air pump at each stroke of the engine renders a larger capacity unnecessary. In some cases the flow of spring has been increased by fitting a tight cover over the well above the water level when the exhaustion of the air causes an increased flow from the underground sources. The vacuum system of power distribution, as used in Paris and London, involves the use of negative reservoirs similar to cylindrical boilers. An important application of vacuum for air and vapor of water is found in the vacuum pans used in sugar refineries. These pans are made in the spherical form, already referred to as most economical of material, the motive in this instance being the high price of copper, of which they are constructed. Gas holders for illuminating gas are reservoirs intended only for very low pressures, the strength of the walls being most

important in the matter of tightness against leakage. These holders are composed of two principal parts, the holder proper, or so-called "bell," often made telescopic, and the tank or reservoir filled with water which acts as a liquid packing; the bell in this case acts as a piston (compare Fig. 948). Similar reservoirs are used in laboratories and chemical works for many kinds of gases. For very large gas holders, in which the internal pressure of the gas is insufficient to sustain the weight, the roof of the holder must be strengthened by internal trussing. Until now the gas holder has had no definite place in construction, but it will be seen from what has already been said, that it, together with various other kinds of reservoirs, belong properly to machine construction, not only because of their character but also because of their intimate connection with the entire subject of mechanical engineering.

§ 364.

OTHER FORMS OF STORAGE RESERVOIRS.

The construction of reservoirs for water has been a most important subject from the earliest times down to the present, many of these being of great extent, although, as has already been said, these have until now been considered rather as belonging to the domain of building construction than to machine construction. To these must also be added the subterranean reservoirs in mines, from the small pump to those of large extent and capacity. Other examples are found in the negative reservoirs which exist in low-lying tracts of land, such as are found in Northern Germany and Holland, intersected by canals. A notable example in Holland is the valley formed by the drainage of the Harlem Lake, the water having been pumped by steam engines out to the level of the sea and the latter kept out by dykes.

Reservoirs for agricultural purposes are often formed by systems of canals, as in Lombardy and in the south of France, where this important subject of irrigation has proved of the greatest benefit to the country. The nature of such systems, considered as reservoirs, is more apparent when the magnitude of the work involves the construction of artificial lakes for water storage. Ancient examples of such storage reservoirs are found in Lake Moeris, of ancient Egypt, and Lake Nitocris, of Babylon, as well as the existing Lake Maineri, in Ceylon, and many others. The mechanical nature of such constructions is more apparent when the reservoir is made by building a dam across a gorge or valley, with weirs to permit the periodical release of the water, the analogy to ratchet action being quite clear.

Finally, another natural form of stored power may be mentioned, one which has not to the writer's knowledge been considered in this light before, yet which possesses the greatest significance in the climatic economy of nature. This is the glacier. The vapor of water, raised from the level of the sea by the heat of the sun, collects in the form of snow about the highest mountain peaks. In the upper valleys the snow packs together, and under gradual pressure forms the glacier ice, and slowly the glacier flows down into the lower and warmer valleys and melts away. The mass of ice, consisting of hundreds of millions of cubic feet, forms a reservoir of stored power, flowing in an irresistible stream of almost uniform strength from the highest snow field to the lower valley. All the actions involved are of a physical and mechanical nature. Taken as a whole the glacier forms a reservoir system of the fifth order: evaporation of the water from the sea by the heat of the sun, transformation of vapor into snow, fusion of the snow into a mass, conversion by pressure into glacier ice, and melting of the ice partly by the friction on its bed and partly by the heat of the sun.

CHAPTER XXVI.

RATCHETS FOR PRESSURE ORGANS, OR VALVES.

§ 365.

THE TWO DIVISIONS OF VALVES.

THE application of the ratchet principle to pressure organs, that is, the periodical interruption of its motion, closely resembles the same principle applied to constructions formed of rigid elements; the principal difference being that the pressure organ is very easily separable into small portions. It might also be remarked that the pressure organ is always confined in a conductor of some kind, but this feature also belongs to some forms of rigid constructions, such as bearings, guides and the like.

Ratchets for pressure organs may be divided into two principal classes, namely, those intended to check the motion in only one direction, and those which check in both directions. The name given to ratchets for pressure organs is valves.*

The difference between the two classes is shown in Fig. 1124. In the form shown at *a*, the pressure organ is checked by the flap valve *b*, from moving in the direction of the arrow at *I*,

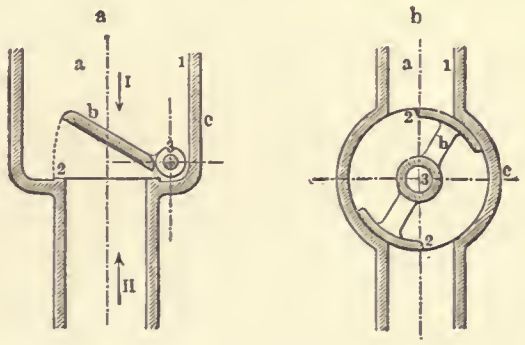

FIG. 1124.

but not against motion in the direction of the arrow at *II*. In the form shown at *b*, the flow is checked in both directions. There is here a close analogy to the two kinds of rigid ratchets, as will be seen in Fig. 1125, which is here reproduced from § 235. The valve *b*, in Fig. 1124 *a*, corresponds to the pawl in a

Running Ratchet,

and the valve in case *b*, to a

Standing Ratchet

for the pressure organ *a*. The difference in construction will also be seen to depend upon the fact that in form *a*, the valve lifts from its seat during the passage of the pressure organ, while in form *b*, the valve slides upon the seat. This permits

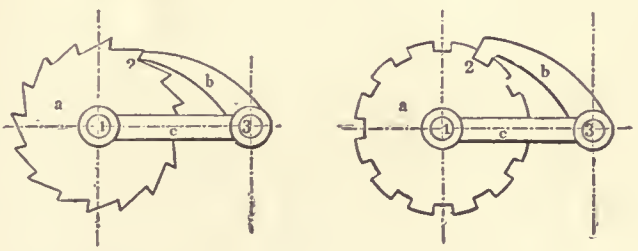

FIG. 1125.

another classification into:

a, Lift valves;
b, Slide valves.

The variety of forms in which valves are constructed is fully equal to that of rachets for rigid elements, as shown in Chapter XVIII., and there is a close analogy existing between the two groups, with one important exception, namely, that the form of rigid rachet which has a tension pawl, has no counterpart among the valves. This exception naturally follows from the fact that the member to be checked is always subject only to compression.

There is also an analogy between the numerous forms of valves and the two classes of toothed and friction rachets, as has already been mentioned in § 319, valves which have but a slight opening, acting like friction rachets (compare § 340), and those with full opening and entire closing like toothed rachets. This circumstance, however, reduces the number of sub-divisions into which valves may be classified, so that the principal basis upon which a classification is made depends upon the character of the motion of the valve, and thence upon the necessary variation in form. This basis of classification has not been used in the case of rigid rachets, the divisions there having been made upon the more practical idea of the variation in form only. We have in rigid ratchets the two forms of pawls, one of which moves about an axis 3 within a finite distance, as in Fig. 1124; and the other in which the axis is removed to an infinite distance. In the first case, every point of the pawl (or valve) moves in circular arc about the axis, while in the second, all points move in straight lines and equally far. In rigid systems these correspond to link pawls and bolt pawls.

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*The author calls attention to the derivation of the German word "ventile," from the mediæval name for valves used for checking wind in church organs. The English word "valve" from the Latin "vulva," meaning hinged doors, is therefore broader and more general — *Trans.*

In addition to the circular and rectilinear motion of valves, there is a third variety possible, although but little used in practice, viz.: those having a spiral motion. We therefore have three sub-divisions of the two main classes of valves, according as the movement is circular, rectilinear, or spiral.

Lift valves may be

1. Hinged or Flap Valves.
2. Disk, Cone, or Ball Valves.
3. Spiral Lift Valves.

Slide valves may be

1. Rotary Valves or Cocks.
2. Rectilinear Slide Valves.
3. Spiral Moving Slide Valves.

Although this sub-classification is not exhaustive, yet it gives a convenient and practical arrangement, the few special forms being placed in the group they most nearly resemble.

A. LIFT VALVES.

§ 366.

HINGED OR FLAP VALVES.

Flap valves are most generally applicable to piston pumps, which, as we have already seen, form fluid escapements, see § 319. Their tightness is often attained by the use of some elastic material, such as leather, rubber, etc., but very generally the joint is made between metallic surfaces, especially when no small hard particles are likely to be found in the passing fluid. It is always difficult to keep the loss due to shock within small limits, this loss being especially marked with flap valves, and indeed in all liquid ratchet systems the loss from this cause is by no means unimportant.

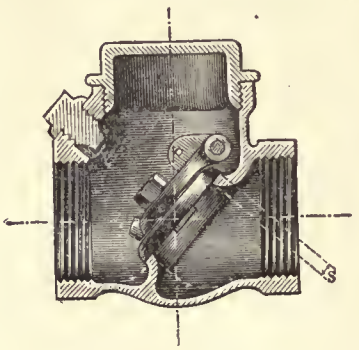

FIG. 1126.

A flap valve with metal seat, which is so constructed as to offer as little obstruction as possible to the flow of liquid, is shown in Fig. 1126*. This is tapped out for the standard pipe thread system described in § 342, the cap gives access to the valve, the screw plug limits the amount of lift, and a flexible connection between the disk and the hinge enables the former to obtain a fair bearing on its seat. The freedom from shock would be somewhat less if the bottom of the case conformed to the shape indicated by the dotted lines.

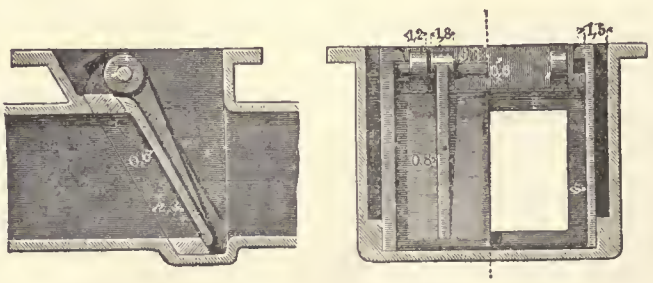

FIG. 1127.

Another form of straight-way flap valve is shown in Fig. 1127. Both valve and seat are made of bronze, the seat being secured in place by two wrought iron keys. The case is closed by a lid shown removed in the illustration. The axis of the valve is made to permit a slight degree of lateral play in order to permit the best bearing on the seat to be obtained. Valves of this sort are used on air pumps for steam engines and for vacuum pans.

The width of bearing s of the valve on its seat is given by the following formula, in which D is the clear opening through the valve.

$$s = \sqrt{D} + 0.16'' \quad \ldots\ldots\ldots\ldots \quad (401)$$

For round valves D is the diameter of the opening; for rectangular openings it is taken as the smaller side of the rectangle.

The blow with which a valve strikes the seat increases in force with the amount of lift (compare § 368), and as the lift depends upon the actual size of the valve, this objectionable feature is reduced by using several valves of smaller size instead of a single large one.

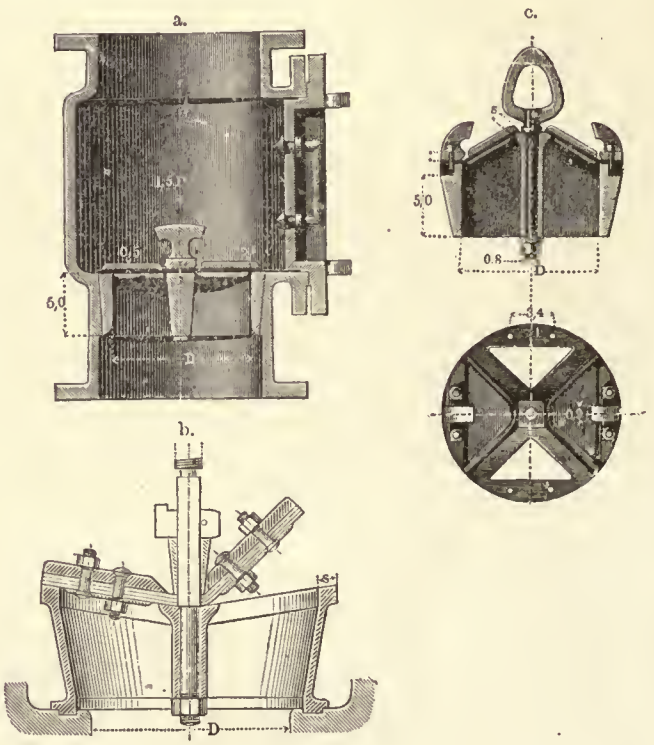

FIG. 1128.

A double flap valve and valve chamber designed for a mine shaft pump, is shown in Fig. 1128*a*. The flaps are formed of pieces of leather between plates of iron, secured either by screws or by rivets. The door by which access is obtained is curved to the shape of the valve chamber in order to avoid excessive dead space, and so reduce the shock, and is supported upon hinges. The stops are so placed that the valves open to an angle of 60°.

Another design for a double flap valve is shown in Fig. 1128*b*, this also being for a shaft pump.† In this instance the valves are formed of three thicknesses of leather. At *c* is shown a quadruple valve. The proportions given are all based upon the unit s, as given by formula (401).

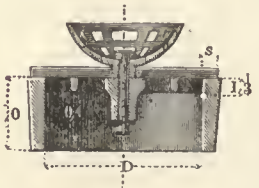

FIG. 1129.

Fig. 1129 shows a circular valve of rubber, this form being much used for air pumps for steam engines. The valve lifts approximately in a circular path, forming a cup, the limit of which is the shape of the guard. On account of the flexibility of the rubber, the bearing of the seat is reinforced by a grating, and the rubber is from ⅞ to 1⅛ inch in thickness. These values are now made also of vulcanized fibre, in which case the thickness need be only about one-third that of rubber disks of the same diameter.‡

Quite similar in principle to the above disk valve, is the leather rolling valve, Fig. 1130*a*, used for water wheel gates, the principal difference being that the bending of the valve takes place at the edge of the valve, as shown in the illustration.

*Pratt's Straight-way Check Valve.

†See Riebler, Indikator versuche an Pumpen und Wasserhaltungs maschinen, p. 34. Munich, 1881.

‡Made by the Vulcanized Fibre Company of New York.

The same principle is ingeniously used in the hanging weir of Caméré,* Fig. 1130*b*. The valve consists of a series of strips of wood, each really forming a separate valve, these being connected and operated by chain links of bronze as indicated in the sketch.

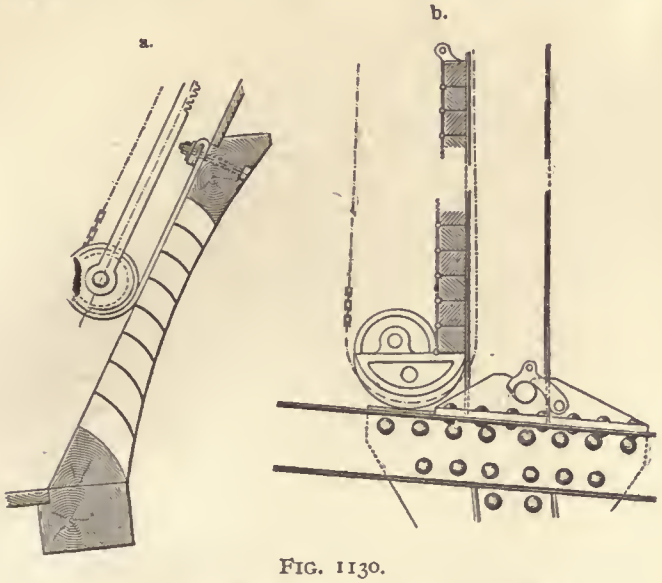

FIG. 1130.

An excellent installation is seen at the sluice gates at Geneva (Passerelle de la machine), where forty such gates are used to dam the right arm of the Rhone. The gates are rolled up by the chains shown, these being connected to suitable windlasses. When a whole section is to be thrown entirely open the supporting posts are also tipped back into the horizontal position, these being jointed at the bottom as shown, and this operation being effected by another chain gearing. Each gate is 3 ft. 8 inches wide; the sets of connecting links are 27½ inches apart, the number of strips is 39, each being about 3 inches wide, the uppermost being 2⅜ inches thick, and bottom one 3⅛ inches.

The weir system at Geneva, of which the above forms only a small portion of the entire work, was completed in 1889, as an intercantonal system to control the level of the lake of Geneva and maintain it between the limits of 1.30 and 1.90 metres (4 ft. 3¼ in. and 6 ft. 2¾ in.) of that of the Rhone. During the year 1888, when the system was not entirely completed, the difference fell to 1.95 metres (6 ft. 4¾ in.) in the drought of June of that year. Between October and May the entire series of gates was kept closed.

§ 367.

ROUND SELF-ACTING VALVES.

Lift valves for small openings are frequently made of conical or spherical form, and in Fig. 1131 two forms are shown which are intended for feed pumps.

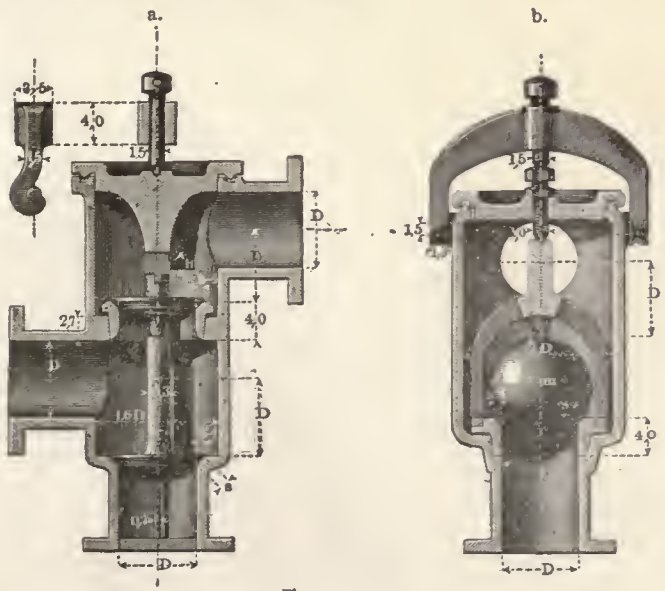

FIG. 1131.

* Chief Engineer of "*Ponts et Chaussées*," of France. The subject of weirs and movable dams has been very skillfully worked out by French engineers.

At *a* is shown a pair of conical valves. The upper valve and seat are made of bronze to avoid rust. The lower one, which is the suction valve, has an iron seat. If it is desired to provide a bronze seat for both valves they may both be made the same size and bevel. The width of bearing, *s*, may be made as in formula (401). If the horizontal projection of the seat is made

$$s_1 = s - 0.16'' = \sqrt{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (402)$$

the smaller valve will have a sharper bevel than the larger one. In designing the valve chamber, it is important to proportion the space over the valves so that the return flow of water shall be high enough over the valves to insure their closing, as it is possible for the return flow to get under the valves and hold them up from closing.† The valves here shown are made without any packing material.

At Fig. 1131*b* is shown a ball valve. In this the width *s* of the seat, and also its projection s_1 are the same as in the preceding. The diameter of the ball is found by drawing lines at right angles to the bevel of the seat from the middle of its width, the intersection of the lines giving the centre of the ball.‡ The high position of the outlet opening is necessary in order to maintain a proper lift to the valve and keep the seat in good condition.

In order that the opening through the valve shall be equal to that of the pipe the lift, *h*, of the valve must equal ¼ *D*. (See § 369).

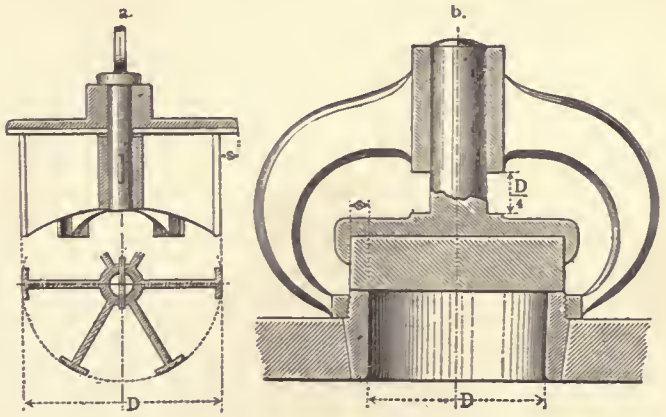

FIG. 1132.

Disk valves are often made with soft packing upon the seat, two examples being given in Fig. 1132. That shown at *a* is a valve for a mine pump, packed with leather. The ribs are shaped so as to form a cylindrical guide for the valve, this construction being also frequently adopted for conical valves. At *b* is a disk valve with rubber packing, similar valves being used on many of the Gaskill pumping engines; all the metallic parts are made of bronze.§ In many instances disk valves are made in the form of a ring, the seat being in two positions, the bearing being on both the inner and the outer edge of the ring.

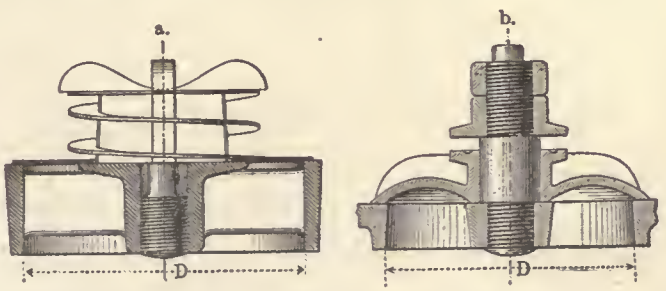

FIG. 1133.

Fig. 1133*a* shows the valve for the air pump of a Corliss engine at Creuzot. In this case the valve is made of a hard material instead of a soft one. The seat is made as usual, and the valve is a ring of phosphor bronze, held down to the seat by a strong flat helical spring. The form shown at *b* is another style of ring valve much used in the air pumps of English marine engines.

† See Zeitschr. Deutscher Ingenieure, 1886, p. 97.
‡ See Uhland, Prakt. Maschinen Konstrukteur, 1870, p. 83. Plate 24.
§ See Engineering and Mining Journal, April, 1886, p. 285.

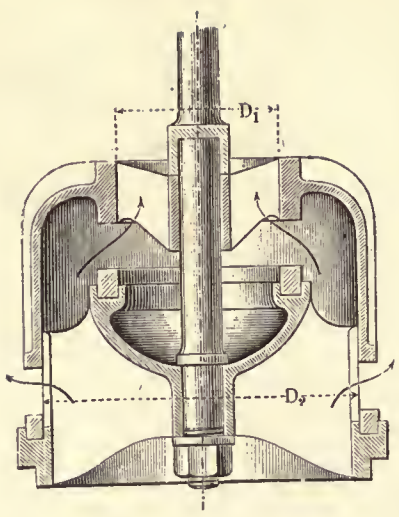

FIG. 1134.

Fig. 1134 is a so-called "bell" valve, used in mine pumps. Here the two seats for the ring of the valve are in different planes. The seats are packed with oak with the end grain up. The outlet in this form is arouud both the inner and outer bearings, in which respect it differs from Fig. 1133*b*. The lift p, which is required to give au area of $\frac{\pi}{4} D^2$ is somewhat less than before, being equal to $\frac{1}{4} \frac{D^2}{D_1 + D_2}$

The necessity for limiting the lift of valves in pumping machinery has led to the use of a large number of small valves in the same valve chamber iu order to obtain the required area with small lift.

A distiuction may be made between two methods of arranging such valves. The first method consists in arranging a number of similar round disk valves each over its own opening in a plate. An example of this is secn in Fig. 1016, in which rubber valves similar to Fig. 1132*b* are arranged in rows. The phosphor bronze valve, Fig. 1133*a*, is also used iu this manner, 38 being placed on the suction side, and 27 ou the discharge side of the air pump.

In a round valve chamber the arrangement of the valves is more difficult, both as to the placing of the valves and to provide guides to control their liftiug and seating.

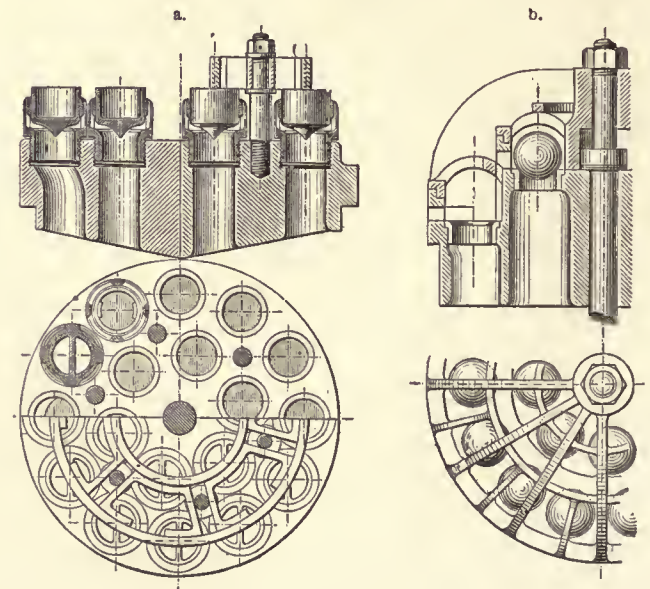

FIG. 1135.

Fig. 1135*a* shows a set of 19 valves as used in the Heidt shaft at Hermsdorf, and Fig. 1135*b* a set of 21 ball valves in the Joseph's snaft at Frohnsdorf.* These are both shown independently of the casing. This system has showu itself so advantageous that it has been extended until sets of several hundreds of ball valves, actiug as a single valve, have beeu put into use. Fig. 1135*b* shows oue feature which must always be taken into account, namely, the relation which the size of the valves and valve casiug bear to the water pipe. Iu this instance the diameters of the casing and pipe are 19⅝ in. and 7¼ iu., and the areas as 7.4 to 1.

The second method of arranging a uumber of valves is suggested by the bell shaped valve of Fig. 1134. In this case the stream which flows toward the centre is above the one which flows outward, thus providing sufficient room for the flow of the upper stream. This idea is also used in the arrangemeut shown in Fig. 1135*b*, the inner circle of balls being placed higher than the outer circle. By extending this idea of superposing the discharge openings of a number of valves we obtain a construction consisting of a number of ring valves, forming what may be called a set or cone of valves,† of which three different forms are shown in Fig. 1136. The form shown at *a* is used in the large pumping engine of the Scharley-Tiefban mine,‡ the pumps being 1 metre diameter (39.37 in.). This consists of a number of ring shaped valves of constantly diminishing diameter, constructed on the bell principle, the seat of each valve being on the one next below.

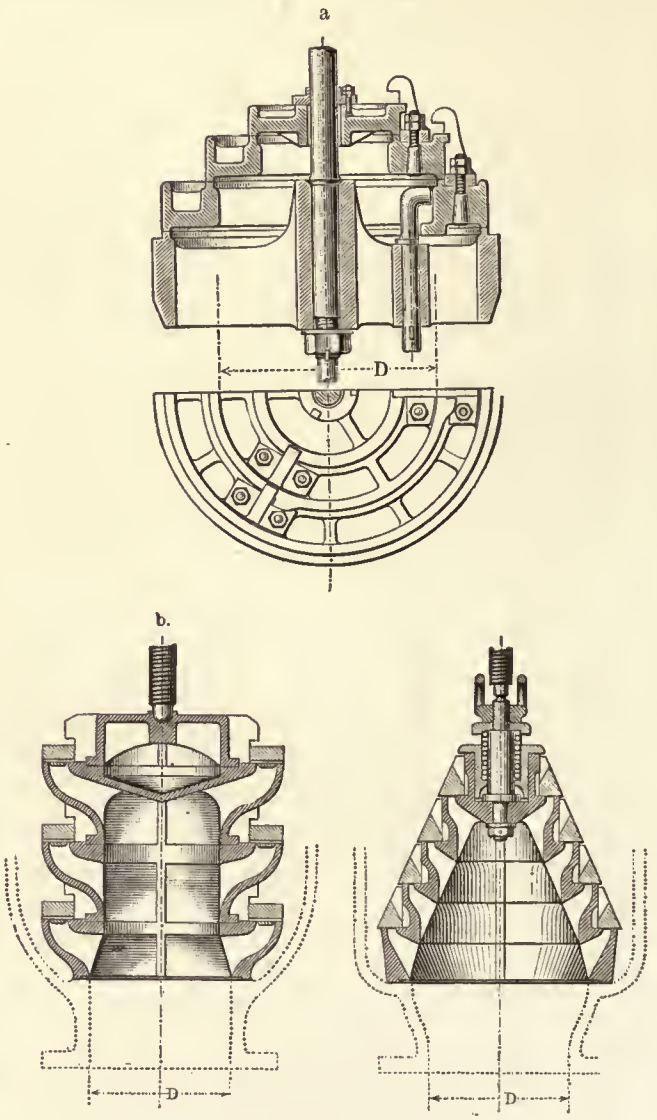

FIG. 1136.

The form at *b* is the design of Thometzek,§ and is very practical. The ring valves are all alike in size and form, each having its own seat, these being built up as high as may be required and held in place by a screw bolt through the lid of the valve casing.

The design *c* is that of the Humboldt Machine Work at Kalk. ‖ The ring shaped valves of bronze are slipped over the succession of seats which form a cone of stepped shape, also of bronze. These seats, as in the system of Thometzek, are separate, and are held together by a screw bolt ou top, with the difference, however, that each valve in lifting strikes against the next, the amount of lift increasing iu an arithmetical ratio from above downward, the uppermost valve being held down by a spring. In this last construction the ratio to D is somewhat smaller than iu form *b*. All of these designs are intended

* See Riedler, Indikator versuche, etc., p. 27, and plate 11.

† German "*Stufenventil*," French "*Etagenventile*."

‡ See Riedler, Indikator versuche, p. 21.

§ To the best of the author's knowledge Director Thometzek, of Bonn, was the first to use ring valves arranged in steps (1875), and his designs have been widely and successfully used in practice.

‖ A very good summary of such valves is found in an article by Engineer Waldästel, entitled, "Ueber Ringveutile für Pumpen und Gebläse," in Z D. Iugenieure, 1886, p. 935.

for water pumps, but an excellent form is designed by the Humboldt Machine Works for blowing engines also, the suction and discharge valves being concentrically arranged.*

§ 368.

Unbalanced Pressure on Lift Valves.

If we assume the joint of contact of a lift valve to he entirely tight and represent the projected area subjected to the pressure of the discharge column hy F_1, the area exposed on the underside being called F, we have at the instant of equilibrium of the two columns as the valve is ahout to lift, $p F = p_1 F_1$, in which p and p_1 are the pressures per unit of area on each side, and the weight of the valve is neglected or counterbalanced. From this we have

$$\frac{p - p_1}{p_1} = \frac{F_1 - F}{F},$$

or of the ratio $\frac{F_1}{F}$ is put $= a$:

$$\frac{p - p_1}{p_1} = a - 1 \quad \ldots\ldots\ldots (403)$$

The pressure $p - p_1$ is the unbalanced pressure on the valve, and the ratio $\frac{p - p_1}{p_1}$ is the ratio of unbalanced pressure.

Upon this question of unbalanced pressure much depends, and many calculations have heen made for various sorts of valves, the pressure tending to close the valve being much reduced in hell shaped valves, such as shown in Fig. 1134. Experimeutal researches, made upon pumps of various sizes, however, have shown that only a small excess of pressure is actually required.† At the same time the preceding formula shows that the question of the unbalanced pressure is hy no means a subject to he neglected.‡

As an instance of the effect of unbalanced pressure may he cited a bell shaped valve, 1 metre clear opening, in the shaft of the Bleyherg mine, of which the seats could not he kept down by their own weight, hut would adhere to the valve, rising and falling with it until secured hy some other means.

Riedler has observed the fact that iu arranging valves in a series in a cone as in Fig. 1136*a*, the uppermost valve which is subjected to the greatest excess of pressure according to (403), lifts first, and is followed by the others, the lowest rising last.

It appears that a thin film of water is retained between the bearing faces of valve and seat, which responds rapidly to the pressure of the lower column p_1, and thus tends to reduce the value given hy the ahove equatiou. If we first make the assumption that such a film exists and acts in the manner indicated, we have for two successive ring valves, arranged for example as in Fig. 1136*a*, the following stresses in the liquid. The weight of the valves, heginning from the top, is indicated by G_1 and G_2, and their projected areas by F_1 and F_2.

$$p' = p_1 + \frac{G_1}{F_1} \text{ and } p'' = p_1 + \frac{G_1 + G_2}{F_1 + F_2} \quad \ldots\ldots (404)$$

Now it appears by examinatiou of the weights and areas that under the circumstances $\frac{G_2}{F_2}$ is greater than $\frac{G_1}{F_1}$, which is then also true for the entire second memher of the value of p'' §, so that p' is the resistance which is overcome first. In the case of the Bleyherg mine F_2 is very much greater than F_1, and p'' hecomes less than p' which explains tne action of the valve seat.

The actual hehavior of the film of liquid between the surfaces of contact may not he so definite as indicated above, hut it approaches to it as an approximation. This is shown hy the very valuable researches made hy Prof. Robinson upon a valve acting under steam pressure.‖ In two extensive series of experiments he investigated the actual weight required to lift a valve under pressure. The results showed that the unbalanced pressure was much less than $p_1 - p$.

The experimental valve, shown in Fig. 1137, had an annular seat of 6 in. outside and 2½ inside diameter, and was subjected to a steam pressure p_1 above, and to the atmospheric pressure p below. In the following table p' indicates the pressure per square inch which would give the equivalent of the actual pressure P required to lift the valve, while a is the area and d the diameter of a circle for which $a(p_1 - p) = P$. This circle Rohinson calls the circle of equilihrium, and it is always smaller than the upper projection of the valve.

The valves under a and d are taken approximately at the nearest values. The unhalanced pressure can readily be determined from the table.

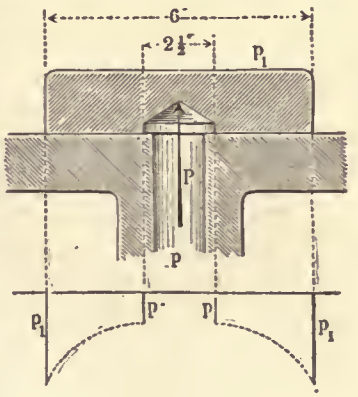

FIG. 1137.

$p_1 - p$ Pounds per Square Inch.	p' Pounds per Square Inch.	a Square Inches.	d Inches.	d' Inches.
5	8	5.6	2 6	2.53
10	17	5.8	2.7	2.85
15	26	6.0	2.8	2.92
20	36	6.2	2.8	3.02
25	46	6.4	2.9	3.09
30	57	6.6	2.9	3.14
35	69	6.8	2.9	3.19
40	81	7.0	3 0	3.22
45	95	7.3	3.0	3.25
50	112	7.8	3.1	3.27
55	129	8.2	3.2	3.29
60	150	8.7	3.3	3.31
65	172	9.2	3.4	3.33
70	198	9.8	3.5	3.34
75	230	10.5	3.7	3.35

If $p_1 - p = 45$ lhs. we have, since $d = 3$ in. $= ½$ 6 in. for the excess pressure, oue-fourth $p_1 - p$; for $p_1 - p = 75$ lbs. it is equal to 0.38 $(p_1 - p)$. The law of reduction of pressure between the surfaces from p_1 to p is not simple. The corresponding curve is convex towards the axis of abscissas, as shown in Fig. 1137. If it is desired to determine the mean pressure p_m we have from the tahle for $p_1 - p = 5$ the value $p_m = \frac{p_1 - p}{4.43}$; for $p_1 - p = 75$ it is $p_m = \frac{p_1 - p}{2.36}$. For a rough approximation we may put $p_m = ⅓ (p_1 - p)$. Prof. Robinson has deduced a theory from these experiments. He assumes that between the surfaces there exists hetween the pressure p_1 at the outer circumference to the pressure p, at the inner circumference, a gradual increase of pressure from p to p_1. Under the assumption that the fluid under consideration is incompressible he obtained by pure analysis the following equation for the value of d:

$$d' = 2r\sqrt{\frac{R}{r} - \frac{p}{p_1}\left(\frac{R}{r} - 1\right)} \quad \ldots\ldots\ldots (405)$$

in which R and r are the iuner and outer radii of the ring of the seat. The values of d' as ohtained from this equation are giveu in the fifth column of the tahle. They increase nearly as the experimental determinatious of d, but with Rohinsbn's assumption of an entirely elastic fluid they are 10 to 15 per cent. too great. Prohahly steam should he considered as midway hetween an elastic and a non-elastic fluid.

The deductions from Rohinson's experiments are hardly applicable to pump valves hecause the lifting of the valve by the action of the lower columu is effected by a varying pressure, while in the experiments p was uniform. If we accept Rohinson's theory we arrive in fact to what has been already stated, namely, that when the value of p increases hetween the surfaces until it reaches p_1, the pressure p_2 will he halanced, since in equation (405) for $p - p_1$ the value of $d' = 2r$, that is, the unbalanced pressure becomes zero. This also agrees with Riedler's indicator tests, since experiments with the indicator failed to show appreciahle unbalanced pressure.

* German Patent, No. 33,103.

† Reference is especia'ly made to the numerous and valuable investigations of Prof. Ried'er.

‡ See the comprehensive papers of Prof. C. Bach, in Zeitschr. D. Ing. for 1886. "Versuche zur Klarstellung der Bewegung Selbstthätiger Pumpenventile."

§ In the case of the arrangement shown in Fig 1136*a*, the ratio of weight and area for the three valves, proceeding from above downwards, is 50: 76: 85.

‖ See Trans. Am. Soc. M. E., Vol. IV, 1882–1883, p. 350.

These experiments appear to indicate that practically the unbalanced pressure cannot be great, and in most cases for self-acting valves it may be neglected. Prof. Robinsou's experiments and theory may serve to determine with considerable accuracy the pressures at which a safety valve begins to lift.

§ 369.

CLOSING PRESSURE OF SELF-ACTING VALVES.

As already shown, a self-acting valve opens whenever the pressure in the under column exceeds that above the valve. As soon as the direction of pressure is reversed the valve should close quickly. This is especially important, as Riedler has shown in the case of suction valves, since when the closing is delayed appreciably after the reversal of the pump piston, the moving column of water is checked with a sudden shock. For this reason the suction valves are given especial attention, as shown in the example already cited from Creuzot, in which there are 38 suction valves and only 27 discharge valves.

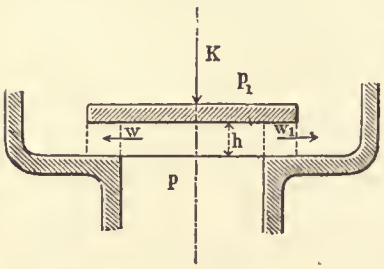

FIG. 1138.

In order that the lift shall not be too great and to insure prompt closing, the valve may be loaded with a definite pressure, K, obtained either from the weight of the valve, or by means of a spring, or by both. This question will here be examined. Referring to Fig. 1138, we have for the lifting pressure due to the under column:

$$p = p_1 + \frac{K}{F_1} = p_1 + q \quad \ldots\ldots\ldots (406)$$

in which $p - p_1 = q$ the closing pressure per unit of area. For a height h, and putting u = the circumference of the cylindrical space inclosing the valve, we have:

$$w_1 h u = Fv$$

w_1 being the velocity of flow at the outer edge of the valve, and v the velocity of flow in the under column, h being in feet. Now if w is the velocity at the inner edge of the valve we have

$$w_1 = w\sqrt{\frac{F}{F_1}}$$

that is:

$$w_1 = \frac{w}{\sqrt{a}}$$

But we also have

$$w = \sqrt{2gh'} = \sqrt{2g \times 2.3\,q}$$

(since the pressure per square inch is equal to $\frac{h'}{2.3}$) and hence:

$$w_1 = \sqrt{\frac{2g \times 2.3\,q}{a}}$$

Substituting, we get:

$$h u \sqrt{\frac{2g \times 2.3\,q}{a}} = Fv$$

Now it is desirable that w and w_1 should not be too great; that is, the ratio of $h u$ to F should be equal to, or less than, unity. If we put $h u = \beta F$, we have:

$$\beta \sqrt{\frac{2g \times 2.3\,q}{a}} = v$$

and, putting for g its value = 32.2, we get:

$$q = \frac{a\,v^2}{148.12\,\beta^2} \text{ or say } = \frac{a}{\beta^2} \cdot \frac{v^2}{150} \quad \ldots\ldots (407)$$

from this formula we get for:

$\beta =$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$q =$	$.006667\,a\,v^2$	$.01185\,a\,v^2$	$.02666\,a\,v^2$	$.1066\,a\,v^2$

in which v is at its maximum value when it equals the velocity of the pump piston. For purposes of numerical calculation we still require the value of a. Taking the width of bearing s, and projection in the case of conical valves s_1 from (401) and (402) we have:

Dia. D =	2 in.	4 in.	6 in.	8 in.	10 in.	12 in.	16 in.
Width of seat s =	0.44	0.56	0.64	0.72	0.80	0.84	0.96
Projection s_1 = . .	0.28	0.40	0.48	0.56	0.64	0.68	0.80
Cone valve a = .	1.65	1.44	1.36	1.31	1.27	1.24	1.21
Flat Valve a = . .	2.17	1.64	1.44	1.39	1.35	1.30	1.25

An example will show how the pressure of closing can be calculated:

Example 1.—For a conical valve whose smallest diameter $D = 4$ inches, and the greatest velocity v of the lower column is 6½ feet per second the area of inlet of valve $hu = F$, and $\beta = 1$, we have a pressure of $q = .006667 \times 1.44 \times (6.5)^2 = 0.4$ lbs. per sq. in. For the total pressure we have

$$K = \frac{\pi}{4}(4 + 2 \times 0.40)^2 \times 0.4 = 7.24 \text{ lbs.}$$

Example 2.—For a flat valve of the same dimensions we have $a = 1.64$ — whence $K = \frac{1.64}{1.44}\,7.24 = 8.24$ lbs.

The method of calculation is similar for ring shaped valves and can readily be applied. The formula (407) can only be cousidered as an approximation as the variations in the jet of water affect the pressure. It is evident, however, that K is often quite large.

In the preceding calculation the momentum of the water column has uot been taken into account. In some cases this is sufficient to hold the valve open until the piston has made a great portion of its return stroke. This is well shown in the case of the pump at the Bleyberg mine (§ 319, note) which apparently showed a discharge of 104 per cent. If this action can be made to exist during the entire stroke by giving the water a sufficient velocity by contracting the tube that the discharge valve does uot close at all, this valve may be entirely omitted. This is the case with the single valved pump of Edmond Henry,* which has only a suction valve and no discharge valve. An analogy to this form of fluid ratchet is found in Langen's fly wheel ratchet train, Fig. 730 and 731. In this case the momentum of the fly wheel is sufficiently great for it to suffer no perceptible loss of velocity during the return stroke of the pawl.

§ 370.

MECHANICALLY ACTUATED PUMP VALVES.

The numerous investigations of recent years have shown that by proper loading of the valves, combined with a reduction of lift, the shock of the water in a pump can be very materially reduced aud kept within practical limits, eveu for high piston speeds. The reduction of lift involves a great multiplication in the number of the valves aud a great increase in dimensions. For this reason another solution of the problem has been attempted, namely, that of abandouing the self-acting feature, and actuating the valves by mechanical means. The best arrangement seems to be that in which the valves are opeued by the action of the water, but closed by a positive gear in advance of the shock. The application of this method enables the size of the valves to be reduced, and as it is principally used for large pumping engines the valves cau be operated by connection to the fly wheel shaft. Professor Riedler has receutly made very valuable investigations upou this system.†

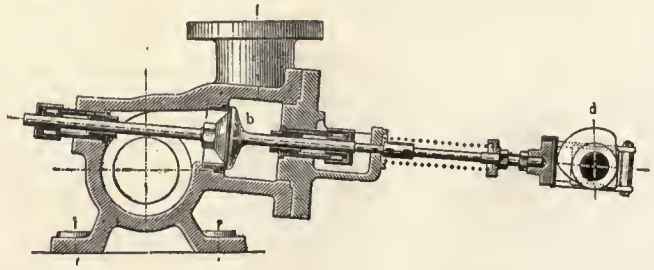

FIG. 1139.

Fig. 1139 shows the valve gear for the Riedler pumping engine at the Wartiuberg mine. The revolving cam d, closes

*See Revue Industrielle, p. 342, September, 1888, where the complete theory of this form of pump is given.

†See Riedler, Mine Pumps with Positive Valve Gear, Zeitschr. D. Ingenieure, 1888, p. 481.

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the valve *b*, just as the plunger is at the end of the stroke, and permits it to open by the action of the water. The valve is held to its seat by a spiral spring. Pumps of this construction operate very smoothly. Further details of this construction are given in the articles already cited. For blowing engines, and especially for air compressors, positively actuated valve gears are much used. A very simple action for the inlet valves is shown in Fig. 1140. The piston rod *c* moves the valve *b*, by means of the friction of the rod in the stuffing box, the action taking place just at the reversal of the stroke. Examples of this construction are to be found in the air pumps for use in physical laboratories.

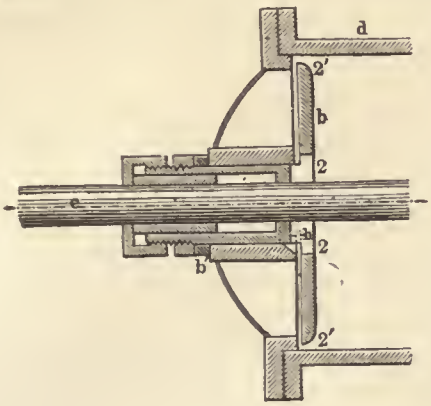


FIG. 1140.

§ 371.

VALVES WITH SPIRAL MOVEMENT.

It is not so convenient to construct a valve so that its motion shall be both rotary and rectilinear axially, and this construction is mainly limited to valves which are operated by hand.

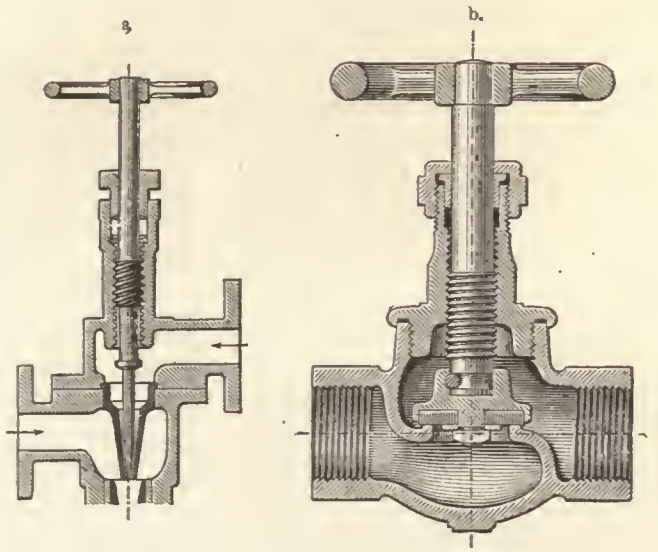


FIG. 1141.

Fig. 1141*a* shows a conical valve with spiral motion, as used on the Giffard injector. This arrangement enables a very fine adjustment of the opening to be obtained; a similar form is also used in the so-called "cataract" for steam engines. The sharp point of the cone has caused valves of this sort to be called "needle" valves, and similar forms, without the spiral action, are found in gas regulators. Stop valves for steam and for water are frequently made with spiral motion. An example is shown in Fig. 1141*b*. When the valve is not in contact with its seat it has both a vertical and a rotary motion. In the particular form shown the valve has a disk of asbestos which forms the surface of contact with the seat. This general form is known as a "globe" valve on account of the form of the body, and such valves are very extensively used for steam and water.

§ 372.

BALANCED VALVES.

Valves which are to be operated by other means than by the action of the fluid, are advantageously made so as to be relieved from fluid pressure, and thus offer less resistance to operation.

Valves of the wing or flap construction are conveniently balanced by combining two valves moving in opposite directions into one valve of the form commonly called "throttle" valve, Fig. 1142.

This is the counterpart of the throttle ratchet shown in § 250, and valves of this sort have been much used with throttling governors for steam engines. The closing of such valves is imperfect, as the edge must be rounded near the hub of the valve, thus giving only a line of contact.*

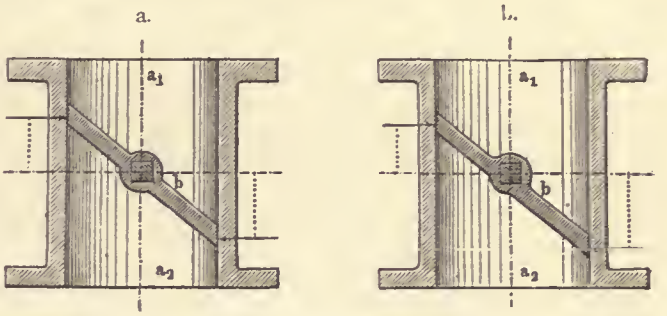


FIG. 1142.

If it is desired to use throttle valves for regulation of water pressure, as the case of turbines, etc., it must not be forgotten that the resistance of the valve will materially affect the efficiency.

For self-acting valves a variety of throttle valve may be used, in which the area of one wing is only about ¾ to ⅞ that of the other wing, thus partially balancing the valve. This form, which is old, appears to be again coming into use. †

Lift valves which are situated in vessels which are not closed at the top may be balanced in a simple manner by making the valve with a tubular continuation which extends above the surface of the water. A balanced valve upon this principle, as used for an outlet valve in a canal lock, as at b_1' and b_2',

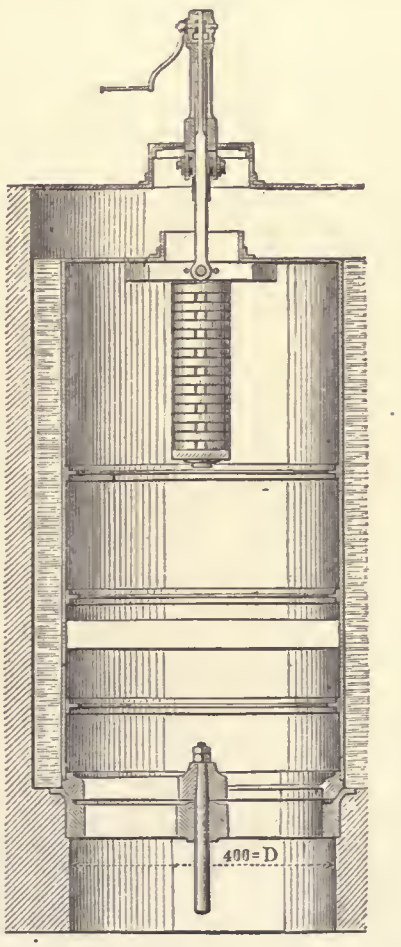


FIG. 1143.

*The form shown at *b* is recommended in Revue Industrielle, p. 205, May 26, 1888, as insuring a better balance, but from Robinson's experiments, already cited, this form would offer too much resistance to opening.

†See Belidor, Architecture Hydraulique, Paris, 1739, Vol. II. These valves were of brass with metallic packing.

Fig. 993, is shown in Fig. 1143. This valve, designed by Constructor Cramer, is made with a cylindrical shell of sheet iron extending to the surface of the water. The diameter of this shell is the same as that of the valve, and the weight of the valve, which is by no means small, is partially counterbalanced, leaving only sufficient to insure proper closing and seating. * If it is desired to apply Cramer's construction to valves which are subjected to high pressure, this may be done by using two stuffing boxes, one external and one internal,

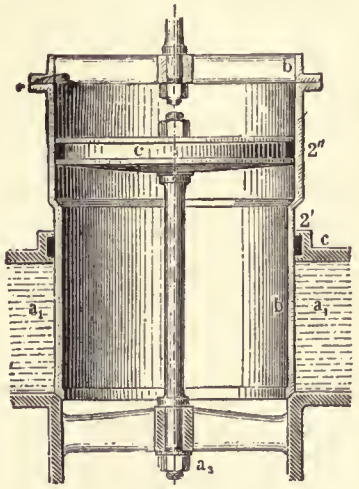


FIG. 1144.

as shown in Fig. 1144, which, however, adds to the complication. For lift valves which are to act under high pressure a better construction is the so-called "double-beat" valve, which, like the throttle valve, consists of two similar valves in which the pressures oppose and neutralize each other. Three forms are shown in the accompanying illustrations. Fig. 1145*a*

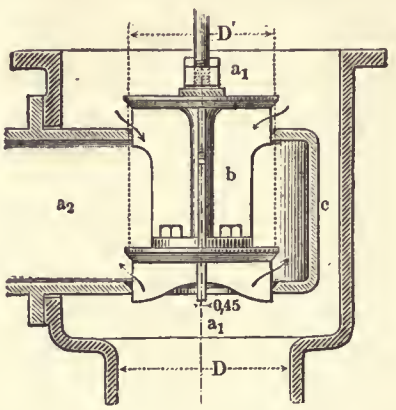


FIG. 1145*a*.

being a double disk valve, and Fig. 1145*b* a tubular valve. Both of these were invented by Hornblower in the latter part of the last century. Fig. 1145*c* is a bell or Cornish valve. These

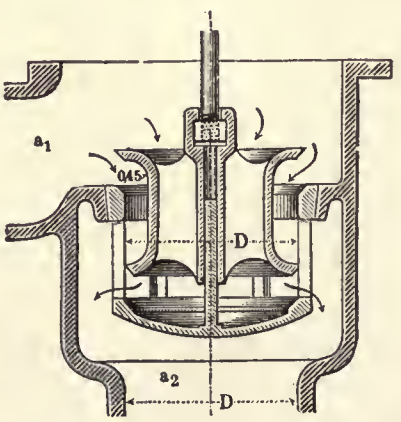


FIG. 1145*b*.

valves each consist of a pair of conical lift valves, the variations appearing in the details of the connections and passages.

* See Annales des Ponts et Chaussées, 6me série, Vol. XII, 1886, II Semestre, p. 248, also Zeitschrift für Bauwesn, 1880, p. 155.

When the projection of one seat falls within that of the other, as in forms *b* and *c*, the unbalanced pressure is that due to the projections of both seats. If so desired, however, these may be

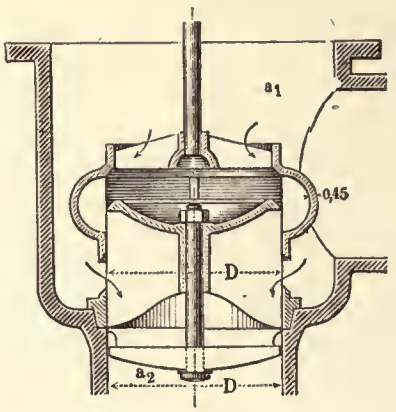


FIG. 1145*c*.

made as Fig. 1145*a*, with one seat directly over the other, in which case the pressure $p_1 - p$ need only be calculated for one seat. For the preceding double seated valves we may make:

$$\left.\begin{aligned} &\text{for the width of seat } s = \tfrac{1}{2}\left(0.2\sqrt{D} + 0.137\right) \\ &\text{and for the projection } s_1 = \tfrac{1}{2}\left(0.2\sqrt{D'}\right) \end{aligned}\right\} \quad . \quad (408)$$

In form *a* the mean diameter D' of the valve is = 0.8 times the diameter D of the pipe, while in forms *b* and *c* the diameters of valve and pipe are the same. For the force required to lift the valve, taking the projection s_1 into account and assuming the pressure between the surfaces to be as in § 368, equal to $\frac{1}{3}(p_1 - p)$, we have, neglecting the weight of the valve:

$$P' = \pi D' s_1' \tfrac{2}{3}(p_1 - p) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (409)$$

while for a single conical valve of the same diameter D it would be:

$$P = \left[\frac{\pi}{4} D^2 + \tfrac{2}{3} s_1 \pi (D + s_1)\right](p_1 - p) \quad . \quad . \quad . \quad (410)$$

P is proportionally very great, while P' is not always unimportant.

Example.—For $D' = 12$, we have for form *a*, $S_1' = \frac{1}{2}\left(0.2\sqrt{12}\right) = 0.346''$. If now $p_1 - p = 60$ pounds per square inch we have:

$$P' = \pi \times 12 \times 0.346 \times \tfrac{2}{3} \times 60 = 521 \text{ pounds.}$$

For a single valve the diameter would be $D = \frac{D'}{0.8} = 15$ inches, and from (402) $s_1 = 0.2\sqrt{15} = 0.77$, whence

$$P = \left[\frac{\pi}{4} 15^2 + \tfrac{2}{3} \times 0.77\, \pi \left(15 + 0.77\right)\right] 60 = 12{,}126 \text{ lbs.}$$

so that P is nearly 24 times P'.

It is very desirable for double seated valves which are to be used for steam, that both valve and seat be made of the same material, in order to avoid unequal expansion.

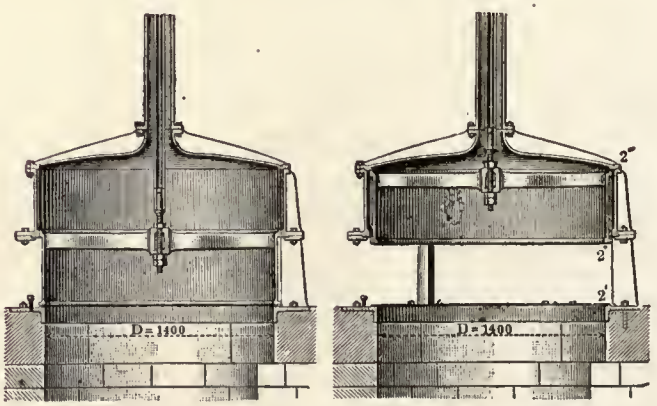


FIG. 1146.

Double seated valves are also used for water. Fig. 1146 shows such a valve arranged for a sluice.

This valve is made with flat seats, the lower seat being faced with rubber, and the upper one packed with leather secured to the housing which is shown over the valve. The valve rod runs through this housing and through a tube above the surface of the water. The diameter D is 1400 mm. = 4 ft. 7 in. This is practically a tubular valve, similar to Fig. 1145*b*, except that the direction of flow is reversed; this arrangement has also been used by Hornblower. The leather packing at 2'' is made flexible, since the projections of the valve seats lie one within the other so as to make a slight tendency for the valve to lift, without entirely overcoming the weight of the valve. Balanced valves of the kind described above are also adapted to large steam engines. In some instances a small balanced valve is arranged so that it is lifted first and admits steam under the main valve before the latter is lifted.

Another device is that shown in Fig. 1147, known as Aitken's automatic steam stop. The main valve b, is closed by being screwed up against its seat by the spindle and hand wheel. Before opening, it is balanced by admitting steam through the by-pass valve b'. The valve itself is loose on the spindle, and if through any breakage in the pipe beyond the valve a sudden or rapid flow of steam should take place, it will be automatically closed by the force of the current.

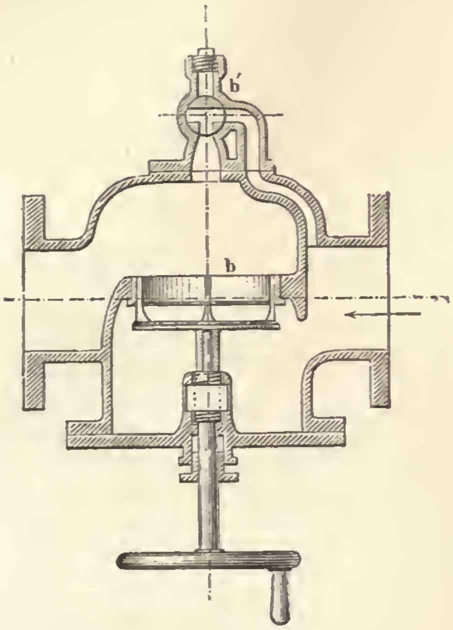


FIG. 1147.

Lift valves may also be balanced by making a balance piston connected with the valve, the pressure of the steam acting upon the piston in the opposite direction to the action on the valve. This construction has also been applied to reducing valves in the place of weighted levers or springs in various ways, but space cannot here be given to the subject.

B.—SLIDING VALVES.

§ 373.

ROTARY VALVES AND COCKS.

For rotary valves the bearing surfaces are conveniently made conical, so that a simple endlong pressure on the valve will hold it firmly to its seat. Valves of this construction are known as cocks.

Fig. 1148 shows two forms of such cocks which are in general use. The opening through the plug of the cock increased in height in order to obtain a full area without requiring the diameter of the plug to be too great; the area of the opening through the plug being made equal to the area of the pipe, *i. e.*,

$$= \frac{\pi}{4} D^2.$$

According to the experiments of Edwards, a good taper for the plug is $\frac{1}{8}$ on each side. For the thickness δ of the metal in the body of the cock formula (319) may be used when the material is of cast iron, which gives $\delta = 0.472'' + \frac{D}{50}$; for bronze the thickness may be made one-half to two-thirds this value. The design shown in Fig. 1148*b* has the plug entirely inclosed in the body, and is made with two stuffing boxes, one for the plug and one for the spindle. The management of screw cap and jam nut enables a fine adjustment to be obtained.*

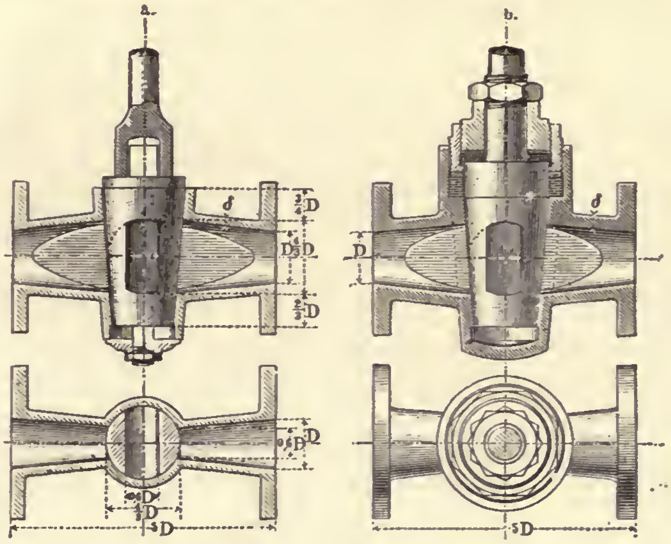


FIG. 1148.

Fig. 1149 shows two forms with hollow plugs, these being much used for injection cocks for jet steam condensers.

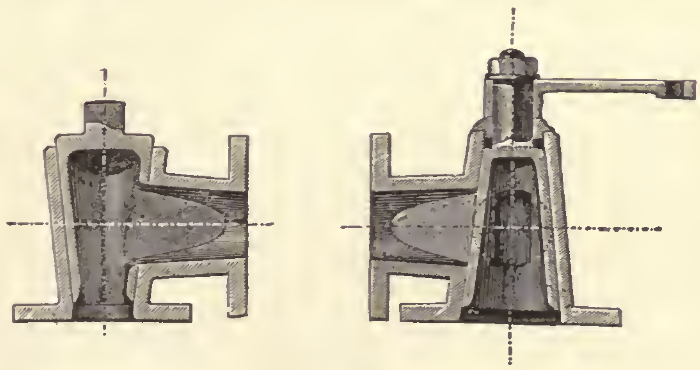


FIG. 1149.

When the angle of the apex of the cone becomes 180° the plug becomes a flat disk, and this form is often found in the throttle valves of locomotives, and less frequently in the valve gear of engines. True cylindrical plugs, *i. e.*, those in which the angle of taper is equal to zero, are rarely used, although recommended by some. This form is better made in a portion of a cylinder, and operated by an oscillating motion, as in the Corliss and similar valves. A starting valve of this type, used as the steam admission valve for a triple expansion engine is shown in Fig. 1150.

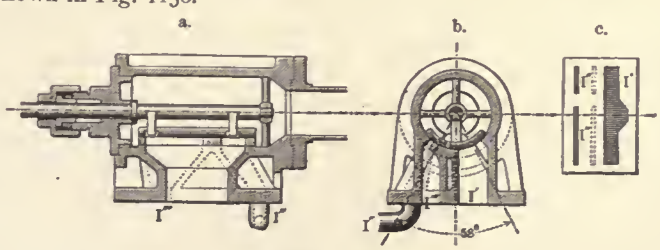


FIG. 1150.

At a is a longitudinal section, b a cross section, and at c is shown the seat looked at from above. In the one seat three passages are controlled at I', I'' and I'''. All three are closed when the valve is in the position shown at b, but open at the same time when the valve is moved to the left. The trapezoidal opening in I' admits a small amount of steam to the high pressure cylinder at the same time that a little live steam is admitted through I'' and I'' to the intermediate and low pressure cylinders, so that the engine is sure to start. The valve is then thrown all the way over, closing I'' and I''' and throwing I' wide open.†

* Mosler's German Patent, No. 33,912.

† See Zeitschr. D. Ingenieure, p. 509, 1886, Meyer, Triple Expansion Marine Engine.

§ 374.

GATE VALVES FOR OPEN AND CLOSED CONDUCTORS.

A great variety of valves has been devised for open water conductors in the form of gates by which the flow can be regulated. Such gates have been preferably made of wood with the exception of the operating mechanism. At the present time iron is beginning also to be used for the gates, and as in the case of other branches of work, wood is likely to be less and less used, being limited to a few special cases. For very broad streams the construction of such gates is now sometimes made upon the principle of subdivision. In such cases the breadth of the stream is subdivided into a number of smaller streams, each with a separate gate, thus keeping the gates small enough to be movable by hand.

A weir which is placed in a stream is both in principle and in construction a valve. When the water in the stream is low the flow is entirely checked; for the mean flow the stream passes through the reduced opening with a velocity due to the reduction in section, while for high water the entire width of the dam is overflowed. Movable weirs are plainly examples of regulating valves. French engineers have given much attention to moveable dams with excellent results. A new design for a moveable dam by Schmick is shown in Fig. 1151.*. This dam consists of a number of pontoons, each three of which are

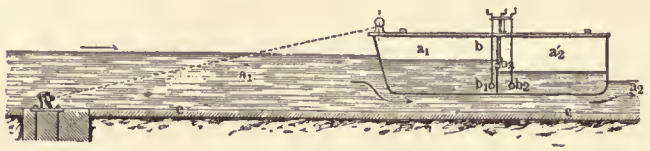


FIG. 1151.

secured together by a yoke and anchored by a chain to a point up the stream. All three pontoons of each set are arranged with variable water ballast in two or more compartments, a_1' and a_2'. An adjustable valve b_1 enables communication to be made with the upper water level, and the compartment a_1', and a similar valve b_2 connecting the compartment a_2' with the lower level, while a third valve b_2 enables communication to be made between the two compartments. By varying the openings of the valves the pontoons can be caused to regulate the difference of water level above and below the pontoons, while if all three valves are closed the pontoons will rise and fall with the variations in the level of the stream.

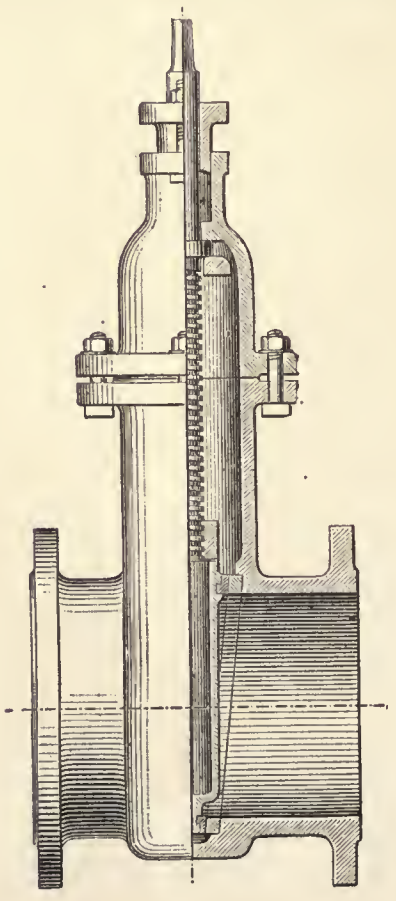


FIG 1152a.

Gate valves are much used for water mains, and an example of the many varieties used for the purpose is shown in Fig. 1152a. The gate or disk of the valve is made of bronze, and is wedge shaped, in order that it may be firmly pressed against its seat when the screw is tightened (this forms a pressure of the second order) while the pressure is immediately relieved at the commencement of opening. The screw is in this case made of "sterro-metal" to avoid rusting.

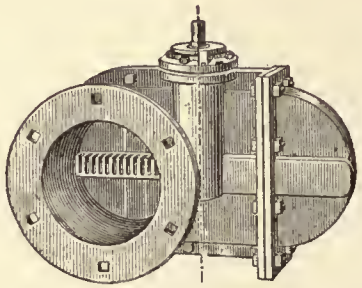


FIG. 1152b.

Gate valves are also used for gas mains, and a valve for this service is shown in Fig. 1152b. In this instance the valve is operated by means of a rack and pinion. The motion is made in the horizontal direction so that the valve will remain in any position, the only resistance being that of friction.

§ 375.

SLIDE VALVES.

Slide valves are mainly used for the purpose of effecting the distribution of steam in steam engines. This is such an important subject that all the forms in general use will here be noticed.

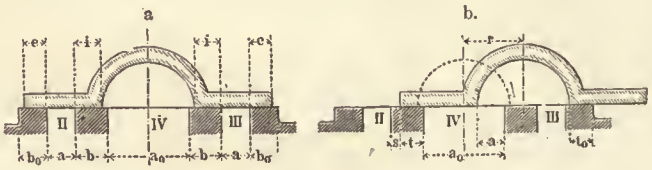


FIG. 1153.

1. Plain D valve, Fig. 1153. This is the most important form of all. The action of this valve has already been discussed in § 328, and hence the dimensions will only be considered here. The width a of the steam ports is kept as small as is practicable, while the length at right angles to the plane of the drawing is made quite large. When a is given, the dimensions to be determined are the outside and inside lap e and i, the bridges b, the width of face b_o beyond the ports, the width a_o of the exhaust port IV, the travel r, the length of the valve l, and of the valve seat l_o. The laps e and i, and under some circumstances two valves e_2 and e_3 for e are determined according to the method given in Figs. 1024 and 1025. In the same manner also is found the greatest distance s, Fig. 1153b, in which the edge of the valves passes the edge of the port. This gives the width of bearing t of the valve upon the bridge, since $b = s + t$. The value of t varies greatly, the least permissible value is $t = \frac{3}{16}''$, and it is more frequently made $\frac{3}{8}''$ to $\frac{1}{2}''$. Approximately, for we have, after assuming t as just given, $a_o + t - (e + a + i) = a$, in which e is taken as a mean between e_2 and e_3. We then have:

$$\left.\begin{aligned} a_o &= 2a + l + i - t \\ \text{whence } r &= a + e + s \\ \text{and } l &= 4a + 3l + i + 2s + t \end{aligned}\right\} \quad \ldots\ldots\ldots (411)$$

The valve face must have an inner width of bearing t_o Fig. b at least equal to t, whence for the total width of the valve face we have the value

$$\left.\begin{aligned} &a_o + 2b + 2a + 2b_o, \text{ or} \\ l_o &= 4a + 3e - i + 4s + t + 2t_o \end{aligned}\right\} \quad \ldots\ldots (412)$$

The thickness of metal in the valve itself, when made of cast iron should be about $= \frac{D}{200} + 0.4''$, which is about half the

*S. Schmick, Prahmwehr (Pontoon Dams), Zeitschrift für Bankunde, Munich, 1884, p. 502.

thickness of the metal of the steam cylinder as given by formula (320). If the valve is faced with white metal the body of the valve should be of bronze, the white metal itself not being strong enough.

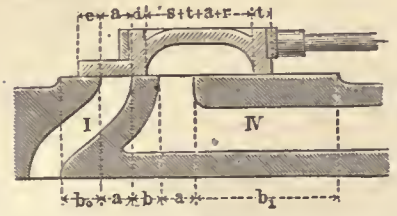


FIG. 1154.

2. Double *D* valve, Fig. 1154. In this form the four valves which in the plain *D* valve are united in one piece, are separated into two portions, connected by a rod. This construction is adopted to shorten the steam passages *I* and *III*, the width of each valve is $= 3a + 2e + 2s + l + l_o$ or of both together $= l = 6a + 4e + 4s + 2l + 2l_o$.

3. Pipe Valve, Fig. 1155. This form is also intended to reduce the length of the ports *II* and *III*, which is often an im-

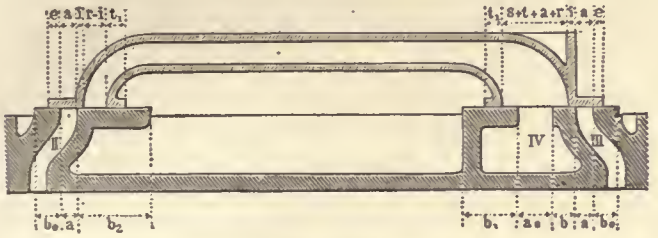


FIG. 1155.

portant consideration in engines of long stroke. The total length of valve bearing surface is $l = 6a + 5e + 3s + i + 2l_o$.

Example 1.—If $a = \frac{3}{4}''$, $e = \frac{3}{4}$, $i = \frac{1}{16}''$, $s = \frac{3}{8}''$, $l = l_o = \frac{3}{16}$ we have for a plain *D* valve the width $l = 4 \times 0.75 + 3 \times 0.75 + 0.6875 + 2 \times 0.375 + 0.1875 = 6.875''$.

For the double *D* valve we have $l = 6 \times 0.75 + 4 \times 0.75 + 4 \times 0.375 + 4 \times 0.1875 = 8.75''$ and for a pipe slide valve as Fig. 1155, $l = 6 \times 0.75 + 5 \times 0.75 + 3 \times 0.375 + 0.6875 + 2 \times 0.1875 = 10.4375''$. The work of friction in moving the valves is directly in proportion to the above widths, since the travel is the same in all three cases, being: $2r = 2e + 2a + 2s = 2 \times 0.75 + 2 \times 0.75 + 2 \times 0.375 = 3.75''$.

In order to reduce the work of friction in slide valves the multiplication of valves has been resorted to, much as has already been shown in the case of lift valves. A division of the valve system into two parts has also been made for marine engines with oscillating cylinders, the object being to place one portion on each side of the cyliuder and thus keep the entire mass symmetrical with regard to the axis of oscillation. In this arrangement the two slide valves correspond to eight separate valves. In these as also in engines, with stationary cylinders, the valves may be combined into one. This may be accomplished by using two or more sets of steam passages which unite at one point and by making corresponding divisions in valve and valve seat. The combination of several valves so as to act as one is not limited to lift valves, as many useful forms of slide valves are made on this principle, some of the best forms being here shown.

4. Penn's Gridiron Valve, Fig. 1156. In this the steam port *a* is divided into two ports, each having a width $= \frac{a}{2}$. To de-

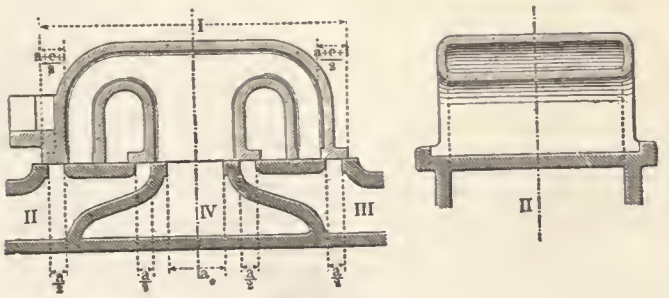


FIG. 1156.

termine the total width of valve as in the previous cases, we have: $l = 5.5a + 3.5e + 3s + l + 2l_o + \frac{1}{2}i$, and for the travel: $2r = a + e + s$, that is half as much as before. It is evident that the laps $\frac{e}{2}$ and $\frac{i}{2}$ must bear the same relation to $\frac{a}{2}$ as the diagram gives for $a : e : i$, in the preceding forms.

Example 2.—For $\frac{a}{2} = \frac{3''}{8}$, $\frac{e}{2} = \frac{3''}{8}$ $\frac{i}{2} = \frac{5''}{16}$, $\frac{s}{2} = \frac{3''}{16}$ $l = l_o = \frac{3''}{16}$ we have $l = 5.5 \times 0.75 + 3.5 \times 0.75 + 3 \times 0.375 + 0.1875 + 2 \times 0.1875 + 0.3125 = 8\ 75''$

$$2r = 0.75 + 0.75 + 0.375 = 1.875.$$

This gives for the work of friction of such a valve as compared with an equivalent plain slide valve:

$$\frac{6.875 \times 375}{8.75 \times 1.875} = 1.57.$$

which is an important gain.

5. Borsig's Gridiron Valve, Fig. 1157. This is the same in principle as the preceding, and differs only in construction, the

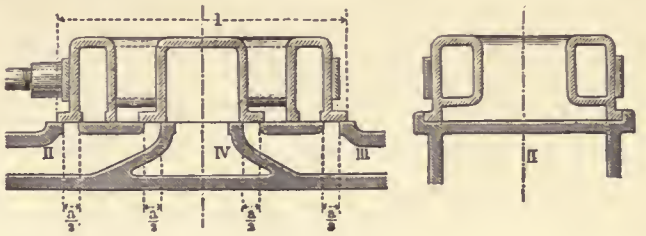


FIG. 1157.

exhaust passages being carried on each side of the valve instead of above, as in Penn's construction.

6. Hick's Double Valve, Fig. 1158. This is intended for use with compound engines with parallel cylinders (Hornblower and Woolf), the ports *II'* and *III'* are for the high pressure cylinder, and *II''* and *III''* for the low pressure cylinder. The width *l* of the valve is:

$l = 5a + 3a_1 + 6e + 4s + e_1 + i_1 \times l_o$.

Usually a_1 is made equal to *a*, which reduces the value of *l* somewhat.

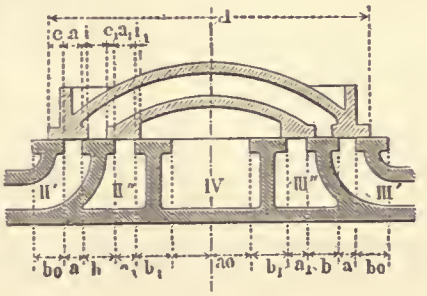


FIG. 1158.

7. Allan's Double Valve, Fig. 1159, is a valve for compound

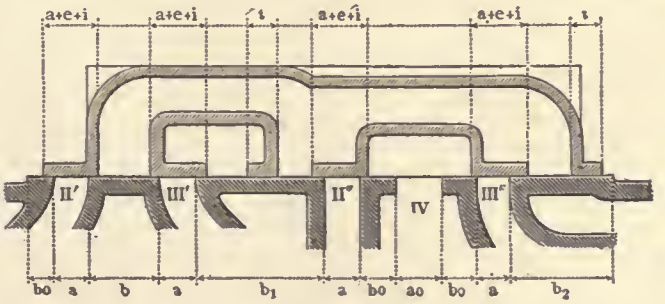


FIG. 1159.

engines with tandem cylinders. The value of *l* is

$$l = 10a + 7e + e_1 + 6s + i + i_1 + 3l_o.$$

This construction not only economizes the work required to operate the valve, but also gives a very simple arrangement of steam passages.

8. The *E* Valve, Fig. 1160, is used to advantage in place of the plain *D* valve when the use of a valve gear actuated directly from the piston rod requires that the valve shall move in the same direction as the piston. (See Fig. 1006 and 1008). This valve consists of two *D* valves cast together, and the over travel beyond the valve seat gives the admission.

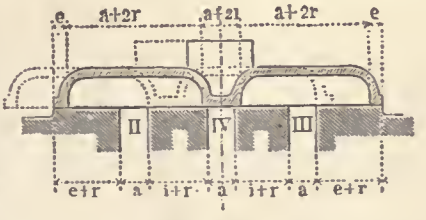


FIG. 1160.

We have as before: $r = a + e + s$, and

$$\left.\begin{aligned} b &= i + r = a + e + i + s \\ b &= e + r = a + 2e + s \\ a_o &= a \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (413)$$

This gives for the width of the valve:

$$\left.\begin{aligned} l &= 3a + 2b + 2b_o + 2t \text{ or:} \\ l &= 7a + 6e + i + 4s + 2t \end{aligned}\right\} \ldots\ldots\ldots (414)$$

which is considerably greater than for an ordinary D slide valve.

Example 3.—If as in Example 1, we make $a = e = 0.75''$, $i = \frac{11''}{16}$, $s = 0.375''$, $t = 0.1875''$, we then have $l = 13 \times 0.75 + 0.6875 + 4 \times 0.375 + 2 \times 0.1875 = 12.3125''$ against $6.875''$ for the plain D slide valve. It will be evident that the E valve is only available for small port widths and small laps, as will also be seen in Figs. 1006 and 1008. The principal value of this valve lies in the use of the outer edge of the valve seat as the edge of opening, which principle also has a valuable application in the following valve.

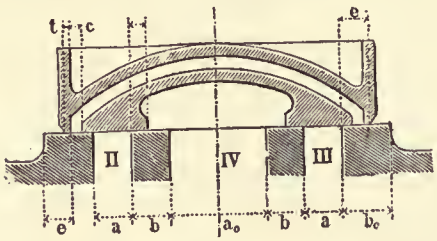


FIG. 1161.

9. Trick's Valve, Fig. 1161.* This is a double valve and consists of one D valve over another, with a steam passage between. As before, we have $r = a + e + s$, and also make $b_o = 2e - t$, *i. e.*, the inner edge of the outer valve when the valve is in mid-position, is at a distance $= e$ from the edge of the valve seat. The consequence is that when the valve is moved a distance equal to e, say to the right, the passage through the valve opens to admit steam at the same instant as does the edge of the valve on the left. This gives a steam admission twice as quickly and an opening twice as great as would otherwise be the case. The following positions from *a* to *f*, Fig. 1162, will show the successive actions, the exhaust ports being omitted for simplicity.

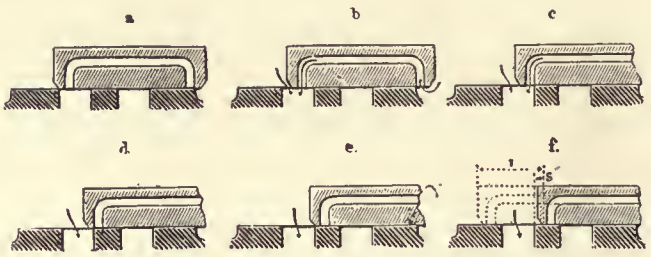


FIG. 1162.

a. The admission is just about to take place both from the edge of the valve on the left and through the passage in the valve. If we apply Zeuner's diagram (compare Fig. 1025) see

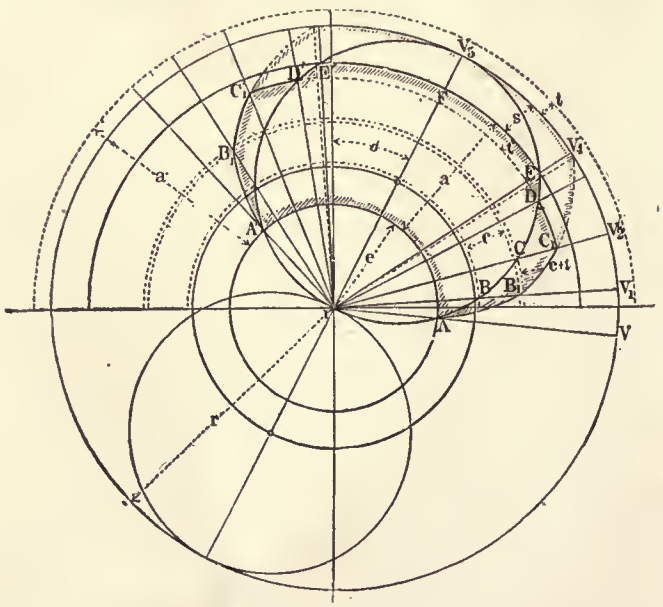


FIG. 1163.

Fig. 1163, we must from the point A, which indicates the port opening, double the width given by the Zeuner circle until the entrance to the passage in the valve is wide open, as at *b*. By thus doubling the opening in the diagram we obtain the curve $A\ B_1$.

b. From this position on, the opening at the left continues to grow wider, but that through the valve on the right does not, hence on the Zeuner diagram from this point we return to the opening which the regular valve circle gives, to which is added the constant opening $c = B\ B_1 = C\ C_1$ indicated by the curve $B_1\ C_1$. This continues until the inner edge of the opening of the valve passage on the left reaches the edge of the bridge as at *c*.

c. As the valve continues to move the passage through it is gradually closed, but the steam port is opened to the same amount, and hence the actual port opening remains constant. This continues until the position *d* is reached, when the passage through the valve is entirely shut off. This is indicated in the diagram by the arc $C_1\ D$, struck from the centre at 1.

d. The valve continues to move to the right until it is entirely upon the bridge, the corresponding portion of the diagram being the arc $D\ E$ of the valve circle.

e. The valve from this position moves on the bridge beyond the port until it has traveled a distance equal to s, as shown at *f*, during which time the port opening remains constant, as indicated in the diagram by the arc $E\ E'$ struck from the centre 1. From this point the same actions take place successively in the reversed order.

It will be seen that Trick's valve gives a much quicker opening and also a much longer duration of the full opening than does the plain slide valve. It remains to be seen how these features can be used to the best advantage. According to Trick's practice this is best done by making the value of s negative, and also $\geqq t$. This makes the port opening from C_1 to C_1' in the diagram constant, as shown in the diagram.

In order that the apparent contraction of the ports by the change in the sign of s shall not occur, the value of a is made greater than would otherwise be the case. Under these conditions we have for the exhaust port a_o, the equation:

$$a_o + t - e_1 - a - i = a - s,$$

in which s is given the maguitude equal to the distance which the edge of the valve is moved beyond the edge of the bridges. (See Fig. 1162 *f*). We then have:

$$\left.\begin{aligned} &\text{For the exhaust port,} && a_o = 2a + e_1 + i - s - t \\ &\text{For the bridge,} && b = e - e_1 + s - t \\ &\text{For the passage through valve} && c = e - t - e_1 \\ &\text{For the total valve,} && l = 4a + 4e - e_1 + i - 3s + t \end{aligned}\right\} (415)$$

Example 4.—Making $s = -\frac{3''}{16}$ and negative also $t = \frac{3}{16}$ and the other data the same as the plain slide valve of Example 1, and we have:

$a = 0.75 + 0.1875 = 0.9375''$; $e = 0.75$; $i = 0.6875$ $e_1 = 0.078''$, whence:

$$r = a + e - s = 0.9375 + 0.75 - 0.1875 = 1.5''$$

$$a_o = 2a + e_1 + i - s - t = 2 \times 0.9375 + 0.078 + 0.6875 - 0.1875 - 0.1875 = 2.2655''$$

$$b = e - e_1 + s - t = 0.75 - 0.078 + 0.1875 - 0.1875.$$

$$c = e - t - e_1 = 0.4845''$$

and

$$l = 4a + 4e - e_1 + i - 3s + t = 3.75 + 3 - 0.078 + 0.6875 - 0.5625 + 0.1875 = 6.9835''$$

as against $6.875''$ for the plain slide valve, which compares very favorably.

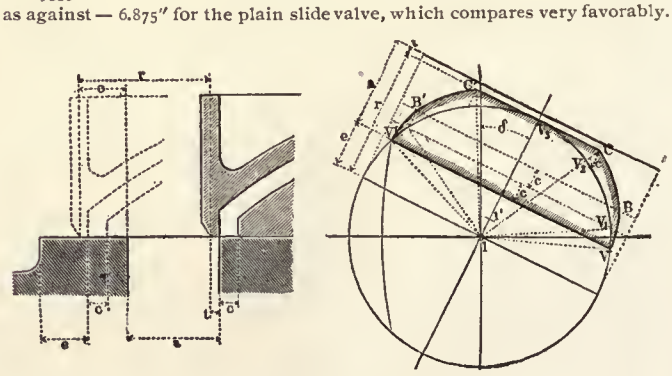


FIG. 1164.

Fig. 1164 shows the application of the author's valve diagram, already shown in Fig. 1024. The action of the inner portion of the valve is the same as with the ordinary slide valve.

For comparison the following dimensions of an executed valve by Trick, are given:

$a = 1.77''$ (45 mm.), $e_1 = i = 0.078''$ (2 mm.), $t = 0.216''$ (5.5 mm.).

*This valve was invented and made by Trick at Esslingen in 1857, and by Allan in England in 1858–1860; in the United States it is correctly known as Trick's valve.

$b = 1''$ (25.5 mm.), $e = 0.846''$ (21.5 mm.), $c = 0.55''$ (14 mm.).

$s = -0.374$ (9.5 mm.), $r = 2.24''$ (57 mm.), $l = 5.27''$ (134 mm.).

$b_o = 1.45''$ (37 mm.), $l_o = 5.83''$ (148 mm.), $\delta = 30°$.

Trick's valve is especially well adapted for use on compound marine engines, and has recently been used in the forms of double and gridiron valves, as in Nos. 4 to 7 preceding.*

§ 376.

Balanced Slide Valves.

The resistance to motion due to the pressure is not so great with a slide valve as with a lift valve of the same area, because in the former case it is only necessary to overcome the friction between the valve and its seat. For large valves, however, it becomes so great as to render some method of balancing necessary. It is desirable that even small slide valves should be balanced, as by this means the wear upon valve face and seat can be greatly reduced. Balancing is most important for steam engine valves, and the following examples belong to this class; for other kinds of service it is unimportant.

But few researches have been made upon the subject of valve friction, but from such as have been made, and for such good bearing surfaces as are used, the coefficient of friction may be taken at 0.05 to 0.04.

American engineers, as we have already seen, enter into very practical investigations and prosecute them with patience and success, and to one of these, Mr. C. M. Giddings, we owe the following results.†

Balanced Valve.—Cylinder 6¾″ × 10″.

Revolutions per Minute.	Pressure Pounds.	H. P. of Engine—N.	H. P. N' for Valve.	Ratio of preceding.	Ratio $a\,P'$
125	10	3	$\frac{1}{15}$	2 per ct.	48
175	30	9	$\frac{1}{9}$	1.2 "	61
200	40	13.5	$\frac{1}{5}$	1.4 "	91

Unbalanced Valve.—Cylinder 9″ × 12″. $n = 100$.

H. P. by Brake.	Ratio $\psi = \frac{N'}{N}$	Ratio $a\,P'$
4.5	4.5 per cent.	247
7.0	3.5 "	245
8.25	4.0 "	330
8.9	6.0 "	534
11.1	7.3 "	810

Balanced Valve.—Cylinder 9″ × 14″. $n = 100$.

H. P. by Brake.	Ratio $\psi = \frac{N'}{N}$	Ratio $a\,P'$
11.4	1.2 per cent.	137
13.5	1.1 "	149
14.0	1.0 "	140
15.6	1.0 "	156

The last column in each of the three tables has beeu added by the author of this work, aud is obtained as follows: If N and N' are the values in horse power of the engine and of the resistance of the valve, v and v', the corresponding mean velocities of pistou and valve, and P and P' the force upon each, the experiments give the relation $\frac{N'}{N} = \psi$ or $P'v' = \psi Pv$. Hence it follows for the force required to move the valve: $P = \frac{\psi P v}{v'}$. Now for a given engiue v' bears a constant relation to the number of revolutions n, so that we may put $P' = \frac{\psi N}{a\,n}$ whence $a\,P' = \frac{\psi N}{n}$. The value $a\,P'$ shows the increase in power required to operate the valve. It is evident that P' increases more slowly than the increase in steam pressure, but the resistance becomes quite great for unbalanced valves. The present difficulty lies in the limi'ed number of engines upon which experiments have been made. Fig. 1165 shows the character of diagram made by Gidding's apparatus, and it will be seen that the greatest resistauce occurs at the beginning of the stroke, diminishing toward the end to nearly zero. The inequality between the resistance of the back and forward strokes is due to the action of the steam pressure upon the area of the valve rod.

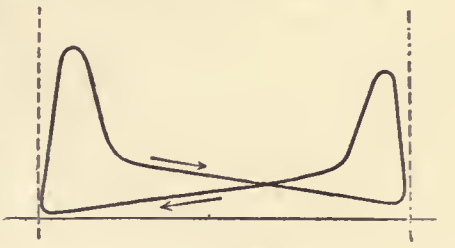


Fig. 1165.

In considering the pressure upon unbalanced slide valves the consideration mentioned already in connection with Robinson's experiments on lift valves is that there exists a counter pressure between the valve and seat which overcomes an important portion of the pressure on the valve. As a rough approximation we may take this pressure between the surfaces as $\frac{1}{3}(p_1 - p)$. Gidding's experiments show that the coefficient of friction is not constant, but diminishes with increased speed. More extensive experiments are much to be desired.

The method of balancing slide valves may be divided into three classes:

a. Removal of pressure from the back of the valve.

b. Opposing the pressure on the valve by counter-pressure.

c. Equalization of pressure on all sides.

Typical examples of these three systems will here be given:

a. Removal of pressure from back of valve.

1. The so-called long *D*-valve, invented by Murray, and used on engines built by Watt, the pressure was relieved from the back by a form of stuffing box, which answered well, but was uot adapted for high pressures.

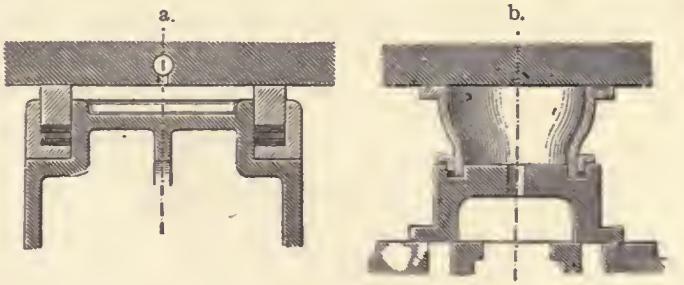


Fig. 1166.

2. Fig. 1166*a* shows the balance ring of Boulton & Watt. The under side of the steam chest lid is finished parallel to the valve face. Against this surface a ring of soft cast iron or bronze is fitted steam tight, this ring being fitted to the valve by an elastic packing aud moving back and forth with it. The space withiu the ring is subject only to the exhaust pressure. This form was used ou the Great Eastern.

Fig. 1166*b* shows the balance ring of Kirchweger, much used for locomotive engines. In this form the ring is pressed against the lid by steam pressure instead of spring packing. Both of these devices, as well as the similar ones of Penn, Borsig and others, leave too great a portion of the steam pressure unbalanced (at least 30 per cent. being left), and also prevent the valve from leaving its seat should water be carried into the cylinder ‡

b. Balancing by counter-pressure.

3. Cavé's Valve with Balance Piston.

The valve in this form, Fig. 1167, is connected by a link to a piston, which works iu a cylinder formed in the steam chest lid, and is subjected on the outside only to atmospheric pressure. Bourne's method of balancing is similar, except that the other side of the balance piston is in communication with the exhaust.

4. Valves with Rolling Support. Fig. 1168.

At *a* is shown Lindner's valve. The top of the valve itself is formed into a piston, sliding up and down in the valve and supported by two segmental rollers.

* See Zeitschr. D. Ing. 1888, p. 509, Triple Expansion Engine by G. L. C. Meyer, of Hamburg.

† See Trans. Am. Soc. Mec Engrs., Vol. VII, p. 631: C. M. Giddings, Description of a Valve Dynamometer for measuring the power required to move a slide valve at different speeds and pressures.

‡ An elegant construction for this form of balanced valve is that of Robinson. (See Trans. Am. Soc. Mech. Engrs., Vol. IV, p. 375.

The degree of balancing is dependent upon the size of the piston. At *b* is shown Armstrong's roller supported valve. In this case the valve is closed at top as usual, and the construction is very simple and practical. At *c* is Bristol's valve, in

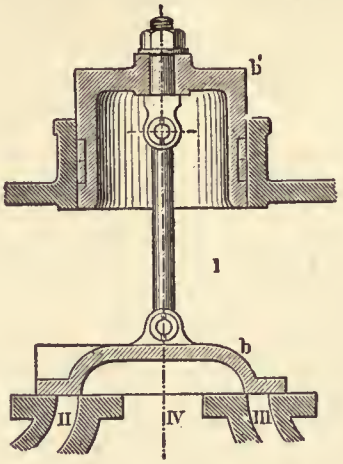


FIG. 1167.

which the valve is supported on a system of friction rollers. This has been used by the works at Seraing for large marine engines. To this class of methods of balancing belongs also that used by Worthington, in which a cylinder is formed in the top of the valve, as shown in Fig. 1016.

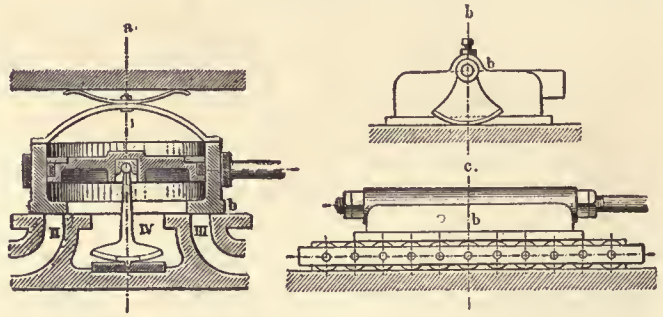


FIG. 1168.

5. Cuvelier's Valve, with pressure beneath,* Fig. 1169*a*. The ordinary slide valve is here combined with another, both being made in one piece, and the combined valve held down to its seat by pressure rollers. Live steam is admitted through the passage *I* into the space between the two valves. At *b* is Fitch's valve, also with pressure beneath. In this form the

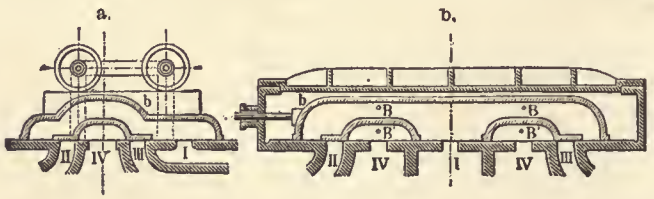


FIG. 1169.

pressure rollers are omitted, and the valve is held down by live steam pressure in the steam chest. The steam is admitted through very small holes *B B*, and escapes to the exhaust through similar holes *B' B'*, so that the supply is about equal to the loss by condensation. An objection to the use of valves with pressure beneath is the large area of valve seat which is required.

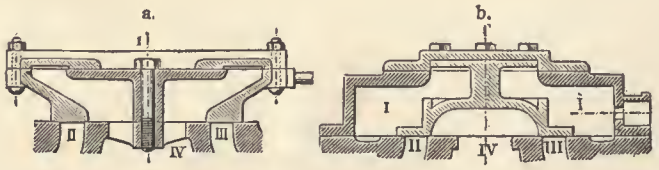


FIG. 1770.

6. Double Seated Valves, Fig. 1170. At *a* is Brandau's valve, and at *b* is Schaltenbrand's valve; the former is analogous to Hornblower's lift valve, Fig. 1145*a*, and the latter to the bell valve of Gros, Fig. 1145*c*. In neither form is the degree of balancing so complete as is desirable.

c. Equalization of Pressure on all Sides.

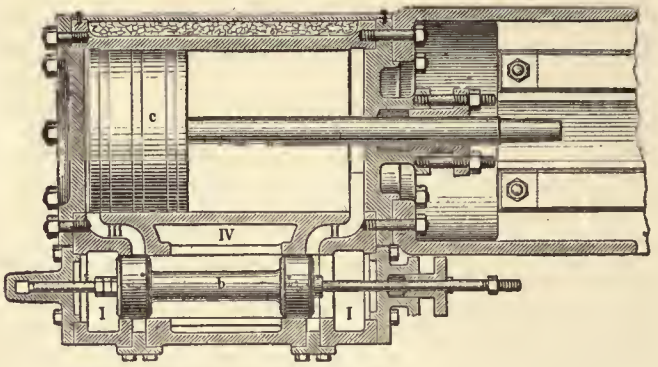


FIG. 1171.

7. A very complete equalization of pressure is obtained by making the valve in the form of a piston. Fig. 1171 shows a recently designed piston valve with its steam cylinder. The flat valve seat here becomes a cylinder, and the valve a double piston, the flat sides of the valve disappearing. The valve pistons are each fitted with a single ring behind which steam is admitted through a small hole, thus rendering springs unnecessary. The principal defect in piston valves is the question of wear. The best results appear to be obtained by making the piston valve solid, and very accurately turned and polished, and made about $\frac{1}{1000}$'' smaller in diameter than the bore of the valve cylinder, both valve and valve cylinder being made of the same material. Piston valves fitted in this manner last a long time.

8. Rotary Valves: For steam hammers, in which valve gear operated by hand has been found preferable to automatic movements, valves with rotary movement, formed like cocks, are used to advantage. These have been well designed by Wilson, the superintendent of Nasmyth's works.

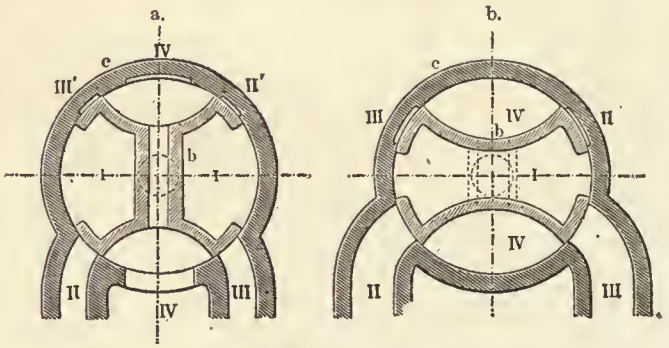


Fig. 1172.

Fig. 1172*a* shows an oscillating valve by Wilson. Opposite to the ports *II*, *III*, *IV*, are false ports or recesses of shallow depth. The steam enters at the end of the valve into the symmetrical spaces *I*, *I*. The unbalanced area of the steam of the valve causes a corresponding endlong pressure which is received by a thrust bearing. If we neglect the slight pressure due to the steam in the false ports when expansion takes place in *II* and *III*, the valve is balanced on all sides. Very large oscillating valves of this sort are easily moved by hand.†

A modification of this valve enables it to be operated by rotation instead of oscillation, as shown in Fig. 1172*b*. Here the parts are symmetrically arranged, as was not the case with the old four way cock of Fig. 987. The exhaust passages *IV* connect with one end of the valve, and the admission *I*, with the other end. There remains here also an unbalanced end-long pressure which is received by a thrust bearing. With this exception the valve is entirely balanced, and when well made the thrust bearing offers but little resistance. The construction of such valves demands a high degree of accuracy, and a specialty of this form is made by the establishments of Dingler of Zweibrücken and of Pfaff in Vienna.

The brief examination which we have given to the preceding methods of balancing does not include a method which, while offering great difficulties of construction, appears to be gradually coming into use. This method consists in surrounding the ordinary flat side valve with equalizing pressure plates. Several practical illustrations of this method will be given.

* Compare Fig. 1034.

† See Zeitschr. D. Ing., 1868, Vol. II, p. 207.

9. Wilson's Balanced Valve. Fig. 1173. (First shown at the London Exhibition of 1862.)

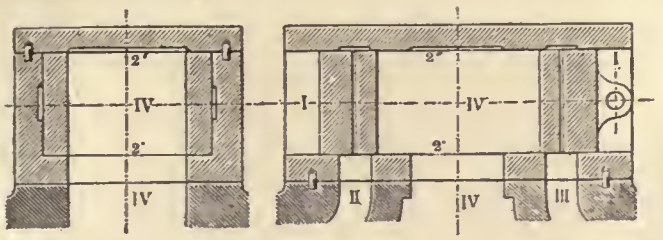


FIG. 1173.

The valve is symmetrical, and slides between two parallel and similar faces, the lower face having openings corresponding to the ports, the upper face having similar false ports. The close fitting and accurate parallelism of the surfaces was depended upon to obtain the balancing. In practice it was found that the balance plates would spring under the pressure of the steam unless made very stiff and strong, and that the weight of the valve caused much friction and wear.

Both of these difficulties have been met in more recent designs, as will be seen below.

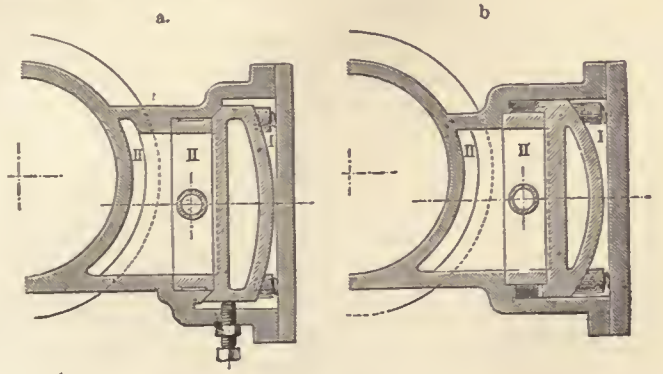


FIG. 1174.

10. Fig. 1174*a* shows the balancing of the valves of the Porter-Allen engine.* The pressure plate is made very deep and stiff and formed with inclined plane bearings and set screws, by which the pressure can be very closely regulated.

Fig. 1174*b* is Sweet's balanced valve. The pressure plate is here also made heavy and stiff, and is supported on longitudinal wedge bearings on each side, adjustable at the ends by screws. In both forms the pressure plate is fitted with springs to allow the plate to yield in case of water getting in the cylinder. These forms of balanced valve have the objection that the ignorant mechanic may render the balancing ineffective by improper adjustment of the screws, permitting the full pressure of the live steam to act upon the valve.

§ 377.

FLUID VALVES.

Valves may be formed of fluids, or, more generally speaking, may be constructed of pressure organs. Ratchets adapted to pressure organs, as fluid valves are properly called, are in extensive use, but have not generally been recognized as valves.

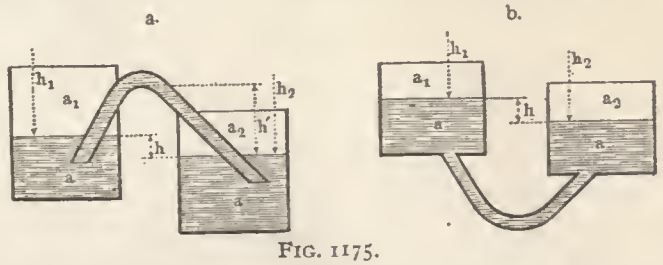


FIG. 1175.

They are all reducible to one of two principal forms, either the direct or inverted siphon. Fig. 1175*a* and *b* (compare § 312).

A direct siphon connects two quantities of the same fluid *above* the level of both portions, these levels differing, for example by a height h; an inverted siphon is similar, but connects them below the surface levels. Let a_1 and a_2 be liquids, which do not combine with *a*.† If the pressures of a_1 and a_2 are equal, the fluid *a* will flow from the higher to the lower level under a pressure due to the height h. In the inverted siphon the flow is constant, but with the direct siphon the flow is stopped, and the siphon empties as soon as the level falls below the short end of the siphon.‡ If the upper vessel is again filled, the flow will begin as soon as the fluid attains the height h' of the bend in the siphon. The fluid in the siphon therefore forms a valve, which converts a continuous flow into the upper vessel into a periodical flow into the lower one (see the example in § 324, where a similar action takes place with a rigid valve). This action of the siphon has recently been applied to excellent advantage.

When the pressures in a_1 and a_2 are different, as represented by the heights h_1 and h_2, as is frequently the case, we have for both forms for the height to which the flow is due, $h_1 + h - h_2$ for the height in an equivalent column of the fluid *a*.

If this valve is positive, there will be an outflow, if it is zero, the fluid will be stationary, and if it is negative, there will be a reverse flow. In cases in which $h_1 + h - h_2 = O$, h represents the measure of the difference between h_2 and h_1. It therefore follows that by means of fluid valves the relation between the fluids a_1 and a_2 can be checked or controlled as may be desired.

Applications of fluid valves are very numerous, as the following examples will indicate:

Fig. 1176*a* shows a water trap in a pipe. This is a fluid valve (inverted siphon) which checks a gas a_2 from mingling with a gas a_1 so long as $h_1 - h_2$ is less than twice the height s of the branches of the siphon. If the pressure from above upon *a* increases the overflow runs off through a_2. This latter pipe must not be too small, however, or a siphon action will occur, and all the water will be drawn off. This device is much used in gas works, chemical works, laboratories, etc.

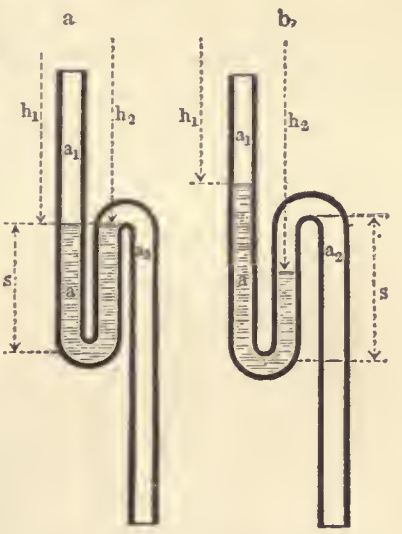


FIG. 1176.

Fig. 1176*b* shows the same arrangement used as a barometer, manometer, vacuum gauge, etc., the difference of level indicating differences of pressure $h_2 - h_1$ for valves below 2 *s*. Applications of this principle are very numerous, from the largest forms to the most delicate physical instruments.

Fig. 1177*a* is an open stand-pipe, used on certain forms of low pressure boilers. This is practically an inverted siphon, of which the boiler shell forms one branch. The fluid valve checks the steam a_2 against the atmosphere a_1. If the pressure becomes so great that $h_2 > h_1 + h'$ the fluid valve will be thrown out at the top of the pipe, the arrangement thus forming a safety valve against an excess of pressure in a_2. This device was for a long time in use for low pressure boilers, Brindley's feeding device, Fig. 1000, being constructed on this principle. Natural stand-pipes with periodical discharge exist as geysers.

Fig. 1177*b* is a closed stand pipe for steam boilers. The pipe which has first been filled with steam gradually fills with water as the steam condenses. If the water level in *a* sinks below the end of the pipe the water runs out and live steam fills the pipe again. This action is utilized in safety devices by Black and Warner, and by Schwartzkopf.

In the blast furnace the fluid iron with the slag floating upon it forms an inverted siphon which checks the blast. In the Bessemer converter the air pressure is so great that the iron is kept in agitation by the air bubbling through it.

*See Trans. Am. Soc. Mech. Engrs, Vol. IV, p. 268. C. C. Collins, Balanced Valves.

† In this statement is included such fluids as do not mingle by simple contact. In this sense steam and water will not mingle, and if they are not of the same temperature the warmer will be transferred to the other. Air and water will not mingle because the water has become saturated with air. According to the researches of Colladon & Sturm (Memoire sur la compression des liquides, 1827, reprinted by Schuchart, Geneva, 1887), the saturation of water with air appears to partake of the nature of an internal, chemical combination. As might be expected, water which is saturated with air shows a smaller compressibility in the Piezometer than water which is free from air, being 48.65 millionths to 49.65 millionths. The combination of air with water ceases upon heating to the boiling point.

‡ Natural inverted siphons with branches of varying levels exist in the case of artesian wells.

In gas holders the water in the tank forming the seal is a fluid valve of the inverted siphon type (compare Fig. 948*b*),

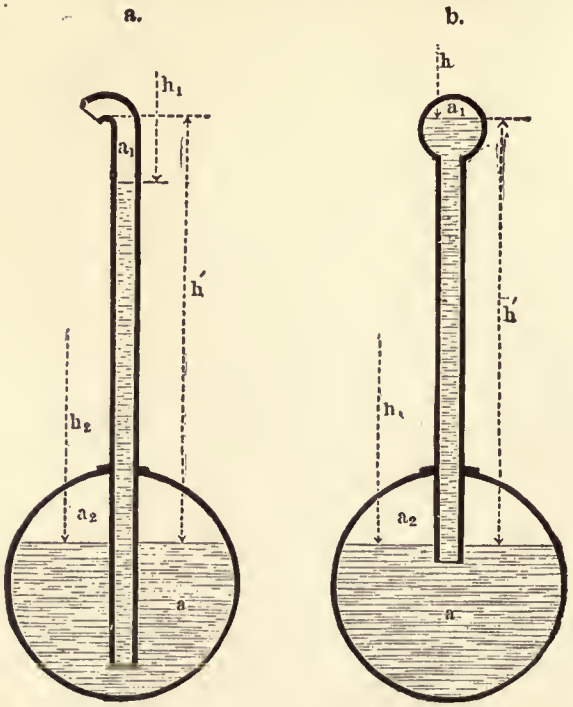


FIG. 1177.

and a similar device is used with sand instead of water in Hoffman's furnace, Fig. 1178, in which a_1 is air, and a_2 smoke, the the bell-shaped lid being sealed with an annular valve of sand.

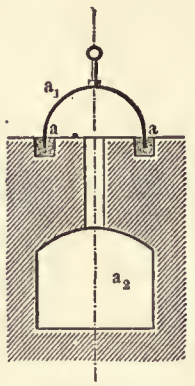


FIG. 1178.

Fig. 1179 is Wilson's water gas furnace.* In this a mixture of waste-slack and water forms a fluid valve. The mixture is propelled by an endless screw and discharged at the end. The atmosphere is at a_1 and the gas at a_2, the latter being kept under pressure by a steam jet.

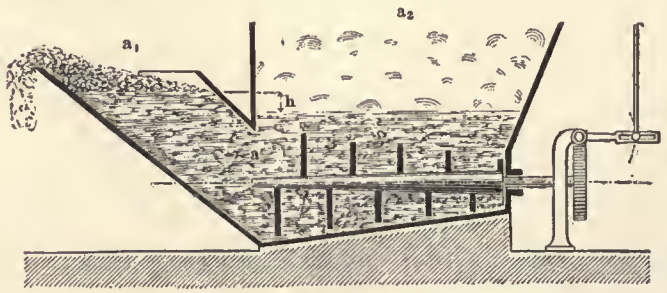


FIG. 1179.

Hero's Fountain, Fig. 1180, consists of two inverted siphon valves, in which a_1 and a_3 have air at the atmospheric pressure, a_2 is air under pressure, and a is water (often Cologne water). The action continues until the column $h'_2 = h_2$.

*See A. Wilson, "Generation of Heating Gas, etc., Journal of Soc. of Chemical Industries, Manchester, Nov., 1883.

A practical application of the principle of Hero's fountain is the water trap of Morrison, Ingram & Co., Fig. 1181.† In this device there is a periodical action of fluid valves as follows: a stream of water flows into the tank F at E, gradually filling it, Fig. 1181*a*. The inner tube C, and fixed bell D, form an inverted siphon, the shorter branch of which is the space between

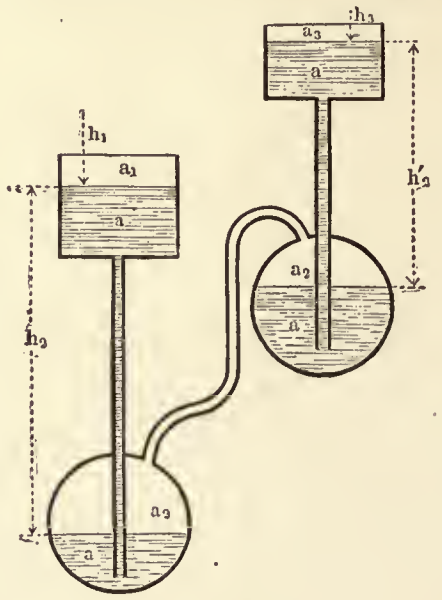


FIG. 1180.

C and D. As soon as the level of the water in the tank F rises above the top of C an overflow begins, filling the cup B, at the foot of the pipe C, and forming there a second siphon and making a seal between a_3 and a_2, Fig. 1181*b*. The two siphons now form a Hero's fountain, in which the contiuuing flow at E

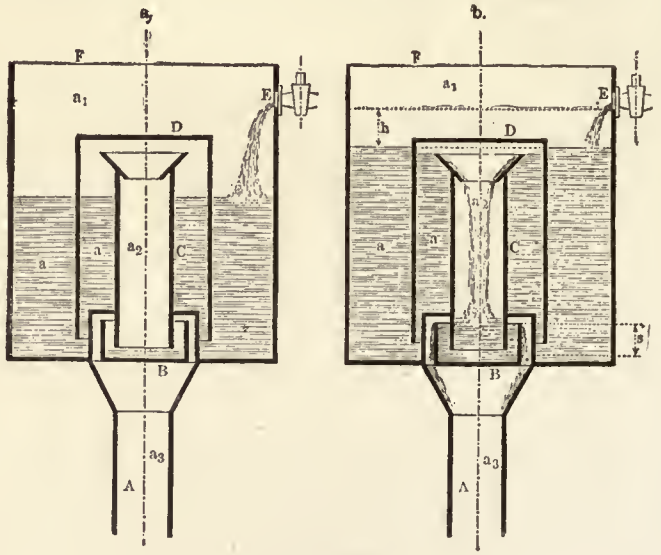


FIG. 1181.

causes an outflow into the discharge pipe A. As the level continues to rise iu F, the air in a_2 becomes more and more compressed, until finally the pressure column h becomes greater than the difference in level of the lower siphon, causing its discharge and consequent opening of the fluid valve into a_3. This relieves the pressure on the air iu a_2, thus permitting the upper siphon to act, and causing an immediate and rapid discharge of the contents of F. By adjusting the rate of flow at E this action can be regulated so as to take place periodically at any desired intervals of time.

Richard's manometer, Fig. 1182, consists of alternate direct and inverted siphons; a is quicksilver. a_1 steam, a_2 water and a_3 atmospheric air.

The spiral pump and the Cagniardelle shown in Fig. 966*a* and *b* contain successive fluid valves iu the same pipe, alternately direct and inverted.

Langen's device for discharging bone furnaces of the hot granular burnt bone, is a ratchet system iuvolviug valves con-

†See Revue Industrielle, June, 1888, p. 226.

sisting of a granular pressure organ, Fig. 1183. The discharge pipe *d* of the furnace is closed at the bottom by the sliding plate *c* which is given a reciprocating movement (in this instance operated by a small hydraulic motor). This plate *c* is made with a step as shown in the figure at *a*, which receives a layer of the material, and on the return stroke, as shown at *b*,

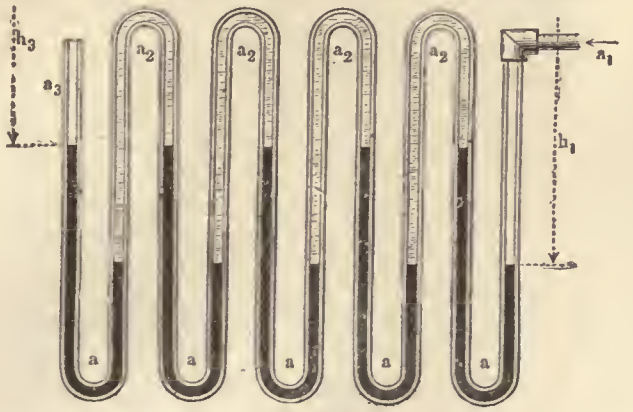


FIG. 1182.

this layer is discharged on the plate. This layer forms a suction valve when acting as at *a*, and a discharge valve, as at *b*, while the plate *c* corresponds to a single acting piston, considering the whole as a pump. If the plate *c* is made with a middle rib, as shown in Fig. 1190*c*, it works both ways and

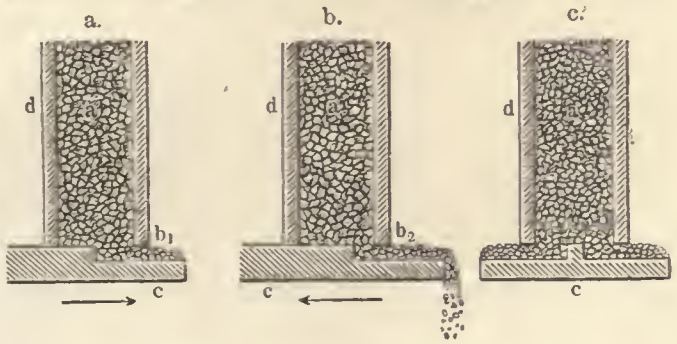


FIG. 1183.

becomes a double-acting pump. This is an illustration of the fluid valve in its most general form as applied to a pump.

In many instances fluid valves are as good and sometimes even better than valves composed of rigid materials. Especially is this the case when they act continuously in one direction in in a free, open pipe, for which purpose they excel all other forms of valves, as in jet pumps and the like (see Fig. 972).

§ 378.

STATIONARY VALVES.

We have thus far considered valves as ratchets for pressure organs, when they operate so as to check the motion of the fluid at the intervals of time (see § 365). If we consider this definition in its most general sense we may take it to include certain kinds of fastenings for closing apertures, and call these also valves. These we may distinguish from ordinary valves by the fact that they are not operated by the motion of the machine, and hence to them may be given the name of "stationary valves."

Stationary lift valves are found in the lids of steam cylinders, these belonging to the class of disk valves. These are required to resist internal pressure, and must therefore be securely bolted in place, the pressure being generally great, and resisted by the bolts. Steam chest covers are generally rectangular, flat, stationary valves, and an example of a stationary flap valve is seen in the valve chest door shown in Fig. 1128, this also being secured by means of bolts. Furnace doors, such as shown in Fig. 763, also belong to this class. The more readily such a valve is opened and closed the more nearly it approaches in construction to the movable valves, and packing is sometimes omitted in order to facilitate opening and closing. The valve chest lids, shown in Fig. 1131, are readily recognized, these being readily slipped into place and held by a yoke, or so-called "gallows screw." Numerous forms of stationary valves are also found in various kinds of bottle stoppers, these being effective substitutes for the older cork stoppers which often were held in place only by friction. Stationary fluid valves are also occasionally still found in use for bottle stoppers in parts of Italy and Greece.

In all the cases thus far mentioned the fastening by which the stationary valve is held in place must be at least slightly stronger than the pressure beneath the valve.

As a stationary valve in which this is not the case, we have the ordinary manhole plate as used in steam boilers, Fig. 1184. In this the pressure acts to hold the plate to its seat. Other examples are found in the spring valves used in the so-called siphons of soda water, and the particular form of bottle stopper which consists of a small ball valve held up to the mouth of the bottle by the pressure within. Stationary slide valves are less frequently used than lift valves, as the conditions are less favorable for proper packing, but examples are to be found. It will be seen by the instances already given how far reaching into all branches of machine design the use of ratchets for pressure organs extends.

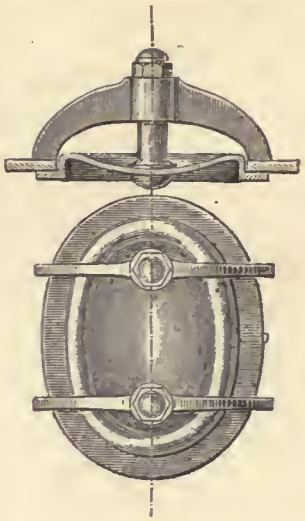


FIG. 1184.

§ 379.

STATIONARY MACHINE ELEMENTS IN GENERAL.

It is not a peculiarity of valves alone to be used conveniently in the "stationary" form in the sense discussed in the preceding section. Here, as we have arrived at the close of the book, it is desirable to review the preceding pages in this respect. In the first four chapters of Section III the subjects considered are nearly always used as stationary elements.

Rivets do not differ in form from cylindrical journals, but they are generally stationary because of two conditions; because of the firm binding of the surrounding metal, and because there are generally two or more rivets placed side by side. If only single rivet is used and no impediment to movement introduced, the binding of the metal would soon give way to any forces tending to cause rotation.

Forced connections resemble journals and their bearings in form. The force by which the external piece grasps the internal one effectively resists all forces acting to produce rotation. Keyed connections are especially adapted for stationary service. The particular examples shown in Figs. 618 and 619 are in fact stationary keys in form, although really special cases of spiral gear wheels. Screws, in by far the greater number of cases, are used as stationary elements, probably in a greater variety of applications, broadly considered, than any other machine element. In § 86 a glance is given at the use of the screw as an active machine element.

Journals are frequently conveniently used as stationary elements, as in the examples illustrated in Figs. 251, 252, 253, 256, 257 and 258. In § 90 we have already distinguished between "journals at rest" and "running journals," the former corresponding to the definition of stationary elements. Roller bearings for bridge truss supports, § 198, are also stationary elements.

Crank connections are found in the bottle stoppers already mentioned, and in numerous other applications such connections are properly considered as stationary elements, Here wheels are rarely used as stationary elements, but such applications are frequently found of ratchet wheels. Longitudinal keys used to secure hubs upon their axles are almost invariably stationary elements, practically corresponding to "stationary ratchets," as a comparison between Figs. 188 and 654 will show. Ratchets also find numerous applications in stationary mechanism for securing bolts, keys and the like. An examination of Figs. 237 to 243 and 246 to 248 will illustrate this point. In the couplings shown in Figs. 423 to 430 we also have a number of stationary ratchets (see also Fig. 678).

In § 309 I have referred to the possibility of using pressure organs as standing or "stationary" elements, but these are as yet unimportant. The pipes used as conductors for pressure organs, however, furnish numerous instances of pressure organs.

The above distinctions are by no means merely theoretical, but are of a highly practical nature. Every means which will enable us to obtain a clearer and better comprehension of the use of machine elements should be most welcome.

In the preceding arrangement the stationary elements have therefore been grouped together for this end. It follows that those forms which as "stationary" or "passive" elements are extensively used in building and civil works, as well as in machine design, forming the connecting links between the works of the civil and the mechanical engineer.

SECTION IV.

MATHEMATICAL TABLES.

§ 380.

Tables of Curves, Areas and Volumes.

The following tables give in convenient form the most important geometrical and mechanical properties of the more useful curves, areas and volumes. The significance of the letters used in the formulæ will be found indicated on the diagrams. The following remarks are also to be noted.

By the rectification of a curve is meant the length s of that portion of the curve from the origin to the point x y, corresponding to the angle ϕ; and by S is meant the entire length of the curve.

In the moment of inertia the mass of the body is assumed $= 1$, in order to reduce the number of letters. In view of the importance of this subject a few points are here given. The moments of inertia for surfaces are both equatorial and polar, each referred an axis of moments. This latter is called an equatorial axis when it lies in the plane of the surface, and a polar axis when it is at right angles to the surface. Each equatorial which passes through the centre of gravity is especially termed an equator-axis, and a polar axis which passes through the centre of gravity is called a pole axis. Every surface, therefore, has but one pole-axis, and an infinite number of equator axes. The moment of inertia is called equatorial or polar, according to the axis to which it is referred.

The moment of inertia J_p for any surface referred to the polar axis is found by adding together the two equatorial moments of inertia J_{q1} and J_{q2}, the axes of which intersect each other at right angles in the polar axis:

$$J_p = J_{q1} + J_{q2} \quad \ldots\ldots\ldots\ldots\ldots (416)$$

The moment of inertia J' of a surface, referred to any axis situated at a distance a, from the centre of gravity S, is found from the moment of inertia J referred to a parallel axis through S, by the following relation:

$$J' = J + a^2 F \quad \ldots\ldots\ldots\ldots\ldots (417)$$

in which F is the area of the surface. This relation also holds good for solids, if the mass of the body is substituted for F.

For solids one of the preceding conditions does not hold. For each different shape one of the axes which passes through the centre of gravity, is taken as the pole-axis for all sections normal to it, and the section at right angles to this axis which passes through the centre of gravity is called the Equatorial Section, whence the equatorial and polar moments of inertia are in these cases distinguished according to their position with regard to this equatorial section. In all the examples of solids here given, the actual equatorial and polar axes are meant.

For a right prism, of any given base having as the polar moment of inertia i_p and the half-height $= l$, the polar moment of inertia is:

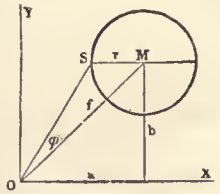
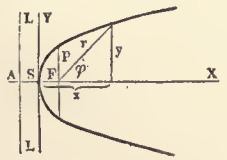
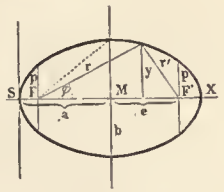
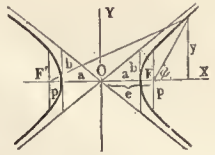
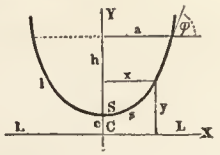
$$J_p = 2\, l\, i_p \quad \ldots\ldots\ldots\ldots\ldots (418)$$

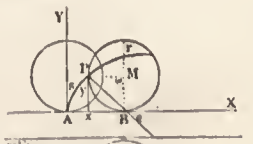
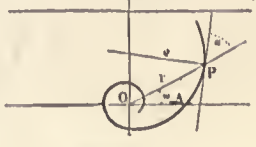
and the moment of inertia referred to an equatorial axis:

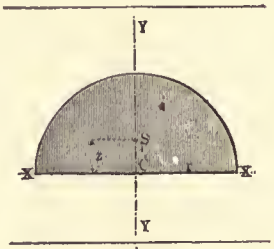
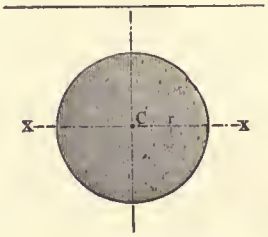
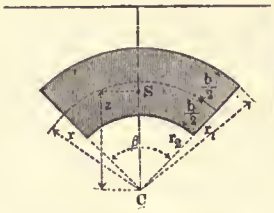
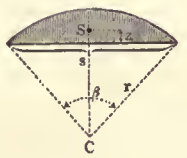
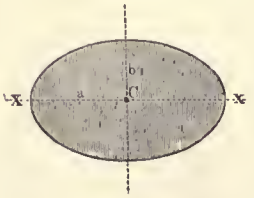
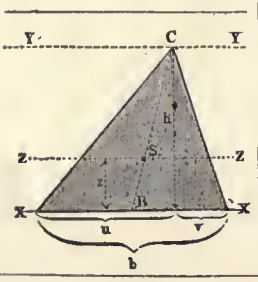
$$J_p = \tfrac{2}{3} f l^3 + 2\, l\, i_q \quad \ldots\ldots\ldots\ldots\ldots (419)$$

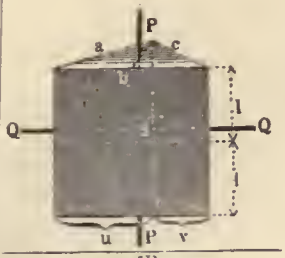
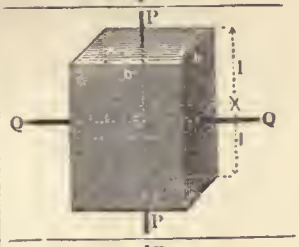
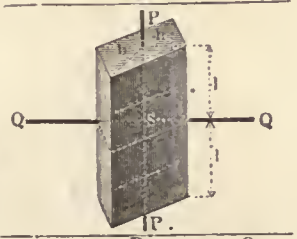
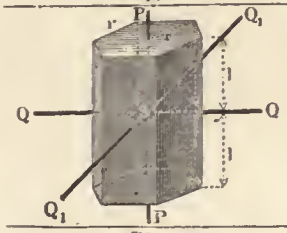
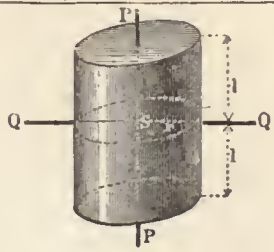
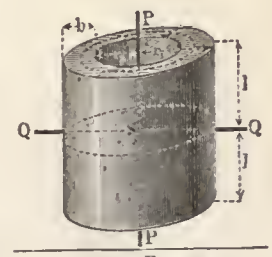
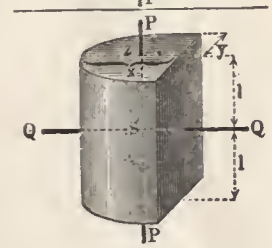
in which f is the area of the cross section, and i_q the equatorial moment of inertia of the cross section referred to the same axis as J_q.

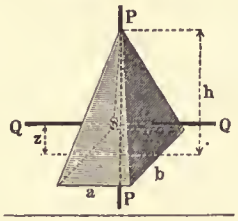
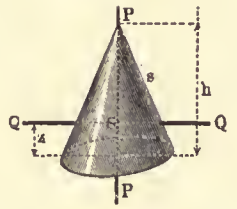
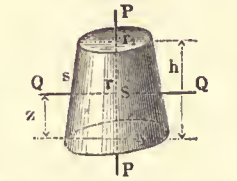
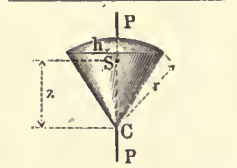
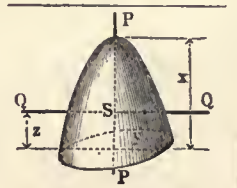
The centre of gravity and the moment of inertia for a surface of irregular form is often readily obtained by grapho-static methods, with sufficient numerical accuracy. For this purpose the force and cord polygons are applicable according to the methods already described in Section II.

No.	Curve.	Rectangular Equation.	Special Properties.	Polar Equation.	Radius of Curvature.	Rectification.
I. Circle.		From O: $(x-a)^2+(y-b)^2=r^2$. From S: $y^2=2rx-x^2$. From M: $x^2+y^2=r^2$.	Approximately, when $\frac{x}{y}$ is small: $\frac{r}{y}=\frac{y}{2x}$	From O: $\rho^2+f^2-2\rho f\cos\varphi=r^2$. From S: $\rho=2r\cos\varphi$.	$\rho=r$.	$s=r\varphi$ $S=2\pi r$
II. Parabola.		From S: $y^2=2px$.	Semi-parameter $=p$ $AS=SF=\frac{p}{2}$ $LL=$ Directrix. $F=$ Focus.	From F: $r=\frac{\rho}{2}+x$ $r=\frac{\rho}{2\sin^2\frac{\varphi}{2}}$	$\rho=p\sqrt{\left(1+2\frac{x}{p}\right)^3}$	$s=\frac{p}{2}\left[\sqrt{\frac{2x}{p}\left(1+\frac{2x}{p}\right)}+\log nat\left(\sqrt{\frac{2x}{p}}+\sqrt{1+\frac{2x}{p}}\right)\right]$ Approximately, when $\frac{x}{y}$ is small: $s=y\left[1+\frac{2}{3}\left(\frac{x}{y}\right)^2-\frac{2}{5}\left(\frac{x}{y}\right)^4\right]$
III. Ellipse.		From M: $y^2a^2+x^2b^2=a^2b^2$. $y^2=\left(\frac{b}{a}\right)^2(2ax-x^2)$ From S: $y^2=2px-\frac{p}{a}x^2$.	Eccentricity $=e=\sqrt{a^2-b^2}$ Numerically $=\varepsilon=\frac{e}{a}$ Semi-parameter: $p=a(1-\varepsilon^2)=\frac{b^2}{a}$.	From F: $r=\rho+ex=\frac{a^2-e^2}{a-e\cos\varphi}$ $=\frac{p}{1-\varepsilon\cos\varphi}$ Radii: $r=a+\varepsilon x$; $r'=a-\varepsilon x$	$\rho=\frac{(rr_1)^{\frac{3}{2}}}{ab}$ For S: $\rho=\frac{b^2}{a}$ For A: $\rho=\frac{a^2}{b}$	$S=\pi(a+b)\left(1+\frac{n^2}{4}+\frac{n^4}{64}+\frac{n^6}{256}+\ldots\right)$, in which $n=\frac{a-b}{a+b}$
IV. Hyperbola.		From O: $-y^2a^2+x^2b^2=a^2b^2$.	Eccen. $=OF=e=\sqrt{a^2+b^2}$ Numerically $\varepsilon=\frac{e}{a}$. Axis $b=a\sqrt{\varepsilon^2-1}$ Semi-param. $=a(\varepsilon^2-1)=\frac{b^2}{a}$.	From F: $r=\rho+\varepsilon x=\frac{e^2-a^2}{a-e\cos\varphi}$ $=\frac{p}{1-\varepsilon\cos\varphi}$ Radii: $r=\varepsilon x-a$, $r'=\varepsilon x+a$.	$\rho=\frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4}$	Very complicated.
V. Catenary.		From O: $y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right)$	$\tan\phi=\frac{l}{c}\sqrt{\frac{h^2+2hc}{c^2}}$ $LL=$ Directrix.	—	$\rho=\frac{y^2}{c}$	$s=\frac{c}{2}\left(e^{\frac{x}{c}}-e^{-\frac{x}{c}}\right)$ $=\sqrt{y^2-c^2}$, $l=\sqrt{h^2+2hc}$

No.	Curve.	Rectangular Equation.	Rectification.	Radius of Curvature.	Remarks.
VI. Cycloid.		$x = r\left(\omega - \frac{r'}{r}\sin\omega\right)$ $y = r\left(1 - \frac{r'}{r}\cos\omega\right)$	$s = 4r\left(1 - \cos\frac{\omega}{2}\right)$ $S = 8r$	$\rho = 4r\sin\frac{\omega}{2}$	r' is the radius to the describing point. When $r = r'$, as here assumed in the rectification and curvature formulæ the equations become those of the common orthocycloid, epicycloid, hypocycloid and pericycloid. Also in the equation of the evolute R' is the distance of the describing point from O when $\vartheta = O$; if $R' = R$ the curve becomes the common evolute. The length of the common cardoid is: $S = 8 \times 2R = 2.546\ (2\pi R)$ Or approximately $0.81\ \pi\ (2\pi R)$
VII. Epicycloid.		$x = (R+r)\cos\frac{r}{R}\omega - r'\cos\frac{R+r}{R}\omega$ $y = (R+r)\sin\frac{r}{R}\omega - r'\sin\frac{R+r}{R}\omega$	$s = 4r\frac{R+r}{R}\left(1 - \cos\frac{\omega}{2}\right)$ $S = 8r\frac{R+r}{R}$	$\rho = 4r\frac{R+r}{R+2r}\sin\frac{\omega}{2}$	
VIII. Hypocycloid.		$x = (R-r)\cos\frac{r}{R}\omega + r'\cos\frac{R-r}{R}$ $y = (R-r)\sin\frac{r}{R}\omega - r'\sin\frac{R-r}{R}\omega$	$s = 4r\frac{R-r}{R}\left(1 - \cos\frac{\omega}{2}\right)$ $S = 8r\frac{R-r}{R}$	$\rho = 4r\frac{R-r}{R-2r}\sin\frac{\omega}{2}$	
IX. Pericycloid.		$x = (r-R)\cos\frac{r}{R}\omega - r'\cos\frac{r-R}{R}\omega$ $y = (r-R)\sin\frac{r}{R}\omega - r'\sin\frac{r-R}{R}\omega$	$s = 4r\frac{r-R}{R}\left(1 - \cos\frac{\omega}{2}\right)$ $S = 8r\frac{r-R}{R}$	$\rho = 4r\frac{r-R}{2r-R}\sin\frac{\omega}{2}$	
X. Evolute.		$x = R'\cos\vartheta + R\vartheta\sin\vartheta$ $y = R'\sin\vartheta - R\vartheta\cos\vartheta$	$s = \frac{R\vartheta}{2}\vartheta$	$\rho = R\vartheta$	s, for 1 revolution $= \pi\ (2\pi R)$ s, for 2 revolutions $= 4\pi\ (2\pi R)$ s, for n revolutions $= n^2\pi\ (2\pi R)$
XI. Archimedian Spiral.		$r = R\omega$	$s = \frac{R}{2}[\omega\sqrt{1+\omega^2} + log\ nat\ (\omega + \sqrt{1+\omega^2})]$	$\rho = \frac{(r^2+a^2)^{\frac{3}{2}}}{2(r^2+2a^2)}$	The Archimedian spiral is the prolonged evolute of R. s, for 1 revolution $= 1.08\pi\ (2\pi R)$ s, for 2 revolutions $= 4.09\pi\ (2\pi R)$ s, for n revolutions $= n^2\pi\ (2\pi R)$ approximately.
XII. Logarithmic Spiral.		$r = a^{\omega}$	$s = r\frac{\sqrt{1+(log\ nat\ a)^2}}{log\ nat\ a}$	$\rho = r\sqrt{1+(log\ nat\ a)^2}$ $= \frac{r}{\sin a}$, whence $cot\ a = log\ nat\ a.$	In the logarithmic spiral the tangent at any point P makes the angle a with OP, whence $cot\ a = log\ nat\ a$.

No.	Form.	Area.	Centre of Gravity.	Moment of Inertia.
XIII. Sector.		$F = r^2 \frac{\beta}{2}$	$z = \frac{4}{3} r \frac{\sin \frac{\beta}{2}}{\beta}$ $= \frac{s\, r^2}{3\, F}$	For polar axis through C: $J_c = \frac{\mu}{\beta} r^2 = \frac{r^4 \beta}{4}$ For Pole axis through centre of gravity S: $J_s = \frac{\mu}{2} r^2 \left(1 - \frac{1 - \cos \beta}{\beta^2}\right) = \frac{r^4}{4}\left(\beta - \frac{1 - \cos \beta}{\beta}\right)$
XIV. Semicircle.		$F = \frac{\pi r^2}{2}$	$z = \frac{4}{3\pi} r$	For polar axis through C: $J_c = \frac{\mu}{2} r^2 = \frac{\pi}{4} r^4$ For polar axis through S: $J_s = \frac{\mu}{2} r^2 \left(1 - \frac{2}{\pi^2}\right) = 0.398\, \mu\, r^2$ $= \frac{r^4}{4}\left(\pi - \frac{2}{\pi}\right) = 0.726\, r^4$ For the equatorial axes XX, YY: $J_x = J_y = \frac{\mu r^2}{4} = \frac{\pi}{8} r^4$.
XV. Circle.		$F = \pi r^2$	$z = o$	For Pole axis through C: $J_c = \frac{\mu}{2} r^2 = \frac{\pi}{2} r^4$ For Equator axis XX: $J_x = \frac{\mu}{4} r^2 = \frac{\pi}{4} r^4$
XVI. Portion of Ring.		$F = (r_1^2 - r_2^2)\frac{\beta}{2} = b\, r\, \beta$	$z = \frac{4}{3} \frac{\sin \frac{\beta}{2}}{\beta} \frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}$ $= \frac{\sin \frac{\beta}{2}}{\beta} r \left(2 + \frac{1}{6} \frac{b^2}{r^2}\right)$	For polar axis through C: $J_c = \frac{\mu}{2}\left(r_1^2 + r_2^2\right) = \mu\left(r^2 + \frac{b^2}{4}\right)$ $= \frac{\beta}{4}\left(r_1^4 - r_2^4\right) = \frac{\beta}{4}\left(4\, b\, r^3 + r\, b^3\right)$
XVII. Segment.		$F = \frac{r^2}{2}(\beta - \sin \beta)$	$z = \frac{s^3}{12\, F}$ $= \frac{4}{3} r \frac{\sin^3 \frac{\beta}{2}}{\beta - \sin \beta}$	For polar axis through C: $J_c = \frac{r^4 \beta}{4} - \frac{1}{4}\left[s\, r^3 \cos^3 \beta + \frac{r\, s^3 \cos \beta}{12}\right]$ $= \frac{r^4}{4}\left[\beta - 2 \sin \beta \cos^3 \beta - \frac{2}{3} \cos \beta \sin^3 \beta\right]$.
XVIII. Parabola.		$F = \frac{2}{3} x\, y$	$z_1 = \frac{3}{5} x$ $z_2 = \frac{3}{8} y$	For equatorial axes XX and YY: $J_x = \mu \frac{y^2}{5} = \frac{8}{15} x\, y^3$ $J_y = \frac{8}{35} \mu\, x^2 = \frac{32}{105} y\, x^3$.
XIX. Ellipse.		$F = \pi\, a\, b$	$z = o$	For Equator axis XX: $J_x = \frac{\pi}{4} a\, b^3$ For Pole axis C: $J_c = \frac{\mu}{4}(a^2 + b^2) = \frac{\pi}{4} b\, a^3 \left(1 + \frac{b^2}{a^2}\right)$
XX. Triangle.		$F = \frac{b\, h}{2}$	$z = \frac{h}{3}$	$J_x = \mu \frac{h^2}{6} = \frac{b\, h^3}{12}$; $J_z = \mu \frac{h^2}{18} = \frac{b\, h^3}{36}$ $J_y = \mu \frac{h^2}{2} = \frac{b\, h^3}{4}$; $J_c = \frac{b\, h^3}{4} + \frac{h}{12}(u^3 + v^3)$ $J_b = \frac{b\, h^3}{12} + \frac{h}{4}(u^3 + v^3) - \frac{b\, h}{24}[8(u^2 + v^2) - 3\, b^2]$ $J_s = \frac{b\, h^3}{36} + \frac{h}{12}(u^3 + v^3) - \frac{b\, h}{18}[2(u^2 + v^2) - b^2]$

No.	Form.	Surface.	Volume.	Centre of Gravity.	Moment of Inertia.
XXI. Triangular Prism.		Sides: $F_1 = 2\,l\,(a+b+c)$ One end: $F_2 = \frac{b\,h}{2}$	$V = b\,h\,l$	Centre of Figure.	For Equator axis $Q\,Q$: $J_q = m\left[\frac{l^2}{3}+\frac{h^2}{18}\right] = \frac{b\,h\,l^3}{3}+\frac{b\,l\,h^3}{81}$ For Pole axis $P\,P$: $J_p = m\left[\frac{b\,h^3}{18}+\frac{h}{6}(u^3+v^3)-\frac{b\,h}{9}\right.$ $\left.(2\,[u^2+v^2]-b^2)\right]$
XXII. Rectangular Prism.		Sides: $F_1 = 4\,l\,(b+h)$ One end: $F_2 = b\,h$	$V = 2\,b\,h\,l$	Centre of Figure.	For Equator axis $Q\,Q$: $J_q = m\left(\frac{l^2}{3}+\frac{h^2}{12}\right)$ For Pole axis $P\,P$: $J_p = \frac{m}{12}(h^2+b^2)$
XXIII. Rhombic Prism.		Sides: $F_1 = 8\,l\sqrt{h^2+\frac{b^2}{4}}$ One end: $F_2 = b\,h$	$V = 2\,b\,h\,l$	Centre of Figure.	For Equator axis $Q\,Q$: $J_q = m\left(\frac{l^2}{3}+\frac{h^2}{6}\right)$ For Pole axis $P\,P$: $J_p = m\left(\frac{h^2}{6}+\frac{6^2}{24}\right)$
XXIV. Hexagonal Prism.		Sides: $F_1 = 12\,l\,r$ One end: $F_2 = \frac{3}{2}r^2\sqrt{3} = 2.598\,r^2$	$V = 3\,l\,r^2\sqrt{3} = 5.196\,l\,r^2$	Centre of Figure.	For Equator axes $Q\,Q$ and $Q_1\,Q_1$: $J_q = m\left(\frac{l^3}{3}+\frac{5}{24}r^2\right)$ For Pole axis $P\,P$: $J_p = \frac{5}{12}m\,r^2$
XXV. Cylinder.		Vertical surface: $F_1 = 4\,\pi\,l\,r$ One end: $F_2 = \pi\,r^2$	$V = 2\,\pi\,l\,r^2$	Centre of Figure.	For Equator axis $Q\,Q$: $J_q = m\left(\frac{l^2}{3}+\frac{r^2}{4}\right)$ For Pole axis $P\,P$: $J_p = \frac{1}{2}m\,r^2$
XXVI. Hollow Cylinder.		Vertical surface: $F_1 = 4\,\pi\,l\,(r_1+r_2) = 8\,\pi\,l\,r$ One end: $F_2 = \pi\,(r_1^2-r_2^2) = 2\,\pi\,r\,b$	$V = 2\,\pi\,l\,(r_1^2-r_2^2) = 4\,\pi\,r\,b\,l$	Centre of Figure.	For Equator axis $Q\,Q$: $J_q = m\left[\frac{l^2}{3}+\frac{r_1^2+r_2^2}{4}\right]$ $= m\left[\frac{l^2}{3}+\frac{r^2}{2}+\frac{b^2}{8}\right]$ For Pole axis $P\,P$: $J_p = \frac{m}{2}[r_1^2+r_2^2] = m\left[r^2+\frac{b^2}{4}\right]$
XXVII. Parabolic Prism.		One end: $F_2 = \frac{4}{3}x\,y$	$V = \frac{8}{3}l\,x\,y$	$z = \frac{3}{5}x$	For Equator axis $Q\,Q$: $J_q = m\left[\frac{l^2}{3}+\frac{8}{35}x^2\right]$ For Pole axis $P\,P$: $J_p = m\left[\frac{l^2}{5}+\frac{8}{35}x^2\right]$

No.	Form.	Surface.	Volume.	Centre of Gravity	Moment of Inertia.
XXVIII. Globoid Ring.		$F = 4\pi^2 R r$	$V = 2\pi^2 R r$	Centre of Figure.	For Equator axis $Q\,Q$: $J_q = m\left[\frac{R^2}{2} + \frac{5}{8} r^2\right]$ For Pole axis $P\,P$: $J_p = m\left[R^2 + \frac{3}{4} r^2\right]$
XXIX. Rectangular Pyramid.		Sides: $F_1 = a\sqrt{h^2 + \frac{b^2}{4}} + b\sqrt{h^2 + \frac{a^2}{4}}$ Bottom: $F_2 = a\,b$.	$V = F_2 \frac{h}{3} = \frac{a\,b\,h}{3}$	$z = \frac{h}{4}$	For Equator axis $Q\,Q$: $J_q = m\left[\frac{3}{80} h^2 + \frac{b^2}{20}\right]$ For Pole axis $P\,P$: $J_p = \frac{m}{20}[a^2 + b^2]$
XXX. Right Cone.		Inclined surface: $F_1 = \pi r\sqrt{h^2 + r^2} = \pi r s$ Bottom: $F_2 = \pi r^2$	$V = \frac{\pi r^2 h}{3}$	$z = \frac{h}{4}$ For the surface only $z' = \frac{h}{3}$	For Equator axis $Q\,Q$: $J_q = \frac{3}{20} m\left[r^2 + \frac{h^2}{4}\right]$ For Pole axis $P\,P$: $J_p = \frac{3}{10} m r^2$
XXXI. Truncated Cone.		Sides: $F_1 = \pi(r_1 + r_2)\sqrt{h^2 - (r_1 - r_2)^2} = 2\pi r s$ Ends: $F_2' = r_1^2\pi$, $F_2'' = r_2^2\pi$	$V = \frac{\pi}{3} h[r_1^2 + r_1 r_2 + r_2^2]$	$z = \frac{h}{4}\left(\frac{r_1^2 + 2r_1 r_2 + 3r_2^2}{r_1^2 + r_1 r_2 + r_2^2}\right)$	For Pole axis $P\,P$: $J_p = \frac{3}{10} m \frac{r_1^5 - r_2^5}{r_1^3 - r_2^3}$
XXXII. Sphere.		$F = 4r^2\pi$	$V = \frac{4\pi}{3} r^3$	Centre of Figure.	For Equator axis $Q\,Q$: $J = \frac{2}{5} m r^2$
XXXIII. Sector of Sphere.		Conical surface: $F_1 = a\pi r = \pi r\sqrt{2rh - h^2}$	$V = \frac{2}{3}\pi r^2 h$	$z = \frac{3}{4}\left(r - \frac{h}{2}\right)$	For Pole axis $P\,P$: $J_p = \frac{m}{5}(3rh - h^2)$
XXXIV. Segment of Sphere.		Curved surface: $F_1 = 2\pi r h = \pi(a^2 + h^2)$ Bottom: $F_2 = a^2\pi$, $r = \frac{a^2 + h^2}{2h}$	$V = \pi h^2\left(r - \frac{h}{3}\right)$ $= \frac{\pi}{6} h(3a^2 + h^2)$	$z = \frac{3}{4}\frac{(2r - h)^2}{3r - h}$ For the surface only: $z' = \frac{h}{2}$	For Pole axis $P\,P$: $J_p = m\left[r^2 - \frac{3}{4} rh + \frac{3}{20} h^2\right]\frac{2h}{3r - h}$
XXXV. Spheroid.		——	$V = \frac{4}{3}\pi a b c$	Centre of Figure.	For Equator axis $Q\,Q$, coincident with a: $J_a = \frac{m}{5}(b^2 + c^2)$
XXXVI. Paraboloid of Revolution.		Bottom: $F_2 = \pi y^2$	$V = \frac{\pi}{2} x y^2$	$z = \frac{x}{3}$	For Equator axis $Q\,Q$: $J_q = m\left(\frac{y^2}{6} + \frac{x^2}{18}\right)$ For Pole axis $P\,P$: $J_p = \frac{m}{3} y^2$

§ 381.

Trigonometrical Table.

The following table contains, in convenient form, the sines, cosines, tangents and cotangents for angles from 0° to 90° for every ten minutes, and also the corresponding arcs to a radius of unity. At the foot of the table arcs are also given for small angles and also for some of the more frequently used angles greater than 90°.

Angle. deg.	Angle. min.	arc.	sine.	cosine.	tan.	cot.	arc.	Angle. deg.	Angle. min.
0	0	0.0000	0.0000	1.0000	0.0000	∞	1.5708	90	0
	10	0.0029	0.0029	1.0000	0.0029	343.77	1.5679		50
	20	0.0058	0.0058	1.0000	0.0058	171.89	1.5650		40
	30	0.0087	0.0087	1.0000	0.0087	114.59	1.5621		30
	40	0.0116	0.0116	0.9999	0.0116	85.940	1.5592		20
	50	0.0145	0.0145	0.9999	0.0145	68.750	1.5563		10
1	0	0.0175	0.0175	0.9998	0.0175	57.290	1.5533	89	0
	10	0.0204	0.0204	0.9998	0.0204	49.104	1.5504		50
	20	0.0233	0.0233	0.9997	0.0233	42.964	1.5475		40
	30	0.0262	0.0262	0.9997	0.0262	38.188	1.5446		30
	40	0.0291	0.0291	0.9996	0.0291	34.368	1.5417		20
	50	0.0320	0.0320	0.9995	0.0320	31.242	1.5388		10
2	0	0.0349	0.0349	0.9994	0.0349	28.636	1.5359	88	0
	10	0.0378	0.0378	0.9993	0.0378	26.432	1.5330		50
	20	0.0407	0.0407	0.9992	0.0407	24.542	1.5301		40
	30	0.0436	0.0436	0.9990	0.0437	22.904	1.5271		30
	40	0.0465	0.0465	0.9989	0.0466	21.470	1.5243		20
	50	0.0495	0.0494	0.9988	0.0495	20.206	1.5213		10
3	0	0.0524	0.0523	0.9986	0.0524	19.081	1.5184	87	0
	10	0.0553	0.0552	0.9985	0.0553	18.075	1.5155		50
	20	0.0582	0.0581	0.9983	0.0582	17.169	1.5126		40
	30	0.0611	0.0610	0.9981	0.0612	16.350	1.5097		30
	40	0.0640	0.0640	0.9980	0.0641	15.605	1.5068		20
	50	0.0669	0.0669	0.9978	0.0670	14.924	1.5039		10
4	0	0.0698	0.0698	0.9976	0.0699	14.301	1.5010	86	0
	10	0.0727	0.0727	0.9974	0.0729	13.727	1.4981		50
	20	0.0756	0.0756	0.9971	0.0758	13.197	1.4951		40
	30	0.0785	0.0785	0.9969	0.0787	12.706	1.4923		30
	40	0.0814	0.0814	0.9967	0.0816	12.251	1.4893		20
	50	0.0844	0.0843	0.9964	0.0846	11.826	1.4864		10
5	0	0.0873	0.0872	0.9962	0.0875	11.430	1.4835	85	0
	10	0.0902	0.0901	0.9959	0.0904	11.059	1.4806		50
	20	0.0931	0.0929	0.9957	0.0934	10.712	1.4777		40
	30	0.0960	0.0958	0.9954	0.0963	10.385	1.4748		30
	40	0.0989	0.0987	0.9951	0.0992	10.078	1.4719		20
	50	0.1018	0.1016	0.9948	0.1022	9.7882	1.4690		10
6	0	0.1047	0.1045	0.9945	0.1051	9.5144	1.4661	84	0
	10	0.1076	0.1074	0.9942	0.1080	9.2553	1.4632		50
	20	0.1105	0.1103	0.9939	0.1110	9.0098	1.4603		40
	30	0.1134	0.1132	0.9936	0.1139	8.7769	1.4573		30
	40	0.1164	0.1161	0.9932	0.1169	8.5555	1.4544		20
	50	0.1193	0.1190	0.9929	0.1198	8.3450	1.4515		10
7	0	0.1222	0.1219	0.9925	0.1228	8.1443	1.4486	83	0
	10	0.1251	0.1248	0.9922	0.1257	7.9530	1.4457		50
	20	0.1280	0.1276	0.9918	0.1287	7.7704	1.4428		40
	30	0.1309	0.1305	0.9914	0.1317	7.5958	1.4399		30
	40	0.1338	0.1334	0.9911	0.1346	7.4287	1.4370		20
	50	0.1367	0.1363	0.9907	0.1376	7.2687	1.4341		10
8	0	0.1396	0.1392	0.9903	0.1405	7.1154	1.4312	82	0
	10	0.1425	0.1421	0.9899	0.1435	6.9682	1.4283		50
	20	0.1454	0.1449	0.9894	0.1465	6.8269	1.4254		40
	30	0.1484	0.1478	0.9890	0.1495	6.6912	1.4224		30
	40	0.1526	0.1507	0.9886	0.1524	6.5606	1.4195		20
	50	0.1542	0.1536	0.9881	0.1554	6.4348	1.4166		10
9	0	0.1571	0.1564	0.9877	0.1584	6.3138	1.4137	81	0
	10	0.1600	0.1593	0.9872	0.1614	6.1970	1.4108		50
	20	0.1629	0.1622	0.9868	0.1644	6.0844	1.4079		40
	30	0.1658	0.1650	0.9863	0.1673	5.9758	1.4050		30
	40	0.1687	0.1679	0.9858	0.1703	5.8708	1.4021		20
	50	0.1716	0.1708	0.9853	0.1733	5.7694	1.3992		10

Angle. deg.	Angle. min.	arc.	sine.	cosine.	tan.	cot.	arc.	Angle. deg.	Angle. min.
10	0	0.1745	0.1736	0.9848	0.1763	5.6713	1.3963	80	0
	10	0.1774	0.1765	0.9843	0.1793	5.5764	1.3934		50
	20	0.1804	0.1794	0.9838	0.1823	5.4845	1.3904		40
	30	0.1833	0.1822	0.9833	0.1853	5.3955	1.3875		30
	40	0.1862	0.1851	0.9827	0.1883	5.3093	1.3846		20
	50	0.1891	0.1880	0.9822	0.1914	5.2257	1.3817		10
11	0	0.1920	0.1908	0.9816	0.1944	5.1446	1.3788	79	0
	10	0.1949	0.1937	0.9811	0.1974	5.0658	1.3759		50
	20	0.1978	0.1965	0.9805	0.2004	4.9894	1.3730		40
	30	0.2007	0.1994	0.9799	0.2035	4.9152	1.3701		30
	40	0.2036	0.2022	0.9793	0.2065	4.8430	1.3672		20
	50	0.2065	0.2051	0.9787	0.2095	4.7729	1.3643		10
12	0	0.2094	0.2079	0.9781	0.2126	4.7046	1.3614	78	0
	10	0.2123	0.2108	0.9775	0.2156	4.6382	1.3584		50
	20	0.2153	0.2136	0.9769	0.2186	4.5736	1.3555		40
	30	0.2182	0.2164	0.9763	0.2217	4.5107	1.3526		30
	40	0.2211	0.2193	0.9757	0.2247	4.4494	1.3497		20
	50	0.2240	0.2221	0.9750	0.2278	4.3897	1.3468		10
13	0	0.2269	0.2250	0.9744	0.2309	4.3315	1.3439	77	0
	10	0.2298	0.2278	0.9737	0.2339	4.2747	1.3410		50
	20	0.2327	0.2306	0.9730	0.2370	4.2193	1.3381		40
	30	0.2356	0.2334	0.9724	0.2401	4.1653	1.3352		30
	40	0.2385	0.2363	0.9717	0.2432	4.1126	1.3323		20
	50	0.2414	0.2391	0.9710	0.2462	4.0611	1.3294		10
14	0	0.2443	0.2419	0.9703	0.2493	4.0108	1.3264	76	0
	10	0.2473	0.2447	0.9696	0.2524	3.9617	1.3235		50
	20	0.2502	0.2476	0.9689	0.2555	3.9136	1.3206		40
	30	0.2531	0.2504	0.9681	0.2586	3.8667	1.3177		30
	40	0.2560	0.2532	0.9674	0.2617	3.8208	1.3148		20
	50	0.2589	0.2560	0.9667	0.2648	3.7760	1.3119		10
15	0	0.2618	0.2588	0.9659	0.2679	3.7321	1.3090	75	0
	10	0.2647	0.2616	0.9652	0.2711	3.6891	1.3061		50
	20	0.2667	0.2644	0.9644	0.2742	3.6470	1.3032		40
	30	0.2705	0.2672	0.9636	0.2773	3.6059	1.3003		30
	40	0.2734	0.2700	0.9628	0.2805	3.5656	1.2974		20
	50	0.2763	0.2728	0.9621	0.2836	3.5261	1.2945		10
16	0	0.2793	0.2756	0.9613	0.2867	3.4874	1.2915	74	0
	10	0.2822	0.2784	0.9605	0.2899	3.4495	1.2886		50
	20	0.2851	0.2812	0.9596	0.2931	3.4124	1.2857		40
	30	0.2880	0.2840	0.9588	0.2962	3.3759	1.2828		30
	40	0.2909	0.2868	0.9580	0.2994	3.3402	1.2799		20
	50	0.2938	0.2896	0.9572	0.3026	3.3052	1.2770		10
17	0	0.2967	0.2924	0.9563	0.3057	3.2709	1.2741	73	0
	10	0.2996	0.2952	0.9555	0.3089	3.2371	1.2712		50
	20	0.3025	0.2979	0.9546	0.3121	3.2041	1.2683		40
	30	0.3054	0.3007	0.9537	0.3153	3.1716	1.2654		30
	40	0.3083	0.3035	0.9528	0.3185	3.1397	1.2625		20
	50	0.3113	0.3062	0.9520	0.3217	3.1084	1.2595		10
18	0	0.3142	0.3090	0.9511	0.3249	3.0777	1.2566	72	0
	10	0.3171	0.3118	0.9502	0.3281	3.0475	1.2537		50
	20	0.3200	0.3145	0.9492	0.3314	3.0178	1.2508		40
	30	0.3229	0.3173	0.9483	0.3346	2.9887	1.2479		30
	40	0.3258	0.3201	0.9474	0.3378	2.9600	1.2450		20
	50	0.3287	0.3228	0.9465	0.3411	2.9319	1.2421		10
19	0	0.3316	0.3256	0.9455	0.3443	2.9042	1.2392	71	0
	10	0.3345	0.3283	0.9446	0.3476	2.8770	1.2363		50
	20	0.3374	0.3311	0.9436	0.3508	2.8502	1.2334		40
	30	0.3403	0.3338	0.9426	0.3541	2.8239	1.2305		30
	40	0.3432	0.3365	0.9417	0.3574	2.7980	1.2275		20
	50	0.3462	0.3393	0.9407	0.3607	2.7725	1.2246		10

Angle.	arc.	cosine.	sine.	cot.	tan.	arc.	Angle.	Angle.	arc.	cosine.	sine.	cot.	tan.	arc.	Angle.

ANGLE.		arc.	sine.	cosine.	tan.	cot.	arc.	ANGLE.	
deg.	min.							deg.	min.
20	0	0.3491	0.3420	0.9397	0.3640	2.7475	1.2217	70	0
	10	0.3520	0.3448	0.9387	0.3673	2.7228	1.2188		50
	20	0.3549	0.3475	0.9377	0.3706	2.6985	1.2159		40
	30	0.3578	0.3502	0.9367	0.3739	2.6746	1.2130		30
	40	0.3607	0.3529	0.9356	0.3772	2.6511	1.2101		20
	50	0.3636	0.3557	0.9346	0.3805	2.6279	1.2072		10
21	0	0.3665	0.3584	0.9336	0.3839	2.6051	1.2043	69	0
	10	0.3694	0.3611	0.9325	0.3872	2.5826	1.2014		50
	20	0.3723	0.3638	0.9315	0.3906	2.5605	1.1985		40
	30	0.3752	0.3665	0.9304	0.3939	2.5386	1.1955		30
	40	0.3782	0.3692	0.9293	0.3973	2.5172	1.1926		20
	50	0.3811	0.3719	0.9283	0.4006	2.4960	1.1897		10
22	0	0.3840	0.3746	0.9272	0.4040	2.4751	1.1868	68	0
	10	0.3869	0.3773	0.9261	0.4074	2.4545	1.1839		50
	20	0.3898	0.3800	0.9250	0.4108	2.4342	1.1810		40
	30	0.3927	0.3827	0.9239	0.4142	2.4142	1.1781		30
	40	0.3956	0.3854	0.9228	0.4176	2.3945	1.1752		20
	50	0.3985	0.3881	0.9216	0.4210	2.3750	1.1723		10
23	0	0.4014	0.3907	0.9205	0.4245	2.3559	1.1694	67	0
	10	0.4043	0.3934	0.9194	0.4279	2.3369	1.1664		50
	20	0.4072	0.3961	0.9182	0.4314	2.3183	1.1636		40
	30	0.4102	0.3987	0.9171	0.4348	2.2998	1.1606		30
	40	0.4131	0.4014	0.9159	0.4383	2.2817	1.1577		20
	50	0.4160	0.4041	0.9147	0.4417	2.2637	1.1548		10
24	0	0.4189	0.4067	0.9135	0.4452	2.2460	1.1519	66	0
	10	0.4218	0.4094	0.9124	0.4487	2.2286	1.1490		50
	20	0.4247	0.4120	0.9112	0.4522	2.2113	1.1461		40
	30	0.4276	0.4147	0.9100	0.4557	2.1943	1.1432		30
	40	0.4305	0.4173	0.9088	0.4592	2.1775	1.1403		20
	50	0.4334	0.4200	0.9075	0.4628	2.1609	1.1374		10
25	0	0.4363	0.4226	0.9063	0.4663	2.1445	1.1345	65	0
	10	0.4392	0.4253	0.9051	0.4699	2.1283	1.1310		50
	20	0.4421	0.4279	0.9038	0.4734	2.1123	1.1280		40
	30	0.4451	0.4305	0.9026	0.4770	2.0965	1.1257		30
	40	0.4480	0.4331	0.9013	0.4806	2.0809	1.1228		20
	50	0.4509	0.4358	0.9001	0.4841	2.0655	1.1199		10
26	0	0.4538	0.4384	0.8988	0.4877	2.0503	1.1170	64	0
	10	0.4567	0.4410	0.8975	0.4913	2.0353	1.1141		50
	20	0.4596	0.4436	0.8962	0.4950	2.0204	1.1112		40
	30	0.4625	0.4462	0.8949	0.4986	2.0057	1.1082		30
	40	0.4654	0.4488	0.8936	0.5022	1.9912	1.1054		20
	50	0.4683	0.4514	0.8923	0.5059	1.9768	1.1025		10
27	0	0.4712	0.4540	0.8910	0.5095	1.9626	1.0996	63	0
	10	0.4741	0.4566	0.8897	0.5132	1.9486	1.0966		50
	20	0.4771	0.4592	0.8884	0.5169	1.9347	1.0937		40
	30	0.4800	0.4617	0.8870	0.5206	1.9210	1.0908		30
	40	0.4829	0.4643	0.8857	0.5243	1.9074	1.0879		20
	50	0.4858	0.4669	0.8843	0.5280	1.8940	1.0850		10
28	0	0.4887	0.4695	0.8820	0.5317	1.8807	1.0821	62	0
	10	0.4916	0.4720	0.8816	0.5354	1.8676	1.0792		50
	20	0.4945	0.4746	0.8802	0.5392	1.8546	1.0763		40
	30	0.4974	0.4772	0.8788	0.5430	1.8418	1.0734		30
	40	0.5003	0.4797	0.8774	0.5467	1.8291	1.0705		20
	50	0.5032	0.4823	0.8760	0.5505	1.8165	1.0676		10
29	0	0.5061	0.4848	0.8746	0.5543	1.8040	1.0647	61	0
	10	0.5091	0.4874	0.8732	0.5581	1.7917	1.0617		50
	20	0.5120	0.4899	0.8718	0.5619	1.7796	1.0588		40
	30	0.5149	0.4924	0.8704	0.5658	1.7675	1.0559		30
	40	0.5178	0.4950	0.8689	0.6696	1.7556	1.0530		20
	50	0.5207	0.4975	0.8675	0.5735	1.7437	1.0501		10
30	0	0.5236	0.5000	0.8660	0.5774	1.7321	1.0472	60	0
	10	0.5265	0.5025	0.8646	0.5812	1.7205	1.0443		50
	20	0.5294	0.5050	0.8631	0.5851	1.7090	1.0414		40
	30	0.5323	0.5075	0.8616	0.5890	1.6977	1.0385		30
	40	0.5352	0.5100	0.8601	0.5930	1.6864	1.0356		20
	50	0.5381	0.5125	0.8587	0.5969	1.6753	1.0326		10
Angle.		arc.	cosine.	sine.	cot.	tan.	arc.	Angle.	

ANGLE.		arc.	sine.	cosine.	tan.	cot.	arc.	ANGLE.	
deg.	min.							deg.	min.
31	0	0.5411	0.5150	0.8572	0.6009	1.6643	1.0297	59	0
	10	0.5440	0.5175	0.8557	0.6048	1.6534	1.0268		50
	20	0.5469	0.5200	0.8542	0.6088	1.6426	1.0239		40
	30	0.5498	0.5225	0.8526	0.6128	1.6319	1.0210		30
	40	0.5527	0.5250	0.8511	0.6168	1.6212	1.0181		20
	50	0.5556	0.5275	0.8496	0.6208	1.6107	1.0152		10
32	0	0.5585	0.5299	0.8480	0.6249	1.6003	1.0123	58	0
	10	0.5614	0.5324	0.8465	0.6289	1.5900	1.0094		50
	20	0.5643	0.5348	0.8450	0.6330	1.5798	1.0065		40
	30	0.5672	0.5373	0.8434	0.6371	1.5697	1.0036		30
	40	0.5701	0.5398	0.8418	0.6412	1.5597	1.0007		20
	50	0.5730	0.5422	0.8403	0.6453	1.5497	0.9977		10
33	0	0.5760	0.5446	0.8387	0.6494	1.5399	0.9948	57	0
	10	0.5787	0.5471	0.8371	0.6536	1.5301	0.9919		50
	20	0.5818	0.5495	0.8355	0.6577	1.5204	0.9890		40
	30	0.5847	0.5519	0.8339	0.6619	1.5108	0.9861		30
	40	0.5876	0.5544	0.8323	0.6661	1.5013	0.9832		20
	50	0.5905	0.5568	0.8307	0.6703	1.4919	0.9803		10
34	0	0.5934	0.5592	0.8290	0.6745	1.4826	0.9774	56	0
	10	0.5963	0.5616	0.8274	0.6787	1.4733	0.9745		50
	20	0.5992	0.5640	0.8258	0.6830	1.4641	0.9716		40
	30	0.6021	0.5664	0.8241	0.6873	1.4550	0.9687		30
	40	0.6050	0.5688	0.8225	0.6916	1.4460	0.9657		20
	50	0.6080	0.5712	0.8208	0.6959	1.4370	0.9628		10
35	0	0.6109	0.5736	0.8192	0.7002	1.4281	0.9599	55	0
	10	0.6138	0.5760	0.8175	0.7046	1.4173	0.9570		50
	20	0.0167	0.5783	0.8158	0.7089	1.4106	0.9541		40
	30	0.6 96	0.5807	0.8141	0.7133	1.4019	0.9512		30
	40	0.0225	0.5831	0.8124	0.7177	1.3934	0.9483		20
	50	0.6254	0.5854	0.8107	0.7221	1.3848	0.9455		10
36	0	0.6283	0.5878	0.8090	0.7265	1.3764	0.9425	54	0
	10	0.6312	0.5901	0.8073	0.7310	1.3680	0.9306		50
	20	0.6341	0.5925	0.8056	0.7355	1.3597	0.9367		40
	30	0.6370	0.5948	0.8039	0.7400	1.3514	0.9338		30
	40	0.6400	0.5972	0.8021	0.7445	1.3432	0.9308		20
	50	0.6429	0.5995	0.8004	0.7490	1.3351	0.9279		10
37	0	0.6458	0.6018	0.7986	0.7536	1.3270	0.9250	53	0
	10	0.6487	0.6041	0.7969	0.7581	1.3190	0.9221		50
	20	0.6516	0.6065	0.7951	0.7627	1.3111	0.9192		40
	30	0.6545	0.6088	0.7934	0.7673	1.3032	0.9163		30
	40	0.6574	0.6111	0.7916	0.7720	1.2954	0.9134		20
	50	0.6603	0.6134	0.7898	0.7766	1.2876	0.9105		10
38	0	0.6632	0.6157	0.7880	0.7813	1.2799	0.9076	52	0
	10	0.6661	0.6180	0.7862	0.7860	1.2723	0.9947		50
	20	0.6690	0.6202	0.7844	0.7907	1.2647	0.9018		40
	30	0.6720	0.6225	0.7826	0.7954	1.2572	0.8988		30
	40	0.6749	0.6248	0.7808	0.8002	1.2497	0.8959		20
	50	0.6778	0.6271	0.7790	0.8050	1.2423	0.8930		10
39	0	0.6807	0.6293	0.7771	0.8098	1.2349	0.8901	51	0
	10	0.6836	0.6316	0.7753	0.8146	1.2276	0.8872		50
	20	0.6865	0.6338	0.7735	0.8195	1.2203	0.8843		40
	30	0.6894	0.6361	0.7716	0.8243	1.2131	0.8814		30
	40	0.6923	0.6383	0.7698	0.8292	1.2059	0.8785		20
	50	0.6952	0.6406	0.7679	0.8342	1.1988	0.8756		10
40	0	0.6981	0.6428	0.7660	0.8391	1.1918	0.8727	50	0
	10	0.7010	0.6450	0.7642	0.8441	1.1847	0.8698		50
	20	0.7039	0.6472	0.7623	0.8491	1.1778	0.8668		40
	30	0.7069	0.6494	0.7604	0.8541	1.1708	0.8639		30
	40	0.7098	0.6517	0.7585	0.8591	1.1640	0.8610		20
	50	0.7127	0.6539	0.7566	0.8642	1.1571	0.8581		10
41	0	0.7156	0.6561	0.7547	0.8693	1.1504	0.8552	49	0
	10	0.7185	0.6583	0.7528	0.8744	1.1436	0.8523		50
	20	0.7214	0.6604	0.7509	0.8796	1.1369	0.8494		40
	30	0.7243	0.6626	0.7490	0.8847	1.1303	0.8465		30
	40	0.7272	0.6648	0.7470	0.8899	1.1237	0.8436		20
	50	0.7301	0.6670	0.7451	0.8952	1.1171	0.8407		10
Angle.		arc.	cosine.	sine.	cot.	tan.	arc.	Angle.	

ANGLE. deg.	min.	arc.	sine.	cosine.	tan.	cot.	arc.	ANGLE. deg.	min.
42	0	0.7330	0.6691	0.7431	0.9004	1.1106	0.8378	48	0
	10	0.7359	0.6713	0.7412	0.9057	1.1041	0.8348		50
	20	0.7389	0.6734	0.7392	0.9110	1.0977	0.8319		40
	30	0.7418	0.6756	0.7373	0.9163	1.0913	0.8290		30
	40	0.7447	0.6777	0.7353	0.9217	1.0850	0.8261		20
	50	0.7476	0.6799	0.7333	0.9271	1.0786	0.8232		10
43	0	0.7505	0.6820	0.7314	0.9325	1.0724	0.8203	47	0
	10	0.7534	0.6841	0.7294	0.9380	1.0661	0.8174		50
	20	0.7563	0.6862	0.7274	0.9435	1.0599	0.8145		40
	30	0.7592	0.6884	0.7254	0.9490	1.0538	0.8116		30
	40	0.7621	0.6905	0.7234	0.9545	1.0477	0.8087		20
	50	0.7650	0.6926	0.7214	0.9601	1.0416	0.8058		10
Angle.		arc.	cosine.	sine.	cot.	tan.	arc.	Angle.	

ANGLE. deg.	min.	arc.	sine.	cosine.	tan.	cot.	arc.	ANGLE. deg.	min.
44	0	0.7679	0.6947	0.7193	0.9657	1.0355	0.8029	46	0
	10	0.7709	0.6967	0.7173	0.9713	1.0295	0.7999		50
	20	0.7738	0.6988	0.7153	0.9770	1.0235	0.7970		40
	30	0.7767	0.7009	0.7133	0.9827	1.0176	0.7941		30
	40	0.7795	0.7030	0.7112	0.9884	1.0117	0.7912		20
	50	0.7824	0.7050	0.7092	0.9942	1.0058	0.7883		10
45	0	0.7854	0.7071	0.7071	1.0000	1.0000	0.7854		0
Angle.		arc.	cosine.	sine.	cot.	tan.	arc.	Angle.	

ang. =	0° 1′	0° 5′	135°	180°	225°	270°	315°	360°
arc. =	0.0003	0.0015	2.3562	3.1416	3.9270	4.7124	5.4978	6.2832

TRIGONOMETRICAL FORMULÆ.

1. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
2. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
3. $\sin 2\alpha = 2\sin\alpha\cos\alpha$
4. $\sin 3\alpha = 3\sin\alpha - 4\sin\alpha^3 = \sin\alpha\,(4\cos\alpha^2 - 1)$
5. $\cos 2\alpha = \cos\alpha^2 - \sin\alpha^2 = 2\cos\alpha^2 - 1 = 1 - 2\sin\alpha^2$
6. $\cos 3\alpha = 4\cos\alpha^3 - 3\cos\alpha = \cos\alpha\,(1 - 4\sin\alpha^2)$
7. $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
8. $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
9. $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
10. $\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$
11. $\sin\alpha^2 = \frac{1}{2}(1 - \cos 2\alpha)$
12. $\cos\alpha^2 = \frac{1}{2}(1 + \cos 2\alpha)$
13. $\sin\alpha^3 = \frac{1}{4}(3\sin\alpha - \sin 3\alpha)$
14. $\cos\alpha^3 = \frac{1}{4}(3\cos\alpha + \cos 3\alpha)$
15. $\operatorname{tang}(\alpha \pm \beta) = \frac{\operatorname{tang}\alpha \pm \operatorname{tang}\beta}{1 \mp \operatorname{tang}\alpha\operatorname{tang}\beta}$
16. $\operatorname{cotang}(\alpha \pm \beta) = \frac{\operatorname{cotang}\alpha\operatorname{cotang}\beta \mp 1}{\pm\operatorname{cotang}\alpha + \operatorname{cotang}\beta}$
17. $\operatorname{tang} 2\alpha = \frac{2\operatorname{tang}\alpha}{1 - \operatorname{tang}\alpha^2}$
18. $\operatorname{cotang} 2\alpha = \frac{\operatorname{cotang}\alpha^2 - 1}{2\operatorname{cotang}\alpha}$
19. $\operatorname{tang}\alpha = \sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \frac{\sin 2\alpha}{1 + 2\cos\alpha} = \frac{2\operatorname{tang}\frac{1}{2}\alpha}{1 - \operatorname{tang}\frac{1}{2}\alpha^2}$
20. $\operatorname{cotang}\alpha = \sqrt{\frac{1+\cos 2\alpha}{1-\cos 2\alpha}} = \frac{\sin 2\alpha}{1-\cos 2\alpha} = \frac{\operatorname{cotang}\frac{1}{2}\alpha^2 - 1}{2\operatorname{cotang}\frac{1}{2}\alpha}$
21. $\operatorname{tang}\alpha \pm \operatorname{tang}\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha\cos\beta}$
22. $\operatorname{cotang}\alpha \pm \operatorname{cotang}\beta = \frac{\sin(\beta \pm \alpha)}{\sin\alpha\sin\beta}$
23. $\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\operatorname{tang}\frac{1}{2}(\alpha+\beta)}{\operatorname{tang}\frac{1}{2}(\alpha-\beta)}$

TABLE OF NUMBERS.—I.

n	$\frac{1}{n}$	n^2	n^3	$\sqrt{n}$	$\frac{1}{\sqrt{n}}$	$\sqrt[3]{n}$	$\frac{1}{\sqrt[3]{n}}$	$\sqrt[4]{n}$	$\frac{1}{\sqrt[4]{n}}$
0.30	3.333	0.090	0.027	0.548	1.826	0.669	1.495	0.740	1.351
0.375	2.667	0.141	0.053	0.612	1.633	0.721	1.387	0.783	1.278
0.60	1.667	0.360	0.216	0.775	1.291	0.843	1.186	0.880	1.136
0.625	1.600	0.391	0.244	0.791	1.265	0.855	1.170	0.889	1.125
0.70	1.429	0.490	0.343	0.837	1.195	0.888	1.126	0.915	1.093
0.75	1.333	0.563	0.422	0.866	1.155	0.909	1.100	0.931	1.075
0.875	1.143	0.766	0.670	0.935	1.069	0.956	1.046	0.974	1.024
0.90	1.111	0.810	0.729	0.949	1.054	0.965	1.036	0.987	1.013
1.10	0.909	1.210	1.331	1.049	0.953	1.032	0.969	1.024	0.976
1.2	0.833	1.440	1.728	1.095	0.913	1.063	0.941	1.047	0.955
1.25	0.800	1.563	1.953	1.118	0.894	1.077	0.928	1.057	0.946
1.50	0.667	2.250	3.375	1.225	0.816	1.145	0.874	1.107	0.904
1.75	0.571	3.063	5.359	1.323	0.756	1.205	0.830	1.150	0.869
2.0	0.500	4.0	8.0	1.414	0.707	1.260	0.794	1.189	0.841
2.25	0.444	5.063	11.391	1.500	0.667	1.310	0.763	1.225	0.816
2.50	0.400	6.250	15.625	1.581	0.632	1.357	0.737	1.257	0.795
2.75	0.364	7.563	20.797	1.658	0.603	1.401	0.714	1.288	0.777
3.0	0.333	9.0	27.0	1.732	0.577	1.442	0.693	1.318	0.759
3.25	0.308	10.563	34.328	1.803	0.555	1.481	0.675	1.342	0.745
3.50	0.286	12.250	42.875	1.871	0.535	1.518	0.659	1.368	0.731
3.75	0.267	14.063	52.734	1.936	0.516	1.554	0.644	1.392	0.719
4.0	0.250	16.0	64.0	2.0	0.500	1.587	0.630	1.414	0.707
4.5	0.222	20.250	91.125	2.121	0.471	1.651	0.606	1.457	0.687
5.0	0.200	25.0	125.0	2.236	0.447	1.710	0.585	1.495	0.669
5.5	0.182	30.250	166.375	2.345	0.426	1.765	0.567	1.531	0.653
6.0	0.167	36.0	216.0	2.449	0.408	1.817	0.550	1.565	0.639
6.5	0.154	42.25	274.625	2.550	0.392	1.866	0.536	1.597	0.626
7.0	0.143	49.0	243.0	2.646	0.378	1.913	0.523	1.627	0.615
7.5	0.133	56.250	421.875	2.739	0.365	1.957	0.510	1.655	0.604
8.0	0.125	64.0	512.0	2.828	0.354	2.0	0.500	1.682	0 595
8.5	0.118	72.250	614.125	2.915	0.343	2.041	0.490	1.707	0.586
9.0	0.111	81.0	729.0	3.000	0.333	2.080	0.481	1.732	0.577
9.5	0.105	90.250	857.375	3.082	0.324	2.118	0.472	1.756	0.570
10	0.100	100.0	1000.0	3.162	0.316	2.154	0.464	1.778	0.562
11	0.091	121.0	1331.0	3.317	0.302	2.224	0.450	1.821	0.549
12	0.083	144	1728	3.464	0.289	2.289	0.431	1.861	0.537
13	0.077	169	2197	3.606	0.277	2.351	0.425	1.899	0.527
14	0.071	196	2744	3.742	0.267	2.410	0.415	1.934	0.517
15	0.067	225	3375	3.873	0.258	2.466	0.405	1.968	0.508
16	0.063	256	4096	4.000	0.250	2.520	0.397	2.000	0.500
17	0.059	289	4913	4.123	0.243	2.571	0.389	2.031	0.492
18	0.056	324	5832	4.243	0.236	2.621	0.381	2.060	0.485
19	0.053	361	6859	4.359	0.229	2.668	0.375	2.088	0.479
20	0.050	400	8000	4.472	0.224	2.714	0.368	2.115	0.473
50	0.020	2500	125000	7.071	0.141	3.684	0.271	2.659	0.376
100	0.010	10000	1000000	10.0	0.10	4.642	0.215	3.162	0.316
1000	0.001	1000000	1000000000	31.623	0.032	10.0	0.100	5.623	0.178
$\pi = 3.142$	0.318	9.870	31.006	1.772	0.564	1.465	0.683	1.331	0.751
$2\pi = 6.283$	0.159	39.478	248.050	2.507	0.399	1 845	0.542	1.583	0.632
$\frac{\pi}{2} = 1.571$	0.637	2.467	3.878	1.253	0.798	1.162	0.860	1.120	0.893
$\frac{\pi}{3} = 1.047$	0.955	1.097	1.148	1.023	0.977	1.016	0.985	1.012	0.989
$\frac{4}{3}\pi = 4.189$	0.239	17.546	73.496	2.047	0.489	1.612	0.622	1.431	0.699
$\frac{\pi}{4} = 0.785$	1.274	0.617	0.484	0.886	1.128	0.923	1.084	0.941	1.062
$\frac{\pi}{6} = 0.524$	1.910	0.274	0.144	0.724	1.382	0.806	1.241	0.851	1.176
$\pi^2 = 9.870$	0.101	97.409	961.390	3.142	0.318	2.145	0.466	1.772	9.564
$\pi^3 = 31.006$	0.032	961.390	29809.910	5.568	1.796	3.142	0.318	2.360	0.424
$\frac{\pi}{32} = 0.098$	10.186	0.0095	0.001	0.313	3.192	0.461	2.168	0.560	1.782
$\frac{3\pi}{16} = 0.589$	1.698	0.347	0.204	0.768	1.303	0.838	1.194	0.876	1.142
$g = 32.2$	0.031	1036.84	33386.24	5.674	0.176	3.181	0.314	2.381	0.419
$2g = 64.4$	0.015	4147.36	267090	8.025	0.125	4.007	0.249	2.833	0.337

TABLE OF NUMBERS.—II.

n	$\sqrt{n}$	$\sqrt[3]{n}$	n	$\sqrt{n}$	$\sqrt[3]{n}$	n	$\sqrt{n}$	$\sqrt[3]{n}$	n	$\sqrt{n}$	$\sqrt[3]{n}$
0.01	0.10000	0.21544	0.26	0.50990	0.63825	0.51	0.71414	0.79896	0.76	0.87178	0.91258
0.02	0.14132	0.27144	0.27	0.51962	0.64633	0.52	0.72111	0.80415	0.77	0.87750	0.91657
0.03	0.17321	0.31072	0.28	0.52915	0.65421	0.53	0.72801	0.80927	0.78	0.88318	0.92052
0.04	0.20000	0.34200	0.29	0.53852	0.66191	0.54	0.73485	0.81433	0.79	0.88882	0.92443
0.05	0.22361	0.36840	0.30	0.54772	0.66943	0.55	0.74162	0.81932	0.80	0.89443	0.92832
0.06	0.24495	0.39149	0.31	0.55678	0.67679	0.56	0.74833	0.82426	0.81	0.90000	0.93217
0.07	0.26458	0.41213	0.32	0.56569	0.68399	0.57	0.75498	0.82913	0.82	0.90554	0.93599
0.08	0.28284	0.43089	0.33	0.57446	0.69104	0.58	0.76158	0.83396	0.83	0.91104	0.93978
0.09	0.30000	0.44814	0.34	0.58310	0.69795	0.59	0.76811	0.83872	0.84	0.91652	0.94354
0.10	0.31623	0.46416	0.35	0.59161	0.70473	0.60	0.77460	0.84343	0.85	0.92195	0.94727
0.11	0.33166	0.47914	0.36	0.60000	0.71138	0.61	0.78102	0.84809	0.86	0.92736	0.95097
0.12	0.34641	0.49324	0.37	0.60828	0.71791	0.62	0.78740	0.85270	0.87	0.93274	0.95464
0.13	0.36056	0.50658	0.38	0.61644	0.72432	0.63	0.79373	0.85726	0.88	0.93808	0.95828
0.14	0.37417	0.51925	0.39	0.62450	0.73061	0.64	0.80000	0.86177	0.89	0.94340	0.96190
0.15	0.38730	0.53133	0.40	0.63246	0.73681	0.65	0.80623	0.86624	0.90	0.94868	0.96549
0.16	0.40000	0.54288	0.41	0.64031	0.74290	0.66	0.81240	0.87066	0.91	0.95394	0.96905
0.17	0.41231	0.55397	0.42	0.64807	0.74889	0.67	0.81854	0.87503	0.92	0.95917	0.97259
0.18	0.42426	0.56462	0.43	0.65574	0.75478	0.68	0.82462	0.87937	0.93	0.96437	0.97610
0.19	0.43589	0.57489	0.44	0.66332	0.76059	0.69	0.83066	0.88366	0.94	0.97954	0.97959
0.20	0.44721	0.58480	0.45	0.67082	0.76631	0.70	0.83666	0.88790	0.95	0.97468	0.98305
0.21	0.45826	0.59439	0.46	0.67823	0.77194	0.71	0.84261	0.89211	0.96	0.97980	0.98648
0.22	0.46904	0.60368	0.47	0.68557	0.77750	0.72	0.84853	0.89628	0.97	0.98489	0.98990
0.23	0.47958	0.61269	0.48	0.69282	0.78297	0.73	0.85440	0.90041	0.98	0.98995	0.99329
0.24	0.48990	0.62145	0.49	0.70000	0.78837	0.74	0.86023	0.90450	0.99	0.99499	0.99666
0.25	0.50000	0.62996	0.50	0.70711	0.79370	0.75	0.86603	0.90856	1.00	1.00000	1.00000

$sin\ 30° = cos\ 60° = \frac{1}{2}$; $cos\ 30° = sin\ 60° = \frac{1}{2}\sqrt{3} = 0.8660$.

$sin\ 75° = cos\ 15° = 0.9659$; $tan\ 30° = cot\ 60° = \frac{1}{3}\sqrt{3} = 0.5774$;

$cos\ 75° = sin\ 15° = 0.2588$; $cotan\ 30° = tan\ 60° = \sqrt{3} = 1.7321$.

$log\ \pi = 0.4971499$. $log\ g = 1.507856$.

ALPHABETICAL INDEX.

ACCUMULATORS 264
" Hoppe's 264
" Hydraulic 218
" Tweddells' 265
Action of Gear Teeth 129
Adamson's Stiffening Ring 269
Addition and Subtraction of Forces 26
Addition, Graphical 26
Adjustable Escapements 170
" Gears for Rotative Motors 237
" Hangers 74
" Power Escapements ... 236
" Pump Gears 236
Admiralty Chain 182
Adyman's Coupling 215
Agudio's Cable Locomotive 176
Air Compressors, Riedler's 279
Air Pump, Bunsen's 222
" Von Gerike's 225
" Watt's 225
Air, Reservoir for Compressed 272
Allan's Link Motion 235
Almgren's Researches on Steam Boilers 271
Althaus' Furnace Hoist 173
Althaus' Pump 223
American Standard Car Bearing 75
Amos & Smyth's Pump 224
Anchor Escapement, Free 168
" Bolts 56
" Ratchet 155
Anemometers 239
Angle and T Iron Columns 83
Angle of Torsion, Determination of 93
Angle of Rotation in Torsion 11
Angström's Valve Gear 233
Anti-Friction Wheels 123
Anti-parallel 22
Anti-projection 23
Application of Tension Organs 172
Archimedes, Tympanon of 221
Archimedian Screw 221
Area of Polygons 23
" Quadrilaterals 24
" Triangles 23
Arithmography 22
Arm Sections, Table for Transforming 103
Arms of Gear Wheels 149
Armstrong Hydraulic Crane 228
Artificial Draft 272
Atmospheric Railway 227
Attachment of Journals 67
Audemar's Pump 224
Automatic Coupling 101
" Friction Brake 170
" Steam Stop 281
Axis, Neutral 3
Axle, American Railway Standard .. 89
" Prussian Railway Standard ... 89
" Simple Crank 106
Axles 85
" for Water Wheels 91
" Graphical Calculation of 86
" Loaded at Two Points 87
" Axles 107
" Non-Symmetrical 86
" Proof Diagrams of 87
" Proportions of 86
" Railway 88
" Symmetrical 85
" with Circular Section 85
" with Cruciform Section 90
" with Inclined Loads 90
Axles with Three or more Bearings. 89
" Wooden 92

BAG PUMP 217
Baker's Blower 221
Balanced Valves 279
Balance Wheel 167
Balancing of Pulleys 194
Balanced Slide Valves 285
Balanced Valve, Cramer's 280
Ball Bearings 127
Ball Joints for Pipes 249
Band Saws 177
Base Figures for Hyperboloidal Wheels 136
Bastard Gears 135
Beale's Gas Exhauster 226
Beams 3
" Double Trussed 35
" Sections, Table of 5–7
" Force Plans for Framed 38
" Multiple Trussed 35
" Scale 111
" Simple Trussed 35
" Triple Trussed 35
" Walking 110
" with Common Load 11
Bearings, Ball 127
" Design and Proportion of 68
" Independent Step 75
" Lateral 68
" Metaline 179
" Multiple Collar 77
" Multiple Supports for 80
" Pedestal 71
" Roller 126
" Roller for Bridges 126
" Simple Supports for 79
" Standard American Car ... 75
" Standard Prussian Car ... 75
" Special Forms of 74
" Step 75
Bearing Supports, General Principles 82
Bearings, Thrust 65, 68, 75
" Thrust with Wooden Surface 76
" Supports for 79
" Wall 68, 71
" Wall Step 75
" with Three-part Boxes ... 70
" Yoke 72
Becker's Clutch 101
Behren's Chamber Gear Train 220
Belidor's Water Pressure Engine . 229
Bell Crank 110
Bellegarde, Rope Transmission at .. 205
Belleville Elastic Washers 212
Bellows Pump 217
Bell Pipe Connections 248
Bell Valve 276
Belt Connections 191
" Fastening, Bötter's 193
" Fastening, Moxon's 193
Belting 186
" Cement for 193
" Efficiency of 194
" Specific Capacity of 190
" Stress on 191
" Various Examples of 187
" Table of Examples 192
Belt Lacing 191
Belts, Capacity of 190
" Creep of 194
" Cross Section of 190
" Path of 186
Belts, Polishing 177
" Quarter Twist 186
Belt Shifters 188
Belt Shifter, Zimmerman's 189
Belts, Stiffness of 194
Belt Transmission, Examples of 191
Belts, Transporting 221
Bending, Bodies of Uniform Resistance to 8
" Load 3
" Moment 3
" Resistance to 2
Bergner's Drawing Board 172
Berlin, Sewerage System of 219
Bevel Friction Wheels 124
" Friction Wheels, Minotto's ... 125
" Gears 135
" " Construction Circles for. 135
" " Spiral 141
" " Stepped 141
Beylich's Universal Gears 136
Biquadratic Parabola 10
Blake's Steam Pump 230
Bleichert's Cable Tramway System .. 175
Blower, Baker's 221
Blower, Root's 221
Blowers, Fan 222
Bloxam's Gravity Escapement 168
Boat, Sail 223
Boat, Chain Propulsion of 183
Bodies of Uniform Strength 2
Bogardus Mill 126
Boiler Construction, Economy in Combustion. 270
" " Economy of Material in 270
" " Improvements in Heating Surface ... 270
Boiler Details 266
" Feeder, Brindley's 228
" Feeders 228
" Flues 269
" Flues, Corrugated 269
" Riveting 42
Boilers, Almgren's Researches on .. 271
" Circumferential Seams of .. 268
" Classified 265–266
" Flat Surfaces of 268
" for Swedish State Railway. 272
" Longitudinal Seams of 267
" Thickness of 266
" Spherical Details 268
" Steam 265
Boiler Tubes 270
Boiling Water, used for Shrinking .. 47
Bolt Connections, Unloaded 60
" Dead 166
" Gerber's 57
" Heads 54
" Latch 166
Bolts, Anchor 56
" and Nuts, Metric 55
" and Screws 50
" Maudslay's Method of Securing 58
" Parsons' 57
" Penn's Method of Securing .. 57
" Special Forms of 55
Borda Turbine 220
Bored Guides 122
Bötter's Belt Fastening 193
Boxes, Various Forms of Journal ... 69
Brace, Weston's Ratchet 154
Bracket Support for Bearings 79

Brackets, Wall ... 72
Brake, Automatic Friction ... 170
Brake, Napier's Differential ... 214
Brakes, Chain ... 215
" Clamping ... 214
" Internal Strap ... 215
" Sliding ... 215
" Strap ... 211–214
Brake, Toggle Friction ... 162
Bramah Lock ... 166
Brasses for Connecting Rod ... 112
Brauer's Intermittent Gearing ... 165
Breaking Load ... 1
Bridge Bolts ... 59
" Flying ... 222
" Roller Bearings ... 126
" St. Louis ... 60
Briggs' System of Pipe Threads ... 250
Brindley's Boiler Feeder ... 228
Britton's Steering Gear ... 238
Brown's Valve Gear ... 235
Brown's Windlass ... 173
Buckling, Resistance to ... 13
Buckling Strains, Table of ... 14
Buffer Couplings ... 181
Built up Screw Propellers ... 57
Bunsen's Air Pump ... 222
Butler's Coupling ... 96

CABLE, Arrangement of Pulleys for 202
" Drum, Fowler's ... 173
" Ferry System, Hartwich's ... 175
" Grip Pawl ... 185
" Haulage Systems ... 174
" Incline at Lucerne ... 173
" Locomotive, Agudio's ... 176
" Railways ... 173
" Rhenish Railway ... 174
" Road, Kahlenberg ... 173
Cables, Table for Tightened ... 200
Cable System for Canals, Schmick's ... 175
" System, Riggenbach's ... 174
" Tramway, Chicago ... 175
" Tramway, Overhead ... 175
" Tramways, San Francisco ... 174
" Transmission, Ring System 208–211
" Transmissions, Short Span ... 200
" Transmission with Inclined ... 200
Cadiat Turbine ... 220
Cagniardelle ... 221
Cail & Co., Valve Gear ... 162
Calculating Machine, Thomas' ... 153, 156
Calculation of Springs ... 18
Calculations for Chains ... 183
Cambon's Roller Bearing ... 127
Cam Valve gears ... 236
Canal Cable System, Schmick's ... 175
" Lift at Les Fontinettes ... 227
" " Green's ... 227
" " La Louviere ... 227
" " Mersey ... 227
" Locks ... 227
Cannon, Thickness of ... 15
Capacity of Belts ... 190
Capstan, David's ... 173
Car Bearing, American Standard ... 75
Car Bearings, Prussian Standard ... 75
Cardan's Coupling ... 97
Cardioide ... 92
Casting ... 240
Cast Iron Cranks ... 105
Central Curve of Valve Gear ... 235
Centre of Gravity Graphically Determined ... 33
Centrifugal Force ... 177
" Force of Wire Rope ... 197
" Pumps ... 222
Chain, Admiralty ... 182
" Brakes ... 215
" Couplings ... 184
" Drums ... 185
" Flat Link ... 183
" Gemorsch ... 182
" Madgeburg-Bodenbacher ... 183
" Open Link ... 182
" Pawls for ... 185
" Pitch ... 183
Chain Propeller, Heuberger's ... 176
" Propelling Gear ... 187
" Proportions of ... 183
" Propulsion of Boats ... 183
Chains, Calculations for ... 183
Chain Sheaves ... 185, 211
Chain, Specific Capacity of ... 211
Chains, Running ... 182
" Stationary ... 182
" Tests for ... 183
Chain Strippers ... 185
" Swivels ... 184
" Transmission ... 211
" Transmission, Efficiency of ... 213
" " in Mines ... 213
" " Decido Mines ... 212
" Weight of ... 183
Chamber Gear Train, Behren's ... 220
" " " Eve's ... 220
" " " Repsold's ... 220
" Wheel Trains ... 219
Channeled Connecting Rods ... 117
Checking Ratchets ... 150, 163
Check Valves ... 274
Cheese Coupling ... 99, 151
Chemical Ratchet Trains ... 171
Cheret's Press, Friction Gear of ... 125
Chicago Cable Tramway ... 175
Chronometer Escapement ... 167
Chubb Lock ... 166
Circular Plate, Deflection of ... 15
Circumference Scale ... 128
Clamp Coupling ... 95
Clamping Brakes ... 214
Clamp Pulley, Fowler's ... 203
Clamp Ratchet ... 160
Clark's Canal Lift ... 227
Clerk, Method of Shrinking Rings ... 45
Clocks, Striking Mechanism for ... 169
Close Link Chain ... 182
Clutch, Becker's ... 101
" Cone ... 99
" Couplings ... 95, 98
" Coupling, Fossey's ... 100
" Dohmen-Leblanc's ... 101
" Forks ... 99
Clutches, Friction ... 99
Clutch, Garand's ... 101
" Jackson's ... 101
" Koechlin's Friction ... 100
" Napier's ... 101
" Reuleaux's Friction ... 100
" Schurmann's ... 101
" Toothed ... 98
" Weston's Friction ... 101
Coating Operations ... 241
Cock, Four Way ... 225
Cocks ... 281
Coefficients of Resistance ... 1
Coefficients of Safety ... 1
Cold Forcing ... 17
" Forcing, Dimensions for ... 47
" Hooping ... 45
Collar Thrust Bearings ... 66
Columns, Calculations for Iron ... 82
" Fluted ... 83
" Forms for Iron ... 84
" Grouped ... 84
" Hollow ... 83
" of Angle and T Iron ... 83
" of Uniform Resistance ... 13
" Strength of Cast Iron ... 83
" Stresses in ... 82
Combined Levers ... 110
Compound Escapements ... 168
" Link as Thrust Bearing ... 67
" Strains, Table of ... 15
" Stresses ... 13
Compressed Air for Power Distribution ... 219
Compression, Resistance to ... 2
Condenser, Watt's ... 230
Conditions of Equilibrium ... 29
Conductors for Pressure Organs ... 242
Conduits for Pressure Organs ... 216
Cone Clutch Coupling ... 99
Cone Coupling, Reuleaux's ... 96
" Pulleys ... 189
" " Diagram for ... 190
" " for Crossed Belts ... 189
" " for Open Belts ... 189
Conical Gear Wheels ... 135
Connecting Rod Brasses ... 112
" " End, Cast Iron ... 113
" " " Krauss' ... 113
" " " Penn's ... 113
" " " Polonceau's ... 114
" " Porter-Allen ... 117
" Rods ... 112
" " Channeled ... 117
" " Forms of ... 118
" " Locomotive ... 116
" " Rectangular Section 117
" Rod, Solid End for ... 113
" Rod, Solid End for Locomotive ... 113
" Rods, Ribbed ... 117
" Rods, Round ... 116
" Rod, Strap End for ... 112
" Rod, Whip Action of ... 116
Connections for Belting ... 191
" " Cast Iron Pipes ... 248
" " Crank Axles ... 115
" " Lead Pipe ... 251
" " Neck Journals ... 114
" " Wrought Iron Pipes 249
Construction Circles for Bevel Gears 135
Construction of Machine Elem'nts. 39–289
" Pulley Stations ... 204
" Rope Curve ... 202
" Rope Pulleys ... 202
" Screw Thread ... 50
Continuous Ratchets ... 150
" Ratchets with Locking Teeth ... 165
" Running Ratchets ... 164
Copper Pipes ... 246
Cord Friction ... 177
Cord Polygon ... 26
Corliss Valve ... 236
Corliss Valve Gear ... 162
Cornish Valve ... 280
Cornish Valve Gear ... 153, 163
Corrugated Boiler Flues ... 269
Corrugated Fire Box ... 269
Cotton Rope ... 179
Cotton Rope Transmission ... 196
Counterbalance, Oeking's ... 217
Countershaft Hanger, Sellers' ... 74
Counting Gear for Gas Meter ... 165
Couples, Force ... 29
Coupling, Adyman's ... 215
" Butler's ... 96
" Cardan's ... 97
" Cheese ... 99, 151
" Clamp ... 95
" Cresson's ... 96
" Drag Link ... 97
" Hooke's ... 97
" Muff ... 95
" Oldham's ... 96
" Plate ... 95
" Pouyer's ... 101
" Prentiss ... 216
" Ramsbottom's Friction ... 99
" Reuleaux's Conc ... 96
Couplings ... 95
" Automatic ... 101
" Buffer ... 181
" Schurman's Friction ... 215
" Clutch ... 98
Coupling, Sellers' ... 96
Couplings, Flexible ... 96
" for Chain ... 184
" for Propeller Shafts ... 95
" Sharp's ... 96
" Link ... 98
Coupling, Uhlhorn's ... 101
Cramer's Balanced Valve ... 280
Crane Hook, Proportions for ... 184
" Pillars ... 89
" Ramsbottom's ... 176

Cranes, Cotton Rope Driven..... 196
" Graphical Calculation..... 27
" Hydraulic..... 228
" Squaring Device for..... 172
" Varieties of..... 173
Crane, Tangye's..... 176
Crane, Towne's..... 176
Crank Axle, Graphostatic Calculation of..... 106
" Axles, Connections for..... 115
" Axle, Simple..... 106
" Graphostatic Calculation for Return..... 105
" Graphostatic Calculation for Single..... 104
" Pin, Tangential Pressure on. 233
" Pins..... 61
" Pins, Connections for..... 112
" Return..... 105
Cranks, Cast Iron..... 105
Cranks, Classified..... 104
Crank Shaft, Graphostatic Calculation of..... 107
Cranks, Hand..... 109
Crank, Sliding..... 226
Cranks, Single Wrought Iron..... 104
Creep of Ropes..... 196
Creep of Belts..... 194
Cresson's Coupling..... 96
Crossed Belts, Cone Pulleys for..... 189
Cross Heads..... 118
" " for Guides..... 119
" " for Link Connections... 119
" " for Locomotives..... 121
" " for Marine Engines... 120
" Head, Slipper..... 121
" Head, Superficial Pressure on. 120
" Keyed Connections..... 48
" Section of Belts..... 190
" " Hemp Rope..... 195
" " Wire Rope..... 196
Crown of Pulleys..... 186
" Ratchet..... 154
" Wheel Escapement..... 169
Cup Packing..... 253
Current Motor..... 223
Curve, Elastic..... 3
Curves, Velocity..... 233
Cycloidal Curves..... 130
" Curves, Generation of..... 130
" Sinoide..... 13
Cycloid, Spherical..... 135
Cylinder Escapement..... 169
" Ratchet..... 156
" Ratchet Gearing..... 165
Cylinders..... 216
Cylinders for Hydraulic Presses..... 243
Cylindrical Spiral Gears..... 138
Cylindrical Vessels..... 15

DANAIDE..... 220
Darcy, Formula for Friction of Water..... 246
David's Capstan..... 173
Davis & Co., Steering Gear..... 238
Dead Bolt..... 166
Dead Ratchet Tooth..... 152
Deane's Steam Pump..... 230
Decido Mines, Chain Transmission of 212
Decomposition into Parallel Forces. 31
Deflection..... 3
" in Bodies of Uniform Resistance..... 8
" of Circular Plate..... 15
" of Shafting..... 94
" of Shafting, Torsional..... 92
" of Wire Ropes..... 198
Delisle's Screw Thread Systems..... 53
Dennison's Escapement..... 168
Design and Proportion of Bearings. 68
Diagram for Cone Pulleys..... 190
Diametral Pitch..... 128
Diaphragm Pump..... 217
Differential Brake, Napier's..... 214
" Hydraulic Lever..... 218
" Pulley Block, Weston's. 174
Differential Windlass..... 173
Dimensions of Gear Wheels..... 147
Disk Friction Wheels..... 124
" Valves, Flat..... 275
" Wheels with Pin Teeth..... 133
Distribution of Weight..... 3
Division by Lines..... 23
Division of Gear Wheels, Circumferential..... 128
Dobo's Ratchet..... 160
Dohmen-Leblanc's Clutch..... 101
Donnadieu's Pump..... 223
Door Locks..... 166
Double Acting Pumps..... 224
" Arm Pulleys..... 193
" Beat Valve..... 280
" Friction Ratchets..... 160
" Spiral Gears..... 141
Douglas & Coulson's Steering Gear. 238
Downton's Pump..... 224
Draft, Artificial..... 272
Draft Keys..... 48
Drag Link Coupling..... 97
Drawing Board, Bergner's..... 172
Driving by Tension Organs..... 173
Drop Hammer, Friction..... 176
Drop Hammer, Merrill's..... 123
Drums for Chain..... 185
Dry Gas Meter..... 240
Ducommun & Dubied's Planing Machine..... 176
Dunning & Boissière's Steering Gear 238
Duplex Escapement..... 167
" Pump, Mazelline's..... 231
" Pump, Worthington's..... 231

ECCENTRICS..... 109
Eccentric Straps..... 115
Edge Keys..... 49
Efficiency of Belting..... 194
" of Chain Transmission... 213
" of Rope Transmission... 205
Equalizing Levers..... 32, 111
Equalizer Worthington's..... 232
Equation of Elastic Curve..... 3
Equatorial Section Modulus..... 5
Equilibrinm, Conditions of..... 29
" of External Forces.... 27
" of Forces..... 22
" of Internal Forces.... 28
" of Three Parallel Forces..... 30
Elastic Curve..... 3
Elastic Curve, Equation of..... 3
Elasticity and Strength of Flexure. 2
Elasticity, Modulus of..... 1, 13
Elastic Limit..... 1, 3
" Limit in Beams..... 8
" Line..... 92
" Washers, Belleville..... 212
Elbe, Chain Propelling Gear on.... 185
Elbow Fittings..... 251
Elbow Fittings, Friction in..... 251
Electric Signals, Siemens & Halske. 166
Elements of Graphostatics..... 22 38
Elevator, Hydraulic,..... 227
Elevator Safety Devices..... 164
Emery Weighing Machine..... 173
Enderlein's Escapement..... 168
Engine, Porter-Allen..... 236
Engines, Rotative Pressure..... 233
Engines, Valve Gear for Rotative... 234
Enlarged Screws..... 58
Epicycloidal and Evolute Teeth Compared..... 135
Erhardt's Flange Joint..... 47
Escapement, Bloxam's Gravity..... 168
" Chronometer..... 467
" Crown Wheel..... 169
" Cylinder..... 169
" Dennison's..... 168
" Duplex..... 167
" Enderlein's..... 168
" Frec Anchor..... 168
" Hipp..... 168
" Graham's..... 169
Escapement, Lamb's..... 168
" Lepante's..... 169
" Mudge's..... 168
" Reuleaux's..... 168
Escapements..... 150, 167
" Adjustable..... 170
" Compound..... 168
" for Measurements of Fluids..... 239
" for Measurements of Volume..... 239
" for Moving Liquids, Pressure..... 228
" for Pressure Organs..... 226
" Isochronous..... 167
" Periodical..... 169
" Periodical Pressure... 229
" Period of..... 167
" Power..... 169
" Power Adjustable..... 237
" Range of..... 167
" Simple..... 167
" Time of Oscillation.... 167
" Uniform..... 167
Escapement, Tiede's..... 168
Eve's Chamber Gear Train..... 220
Evolute Rack Teeth..... 132
Evolute Teeth for Interchangeable Gears..... 131
Examples of Belting, Table..... 192
" of Belt Transmission..... 191
" of Gearing..... 147
" of Journals..... 62
" Thrust Bearings..... 78
Expansion Gear, Farcot's..... 236
" " Gonzenbach's..... 236
" " Meyer's..... 236
" Joints..... 245
" Valves..... 236
External Forces, Equilibrium of... 27
Extraction of Roots..... 26
Eytelwein's Formula for Stiffness of Ropes..... 181, 196

FABRY'S VENTILATOR..... 221
Factor of Safety..... 1
Fairbairn, Experiments on Boiler Flues..... 269
Fan Blowers..... 222
Farcot's Stuffing Box..... 254
Farcot's Valve Gear..... 236
Fast and Loose Pulleys..... 188
Felbinger's Postal Tube..... 227
Ferules for Boiler Tubes..... 270
Fink's Link Motion..... 235
Fire Box, Corrugated..... 269
Fire Box, Kaselowsky's..... 269
Fish Torpedo..... 171
Flange Connections for Lead Pipe. 252
" Joints..... 58
" Joint, Erhardt's..... 47
" Joints for Pipes..... 248
Flanges for Riveted Pipes..... 249
Flap Riveted Joints..... 40
Flap Valves..... 274
Flat Hemp Rope..... 178
" Link Chain, Neustadt's..... 183
" Link Chain, Table of..... 183
" Pivot Bearings..... 66
" Ropes..... 181
Flexible Couplings..... 95, 96
" Pipes..... 252
" Rod Connection..... 114
Flexure, Elasticity and Strength of. 2
Flexure, Strains of..... 3
Flow of Metals..... 240
Fluid Escapements for Transportation..... 227
" Running Ratchet Trains..... 223
" Valves..... 287
Fluted Columns..... 83
Flying Bridge..... 222
Fly Wheel, Oscillating..... 233
Fly Wheels..... 233
Force, Centrifugal..... 177
Force Couples..... 29

Forced Connections, Examples of... 46
Forced Draft... 272
Force Plans for Framed Structures. 34
" Plans for Roof Trusses... 36
" Polygon... 26
Forcing Fit... 17
Forces, Addition and Subtraction of 26
" Equilibrium of... 22
" Resultant of Several... 26
Forcing... 45
Forks, Clutch... 99
Fork Journals... 63
Fork Journal, Stub End for... 114
Forms for Iron Columns... 84
Fossey's Coupling... 100
Foundation Bolts, Keying for... 48
Fourneyron Turbine... 220
Four Way Cock... 225
Fowler's Cable Drum... 173
Fowler's Clamp Pulley... 203
Friction Brake, Automatic... 170
" Brake, Toggle... 162
" Clutches... 99
" Clutch, Koechlin's... 100
" " Ramsbottom's... 99
" " Reuleaux's... 100
" " Westons... 101
" Cord... 177
" Coupling, Schurman's... 215
" Drop Hammer... 176
" Feed, Sellers'... 126
" Gear of Cheret's Press... 125
" Gear, Robertson's... 125
" in Elbow Fittings... 251
" in Spur Gearing... 134
" in Stuffing Box... 254
" of Chain Transmission... 213
" of Journals... 64
" of Pivot Bearings... 66
" of Screw Thread... 50
" of Spiral Gear Teeth... 140
" of Water in Pipes... 246
Friedmann's Jet Pump... 222
Friction Pawls... 159
" Pawl, Saladin's... 161
" Pawls, Release of... 161
" Ratchets... 158
" " Double... 160
" " Rod... 163
" " Running... 160
" " Stationary... 161
" Rollers, Mechwart's... 127
" Trains, Special... 126
" Wheels... 122
" " Bevel... 124
" " Construction of... 123
" " Disk... 124
" " for Inclined Axes... 124
" " for Parallel Axes... 123
" " Material for... 123
" " Minotto's... 125
" " Robertson's... 125
" " Sellers'... 126
" " Two Applications of 123
" " Wedge... 125, 160
Francis' Turbine... 220
Framed Beams, Force Plans for... 38
Framed Structures, Force Plans for 34
Frankfurt on Main, Water Supply of 218
Free Anchor Escapement... 168
Free Cross Heads... 119
Freiburg, Rope Transmission at... 205
French Lock... 166
Front Bearings... 68
Furnace Hoist, Althaus'... 173
Furnace, Wilson's Water Gas... 288
Future Possibilities of Boiler Construction... 270

GANNOW MINE... 214
Garand's Clutch... 101
Gases, Reservoirs for... 219
Gas Exhauster, Beale's... 226
" Holders... 272
" Meter, Counting Gear for... 165
" " Dry... 240
" " Sanderson's... 239
Gas Meter, Wet... 239
Gate Valves... 282
Gearing, Brauer's Intermittent... 165
" Calculation of Pitch and Face... 144
" Cylinder Ratchet... 165
" Double Pin... 132
" Examples of... 147
" Friction in Spur Tooth... 134
" Fundamental Formula for. 128
" Globoid Worm... 143
" Hawkin's Worm... 143
" Jensen's Worm... 143
" Ratchet... 150
" Shield... 133
" Gearing, Step... 141
" Toothed... 127
" Worm... 139
Gears, Bastard... 135
" Bevel... 135
" Bevel Spiral... 141
" Beylich's Universal... 136
" Cylindrical Spiral... 148
" Double Spiral... 141
" Examples of Spiral... 140
" Globoid Spiral... 142
" Hoisting... 144
" Parallel... 133
" Precision... 139
" Single Tooth... 165
" Spiral... 138
" Stepped Bevel... 141
" Table of Cast Iron... 144
" Teeth for Hyperboloidal... 138
" Transmission... 144
Gear Teeth, Action of... 129
" " Construction of Spur.. 128
" " Epicycloidal and Evolute Compared... 135
" " Evolute Interchangeable... 131
" " Friction of Spiral... 140
" " Interchangeable... 130
" " Internal... 131
" " Line of Action of... 129
" " Loss in Determined Geometrically... 134
" " of Circular Arcs... 131
" " Pin... 132
" " Pressure on... 146
" " Section of... 144
" " Stress in... 145
" " Thumb Shaped... 134
" " Wear on... 134
" Tooth Outlines, General Solution... 129
" Wheel Arms, Table of... 149
" " Hubs... 150
" " Plane... 136
" Wheels, Arms of... 149
" " Circumferential Division of... 128
" " Classified... 127
" " Conical... 135
" " Dimensions of... 147
" " Diametral Pitch of... 128
" " Hyperboloidal... 136
" " Interchangeable... 128
" " Pitch of... 144
" " Pitch Radius of... 128
" " Rim of... 147
" " Weight of... 150
Gemörsch Chain... 182
General Form of Toothed Ratchets. 158
General Remarks upon Ratchet Mechanism... 171
Generation of Cycloidal Curves... 130
Geneva, Sluice Gates at... 275
Geneva Stop... 165
Gerber's Bolt... 57
Geyser Pump, Siemens'... 222
Gidding's Slide Valve Experiments. 285
Giffard's Injector... 222
Girard Turbine... 220
Githen's Rock Drill... 231
Globe Valve... 279
Globoids... 142
Globoid Spiral Gears... 142
Globoid Worm Gearing... 143
Gooch's Link Motion... 235
Goodwin's Split Pulley... 194
Gonzenbach's Expansion Gear... 236
Graham's Escapement... 169
Graphical Addition... 26
" Calculation of Axle... 86
" " of Powers... 24
" " of Shafting... 95
" " of Crank Axle 106
" " of Multiple Crank Shaft 107
" " Return Crank 105
" " Single Crank. 104
Graphostatics, Elements of... 22–38
Green's Canal Lift... 227
Greindl's Pump... 221
Gresham's Injector... 222
Grip Pawl for Cables... 185
Grooves for Rope Transmission... 195
Grooved Fly Wheels... 195
Grouped Columns... 84
Group Riveting... 41
Guide Mechanism for Pressure Organs... 217
Guide Pulleys for Belting... 186
Guides and Guide Bars... 121
" Bored... 122
" for Marine Engines... 122
Guide Sheaves... 185
Guides, Locomotive... 122
Guiding by Pressure Organs... 216
Guiding, Tension Organs for... 172
Gun Lock Mechanism... 166
Gun Locks... 163
Guns, Hooping of... 16

HAIR SPRING... 169
Hair Trigger... 168
Half Journals... 64
Hammer, Merrill's Drop... 123
Hand Cranks... 109
Hanger Boxes, Sturtevant's... 74
Hangers... 68
Hangers, Adjustable... 74
Hanger, Sellers'... 74
Hanger, Sellers' Countershaft... 74
Hangers for Rope... 181
" Post... 73
" Proportion of... 73
" Ribbed... 73
Harlow's Valve Gear... 231
Hartwich's Cable Ferry System... 175
Hastie's Steering Gear... 238
Hauling System, Riggenbach's... 172
Haulage Systems, Pennsylvania... 174
Hawkin's Worm Gearing... 143
Helfenberger's Regulator... 236
Hemp Rope... 178
" " Transmission, Specific Capacity of... 195
" " Wear on... 196
" " Weight of... 178
Hero's Fountain... 288
Heusinger's Link Motion... 235
Heuberger's Chain Propeller... 176
Hick, Experiments on Stuffing Box Friction... 254
Hick's Stiffening Ring... 269
High Duty Pumping Engine, Worthington... 232
Hipp Escapement... 168
Hirn's Experiments on Journals... 64
Hodgkinson's Experiments... 13
Hofmann's Valve Gear... 163
Hoist, Althaus'... 173
Hoisting Devices... 172
Hoisting Gears... 144
Hollow Columns... 83
Hollow Journals... 62
Hooke's Coupling... 97
Hooks... 184
Hooping... 45
" by Shrinkage... 45
" of Guns... 16
Hoppe's Accumulator... 264

Hose 252
Howaldt's Metallic Packing 254
Hubs for Rock Arms 102
Hubs of Gear Wheels 150
Hunting Valve 238
Hurdy Gurdy Wheel 220
Hydraulic Accumulators 218, 264
" Cranes 228
" Elevator 227
" Lever 217
" Lever, Differential 218
" Parallel Motion 218
" Power Distribution 219
" Power Distribution, Ring System 256
" Presses, Cylinders for 243
" Press, Thickness of Cylinder 16
" Hydraulic Ram, Montgolfier's 232
" Riveting Machine, Tweddell's 228
" Steering Gear 237
" Tools 228
" Transformer 218
Hyperboloidal Gear Wheels 136
" Gears, Teeth for 138
" Wheels, Base Figures for 136

IDEAL BENDING MOMENT 13
Ideal Twisting Moment 13
Impact Water Wheels 220
Inclined Cable, Transmission with 200
Independent Step Bearings 76
Inertia, Moment of 5-7
Influence of Pulley Diameter on Wire Rope 197
Influence of Weight of Wire Rope 180
Injector, Giffard's 222
Injector, Gresham's 222
Interchangeable Gear Teeth 130
Interchangeabe Gear Wheels 128
Intermittent Gearing, Brauer's 165
Internal Flow 240
" Forces, Equilibrium of 28
" Gear Teeth 131
" Ratchet Wheels 151
" Strap Brakes 215
Intze's Discussion of Tanks 261
Inverted Siphon 244
Iron Columns, Calculations for 82
Iron, Weight of Round 55
Isochronous Escapements 167
Isolated Forces in One Plane 26
Isolated Forces, Resultant of 29

JACKSON'S CLUTCH 101
Jacquard Loom 163
James Watt & Co., Thrust Bearing 77
Jam Nut 56
Jaw Clutch 98
Jensen's Worm Gearing 143
Jet Action 241
" Mechanism 240
" Propeller 223
" Pump, Friedmann's 222
Joints, Expansion 245
" Flange 58
" Strength of Riveted 40
Joint, Universal 97
Jopling's Water Meter 239
Journal Boxes, Various forms 69
" Friction in Rope Transmission 195, 205
" " of Chain Transmission 213
Journals, Attachments of 67
" Dimensions 61
" Examples of 62
" Fork 63
" for Levers 101
" for Shafting 94
" Friction of 64
" Half 64
" Hollow 62
Journals, Lateral 61
" Lubrication of 61
" Multiple 63
" Neck 62
" Overhung 61
" Pressure on 61
" Proportions of 61
" Stress on 61
" —Various Kinds 60

KAHLENBERG CABLE ROAD 173
Kaselowsky's Fire Box 269
Kennedy's Water Meter 239
Kernaul's Key 49
Keyed Connections 47
Keying 47
" Foundation Bolts 48
" Peters' Method 102
" Screw Propellers 49
Key, Kernaul's 49
Keys, Concave 48
" Draft 48
" Edge 49
" Flat 48
" for Rock Arms 102
" Longitudinal 48
" Methods of Securing 50
" Recessed 48
" Stresses on 48
" Taper of 47
" Unloaded 49
Kirchweger's Steam Trap 228
Kirkstall Forge Rolling Mill 126
Kirkstall Forge, The 94
Kley's Pumping Engine 233
Klug's Valve Gear 235
"Knot" in Cord Polygon 27
Koechlin's Friction Clutch 100
Krauss' Connecting Rod End 113
Krauss' Piston 252
Krüger, Investigation of Rivets 39

LACING, BELT 191
Lagarousse Ratchet 164
La Louviere Canal Lift 227
Lamb's Escapement 168
Langen Gas Engine 161
Lap Joints, Riveted 40
Lap of Slide Valve 225
Latch Bolt 166
Lateral Bearings 68
Lateral Journals 61
Lead Pipe Connections 251
Lémielle's Ventilator 82
Lepaute's Escapement 169
Levasseur's Metallic Tubing 252
Lever Arms, Calculation of 103
" Arms of Combined Section 103
" Differential Hydraulic 218
" Hydraulic 217
Levers, Combined 110
" Equalizing 32
" Journals for 101
" Simple 101
Lifting Frame for Screw Propeller 151
Lift Valves 223, 273
Liquids, Pressure Escapement for Moving 228
Limit of Elasticity 1
Limit of Elasticity in Beams 8
Line of Action of Gear Teeth 129
Line Shafting 93
Link Couplings 98
" Motion, Allan's 235
" " Fink's 235
" " Gooch's 235
" " Heusinger's 235
" " Stephenson's 234
Load, Breaking 1
Load Length of Wire Rope 180
Lock, Bramah 166
" Chubb 166
" French 166
Locking Ratchets 150
Locomotive Axles 107
" Connecting Rods 116
Locomotive Connecting Rod, Solid End for 113
" Cross Heads 120
" Guides 122
" Springs, Screws for 58
Locks, Canal 227
" Door 166
" Gun 162
Lock, Yale 166
Logarithmic Spiral 26
Long Distance Fluid Transmission 233
Long Distance Power Transmission 259
Longitudinal Keys 48
Loom, Jacquard 163
Loss in Gear Teeth Determined Geometrically 135
Loss in Hemp Rope Transmission 195
Lubrication of Journals 61
Lucerne, Cable Incline at 173

MACHINE ELEMENTS, CONSTRUCTION OF 39-289
Machine Riveting 39
Mackay & McGeorge, Riveting Machine 111
Magdeburg-Bodenbacher Chain 183
Maltese Cross Gear 165
Manholes 289
Mannesmann Tubing 243
Marine Cross Heads 121
" Engine Guides 122
" Propulsion 222
Marshall's Valve Gear 235
Materials—Strength of 1-21
Mathematical Tables 291
Maudslay, Method of Securing Bolts 58
Maudslay, Thrust Bearing by 77
Mauser's Revolver 165
Mazelline's Duplex Pump 231
Measurement of Fluids, Escapements for 239
Measuring Devices, Running 239
Mechwart's Friction Rollers 127
Medart Pulley 193
Merrill's Drop Hammer 123
Metaline Bearings 179
Metallic Piston Packing 253
Metallic Tubing 252
Metals, Weight of Sheet 43
Meter for Alcohol, Siemen's 239
Methods of Securing Bolts 57
Methods of Securing Pawls 153
Metrical Screw Systems 52
Metric Bolts and Nuts 55
Metric Pipe Thread System 250
Meyer's Valve Gear 236
Mill, Bogardus 126
Minotto's Bevel Friction Wheels 125
Mixed Tooth Outlines 133
Mines, Chain Transmission in 213
Modulus of Elasticity 1, 13
" Resistance 1
" Rupture 1, 2
" Transmission 208
Moliuos & Pronnier, Speed of Riveting 39
Moment of Inertia 3, 5, 7
Moment of Inertia, Polar 11
Mont Cenis Air Compressors 232
Montejus 228
Montgolfier's Hydraulic Ram 232
Morin's Experiments on Journals 64
Motors for Pressure Organs 219
Moulding 240
Moxon's Belt Fastening 193
Mudge's Escapement 168
Muff Coupling 95
Mule Post 188
Mule, Spinning 169
Multiple Collar Bearings 77
" Collar Thrust Bearings 66
" Crank Shafts 107
" Journals 63
" Ratchets 154
" Supports for Bearings 80
" Trussed Beams 35

REESE LIBRARY OF THE UNIVERSITY

Multiple Valves..... 276
Multiplication and Division Combiued..... 23
Multiplication by Lines..... 22
Murdock's Slide Valve..... 234
Muschenbroeck's Pump..... 223

NAGEL TURBINE..... 220
Napier's Clutch..... 101
Napier's Differential Brake..... 214
Natural Reservoirs..... 218
Neck Journals..... 62
Neck Journals, Connections for..... 114
Negative Reservoirs..... 219
Neutral Axis..... 3
Neutral Plane..... 10
Neustadt's Chain..... 183
Newcomen Engine..... 163
Normandy's Pipe Joint..... 249
Norton's Pump..... 225
Nut, Jam..... 59
Nut Locks..... 56
Nuts, Washers and Bolt Heads..... 54

OBELISK, FORCES IN RAISING 28
Oberursel, Rope Transmission at.203,205
Oeking's Water Counter-balance... 217
Oil Tanks..... 218
Oldham's Coupling..... 96
Open Belts, Cone Pulleys for..... 189
Open Link Chain..... 182
Ordway, Experiments on Pipe Covering..... 245
Oscillating Fly Wheel..... 233
Oscillating Pumps..... 226
Oscillation of Escapements, Time of 167
Osterkamp's Rope Hanger..... 181
Otis Elevator..... 228
Overhead Cable Tramway..... 175
Overhung Journals..... 61

PACKING FOR HYDRAULIC Press..... 253
" for Pump Pistons..... 255
" Howaldt's Metallic..... 254
" Piston..... 216
" Standard Prussian Railway..... 255
Pagel's Elastic Washer..... 57
Pallets..... 168
Pappenheim Chamber Wheel Train. 219
Parabola, Biquadratic..... 10
Parallel Forces—Equilibrium of. .30, 31
" Gears..... 133
" Motion, Hydraulic..... 218
" Rods for Locomotive Engines..... 117
Parson's Bolts..... 57
Pattison's Pump..... 226
Pawl, Cable Grip..... 185
Pawl, Saladin's Friction..... 161
Pawls for Chains..... 185
" Friction..... 159
" Methods of Securing..... 153
Pawl, Spring..... 153
Pawls, Release of Friction..... 161
Pawl, Thrust upon..... 152
Pawl, Thumb Shaped..... 160
Payton's Water Meter..... 220
Pedestal Bearings..... 71
Penn's Connecting Rod End..... 113
" Method of Securing Bolts.. 57
" Piston..... 252
Pennsylvania Haulage Systems.... 174
Periodical Escapements..... 169
Periodical Pressure Escapements... 229
Peters' Method of Keying..... 102
Petit's Pipe Joint..... 248
Pfalz-Saarbruck Screw Thread System..... 53
Phœnix Column..... 59, 83
Physical Ratchet Train..... 171
Pickering's Steam Pump..... 230
Pillow Blocks..... 68
" Blocks, Adjustable.....69, 70
" Block, Sellers'..... 70
Pillow Blocks, Large..... 69
" Blocks, Proportional Scale for 68
" Block, Sturtevant's..... 70
Pin Gearing, Double..... 132
" Gear Teeth..... 132
" Ratchet Wheel..... 152
Pins, Crank..... 61
Pin, Split..... 56
Pipe Connections, Socket..... 248
" Coverings..... 245
" Fittings..... 249
" Joint, Normandy's..... 249
" " Petit's..... 248
" " Riedler's..... 249
" Riveted..... 244
Pipes, Ball Joints for..... 249
Pipes, Connections for Cast Iron... 248
" Connections for Wrought Iron 249
" Copper..... 246
" Flange Joints for..... 248
" Flexible..... 252
" for High Pressures..... 242
" Resistance of Bends in..... 247
" Resistance to Flow in..... 246
Pipe Sockets..... 250
Pipes, Steam..... 245
Pipe, Steel..... 243
Pipes, Thickness of Cast Iron..... 242
Pipe Threads, Briggs' System..... 250
" Thread System, Metric..... 250
" Weight of Cast Iron..... 242
" Wrought Iron..... 243
Piston, Krauss'..... 252
" Packing.....216, 253
" Packing, Metallic..... 253
" Penn's..... 252
" Pumps..... 223
" Rods..... 255
Pistons..... 216, 252
Piston, Swedish..... 253
Pistons with Valves..... 286
Piston Valves..... 286
Pitch and Face of Gearing, Calculation of..... 144
" " Hoisting Gears.. 144
" " Transmission Gears..... 144
" Chain..... 183
" Circles, Table of..... 128
" of Gear Wheels..... 144
" Radius of Gear Wheels..... 128
Pivot Bearings, Flat..... 66
Pivots, Formula for..... 65
" Pressure on..... 65
" Proportions of..... 65
Plain Slide Valve..... 282
Plain Slide Valve Gear..... 234
Plane Gear Wheel..... 136
Planing Machine, Ducommun & Dubied's..... 176
" " Sellers..... 176
" " Shanks'..... 163
Plate Coupling..... 95
Plungers.....216, 253
Plunger Pumps..... 223
Pneumatic Power Distribution..... 257
Pneumatic Tube..... 227
Polar Moment of Inertia..... 11
Polishing Belts..... 177
Polonceau's Connecting Rod End.. 114
Polygons, Area of..... 24
Poncelet's Chain..... 173
Poncelet's Water Wheel..... 220
Porter-Allen Connecting Rod..... 117
Porter-Allen Engine..... 236
Post Hangers..... 73
Pouyer's Coupling.....101, 152, 153
Powel Valve Gear..... 163
Power Distribution, Compressed Air 219
" " Hydraulic..... 219
" " Hydraulic Ring System..... 256
" " Pneumatic..... 257
" " Steam..... 219
" " Systems..... 219
" " Vacuum..... 219
Power Escapements..... 169
Powers, Graphical Calculation of... 24
Powers of Trigonometrical Functions..... 25
Power Transmission by Superheated Water..... 219
Practical Resistance..... 1
Precision Gears..... 139
Precision Ratchets..... 157
Prentiss' Coupling..... 216
Pressure Escapements for Moving Liquids..... 228
" on Gear Teeth..... 146
" Journals..... 61
" Lift Valves..... 277
" Pivots..... 65
" Screw Threads..... 58
" Organs..... 216
" " Conductors for..... 242
" " Conduits for..... 216
" " Escapements for... 226
" " Guiding by..... 216
" " Guide Mechanism for..... 217
" " Motors for..... 219
" " Ratchet Mechanism for..... 223
" " Reservoirs for. .218, 260
" " Running Mechanism for..... 219
" " Technological Applications of..... 250
" Superficial..... 1
" Transmission, Specific Capacity of..... 255
Proof Diagrams of Axles..... 87
Propeller Bearing, Ravenhill & Hodgson's..... 74
" Jet..... 223
" Lifting Frame for..... 151
" Screw..... 223
" Shafts, Couplings for..... 95
Propelling Chain..... 185
Proportions of Axles..... 85
" Chain..... 183
" Flange Joints..... 59
" Hooks..... 184
" Journals..... 61
" Pivots..... 65
" Pulleys..... 193
Propulsion, Marine..... 222
Prussian Standard Car Bearing..... 75
Pulley Block, Weston's Differential. 174
" by Walker Mfg. Co..... 193
" Diameter, Influence on Wire Rope..... 197
" Fowler's Clamp..... 203
" Medart..... 193
Pulleys, Balancing of..... 194
" Cone..... 189
" Construction of Rope..... 202
" Crown of Face..... 186
" Double Arm..... 193
" Guide..... 186
" Fast and Loose..... 188
" for Cable, Arrangement of. 202
" Proportions of..... 193
" Split..... 193
Pulley Stations, Construction of ... 204
Pulleys, Tightening..... 186
Pulley, Sturtevant's..... 194
Pulleys, Umbrella..... 207
Pulleys, Vertical Supporting..... 188
Pulsometer..... 232

Pump, Althaus'..... 223
" Amos & Smyth's..... 224
" Bag..... 217
" Bellows..... 217
" Diaphragm..... 217
" Donnadieu's..... 223
" Downton's..... 224
" Friedmann's Jet..... 222
" Gears, Adjustable..... 236
" Greindl's..... 221
" Engine, Kley's..... 223

Pumping Engine, Worthington's High Duty..... 232
Pumping Machinery..... 229
Pump, Mazelline's Duplex..... 231
" Muschenbroeck's..... 233
" Norton's..... 225
" Pattison's..... 226
" Pistons..... 253
" Pistons, Packing for..... 255
" Regulator, Helfenberger's . 236
" Repsold's..... 221
" Rittinger's..... 223
Pumps, Centrifugal..... 222
" Considered as Ratchet Trains..... 223
" Double Actiug..... 224
Pump, Siemen's Geyser..... 222
Pumps, Oscillating..... 226
Pump, Spiral..... 221
Pumps, Piston..... 223
" Plunger..... 223
" Rotary..... 226
" Stolz's..... 224
Pump, Stone's..... 224
" Valve Gear..... 225
" Valves, Riedler's..... 278
" Vose's..... 224
" Worthiugton's Duplex..... 231

QUADRANTS..... 153
Quadrilateral Figures, Area of..... 25
Qnarter Twist Belts..... 186
Quarter Twist Belt, Shifter for..... 189

RACK, RATCHET..... 151
Rack Teeth, Evolute..... 132
Railway Axles..... 88
Ramsbottom's Crane..... 176
Ramsbottom's Friction Clutch,..... 99
Ratchet, Anchor..... 155
" Brace, Weston's..... 154
" Clamp..... 160
" Crown..... 154
" Cylinder..... 156
" Dobo's..... 160
" Gearing..... 150
" " Cylinder..... 156
" " Dimensions of Parts of..... 158
" Gears, Toothed Running.. 150
" Lagarousse..... 164
" Mechanism for Pressure Organs..... 223
" " General Remarks upon 171
" " Kinematically Discussed.. 171
" Rack..... 151
" Rod Friction..... 163
Ratchets, Checking..... 150
" Continuous..... 150
" Continuous Running..... 164
" Double Friction..... 160
" Friction..... 158
" General Form of Toothed 158
" Locking..... 150, 166
" Multiple..... 154
" of Precision..... 157
" Releasing..... 150, 162
" Running..... 150
" Running Friction..... 158
" Silent..... 153
" Spring..... 153
" Stationary..... 150, 156
" Stationary Friction..... 161
" Step..... 155
" Step Anchor..... 157
" Throttle..... 161
" with Locking Teeth, Continuous..... 165
" Teeth, Flanks of..... 153
" Teeth, Form of..... 150
" Tension Organs..... 185
" Tooth, Dead..... 152
" Train, Physical..... 171
Ratchets, Trains, Chemical..... 171
" Wheels, Internal..... 151
" Wheels, Special Forms... 154
" Wilber's..... 153
Ravenhill & Hodgson, Propeller Bearing..... 74
Ravenhill & Hodgson, Thrust Bearing by..... 77
Reciprocating Valve Gears..... 234
Regulator, Guhrauer & Wagner's.. 237
Regulator, Rigg's..... 236
Reichenbach's Water Pressure Engine..... 229
Releasing Ratchets..... 150, 162
Releasing Valve Gears..... 162
Release of Friction Pawls..... 161
Rennie, Experiments on Journals... 64
Repeating Watches..... 169
Repsold's Chamber Gear Train..... 220
Repsold's Pump..... 221
Reservoirs for Air and Gas..... 272
" for Gases..... 219
" for Pressure Organs. 218, 260
" Natural..... 218
" Negative..... 219
Resistance, Coefficients of..... 1
" Modulus of..... 1
" of Bends in Pipes..... 247
" of Valves in Pipes..... 247
" Practical..... 1
" Theoretical..... 1
" to Bending..... 2
" Buckling..... 13
" Flow in Pipes..... 246
" Shearing..... 2
" Torsion..... 11
Resultant of Isolated Forces..... 29
" Load on Water Wheel determ'd Graphically. 34
" Several Forces..... 26
Return Crank..... 105
Return Crank, Graphostatic Calculation for..... 105
Renleaux's Coupling..... 96
" Escapement..... 168
" Friction Clutch..... 100
" System of Rope Transmission..... 206
" Valve Diagram..... 234
" Winding Drum..... 173
Reversing Gear, Globoid..... 143
Revolver, Mauser..... 165, 166
Rhenish Railway Cable..... 174
Ribbed Axles..... 91
Rib Profiles, Construction of..... 91
Richard's Manometer..... 288
Rider's Valve Gear..... 236
Riedler's Air Compressors..... 279
" Pipe Joint..... 249
" Valve Gear..... 278
Riggenbach's Cable System..... 174
Riggenbach's Hauling System..... 172
Rigg's Regulator..... 236
Rigid Couplings..... 95
Rim of Gear Wheel..... 149
Ring System of Cable Transmission..... 208–211
Rittinger's Pump..... 223
Riveted Joints, Construction of Angles..... 44
" " Junction of Plates.. 43
" " Proportional Scale for..... 41
" " Reinforcement of Plates..... 44
" " Special Forms..... 43
" " Strength of..... 40
" " Table of..... 40
" Pipe..... 244
" Pipes, Flanges for..... 249
Rivet Heads, Proportions of..... 39
Riveting..... 39
" Boiler..... 42
" Group..... 41
" Machine..... 39
" " Mackay & McGeorge 111
Riveting Machine, Tweddell's Hydraulic..... 228
Riveting, Speed of..... 39
Rivets..... 39
Robinson's Experiments on Lift Valves..... 277
Robertson's Friction Wheels..... 125
Rock Arms..... 162
Rock Drill, Githen's..... 231
Rod Connection, Wiedenbruck's..... 50
Rod, Friction Ratchet..... 163
Rods, Connecting..... 112
Rolled Shafting..... 94
Roller Bearing, Cambon's..... 127
" Bearings..... 126
" Bearings for Sheaves..... 179
Roof Trusses, Force Plans for..... 36
" Truss, Polygonal..... 37
" with Simple Principals..... 36
" with Trussed Principals..... 36
Root's Blower..... 221
Roots, Extraction of..... 26
Rope, Centrifugal Force of Wire.... 197
" Connections..... 181
" Cotton..... 179
" Cross Section of Wire..... 196
" Curve, Construction of..... 202
" Hanger, Osterkamp's..... 181
" Hangers..... 181
" Hemp..... 178
" Influence of Pulley Diameter on..... 197
Ropes of Organic Fibres..... 178
Rope Pulleys, Construction of..... 202
Ropes, Creep of..... 202
" Deflection of Wire..... 198
" Flat..... 178, 181
" Loss from Stiffness..... 196
Rope, Specific Capacity of Wire.... 196
Rope Splice..... 181
Ropes, Stiffness of..... 181
" Tightened Driving..... 200
" Ziegler's Experiments on... 181
Rope Transmission..... 194
" " at Bellegarde.... 205
" " at Freiburg..... 205
" " at St. Petersburg 205
" " at Schaffhausen. 204
" " at Znrich..... 205
" " Cotton..... 196
" " Cross Section for Hemp..... 195
" " Efficiency of.... 205
" " Loss in Hemp... 195
" " Reuleaux's System of..... 206
" " Specific Capacity of Hemp..... 195
" " Wire..... 196
" Weight of Hemp..... 178
" Wire..... 179
Rotary Pumps..... 226
" Valves..... 281
" Valve, Wilson's..... 285
Rotative Motors, Adjustable Gears for..... 237
" Pressure Engines..... 233
" Valve Gears..... 234
Round Connecting Rods..... 116
Round Valves..... 275
Roux's Water Pressure Engine..... 229
Rubber Springs..... 21
Rubber Springs, Werder's Experiments..... 21
Running Chains..... 182
" Friction Ratchets..... 158, 160
" Mechanism for Lifting Water..... 221
" Mechanism for Pressure Organs..... 219
" Ratchet Gears, Toothed... 150
" Ratchets..... 150
" Ratchet Trains, Fluid.... 223
" Tension Organs..... 172
Rupp's Variable Speed Gear..... 124
Rupture, Modulus of..... 1

SAFETY, COEFFICIENT OF... 1
Safety Devices for Elevators... 164
Safety, Factor of... 1
Sail Boat... 223
St. Petersburg, Rope Transmission at 205
St. Louis Bridge... 60
Saint Venant, Friction of Water... 247
Saladin's Friction Pawl... 161
Sanderson's Gas Meter... 239
San Francisco Cable Tramways... 174
Saxby & Farmer, Signal Apparatus. 166
Saw, Zervas' Wire... 177
Saws, Band... 177
Scale Beams... 111
Schaffhausen, Rope Transmission at 204
Schiele Turbine... 220
Schmick's Canal Cable System... 175
Schmid's Water Meter... 240
Schmid's Water Pressure Engine... 236
Schurman's Clutch... 101
Schurman's Friction Coupling... 215
Screw, Archimedian... 221
" Connections... 58
" Propeller... 223
" Propeller, Lifting Frame for 151
" Propellers, Built up... 57
" Propellers, Method of Keying 49
Screws, Enlarged... 58
Screw Thread, Construction of... 50
" " Dimensions of V... 50
" " Friction of... 51
" " Pressure on... 58
" Threads, Special Forms of... 57
" Threads, Trapezoidal... 58
Section of Gear Teeth... 144
Section Modulus... 5, 7, 11
Sections of Uniform Resistance... 8
Secured Bolts... 57
Securing Keys, Methods of... 50
Segner's Water Wheel... 220
Self Guiding Belting... 186
Seller's Coupling... 96
" Friction Feed... 126
" Hanger... 74
" Pillow Block... 70
" Planing Machine... 176
" Screw Thread System... 52
" Wall Bearing... 71
Sewage System of Berlin... 219
Sewing Machine Check... 151
Shafting... 92
" Deflection of... 94
" Dimensions of... 92
" Examples of Torsion in... 94
" Graphical Calculation of... 94
" Journals for... 94
" Line... 93
" Rolled... 94
" Specific Capacity of... 257
" Torsional Deflection of... 92
" Wooden... 94
" Wrought Iron... 93
Shank's Planing Machine... 163
Sharp's Coupling... 96
Sharp's Strap End... 112
Shearing, Resistance to... 2, 10
Shearing Strain... 2
Sheaves, Chain... 185, 211
Sheaves, Roller Bearings for... 179
Shield Gearing... 133
Shifter for Quarter Twist Belt... 189
Shifters, Belt... 188
Shifting Eccentrics... 235
Short Span Cable Transmissions... 200
Shrinkage, Hooping by... 45
Shrinking Fit... 17
" Fits, made with Boiling Water... 47
" Rings, Clerk's Method... 45
" Temperatures... 45
Sickles' Adjustable Valve Gear... 237
Sickles' Valve Gear... 162
Side Wheel Steam Boat... 223
Siemens & Halske, Electric Signals. 166
Siemens' Alcohol Meter... 239
Siemens' Geyser Pump... 222
Signal Apparatus, Saxby & Farmer. 166
Silent Ratchets... 153
Simple Crank Axle... 106
Simple Escapements... 167
Single Acting Steam Engine... 229
Single Tooth Gears... 165
Sinoide... 91
Sinoide, Cycloidal... 13
Siphon, Direct... 287
Siphon, Inverted... 244, 287
Slide Valve, Common... 225
" " Gear, Plain... 234
" " Lap of... 225
" " Murdock's... 234
" Valves... 223, 273, 281, 282
" Valves, Balanced... 285
Sliding Brakes... 215
Sliding Crank... 226
Slipper Cross Head... 121
Sluice Gates at Geneva... 275
Sluice Valve... 281
Snail... 169
Solid End for Connecting Rod... 113
Special Forms of Bearings... 74
" " of Bolts... 55
" " of Ratchet Wheels... 154
" " of Screw Threads... 58
Specific Capacity of Belting... 190
" " of Driving Chains. 211
" " of Hemp Rope Transmission... 195
" " of Pressure Transmissions... 255
" " of Shafting... 257
" " of Wire Rope... 196
Speed Gear, Variable... 124
Spencer & Inglis Valve Gear... 262
Spherical Cycloid... 135
" Journal, Connection for... 115
" Spiral... 142
" Valves... 275
Spiral Bevel Gears... 141
" Gears... 138
" " Double... 141
" " Examples of... 140
" " Teeth, Friction of... 140
" Gears, Globoid... 142
" Pump... 221
" Spherical... 142
" Winding Drums... 181
" Wire Pipe... 252
Spinning Mule... 169, 196
Splice for Ropes... 181
Split Pin... 56
" Pulley, Goodwin's... 194
" Pulleys... 193
Spring, Dudley's... 20
" Pawl... 153
" Ratchets... 153
Springs, Best Material for... 20
" Calculation of... 18
" Table of... 18–19
" Vulcanized Rubber... 21
Spur Gear Teeth, Construction of... 128
Squaring Device for Cranes... 172
Square Thread... 40
Standing Tension Organs... 172
Stand-Pipes... 287
Statical Moment... 3
Statical Moment, Graphically Considered... 33
Starting Valve... 281
Star Pin... 153
Stationary Chains... 182
" Friction Ratchets... 161
" Machine Elements... 289
" Ratchets... 150, 156
" Valves... 289
Steam Boat, Side Wheel... 223
" Boilers... 265
" Distribution of Power... 257
" Engine, Single Acting Steam 229
" Pipes... 245
" Power Distribution... 219
" Pump, Blakes... 240
" " Deane's... 230
Steam Pump, Pickering's... 230
" " Tangye's... 230
" Steering Gear... 238
" Trap, Kirchweger's... 228
Steel Pipe... 243
Steering Gear... 171
" " Britton's... 238
" " Davis & Co.'s... 238
" " Douglas & Coulson's. 238
" " Dunning & Bossiere's 238
" " Hastie's... 238
" " Hydraulic... 237
" " Steam... 238
Steib's Ventilator... 222
Step Anchor Ratchet... 156
" Bearings... 75
" Bearing, Support for... 80
" Bearings, Wall... 75
" Gearing... 141
Stephenson's Link Motion... 235
Stepped Bevel Gears... 141
Step Ratchets... 155
Step Valves... 276
Stévart, Experiments on Springs... 21
Stiffness of Belts... 194
" of Ropes... 181
" " Eytelwein's Formula... 181
" " Loss from... 196
" " Weisbach's Formula... 181
" " Wire Rope... 206
Stolz's Pump... 224
Stone's Pump... 224
Stop, Geneva... 165
Storage Reservoirs, General... 273
Strain, Shearing... 2
Strains of Flexure... 3
Strap Brakes... 211, 215
" Brakes, Internal... 215
" End for Connecting Rod... 112
" End, Sharp's... 112
Straps, Eccentric... 115
Strength of Cast Iron Columns... 83
" of Materials... 1–21
" of Wire Rope... 179
" Tensile... 1
Stress Curve... 87
Stresses, Compound... 13
" in Columns... 82
" on Keys... 48
Stress on Belting... 191
" on Gear Teeth... 145
" on Journals... 61
" S, Value of... 8
Striking Mechanism for Clocks... 169
Stub End, for Fork Journal... 114
Stuffing Boxes... 253
" Box, Farcot's... 254
" Box, Friction in... 254
Sturtevant's Hanger Boxes... 74
" Pillow Block... 70
" Pulley... 194
Superficial Pressure... 1
Superheated Water Transmission... 219
Supporting Pulleys, Vertical... 188
Supports for Bearings... 79
" " General Principles... 82
" " Simple... 79
Supporting Power of Beams... 5
Swedish Piston... 253
Swedish Railway, Boilers for... 272
Sweet's Valve Gear... 235
Swivels... 182
Swivels for Chain... 184
Symmetrical Simple Axles... 83

TABLE OF BEAM SECTIONS. 5, 6, 7
Tables of Curves, Areas and Volumes... 291–296
Table of Numbers... 300–301
Tackle Block... 172
Tangential Pressure on Crank Pin... 233
Tangye's Crane... 176
Tangye's Steam Pump... 230

Tanks, Cast Iron ... 260
" Combination Forms for ... 264
" Intze's Discussion of ... 260–264
" Oil ... 218
" With Concave Bottoms ... 262
" Wrought Iron ... 260
Taper of Keys ... 47
Technological Applications of Pressure Organs ... 240
Technological Applications of Tension Organs ... 177
Tenacity ... 1
Tensile Strength ... 1
Tension Organs ... 172
" " for Driving ... 173
" " for Guiding ... 172
" " for Winding ... 172
" " Ratchet ... 185
" " Running ... 174
" " Technological Applications of ... 177
" Resistance to ... 2
Tests for Chain ... 183
T Fittings ... 251
Theoretical Resistance ... 1
Thickness of Cast Iron Pipes ... 242
Thick Vessels, Walls of ... 16
Thomas' Calculating Machine ... 153, 146
Thometzek's Valve ... 276
Thomson's Turbine ... 220
Three-part Bearings ... 70
Throttle Valves ... 161
Throttle Vaves ... 279
Thrust Bearing by James Watt & Co. 77
" " by Maudslay ... 77
" " by Penn ... 77
" " by Ravenhill & Hodgson ... 77
" " Compound Link as 67
" Bearings ... 65, 68, 75
" " Collar ... 66
" " Examples of ... 78
" " Multiple Collar ... 66
" " with Wooden Surface ... 76
" upon the Pawl ... 152
Thumb Shaped Pawl ... 160
Thumb Shaped Teeth ... 134
Tiede's Escapement ... 168
Tightened Cables, Table for ... 200
Tightened Driving Ropes ... 200
Tightening Pulleys ... 186
Tightening Pulley, Weaver's ... 186
Toggle Friction Brake ... 162
Tools, Hydraulic ... 218
Toothed Gearing ... 127
Tooth Friction in Spur Gearing ... 134
" Outlines, General Solution of 129
" " Mixed ... 133
" " of Circular Arcs ... 131
Torpedo, Fish ... 171
Torsional Deflection of Shafting ... 92
Torsion, Determination of Angle of 93
" Resistance to ... 11
" Table ... 12
" Uniform Resistance to ... 13
Towne Crane ... 176
Transformer, Hydraulic ... 218
Tranforming Arm Sections, Table for 103
Transmission at Long Distance, Fluid ... 233
" Chain ... 211
" Gears ... 144
" Long Distance Power. 259
" Modulus of ... 208
" Rope ... 194
" With Inclined Cable ... 200
Transportation, Fluid Escapement ment for ... 227
Transporting Belts ... 221
Trapezoidal Screw Threads ... 51, 58
Trap, Water ... 287
Triangles, Area of ... 23
Trick's Valve ... 284
Trigger, Hair ... 153
Trigonometrical Formulæ ... 299
Trgionometrical Functions, Powers of ... 25
" Table ... 297–299
Trussed Beams, Double ... 35
" " Simple ... 35
" " Triple ... 35
Tubing, Levasseur's Metallic ... 252
Tumbling Gears ... 163
Turbine, Borda's ... 220
" Cadiat ... 220
" Fourneyron ... 220
" Francis ... 220
" Girard ... 220
" Nagel ... 220
" Schiele ... 220
" Thomson's ... 220
Tweddell's Accumulator ... 265
Tweddell's Hydraulic Riveter ... 228
Twin Link ... 184
Twisting Moments, Graphically Considered ... 33
Tympanon of Archimedes ... 221

UHLHORN'S COUPLING ... 101, 153
Umbrella Pulley ... 207
Uniform Escapements ... 167
Uniformly Distributed Forces ... 32
Uniform Resistance, Columns of ... 13
" " Sections of ... 8
" " to Bending ... 8
" " to Torsion ... 13
" Strength, Bodies of ... 2
Universal Gears, Beylich's ... 136
Universal Joint ... 97
Unloaded Bolt Connections ... 60
Unloaded Keys ... 49
Unperiodic Power Escapements for Pressure organs ... 227

VALVE, ALLAN'S DOUBLE ... 283
" Armstrong's Supported ... 286
" Bell ... 276
" Boulton & Watt's Balanced ... 285
" Brandau's Double Seated ... 286
" Cave's Balanced ... 285
" Corliss ... 236
" Cornish ... 280
" Cramer's Balanced ... 280
" Cuvelier's Underpressure 286
" Double Beat ... 280
" " D ... 283
" " Flap ... 274
" Diagram, Reuleaux's ... 234
" Zeuner's ... 234
" Gear, Angström's ... 235
" " Brown's ... 235
" " Cail & Co ... 162
" " Cam ... 236
" " Corliss ... 162
" " Cornish ... 163
" " for Pumps ... 225
" " Harlow's ... 231
" " Hofmann's ... 163
" " Klug's ... 235
" " Marshall's ... 235
" " Plain Slide ... 234
" " Powel ... 163
" " Rider's ... 236
" Gears for Rotative Engines ... 234
" " Reciprocating ... 234
" " Releasing ... 162
" " Rotative ... 234
" Gear, Sweet's ... 235
" Gear, Wannich ... 162
" Globe ... 279
" Gridiron ... 283
" Hick's Double ... 283
" Injector ... 279
" Kirchweger's Balanced 285
" Lindner's Balanced ... 285
" Plain Slide ... 225, 282
" Porter-Allen ... 287
" Rubber Disk ... 274
Valves ... 279
Valves, Balanced ... 279
Valves, Balanced Slide ... 285
Valve, Schaltenbrand's Double Seated ... 286
Valves, Check ... 274
" Closing Pressure of ... 278
" Conical ... 275
" Considered as Pawls ... 223, 273
" Flap ... 274
" Flat Disk ... 275
" Fluid ... 287
" Gate ... 282
" Gidding's Experiments on ... 285
" Lift ... 223, 273
" Mechanically Actuated ... 278
" Multiple ... 276
" Piston ... 286
" Resistance of ... 247
" Robinson's Experiments on. 277
" Rotary ... 281
" Round ... 275
" Slide ... 225, 273, 281
" Spherical ... 275
Valve, Starting ... 282
Valves, Stationary ... 289
" Step ... 276
" Throttle ... 279
" Unbalanced Pressure on Lift 277
Valve, Sweet's Balanced ... 287
Valves, Width of Seat ... 274
Valve, Thometzek's ... 276
" Trick's ... 274
" Wilson's Balanced ... 287
" Wilson's Rotary ... 286
Value of Stress S ... 8
Vacuum Power Distribution ... 219
Variable Speed Gear ... 124
Variable Speed Gear, Rupp's ... 124
Velocity Curves ... 233
Ventilator, Fabry's ... 221
" Lémielle's ... 82
" Steib's ... 222
Verge Escapement ... 168
Volume, Escapements for Measurement of ... 239
Von Gerike's Air Pump ... 225
Vose's Pump ... 224
V Screw Thread ... 50

WALKER MFG. CO., PULLEY by ... 194
Walking Beams ... 110
Wall Bearings ... 68, 71
" Bearing, Support for ... 79
" Bearing, Sellers' ... 71
" Brackets ... 72
Walls of Vessels, Resistance of ... 15
Wall Step Bearings ... 75
Wannich Valve Gear ... 162
Washers ... 54
Watches, Repeating ... 169
Water Counterbalance, Oeking's ... 217
" Meter, Jopling's ... 239
" " Kennedy's ... 239
" " Payton's ... 220
" " Schmid's ... 240
" Pressure Engine, Belidor's ... 229
" " " Reichenbach's ... 229
" " " Roux's ... 229
" " " Schmid's ... 236
" Reservoir, of Frankfurt on Main ... 218
" Rod Connection ... 233
" Running Mechanism for Lifting ... 222
" Trap ... 287
" Trap, Morrison, Ingram & Co. 288
" Wheel, Poncelet's ... 220
" Wheel, Resultant of Load on 34
" Wheels, Axles for ... 91
" Wheel, Segner's ... 220
" Wheels, Gravity ... 219
" Wheels, Impact ... 220
Watt's Condenser ... 230
Wear on Gear Teeth ... 134

Wear on Hemp Rope..... 196
Weaver's Tightening Pulley..... 186
Wedge Friction Wheels.....125, 160
Weighing Machine, Emery's..... 173
Weight of Cast Iron Pipe..... 242
" of Chain..... 183
" of Gear Wheels..... 150
" of Hemp Rope..... 178
" of Round Iron..... 55
" Sheet Metal..... 43
" of Wire Rope..... 180
Weir, Camere's..... 275
Weisbach, Formula for Friction of Water..... 246
Weisbach's Formula for Stiffness of Ropes..... 181
Werder, Experiments on Springs... 21
Weston's Differential Pulley Block. 173
" Friction Clutch..... 101
" Ratchet Brace 154
Wet Gas Meter..... 239
Wheels, Classification of..... 122
Whip Action of Connecting Rod... 116
Whitehead Torpedo..... 237
Whitworth's Screw System..... 51
Whitworth's Pipe Thread Scale.... 51
Wiedenbruck's Rod Connection.... 50
Wilber's Ratchet..... 153
Wilson's Rotary Valve..... 286
Wilson's Water Gas Furnace..... 288
Winding Drum, Reuleaux's..... 173
" Drums, Spiral..... 181
" Tension Organs for..... 172
Windlass..... 172
" Brown's..... 173
" Differential..... 173
Windmills..... 220
Wind Stresses, Graphically Determined..... 37
Wire Rope..... 179
" " Influence of Weight.... 180
" " Load Length of 180
" " Strength of..... 179
" " Transmission..... 196
" " Weight of..... 180
" Saw, Zervas'..... 177
Wooden Axles, Proportions of..... 92
Wooden Shafting..... 94
Worm and Worm Wheel..... 139
" Gearing, Globoid..... 143
" " Hawkins..... 143
" " Jensen's..... 143
Worthington High Duty Pumping Engine..... 232
Worthington's Duplex Pump..... 231
Worthington's Equalizer..... 232
Wrapping Connections..... 173
Wrenches..... 56
Wrought Iron Cranks, Single..... 104
" " Pipe..... 243
" " Shafting..... 93
" " Walking Beams..... 111

YALE LOCK..... 167
Yoke Bearings..... 72

ZERVAS' WIRE SAW..... 177
Zeuner's Valve Diagram..... 234
Zimmermann's Belt Shifter..... 189
Zuppinger's Water Wheel..... 219
Zurich, Rope Transmission at..... 205

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